Synthesis of a Launch Trajectory of Aircraft Optimal in the Minimum Fuel Consumption Based on Sufficient Conditions of Optimal Control

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Abstract. In the theory of automatic control of complex dynamic objects, the necessary conditions for optimality of control are widely used on the basis of the method of L.S. Pontryagin, necessary and sufficient conditions for optimality of control based on the method of R. Bellman and sufficient conditions for optimality of control based on the method of V.F. Krotova. To date, control objects are becoming more complex and the development of optimal control methods themselves continues. Of particular interest for complex objects is the development of control synthesis methods based on sufficient optimality conditions. A modification of V.F. Krotova-M.M. Khrustaleva method, which consists in a conditional change in the characteristics of the object under study. This technique makes it possible, with minor changes, to apply the already known technique for synthesizing the suboptimal launch trajectory, obtaining better estimates of the final mass in comparison with the known method. In this article, this approach is used to construct an optimal launch-acceleration trajectory of an aircraft under the influence of gravity, aerodynamic forces, and engine thrust that is optimal in terms of fuel consumption. Using the method of mathematical modeling, numerical results have been obtained that confirm the efficiency of the proposed modification of the V.F. Krotova-M.M. Khrustalev method.

Keywords: Optimal control · Sufficient conditions of optimal · Launch trajectory · Minimum fuel consumption · Angle of attack

1 Introduction

In the modern theory of automatic control, fundamental results are actively used, such as the necessary conditions for optimality of control based on the method of L.S. Pontryagin

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[1], necessary and sufficient conditions for optimality of control based on the method of R. Bellman [2], as well as sufficient conditions for optimality of control based on the method of V.F. Krotov [3, 4, 5]. The development of these methods is reflected in the works of many domestic and foreign authors [6–15]. All existing methods for solving optimal problems are focused on certain classes of problems in a formulation that presupposes certain assumptions.

One of the successful options for the development of control theory with the help of sufficient optimality conditions of V.F. Krotov [1, 2, 3] is the method of V.F. Krotov-M.M. Khrustalev [4, 5]. It allows you to find the optimal ascent-acceleration trajectory $V(h)$ in terms of minimum fuel consumption, as well as an estimate of the upper limit of the final mass of the aircraft. By virtue of the latter property, the found trajectory is called estimated. In [4], it was proved that when flying along an optimal trajectory, the final mass turns out to be no more than the estimated one. For some cases of the problem ([4] problem A'), the found estimate corresponds to the exact solution of the boundary value problem.

Method V.F. Krotov-M.M. Khrustaleva has its own limitations. In particular, when substantiating the method [4], the differential connection by the angle of inclination of the trajectory is excluded θ and further θ acts as a control. The dependencies with which the estimated trajectory can be found does not take into account the feasibility of the processes, which, for the known ones h, V, m, β, is determined by the relationship between α and θ. Although, in the same work of V.F. Krotov and M.M. Khrustalev, a possible method for determining α and θ is proposed, which allows one to realize the found optimal trajectory. But at the same time, the search for the very trajectory $V(h)$ to be implemented is based on the dependencies that assume $\alpha = \theta$. That is, first a trajectory close enough to the optimal one is synthesized, and then it is refined.

The undoubted advantage of the method lies in the possibility of obtaining solutions that meet the requirements of design calculations in terms of accuracy with a relatively low cost of computer time, which is especially important when forming the appearance of a future object at the preliminary design stage. It is this quality that prompts researchers [14, 15] to turn to the method of V.F. Krotov-M.M. Khrustaleva. The breadth of application of the method largely depends on the degree of closeness of the results of the synthesis of the estimated trajectory and the exact solution of the boundary value problem. Therefore, it is understandable to strive to refine the results obtained with low additional costs of computer time.

In the proposed work:

- an attempt was made to refine the method of V.F. Krotov-M.M. Khrustalev by introducing into the calculation formulas the reference values of the angle of attack.
  $\alpha_{\text{o}\Pi} = \alpha_{\min}$ at the stage of trajectory synthesis; at the same time, additional costs of processor time are no more than 0.1%;
- an iterative algorithm for the synthesis of a realizable injection trajectory based on the method of V.F. Krotov-M.M. Khrustalev, which allows one to approach the solution of longitudinal and spatial problems of injection from a single point of view, is proposed.

2 Application of the V.F. Krotov-M.M. Khrustalev Method for Finding the Estimated Launch Trajectory

Let us clarify the problem statement [5]. The longitudinal motion of the center of mass of an aircraft in a vertical plane passing through the center of the Earth is considered.

In accordance with the differential equations of motion (1–3), the object is acted upon by the force of gravity, aerodynamic forces and engine thrust. It is necessary to find an ascent-acceleration trajectory that is optimal in terms of the minimum fuel consumption. Initial conditions:

$$t = 0;\quad V_0 = V(0);\quad \theta_0 = \theta(0);\quad h_0 = h(0);\quad L_0 = L(0);\quad m_0 = m(0).$$

End conditions:

time t_1 is not fixed; $V_1 = V(t_1);\quad \theta_1 = \theta(t_1);\quad h_1 = h(t_1)$.

Constraints on control α, β and phase coordinates $h,\ V,\ \theta,\ m,\ L$:

$$0 < V_{min}(h) \le V \le V_{max}(h),\ \ where\ \ h \ge 0;$$

$$0 < \beta_{min}(h, V) \le \beta \le \beta_{max}(h, V),\ \ where\ 0 < \beta_{max}(V, h) < \infty; \tag{1}$$

$$\alpha_{min}(h, V) \le \alpha \le \alpha_{max}(h, V),\quad \text{where } \alpha_{min} \le 0;\ \ \alpha_{max} \ge 0.$$

The functions $a_{min},\ a_{max},\ \beta_{min},\ \beta_{max}$ are continuously differentiable. Also assume that

$$0 \le \theta \le \pi. \tag{2}$$

The need for this requirement near the Earth's surface is obvious [5], and for many types of aircraft this condition must be fulfilled throughout the entire lift-acceleration section. The research carried out by the author of the thesis on the synthesis of optimal launch trajectories shows that the need to violate the requirement (2) is most likely at transonic speeds in the process of breaking the sound barrier, which is caused by a sharp increase in the profile resistance coefficient of the value $P \cdot \cos\alpha - X(\alpha)$. In the remaining sections of the trajectory, the need to violate the requirement (2) is less likely, but may be due to a sharp change in the traction characteristics or the operating mode of the aircraft power plant. In the overwhelming part of the extraction stage, the optimal trajectory satisfies condition (2). Consequently, this assumption does not lead to significant errors in the synthesis of the optimal launch trajectory.

The functions $g = g(h);\quad P = P(V, h, \beta);\quad X = X(V, h,\ \alpha);\quad Y = Y(V, h,\ \alpha)$ are defined and continuously differentiable on the set of admissible values $\Omega_{h,V,\alpha,\beta}$ for all time values. The physical meaning of the problem implies the properties of the listed functions

$$\begin{aligned} &P(V, h,\ 0) = 0;\ \frac{\partial P}{\partial \beta} \ge 0;\ \frac{\partial Y}{\partial \alpha} > 0;\ \frac{\partial X}{\partial V} > 0; \\ &X(V, \infty, \alpha) = 0;\ \ X_0(V, h) = X(V, h,\ 0) \le X(V, h, \alpha). \end{aligned} \tag{3}$$

For existing aircraft, the following conditions are characteristic

$$\frac{\partial}{\partial h}G(V,h) > 0; \quad 0 > \frac{\partial}{\partial V}G(V,h) > -C. \tag{4}$$

where $C = \text{const}$ for all $(h, V) \in \Omega_{V,h}$ satisfying

$$G(V,h) = P(V,h,\ \beta_{max}(V,h)) - X_0(V,h) \geq 0 \tag{5}$$

The boundary conditions satisfy the inequalities

$$G(V,h) > 0; G(V_1,h_1) > 0. \tag{6}$$

Inequalities (6) are certainly valid when the engine thrust does not depend on V and h.

Trajectories satisfying the above conditions will be called admissible, and the set of elements $z = (t_1,\ x(t),\ u(t))$, corresponding to these trajectories will be denoted D. The vector function of the phase coordinates $x(t) = (V(t),\ \theta(t),\ h(t),\ L(t),\ m(t))$ is assumed to be continuous and piecewise differentiable, and the control $u(t) = (\alpha(t),\ \beta(t))$ is piecewise continuous. Both functions are defined on the segment $[0,\ t_1]$. It is required from the number of admissible injection trajectories to choose the one on which the mass of the consumed fuel is minimal, i.e., the functional

$$J(z) = -m(t_1)$$

takes the smallest value on the set of valid elements D. To solve the problem of finding an estimate for the maximum finite mass, we define on the set D the set of triples of arguments D_0 satisfying the inequality

$$P(V,h,\ \beta) - X_0(V,h) > 0. \tag{7}$$

Let us make some remarks about the physical meaning of inequality (7). For this, we write down the expression for the derivative of the energy height with respect to time

$$\frac{dh_\varepsilon}{dt} = Vn_x = \frac{V\left(P(V,h,\beta)\cos\alpha - \left(X_0(V,h) + AC_y^{\alpha^2} qS\alpha^2\right)\right)}{mg} \tag{8}$$

The energy altitude is the reduced total energy of the aircraft

$$h_\varepsilon = \frac{E}{m} = h + \frac{V^2}{2g},$$

that is expression (8) characterizes the rate of energy growth. It is easy to see that the extremum (8) with respect to the angle of attack is attained at $\alpha = 0$. Moreover, the variables $V,\ m,\ g$ are nonnegative. Consequently, condition (7) indicates the requirement that the sign of the derivative (8) be positive, that is, the need for a constant increase in energy in the case of a zero angle of attack. Simulation shows. That condition (7) is satisfied for the optimal trajectory of extraction, as well as for the set of trajectories lying in some of its neighborhood.

Let us define on the set additional functions of the form [5]

$$Q(V, h, \beta) = \frac{\beta}{P(V, h, \beta) - X_0(V, h)} \tag{9}$$

$$Q(V, h) = \min_{\beta} Q(V, h, \beta) \tag{10}$$

$$S(V, h) = \int_{V^*}^{V} \frac{\partial}{\partial h} \cdot Q_{\min}(V, h) \cdot dV - \frac{g}{V} \cdot Q_{\min}(V, h) \tag{11}$$

$$\gamma(h) = Q_{\min}\big(V^*(h),\ h\big)\frac{dV^*(h)}{dh}\ . \tag{12}$$

Here $V^*(h)$ is a function satisfying (7), continuously differentiable. In addition, we require that for all $(h,\ V)$ satisfying the set D_0, the function $Q_{min}(V, h)$ is continuously differentiable. This requirement is being met. $P(V, h, \beta) = C(V, h)\beta$ where C is the outflow rate. In other cases, as a rule, a slight change in the dependency $P(V, h, \beta)$ allows you to achieve its implementation.

An expression similar to (9) for the dynamic relations of the mechanical energy of the aircraft has the following form:

$$Q_\varepsilon(V, h, \alpha, \beta) = -\frac{V(P\cos\alpha - X(\alpha))}{\beta} = \frac{dE}{dm} - gh_\varepsilon\ .$$

Taking into account (9), we can write

$$Q_\varepsilon(V, h, \beta) = \frac{-V}{Q_\varepsilon(V, h, \alpha = 0, \beta)}\ .$$

The extremum of the function Q_ε with respect to α is achieved at $\alpha = 0$, therefore, expression (9) can be interpreted as an indirect indicator of the efficiency of motion in the case when one of the controls α is equal to its optimal value $\alpha = 0$.

According to Theorem 2 from [5], for any admissible trajectory, the optimal value of the final mass $m(t_1)$ does not exceed the estimated value

$$m^* = m_0 \exp\left[\int_{h_0}^{h_1} \max_V S(V, h) \cdot dh - \int_{V_0}^{V_1} Q_{\min}(V, h) \cdot dV\right]. \tag{13}$$

There, it is also proved that the optimal values satisfy $G(V, h, t)$ for all $t \in [0,\ t_1]$, if the condition is satisfied $G(V_0,\ h_0) > 0$, and it is also shown that it is necessary to consider only the trajectories lying in the domain $G(V,\ h) > 0$.

In [5], an estimate of the optimal value of the final mass was obtained in a refined form

$$m^* = m_0 \exp\int_{h_0}^{h_1}\left[\max_V S(V, h) - \gamma(h)\right]dh\ . \tag{14}$$

If we assume that $V^*(h_0) = V_0,\ V^*(h_1) = V_1$, by substituting expression (12) for $\gamma(h)$, dependence (14) can be easily transformed to form (13). Consequently, in the analytical sense, dependencies (13) and (14) are similar, however, the numerical implementation of (14) is much simpler, since it requires fewer calls to the subroutine for finding the mass estimate. In the absence of aerodynamic and gravity forces (14) takes the form of K.E. Tsiolkovsky's formula

$$m^* = m_0 \exp\left[-\frac{1}{C}(V_1 - V_0)\right]. \tag{15}$$

A special case of the formula is also possible, which does not take into account aerodynamic forces, but makes it possible to take into account the energy costs for overcoming gravitational forces

$$m^* = m_0 \exp\left[-\frac{1}{C}\left(\frac{g(h_1 - h_0)}{V_1} + (V_1 - V_0)\right)\right]. \tag{16}$$

Dependences (14–16) were used to synthesize the lift-acceleration trajectory and estimate the final mass of specific aircraft. Note that the above-described method of V.F.Krotov and M.M. Khrustalev [3], as well as a refined version of the method (14) can be used to search not only the final mass, but also the mass of the aircraft at all current points of the trajectory. To do this, it is enough to take the current one $(V,\ h)$ as the final values in the above dependencies (V_1, h_1).

3 Modeling the Method of V.F. Krotov-M.M. Khrustalev for the Synthesis of the Estimated Launch-Acceleration Trajectory

In the works of V.F. Krotov-M.M. Khrustalev [4– 7], two problems of launching an aircraft in a longitudinal channel are considered: A and A'. The problem A' differs from the A fact that it accepts the assumptions $\cos\alpha = 1$ and $C_{xi} = 0$ (equality to zero inductive resistance). To perform a modification of the method of V.F. Krotov-M.M. Khrustalev in order to improve its accuracy, we defined the problem A''. To do this, on the considered flight segment, from the set of admissible angles of attack Ω_α, we select the set of angles that make it possible to realize trajectories belonging to a certain neighborhood of the optimal trajectory. Let's call such values $\alpha \in \Omega_\alpha$ realizable.

Analysis of the behavior of α for a trajectory lying in some neighborhood of the optimal trajectory $V^*(h)$ shows that on most sections of the launch trajectory there is some $\alpha_{\min}$ such that for all $h \in \Omega_h$

$$0 < \alpha_{\min} < \alpha_{\text{o}\Pi\text{T}} < \alpha_{\text{o}\Pi\text{T}}\ . \tag{17}$$

If in our formulation of the problem it was necessary to accept constraint (2), then for steady flight modes it is possible to limit the possible values of the angles of attack from below

$$\alpha_P < \alpha_{\Pi\Gamma} < \alpha_{\min}\ . \tag{18}$$

where $\alpha_{\Gamma\Pi}$ is the angle of attack required for the implementation of steady horizontal flight

$$\alpha_{\Pi\Gamma} = \frac{mg}{P + C_y^{\alpha} qS}\left[1 - \frac{V^2}{rg}\right]. \quad (19)$$

In the case of a choice of altitude or a positive change in the angle of inclination of the trajectory, the angle of attack α_P will be greater than $\alpha_{\Gamma\Pi}$. Violation of conditions (18) is possible only in cases of a sharp decrease in the angle of inclination of the trajectory or an arbitrary decrease θ less than a certain one $\theta_{\min} < 0$, which is unacceptable in the presence of constraint (2). Note that in the injection site, the modes associated with a sharp change θ are extremely rare and short-lived. Thus, with an insignificant distortion of the physical meaning and insignificant errors in the estimate of the mass, it is possible to determine some according $\alpha_{\min}$ to the formula (19) (or another similar dependence) that satisfies the conditions (17, 18). If the specified $\alpha_{\min}$ exists, then it is possible to define the task A'', which differs from the task A by the presence of an assumption $\alpha = \alpha_{\min} = \text{const}$. Let us systematize the signs of tasks $A,\ A',\ A''$ in Table 1.

Table 1. The systematize the signs of tasks $A,\ A',\ A''$

Task	Attack angle	Longitudinal overload
A	$\alpha = \alpha_P = v\alpha r$	$(P\cos\alpha - X(\alpha))/mg$
A'	$\alpha = 0$	$(P - X_0)/mg$
A''	$\alpha = \alpha_{\min} = \text{const}$	$\left(P' - X_0'\right)/mg$

The problem A'' easily turns into A' if, instead of the original object, we consider an object with new traction and aerodynamic characteristics found from the relations

$$P' = P\cos\alpha_{\min}, \quad X_0' = X_0 + C_{=1}(\alpha_{\min})qS \quad (20)$$

where $\alpha_{\min} = \text{const}$.

In this case, all conclusions that are correct for the problem A' will also be true for A''.

In [5], an exact solution A' to the problem was obtained. In this case, the solution to the problem A' can be considered as an approximate solution to the problem A [5]. Due to the fact that $\alpha_{\min}$ it is closer to $\alpha_{o\Gamma\Pi} = \alpha_P$ than $\alpha = 0$, the solution to the problem A'' will be closer to the solution to the problem A than the solution A'. Of all $\alpha \in \Omega_\alpha$, the minimum fuel consumption of the task would be achieved at $\alpha = 0$, if such a regime were possible. However, in real flight during the launching phase $\alpha > 0$.

The proposed modification makes it possible to reduce the difference between the problem A' and the solution of the problem A by passing to the problem A''. The $\alpha_{\min}$ can be determined directly at each step of the trajectory synthesis $V^*(h)$. In general, the methodology for finding the optimal estimated trajectory and searching for the mass estimate remains the same. Constraints and boundary conditions are also unchanged. Let us note the existing differences in the form of the used dependences (5), (7)

$$G(V, h) = P(V, h, \beta_{\max}(V, h)) \cos \alpha_{\min} - X(V, h, \alpha_{\min}) \geq 0 \,, \tag{21}$$

where $X(V, h, \alpha_{\min}) = X_0(V, h) + AC_y^{\alpha^2} qS\alpha_{\min}^2$.

The set D_0 is defined as the set of arguments satisfying

$$P(V, h, \beta) \cos \alpha_{\min} - X(V, h, \alpha_{\min}) > 0 \,. \tag{22}$$

Function (9) is defined as

$$Q(V, h, \beta) = \frac{\beta}{P(V, h, \beta) \cos \alpha_{\min} - X(V, h, \alpha_{\min})} \,. \tag{23}$$

The modified method retains the ability to find the mass estimate at the final and intermediate points. In addition, the described modification can be used to organize an iterative search for a realized suboptimal trajectory and the corresponding mass estimate.

The presented procedure is repeated for each current height value. For the task A'' it is supposed to $\alpha_{\min} = \alpha_{o\Pi}$. For each controversial value $\alpha_{o\Pi} \in \Omega_\alpha$, the optimal values of speed $V_{o\Pi\mathrm{T}}$ and fuel consumption per second are found $\beta_{o\Pi\mathrm{T}}$. From the differential equations written in finite differences, θ and α_{P}, are found, which make it possible to realize the transition of the aircraft to a new point from the previous one.

We are interested in such values $V_{o\Pi\mathrm{T}},\ \alpha_P$ for which the reference value α is simultaneously realizable $\alpha_p = \alpha_{o\Pi}$, that is, when the dependence graph $\alpha_p(\alpha_{o\Pi})$ intersects with the bisector of the angle formed by the coordinate axes.

If there are several intersection points, then we are interested in the one in which the value of α is minimal. It is possible that there is no intersection of the curve $\alpha_p(\alpha_{o\Pi})$ with the bisector of the angle. In this case, it is necessary to go back one step in height, exclude the value found at the previous step $V_{oT\Pi}$ from the number of possible solutions, and repeat the search procedure.

The trajectory synthesis problem can be divided into the following stages:

1. analysis of the feasibility of a particular launching task;
2. synthesis of the implementation trajectory of launch;
3. synthesis of control for the flight of wills of the found reference trajectory.

If at the first stage it turns out that among the admissible trajectories there is no one that will allow the inference task to be completed, then it makes no sense to proceed to stages 2, 3. At the first stage, as a rule, the problem is considered in a somewhat simplified form, but the more accurately the problem is solved on at this stage, the easier it is to perform items 2, 3. The feasibility of the lifting-acceleration task is defined as the ability of an object to bring the payload to a given point in space no less than a given one.

In our case, at the first stage, the method of V.F. Krotova-M.M. Khrustalev or one of its modifications that give an estimate by mass (which makes it possible to judge the feasibility of the injection task) and an optimal trajectory $V(h)$ synthesized without taking into account its feasibility. Refinement of the initial trajectory $V(h)$ in the course of bringing it to a realizable form simultaneously with refining the estimate is carried out at the second stage. Modifications of the V.F. Krotova-M.M. Khrustaleva allows:

1. to clarify the solution sought at the first stage (task A'');

2. choose a compromise between the real and joint solution of the tasks of stages 1 and 2 by establishing the degree of proximity α_P and $\alpha_{o\Pi}$, and, consequently, the number of necessary iterations, depending on the formulation of the research problem.

The validity of the theoretical studies described is confirmed by the results of numerical modeling. The synthesis of the estimated launch trajectory was performed with different reference values $\alpha_{\min} = \text{const}$, $\alpha_{\min} \in [0°,\ 10°]$ for an aircraft with a turbojet engine in the altitude range from 0 to 19 km.

Let us dwell on individual results and features of the algorithm.

Beginning with $\alpha_{\min} = 7°$, a range of heights appears (Table 2) within which the set turns out to be empty, ie, there are no such V for which condition (22) would remain valid.

Table 2. The range of heights appears with different reference values $\alpha_{\min}$

$\alpha_{\min}$	The h range in which $D_o = 0$
7°	4–8 км
10°	1–13 км

This indicates that the use of the same $\alpha_{\min}$ at different heights is not always acceptable and gives obviously worse results than the flexible change algorithm $\alpha_{\min}$ as the altitude changes. In the latter case, there is a real opportunity to obtain an estimate of the mass that completely coincides or is close enough to the exact solution of the boundary value problem. As a result of calculations, a functional dependence of the relative mass on the height and the reference value of the angle of attack $\mu(h,\ \alpha_{o\Pi})$ was found.

The magnitude of the improvement in the weight estimate compared to the unmodified method was also determined

$$\Delta\mu(h,\ \alpha_{o\Pi}) = \mu(h,\ 0) - \mu(h,\ \alpha_{o\Pi}) \tag{24}$$

The studies indicate that in the considered range of heights $h \in [0, 19]$ КМ , the realizable value of the angle of attack in the vicinity of the optimal trajectory is not less 3°–4°. If we focus on such a minimum possible value $\alpha_{\min}$, then the degree of improvement of the estimate in comparison with the base version of the V.F. Krotova-M.M. Khrustalev method fluctuates in a range $\Delta\mu = 0{,}1 - 0{,}2\%$ that corresponds to the absolute mass $\Delta m = 300 - 1200$ КГ for objects with a starting mass of 300 to 600 tons. When the reference value of $\alpha_{O\Pi}$ is varied within the limits $3° - 5°$, an improvement in the estimate for the final mass at an altitude of 19 km can reach, respectively, the values

$$\Delta\mu = 0{,}1 - 0{,}4\%, \quad \Delta m = 300 - 2400 \text{ kg}$$

When driving to high altitudes, the difference $\Delta\mu$ will be significant and can reach a few percent of the relative mass.

It is easy to see that $\Delta\mu(h,\ \alpha_{o\Pi})$ it changes non-monotonically for all values $\alpha_{on} = const$. In addition, for different $\alpha_{o\Gamma}$ types of dependence $\Delta\mu(h,\ \alpha_{o\Pi})$, the quality is also different. In a local sense, the reasons for this phenomenon lie in the imperfection of the interpolation of the original data arrays during printing. In a global sense, the general view of the dependency $\Delta\mu(h,\ \alpha_{o\Pi})$ is inextricably linked with the type of dependency $V(h)$ corresponding to the value used $\alpha_{o\Pi}$.

$$\frac{d\mu}{dh} = \frac{d\mu}{dt}\frac{dh_\theta}{dh}\left[\frac{dh_\theta}{dt}\right]^{-1} = \frac{-\beta\left[1 + \frac{V}{g}\frac{dV}{dh}\right]}{m_0\ V\ n_x} = \frac{-\beta\ \mu\ g\left[1 + \frac{V}{g}\frac{dV}{dh}\right]}{V[P\ \cos\ \alpha_{o\Pi} - X(\alpha_{o\Pi})]} \tag{25}$$

The change causes a change in the type of dependence, which, in accordance with (24), (25), leads to the non-monotonicity of the dependence $\Delta\mu(h,\ \alpha_{o\Pi})$.

4 Conclusion

1. The proposed modification of the methods of V.F. Krotova-M.M. Khrustalev consists in a conditional change in the characteristics of the object under study, which makes it possible, with minor changes, to apply the already known [5] method for synthesizing the suboptimal launch trajectory, while obtaining better estimates of the final mass in comparison with the known method. To apply the modification, information is needed on the possible realizable angles of attack when withdrawing along a trajectory belonging to a certain neighborhood of the optimal one. Based on conditions (17), (18), the dependences for the smallest realizable values of the angle of attack were found by numerical methods on the basis of dependence (19).
2. Based on the modification of the method of V.F. Krotova-M.M. Khrustalev developed an iterative algorithm for synthesizing a realizable injection trajectory, which allows one to approach the solution of the longitudinal and spatial injection problem from one point of view. In addition, the algorithm provides the ability to choose a compromise solution between the accuracy and speed of the algorithm, depending on the formulation of the research problem.
3. At the smallest realizable angles of attack of the aircraft $\alpha = 3°-4°$ (which corresponds to reality), the refinement of the estimate for the relative final mass at an altitude of 19 km varies within the limits $\Delta\mu = 0{,}1-0{,}2\%$, which corresponds to the absolute mass $\Delta m = 300-1200$ kg for objects with a launch mass of 300 to 600 tons. By varying the smallest realizable angles of attack within the limits $\alpha = 3°-5°$, an improvement in the estimate for the final mass at an altitude of 19 km, respectively, can reach $\Delta\mu = 0{,}1-0{,}4\%$, $\Delta m = 300-2400$ kg.
4. When fixed $\alpha_{o\Pi} = \text{const}$, the dependence $\Delta\mu(h)$ has a non-monotonic character, which is explained by a change in the slope of the dependence $V(h)$, the type of which depends on the method of redistribution of the energy entering the system between the kinetic potential components, as well as on the indicators of the efficiency of the excretion process.
5. The change in the qualitative and quantitative characteristics of the dependence $\Delta\mu(h)$ when changing $\alpha_{o\Pi} = \text{const}$ is associated with a change in the type of trajectory $V(h)$ when changing $\alpha_{o\Pi}$. The impact $V(h)$ on efficiency indicators is described above.

6. For the object under study, there is some $\alpha_{\min}$, starting from which the set $D_0 = 0$ turns out to be empty (Table 1). Consequently, for a more complete study with a more accurate estimate of the mass, it is necessary to connect the apparatus of flexible change $\alpha_{o\Pi}$ when changing h.

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