







# Stress Distribution in the Eccentrically Loaded Lapped Adhesive Joint. An Analytical Model

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**Abstract.** The deflected mode problem for the adhesive joint of two rectangular plates in a simplified two-dimensional formulation is solved. The proposed solution takes into account the bending of the base layers in the joint plane. Base layers are considered as Bernoulli beams. The stress and deformation values are assumed to be linearly distributed along the joint width and uniformly in thickness of the joint elements. The problem is reduced to a system of two linear differential equations of the fourth order with respect to transverse (in the joint plane) shifts of layers. An analytical solution to the problem is obtained. The model problem is solved. Comparison of the results obtained with using the proposed method with the results, obtained using finite element modeling is done. Good correspondence of the results obtained by using two different techniques is shown, and also that the proposed model is adequate and has sufficient precision for engineering tasks.

**Keywords:** Adhesive joints · Analytical modeling · Interlaminar stresses · Beam theory

## 1 Introduction

Lapped adhesive joints are widespread in modern technology. There are several basic stress state models for joints, which allow us determine the joint stress state in an analytical form [1–4]. Most of the models are one-dimensional. Base (outer) layers are considered as rods that work in stress-strain, or as beams in the Bernoulli or Timoshenko approximation. The adhesive (connecting) layer is considered as an elastic Winkler base or as a two-parameter elastic base [5–9]. In the latter case, the mathematical model makes it possible to describe with high accuracy the stress state of the adhesive layer at the border of the gluing area. The stress distribution over the base layers thickness is assumed to be linear, and is uniform over the adhesive layer thickness, or stepped, or also linear, depending on the model. In all the listed mathematical models, the stress distribution depends on only one coordinate, i.e. the models are one-dimensional. However, when computing the deflected mode of overlapping adhesive joints, in a number of cases, it is necessary to take into account the nonuniformity of the stress and deformation distribution along the width of the joint. This problem is qualitatively more difficult than

constructing a one-dimensional, even though advanced, mathematical model of a three-layer beam. Various numerical and approximate methods for the numerical solution of this problem are proposed [10–17]. An analytical solution to the two-dimensional joint stress state problem is still unknown. However, several simplified mathematical models have been proposed to study the various effects that arise in the joint. To study the effect of base plate transverse deformations, which are caused by Poisson’s ratios, on the stress distribution in the adhesive, a model based on hypotheses about the uniformity of the applied load and the shear compliance of the base layers is proposed [16, 18]. Often the structure and the loads applied to it are symmetrical relative to the longitudinal axis. In this case, it is possible to use a simplified model, according to which transverse deformations and shifts in the joint plane are assumed to be zero [19]. In papers [20–22] such approach is developed for the different types of boundary equations. The described mathematical models rank an intermediate position between the models of the elasticity theory and more simple structural mechanics models.

As noted above, in the previously created two-dimensional joint stress state models, which take into account the nonuniform stress distribution along the width of the joint, it is assumed that the applied load is symmetric throughout the longitudinal axis of the structure. I.e. the bending of the joined plates in the plane is absent. In this paper, a joint model is proposed, which takes into account the bending of the base layers in the gluing plane. In this case, the base layers are considered as Bernoulli beams, which bend in the gluing plane. Such loading of the structure occurs when the load is applied to the joint by some eccentricity. This problem was investigated using finite element modeling in [23]. In this paper, an analytical solution to the problem is proposed.

## 2 Problem Formulation

The force applied to the eccentric joint, at some distance from the point of application, causes the forces in the rods to be linearly distributed over the width. These forces can be considered as a superposition of uniform stress-strain and skew-symmetric forces due to the bending moment, Fig. 1.

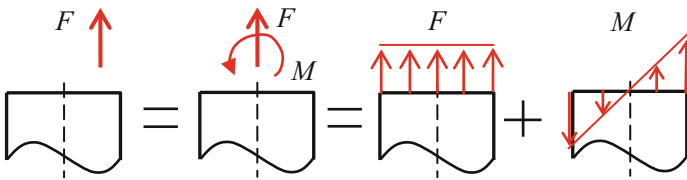
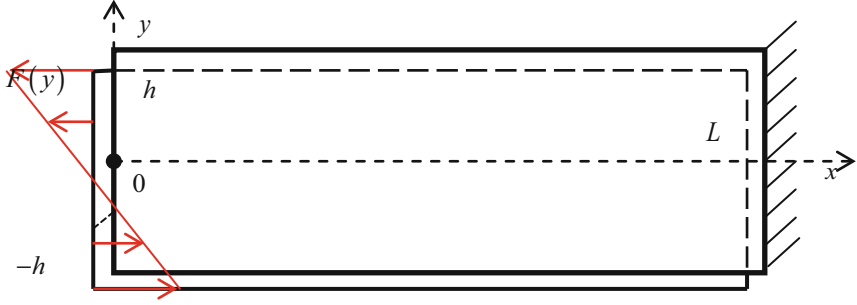


Fig. 1. Load diagram

It is not difficult to find the stress state of the joint, which is due to forces uniformly distributed over the width, since this problem is well known [1, 4]. Whereas the problem of finding the stress state of a joint loaded with a bending moment is new. Therefore, we will focus on this particular task.

Consider an adhesive joint of two rectangular plates (linear dimensions of plates are  $L \times 2h$  and thickness  $\delta_1$  and  $\delta_2$  correspondingly), shown in Fig. 2. The lateral sides of

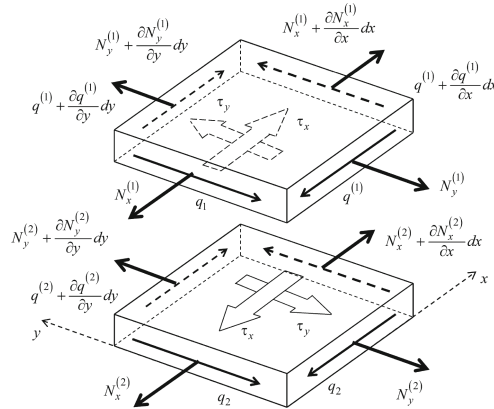
the plates are load free. The plates are deformed only in the joint plane, the adhesive layer acts in shear, the stress distribution is uniform over the layer thickness. Without diminishing the generality of the proposed approach, we assume that the load is applied only to the second layer along the side  $x = 0$ , and the first layer along the side  $x = L$  is rigidly fixed. The load applied to the second layer is assumed to be linearly dependent on the transverse coordinate  $y$ .



**Fig. 2.** Diagram of the structure

The applied load is described by the dependence  $F(y) = K \cdot y$ , where a coefficient  $K = \frac{3M}{2h^3\delta_2}$ ,  $M$  is a bending moment applied to the joint.

The forces acting on the differential elements of the joint base layers are shown in Fig. 3.



**Fig. 3.** Equilibrium of the joint differential element

The equilibrium equations for the base layers are

$$\tau_x + \frac{\partial N_x^{(1)}}{\partial x} + \frac{\partial q^{(1)}}{\partial y} = 0, \quad -\tau_x + \frac{\partial N_x^{(2)}}{\partial x} + \frac{\partial q^{(2)}}{\partial y} = 0 \quad (1)$$

$$\tau_y + \frac{\partial N_y^{(1)}}{\partial y} + \frac{\partial q^{(1)}}{\partial x} = 0, \quad -\tau_y + \frac{\partial N_y^{(2)}}{\partial y} + \frac{\partial q^{(2)}}{\partial x} = 0 \quad (2)$$

where the superscript is the base layer number and the lower is the direction;  $\tau_x$ ,  $\tau_y$  are tangential stresses in the adhesive layer along the corresponding axis;  $q^{(1)}$ ,  $q^{(2)}$  are tangential forces in the corresponding base layers;  $N_x^{(1)}$ ,  $N_y^{(1)}$ ,  $N_x^{(2)}$ ,  $N_y^{(2)}$  are normal forces in the base layers.

Forcess in layers (Poisson deformations are not taken into account)

$$N_x^{(k)} = B_k \frac{\partial u^{(k)}}{\partial x}; \quad q^{(k)} = \delta^{(k)} G^{(k)} \left( \frac{\partial u^{(k)}}{\partial y} + \frac{\partial v^{(k)}}{\partial x} \right), \quad (3)$$

where  $u^{(k)}$ ,  $v^{(k)}$  are shifts of the  $k$ -th layer ( $k = 1, 2$ ) in the longitudinal and transverse direction correspondingly;  $B_k = \delta^{(k)} E_x^{(k)}$ ;  $E_x^{(k)}$ ,  $G^{(k)}$ ,  $\delta^{(k)}$  are a modulus of elasticity, shear modulus and thickness of the corresponding base layer.

The stresses in the adhesive layer are proportional to the shift difference of the plates [16, 18]

$$\tau_x = \frac{G_0}{\delta_0} (u^{(1)} - u^{(2)}); \quad \tau_y = \frac{G_0}{\delta_0} (v^{(1)} - v^{(2)}), \quad (4)$$

where  $G_0$  and  $\delta_0$  are shear modulus and adhesive layer thickness correspondingly.

Homogeneous boundary conditions:

$$\begin{aligned} u^{(1)}|_{x=L} = 0; \quad v^{(1)}|_{x=L} = 0; \quad N_x^{(2)}|_{x=L} = 0; \\ N_x^{(1)}|_{x=0} = 0; \quad q^{(k)}|_{y=\pm h} = 0; \quad N_y^{(k)}|_{y=\pm h} = 0. \end{aligned} \quad (5)$$

Normal force is given on the left lateral side.

$$N_x^{(2)}|_{x=0} = F(y) = K \cdot y \quad (6)$$

We will assume that the bending of the plates in the joint plane is described by the Bernoulli beams theory. In this case, the shifts of the plates are described by the dependences.

$$u^{(k)}(x, y) = -y \cdot \frac{d}{dx} V^{(k)}(x), \quad v^{(k)} = V^{(k)}(x), \quad k = 1, 2. \quad (7)$$

Here  $V^{(k)}(x)$  are transversal shifts of  $k$ -th layer in the joint plane (i.e. elastic line equation), Fig. 4.

If  $V(x)$  is an elastic beam line, then longitudinal point shifts, remote from the beam axis at a distance  $y$  are equal  $u = y \sin \varphi = -y \frac{dV}{dx}$ , since the shifts are assumed to be small (linear theory).

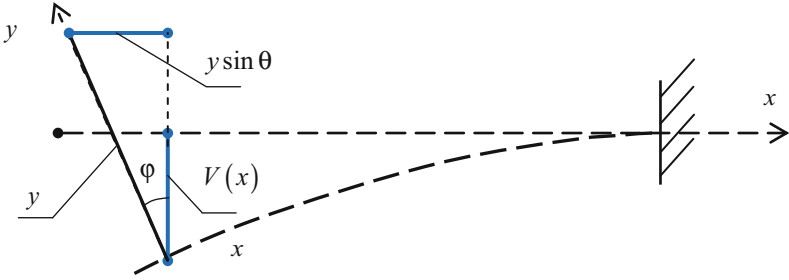


Fig. 4. Shifts of the beam element

### 3 Constructing the Solution

Substituting Eq. (7) in Eq. (4), we get

$$\tau_x = -\frac{G_0}{\delta_0} y \left( \frac{dV^{(1)}}{dx} - \frac{dV^{(2)}}{dx} \right); \quad \tau_y = \frac{G_0}{\delta_0} (V^{(1)} - V^{(2)}) \quad (8)$$

Substituting longitudinal forces Eq. (3),  $N_x^{(k)} = B_k y \frac{d^2 V^{(k)}}{dx^2}$ , in Eq. (1), then integrating by  $y$  and satisfying zero conditions for tangential stresses at the lateral sides of the joint (5), we get.

$$q^{(k)} = (-1)^k \frac{h^2 - y^2}{2} \left[ \frac{G_0}{\delta_0} \left( \frac{dV^{(1)}}{dx} - \frac{dV^{(2)}}{dx} \right) - (-1)^k B_k \frac{d^3 V^{(k)}}{dx^3} \right]. \quad (9)$$

Substituting Eq. (9) in Eq. (2) and integrating by  $y$ , we get

$$N_y^{(k)} = (-1)^k \frac{G_0}{\delta_0} y (V^{(1)} - V^{(2)}) + f^{(k)}(x) + \frac{1}{2} \left( \frac{y^3}{3} - h^2 y \right) \left[ B_k \frac{d^4 V^{(k)}}{dx^4} - (-1)^k \frac{G_0}{\delta_0} \left( \frac{d^2 V^{(1)}}{dx^2} - \frac{d^2 V^{(2)}}{dx^2} \right) \right]. \quad (10)$$

Functions  $f^{(1)}(x)$  and  $f^{(2)}(x)$  we find, using conditions (5)  $N_y^{(k)} \Big|_{y=h} = 0$ :

$$f^{(k)}(x) = -\frac{h^3}{3} B_k \frac{d^4 V^{(k)}}{dx^4} - (-1)^k \frac{G_0}{\delta_0} \left[ \frac{h^3}{3} \left( \frac{d^2 V^{(1)}}{dx^2} - \frac{d^2 V^{(2)}}{dx^2} \right) - h (V^{(1)} - V^{(2)}) \right].$$

Boundary conditions on the second lateral side  $N_y^{(k)} \Big|_{y=-h} = 0$  leads us to the differential equations system

$$A_4 \frac{d^4 V}{dx^4} + A_2 \frac{d^2 V}{dx^2} + A_0 V = 0, \quad (11)$$

where

$$V = \begin{pmatrix} V^{(1)} \\ V^{(2)} \end{pmatrix}, \quad A_4 = \frac{\delta_0}{G_0} \begin{pmatrix} \delta^{(1)} E_x^{(1)} & 0 \\ 0 & \delta^{(2)} E_x^{(2)} \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, \quad A_0 = \frac{3}{h^3} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

The general solution to system (11) can be written as

$$\mathbf{V} = \sum_{n=1}^4 S_n x^{(n-1)} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \sum_{n=1}^4 S_{n+4} e^{\theta_n x} \mathbf{H}_n, \quad (12)$$

where  $\theta_n$  are nonzero roots of a characteristic equation.

$$\det(\mathbf{A}_4 \theta^4 + \mathbf{A}_2 \theta^2 + \mathbf{A}_0) = 0.$$

Vectors  $\mathbf{H}_n$  are solutions of the system

$$(\mathbf{A}_4 \theta_n^4 + \mathbf{A}_2 \theta_n^2 + \mathbf{A}_0) \mathbf{H}_n = 0$$

and are determined up to an arbitrary factor  $S_n$ .

To find eight unknown constants that are included in (12), we formulate eight boundary conditions.

$$\begin{aligned} V_1(L) = \frac{dV_1(x)}{dx} \Big|_{x=L} &= \int_{-h}^h q^{(1)}(0, y) dy = \frac{d^2 V^{(1)}}{dx^2} \Big|_{x=0} = \int_{-h}^h q^{(2)}(0, y) dy = 0; \\ \int_{-h}^h q^{(2)}(L, y) dy &= 0; \quad \frac{d^2 V^{(2)}}{dx^2} \Big|_{x=L} = 0; \quad \frac{d^2 V^{(2)}}{dx^2} \Big|_{x=0} = \frac{K}{E_x^{(2)}}; \end{aligned}$$

The last condition is similar to the classical Bernoulli beam bending equation  $E_x^{(2)} I^{(2)} \frac{d^2 V^{(2)}}{dx^2} \Big|_{x=0} = M$ , where  $I^{(2)} = \frac{2h^3 \delta_2}{3}$  is a moment of inertia.

The above conditions form a system of linear equations with respect to the unknown constants  $S_1, \dots, S_8$ .

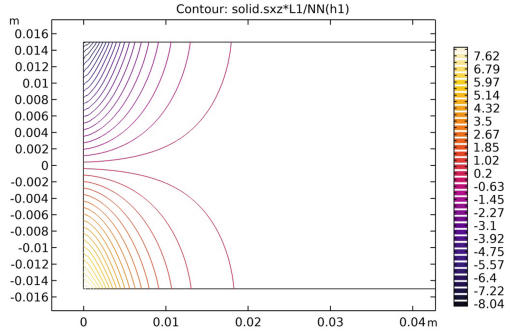
## 4 Model Problem

To analyze the stress state of the joint and verify the proposed analytical model, it was considered the joint of two aluminum ( $E^{(1)} = E^{(2)} = 72$  GPa, Poisson coefficient  $\mu = 0.28$ ) plates of length  $L = 90$  mm and width  $2h = 30$  mm. Thickness of the plates are  $\delta_1 = 2, 5$  mm and  $\delta_2 = 2$  mm. The plates are glued with the adhesive, the elastic parameters of which are  $G_0 = 0.34$  GPa,  $\mu = 0.32$  and thickness is  $\delta_0 = 0.1$  GPa. All materials are isotropic.

To verify the proposed analytical model according to the above parameters, a three-dimensional finite element model was created. The characteristic size of the element in the adhesive layer is equal to the thickness of the adhesive layer.

Stresses in the median plane of the adhesive layer in the longitudinal direction  $\tau_{zx}$ , calculated using finite element modeling are shown in Fig. 5.

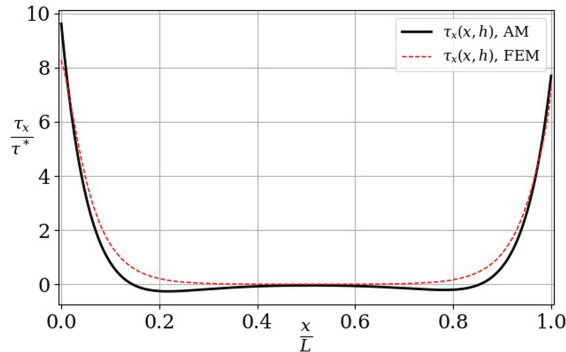
The stresses in the figure are given in dimensionless form, as the ratio of the acting stresses to some hypothetical stresses  $\tau^* = \frac{F(h)}{L}$ , which would arise in the joint provided



**Fig. 5.** Stresses in the adhesive (fragment of the structure)

that the applied linear forces are uniformly distributed along the length of the gluing  $F(h) = K \cdot h$ . These forces represent the maximum values of the forces applied to the joint (5). Therefore, the stress ratio  $\frac{\tau_x(x,y)}{\tau^*}$  is someone similar to the stress concentration coefficient.

The graph shows that on the axis of symmetry the stresses  $\tau_{zx}$  are equal to zero and increase when approaching the lateral sides. In Fig. 6 it is shown the stresses  $\tau_{zx}$  along the lateral side of a joint ( $y = h$ ), computed using finite element modeling (FEM) and using proposed analytical model (AM),  $\tau_x$ , Eq. 4.

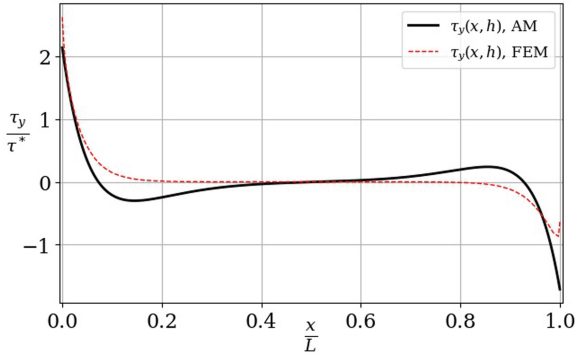


**Fig. 6.** Stresses  $\tau_x$  in the adhesive along the lateral side

The stresses in the graph are also given in dimensionless form.

In Fig. 7 the stresses in the transverse direction,  $\tau_{zy}$ , which appear in the adhesive layer along the lateral side of the joint ( $y = h$ ) are shown. Stresses were calculated using finite element modeling (FEM) and using the proposed analytical model (AM),  $\tau_y$ , Eq. 4.

As we can see, the stress values calculated using the proposed analytical model and using finite element modeling practically coincide. The most loaded are the ends of the joint, while the analytical model gives slightly overestimated values of stresses in comparison with the finite element model. This phenomenon is well known [5, 8] and is due to a set of simplifying hypotheses underlying the model. However, this does not

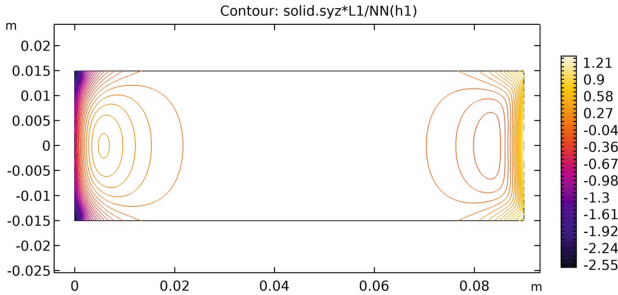


**Fig. 7.** Stresses  $\tau_y$  in the adhesive along the lateral side

reduce the possibility of using the proposed model for solving optimization and joint design problems.

An important difference between the finite element and analytical models is that, according to the proposed analytical model, the stresses in the adhesive in the transverse direction are constant throughout the width of the plates, and change only in the longitudinal direction, since they depend only on the coordinate  $x$ , Eq. (4). Whereas a three-dimensional finite element model allows us to study the stress distribution in the adhesive layer  $\tau_y$  in the transverse direction also.

Stresses in the median plane of the adhesive layer in the longitudinal direction  $\tau_{zy}$ , calculated using finite element modeling are shown in Fig. 8

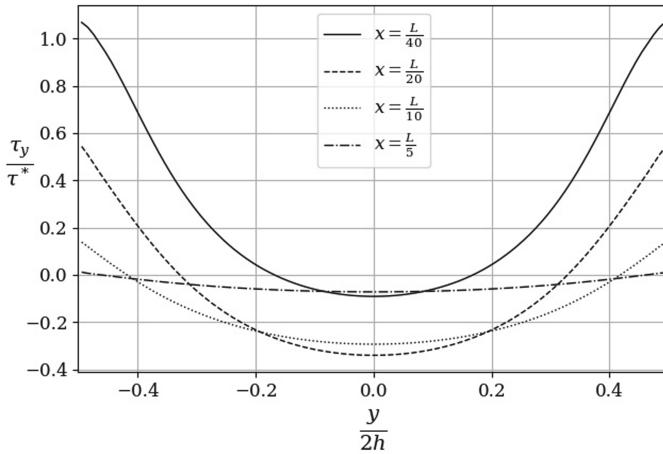


**Fig. 8.** Stresses  $\tau_{zy}$  in the adhesive layer

In Fig. 9 graphs that illustrate the distribution of stresses in the adhesive in the transverse direction ( $\tau_y$ ) throughout the joint width are shown. The given graphs are obtained as a result of finite element modeling. The graphs show the stress distribution  $\tau_y$  throughout the width of the adhesive layer at a distance  $\frac{L}{40}$ ,  $\frac{L}{20}$ ,  $\frac{L}{10}$  and  $\frac{L}{5}$  from the left edge of the joint.

It should be noted that finite element modeling shows that normal stresses in the base layers are distributed across the width almost linearly. And forces  $N_x^{(1)}$  and  $N_x^{(2)}$ , Eq. (3)





**Fig. 9.** Stresses in the adhesive in the transverse direction at different distances from the loaded edge

coincide with the results of finite element modeling. Hence, the application of Bernoulli beams theory to describe the shifts and deformations of the base layers is grounded.

## 5 Conclusions

- 1) A mathematical model of an overlap joint is proposed, which allows us to describe the three-layer structure stress state, caused by the bending in the gluing plane.
- 2) The proposed approach is based on the Bernoulli beam mathematical model and is a development of the classical Volkersen joint model [1]. Therefore, the usage of the proposed analytical model is available if the length of the joint is significantly greater than the width.
- 3) Finite element modeling has shown the high accuracy of the proposed model and the reliability of the hypotheses used.
- 4) The proposed model expands the class of problems to be solved and, together with the previous results [19], makes it possible to find the stress state of joints with an arbitrary load. To do this, it is necessary to represent the applied load as the sum of a linear load of the form (6) and a load that does not create a bending moment. This will allow us to split the problem into two independent problems, the solutions of which are known.
- 5) Further development of the proposed model can be aimed at finding the stress state of the coaxial pipes joints, which are loaded with a bending moment [24, 25].

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