Method of Forming an Updated List of Technical Products Fuzzy Quality Indicators Based on Fuzzy Clustering

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Abstract. At the stage of the technical products' production planning, the key point of the success for meeting the customer's expectations is to determine customer's needs and to convert these needs into product characteristics, which are eventually reflected in the formed product quality indicators list. To monitor quality indicators at the production stage manufacturers often use the theory of fuzzy sets to combine several groups of quality indicators into one indicator, which simplifies the process of monitoring and evaluating product quality, but complicates the process of interpreting these indicators on the principle of "acceptable – not acceptable". To solve the problem of interpretation, an updated list based on fuzzy sets should contain not only information about the types of quality indicators, but also information on evaluation or measurement scales for each product quality indicator. In this paper, we propose a method for forming a gradations scale for product quality indicators based on fuzzy clustering. A method for forming a rank scale used for particular indicators in a two-level product quality optimization model is developed and justified. The proposed method of a fuzzy term set constructing allows to solve the problem of determining an updated list of fuzzy quality indicators.

Keywords: Fuzzy sets · Measure of uncertainty · Two-level optimization model · Fuzzy clustering · Graph

1 Introduction

Formed indicators of technical products quality at the planning stage are necessary to meet the needs of all interested parties. At the same time, product quality management at the production stage often requires the definition of evaluation or measurement scales, not only for the product technical characteristics, but also for economic or organizational indicators. The combination of heterogeneous product quality indicators and the formation of fuzzy quality indicators allow to cover several aspects of production at once. Thus, the introduction of the established Updated List of Fuzzy Quality Indicators (ULFQI) with the use of evaluation or measurement scales allows you to get answers to questions:

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- is the quality optimal taking into account the investments?
- is it needed to develop corrective or preventive measures regarding the product quality?

It should be noted that the use of a multi-criteria or multi-level evaluations based on modern mathematical methods, in particular, optimization theory [1], complicates the problem of ULFQI forming, since it requires to consider a larger amount of information, which leads to the growth of fuzzy rules and the emergence of the so-called dimension problem. In this paper, we will propose a method for forming a rank scale used for partial indicators of a two-level product quality optimization model.

2 Description of the Product Quality Assessment Model

The evaluation of product quality is carried out using a two-level optimization model, which is based on the principle of decentralized management. A decentralized approach from the management standpoint refers to an approach in which the top management transfers part of its functions to subordinates. In such conditions, the low level units get a degree of freedom in their activities, but at the same time they are still under the upper level control.

Concerning the product quality, the decentralized approach provides independent management of the enterprise structural departments, related to product development, economic support, logistics, material supply and production. At the same time, assessment of the effectiveness and efficiency of the listed structural elements activities is provided by the Quality management department. One of the ways to assess the level of quality when implementing a decentralized approach is the theory of optimization, in particular the theory of two-level optimization.

The two-level optimization problem was described in 1973–1974 by J. Bracken and J. McGill [2, 3]. The model for assessing the quality level in these works is formulated through the two-level decentralized model. The solution to the problem of assessing the quality level, based on a given model, is found by solving the problem of two-level optimization, where the upper level is the leader, and the sublevel is the follower. In this setting, the follower makes the decision first, taking into account the wishes of the leader, and after the follower found a decision, the leader considers follower wishes and determines his own optimal solution. The main advantages of the two-level optimization model in solving of quality assessing problems, related to the instrument-making products, are given in authors' papers [4–6].

A detailed description of the objective functions, the partial summands of the objective functions, the system of constraints, and the algorithm for finding the optimal point will be given in the following sections.

The assessment of the instrument-making products quality level will be determined by the set-theoretic model $\langle Q, X, F_i, Y_i \rangle$, where Q is the target function of the main level (the leader's function), X is the area for the numerical values determining of the main level function, F_i are the i-th target functions of sublevels (the followers' functions), Y_i is the area for determining of the i-th target function values.

Target function of the main level Q is determined by a continuous sequence of numbers $a_{11}, a_{12}, ..., a_{ij}$ $(j = 1, 2, ..., n)$, $x_1, x_2, ..., x_i$ $(i = 1, 2, ..., m)$. In this case a_j : $0 \le a_j \le 1$ (for a fixed i-th value of x), and $x_i : \sum_{i=1}^{m} x_i = 1$.

The hierarchy of quality indicators sets additional restrictions on the scope of the Q definition; therefore, the main TF will take the form:

$$Q\ [X, F_i(Y_i)], \tag{1}$$

where $x \in X : Ax \le d$, $|a_{ij}| = A$, sublevels $y \in Y : B_i \cdot y \le d$, $|b_{ij}| = B_i$, $0 \le b_j^k \le 1$ (for a fixed i-th value of y), and $0 \le \sum_{i=1}^{m} y_i \le 1$.

The search for the optimal value of the function (1) is performed from the bottom to top: first, the optimal value of sublevels $F_i(Y^i)$ is determined, after which the founded values of Y^i are substituted in (1), and the function value for the main level is searched.

The optimization problem for the upper level, taking into account the constraints imposed by sublevels, will look like [7]:

$$\min_x \{Q[y(x), x] : G[y(x), x] \le 0, H[y(x), x] = 0, y(x) \in \psi(x)\}, \tag{2}$$

where $y(x) = F_i$, $F_i : y(x) \in \psi(x)$, $\psi(x)$ is a polyhedron, the domain of its constraints is such, that $Q : R^n \times R^m \to R$, $G : R^n \times R^m \to R^k$, $H : R^n \times R^m \to R^l$ (for the k-th indices there are constraints with the sign "$\le$", and for the l-th indices – with the sign "="). Then the optimization problem for the lower level is represented as follows:

$$\min_y \{f(x, y) : g(x, y) \le 0, h(x, y) = 0\}. \tag{3}$$

3 Description of the Problem Area

As mentioned above, the ULFQI is designed to solve the problem of monitoring and evaluating the technical products quality, covered quality indicators of which are based on fuzzy sets, and the evaluation itself is carried out using a multi-level optimization model. In this case, the quality indicators' forming is carried out using a fuzzy inference system based on the Tagaki – Sugeno algorithm [8], in which the fuzzy rules are set as:

$$R_j : u_1(x_1) = a_{1j} AND, ..., AND u_i(x_i) = a_{ij} \to y_j = b_j, \tag{4}$$

where R_j is the singular inference rule, $j = 1, 2, \ldots, n$ (n is the total number of inference rules); $u_i(x_i)$ is the membership function of the fuzzy variable x_i, $i = 1, 2, \ldots, m$ (m is the number of antecedents in the j-th rule); a_{1j} is the fuzzy term evaluating the membership function $u_i(x_i)$; y_j is the fuzzy inference variable; b_j is the fuzzy term evaluating the fuzzy variable y_i; $y_i = b_j$ is the consequent of the j-th rule.

The output linear variables are set as follows:

$$y_j = a_{1j}x_1 + a_{2j}x_2 + ... + a_{ij}x_i + a_0, \tag{5}$$

where a_{1j} is a fuzzy term (in our case, this is a fuzzy term of a linguistic variable); x_i are variables that define the scale of the fuzzy term and its contribution to the resulting value of y_j compared to other fuzzy terms of the j-th rule.

Fuzzy rules allow to determine the output value of a fuzzy system by applying combinations of judgments, where the antecedents act as a combination of judgments, and the output value is the consequents. The content of antecedents and consequents is determined out of the task being solved. Based on the analysis of the existing literature and scientific publications on the topic of fuzzy inference systems modeling, it can be concluded that the accuracy of the fuzzy inference system depends not only on the inference algorithm, but also on the power of the term set and the number of fuzzy rules. The last two problems in the theory of fuzzy modeling are called the “curse of dimensionality”, i.e., when the accuracy of the output of a fuzzy system is affected by the dimensionality of the fuzzy rules set and the power of the term-set. For example, in [9] authors are modeling a fuzzy control system for electronic devices, where they reduce 2500 rules of the standard fuzzy system to 500 rules, using a top-down hierarchical training approach. In [10], new approaches are proposed to solve the dimension problem by measuring the dimension, which is carried out using the particle swarm optimization and differential evolution.

This problem (the dimension problem) is related to the system of quality indicators fuzzy inference as follows. To determine the quality criteria required for regulation, the quantitative scale should contain reference points (divisions), for example, “very high” – “high” – “moderately high” – “medium” – “low” – “very low”. Since the decision-making is grounded on 9 partial criteria, and each criterion can contain more than 10 divisions on a quantitative scale, top-manager will have more than 100 different combinations in the decision-making process, where the columns are private criteria, and the rows are their formalized verbal-numerical values.

The described situation resembles the formation of a fuzzy inference rule base and, as a result, the emergence of a dimension problem when forming a combination matrix containing more than 100 different combinations. To solve this problem, it is needed to:

- determine the cardinality of an extended fuzzy rules set;
- develop a way to reduce the dimension;
- define a refined version of the fuzzy rules set.

Based on the fuzzy sets description, by extended fuzzy set we mean a fuzzy set with linguistic variables that have not three, but more gradations.

It is known that the decision accuracy depends on the result of the evaluation, which, in turn, is the result of comparing the obtained numerical value with so-called quantitative scale. At the same time, depending on the received point position on the quantitative scale (estimated value), one or the other measures are produced. Since the recorded gradations on the quantitative scale carry not only a quantitative expression, but also, as a rule, a description of the obtained numerical value influence on the overall product quality assessment, i.e., some physical meaning. At the same time, with the increasing of gradations’ levels, the degree of a given value specification impact on the overall quality assessment result increases, and, as a sequence, the accuracy of the measures developed to improve or ensure the current quality increases.

To determine factors that affect the change of target functions individual criteria and, as a result, the assessment of product quality, it is necessary to describe the operation of the product quality monitoring and evaluation model.

4 Description of the Product Quality Monitoring and Evaluation Model Operation

To present the approach to the extended fuzzy set formalization, it is needed to define the content of fuzzy sets, namely, to describe linguistic variables. As an example, we consider the content of the quality indicator "The level of usable products' output", given in the authors' work [11] (Table 1).

From Table 1, it can be concluded that linguistic variables are related to the following departments activities: economic, technological, production, quality office. Indicators listed in the Table 1 in the form of linguistic variables are the result of the listed departments activities (services or departments), which means that these linguistic variables express the state of not only the products quality, but also the quality of the production and structural divisions functioning.

Table 1. Description of the indicator "the level of usable products' output" content.

Partial criterion	Name of linguistic variable
The level of usable products' output x1	Percentage of component parts with deviation permits
	Percentage of component parts with acts on defects
	Percentage of component parts with acts on non-conformities
	Percentage of purchased items included in the product

To determine the relationship between the two-level optimization model, linguistic variables, and the state of structural elements (departments), it is necessary to identify their relationships. The structural relationship between listed departments is shown in Fig. 1.

The functions shown by arrows in Fig. 1 implement the following actions:

- function I: coordination of production and output goals and objectives;
- function II: transfer of templates for the accumulation and storage of primary information by departments and structural divisions;
- function III: structuring, grouping and distribution of primary information that characterizes production and manufactured products;
- function IV: distribution of the processed information by structural divisions and departments for the purpose of transmission to the input of the fuzzy inference system;

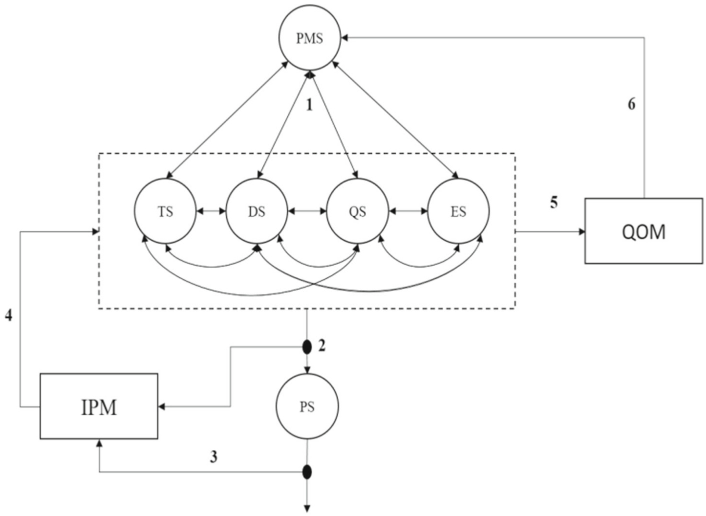

Fig. 1. Structural relationship between departments (PMS - top management, TS - technological department, DS - design department, QS - quality service, ES - economic department, PS - production process, QOM – quality assessment methodology based on two-level optimization, IPM - forms of incoming information).

- function V: transmitting the output values of the fuzzy inference system;
- function VI: providing information that characterizes production and manufactured products in terms of quality.

The work of production structures or departments (according to Fig. 1) it is carried out as follows:

1. In the PMS, vectors $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_n)$, $z = (z_1, ..., z_n)$, $i = 1, 2, ...n$ are formed that determine the target impact $x_i \subset x^t : x^t \Rightarrow x^t = \left[x^-, x^+\right]$, where x, y, z are singular criteria in accordance with Table 1, x^t is the target impact of singular criteria, x^- and x^+ are the tolerance lower and upper limits, respectively.
2. Through the function I, the target vector (target influence) $x_i \subset x^t : x^t \Rightarrow x^t = \left[x^-, x^+\right]$ is transmitted from the PMS to TS, DS, QS, ES and PS, and under its influence, target vectors $y_i \subset y^t : y^t \Rightarrow y^t = \left[y^-, y^+\right]$ and $z_i \subset z^t : z^t \Rightarrow z^t = \left[z^-, z^+\right]$ are formed in TS, DS, QS, ES and PS, which set the tolerance limits of target functions consisting of three groups of variables X, Y, Z, where y and z are singular criteria, y^t and z^t are target values of singular criteria, $\left[y^-, y^+\right]$ and $\left[z^-, z^+\right]$, respectively, the lower and upper tolerance limits. In that case of the optimization problem existing in a convex or concave set, the target constraints will be defined using Jensen's inequalities [12].

   In addition to the target vectors formulating, PMS together with TS, DS, QS, ES and PS form the partial values of target functions $c = (c_1, ..., c_m)$, $d_{(1)} = \left[d_{(1)1}, ..., d_{(1)m}\right]$, $d_{(2)} = \left[d_{(2)1}, ..., d_{(2)m}\right]$ as normals to the hyper planes, composing the direction of the target function growth and decline in the area specified by the matrices A, $B_{(1)}$, $B_{(2)}$ such, that $a_{ij}x_j \in Ax$, $b_{(1)ij}y_j \in B_{(1)}y$, $b_{(2)ij}z_j \in B_{(2)}z$. It should be recalled that the target vectors formation occurs together between the departments TS, DS, QS and ES (Fig. 1).

3. Then, through the function II, the control command is sent to the production, and forms for filling in and maintaining information about the status of the main blocks (PMS, TS, DS, QS, ES and PS) are submitted to IPM.
4. In the production process vectors $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$, $z = (z_1, \ldots, z_n)$, $i = 1, 2, \ldots n$ are sent from PS to IPM through function III, such that $x_i \subset x^r$, $y_i \subset y^t$, $z_i \subset z^t$. The change in numerical values of vectors x, y and z within the boundaries $\left[x^-, x^+\right]$, $\left[y^-, y^+\right]$ and $\left[z^-, z^+\right]$ occurs under the influence of two factors' types: controlled and unmanaged.
5. The processed information is sent from the IPM to TS, DS, QS and ES via the function IV, and further the incoming information is formalized using a fuzzy inference system.
6. The obtained numerical values (partial quality indicators) are sent to QOM via the function V to solve the next optimization problem, the optimal solution of which exists if the conditions [13, 14]:

- there are permissible solutions on the set

$$S = \left\{ (x, y) \in X \times Y_i : Ax + \sum_{i=1}^{k} B_i y \geq d,\ A_i x + B_i y \geq d_j,\ i = 1, 2, \ldots, k,\ j = 1, 2, \ldots, n \right\};$$

- there are permissible solutions for the lower level under constraints imposed by the upper level.

7. Through the function VI, user information that reflects the main problem areas and product quality assessment is transmitted to PMS for decision-making on production and manufactured products. Then, from PMS, the TS, DS, QS, and ES receive control impacts to eliminate problem areas.

From the presented description of the relations between structural units and elements of the fuzzy inference system, it can be seen that the information supplied to the input of functions II and III depends on the work of the structural units and production sites, i.e. these functions depend on arguments reflecting the state of work these structural units. However, to define an extended set, it is necessary to determine how the state of the structural unit will affect both the products quality and the quality of the interrelated departments work. Hence the following problem arises: how to determine the importance of the structural unit work and the necessary number of gradations on the evaluation scale, so that it'll become possible to regulate the products quality flexibly.

As it is known from TQM [15], the quality management system consists of interrelated processes, divided into the main, auxiliary, providing and managerial. At the same time, each process contributes to the consumer product value. Besides, the contribution of the process can be both positive and negative, for example, tighter control at the stage of the production process leads to an increase in the products cost, which reduces attractiveness to potential customers. However, there could be not only external, but also internal customers of the process result. At the same time, the internal customers' satisfaction is directly related to the satisfaction of external ones, for example, the occurrence of defects at the production stage leads to an increase in the production time of finished products, which affects the delivery time of products to the customer. Thus, to determine the evaluation scale, it is necessary to distinguish the final functions of the process, the complexity of the process and customers of the process. Then, based on the selected process features, it is necessary to determine the quantitative measure of the process information, and, as the numerical measure value is greater, the more important this process is. After that, based on the process importance, we must determine the number of gradations on the quantitative scale.

The most effective tool for describing the states of physical and non-physical processes is the theory on the amount of information based on B. Hotling and (or) C. Shannon measure [16]. The need to apply this theory is caused by the following:

a. the state of the quality management system processes, which is difficult for modeling from the perspective of probability theory. For a probability-theoretic processes functioning states' description, it is necessary to have a large amount of statistical information. It is important that the processes functioning depends on such factors as the number of personnel and their qualifications, which is almost impossible to describe using normal, Poisson, exponential and other distribution laws;
b. taking into accounting the heterogeneous information, such as staff qualifications, technologies used, labor intensity;
c. the presence of structural elements that determine the process functioning and its state. With this information, it is possible to assess the process complexity, and, consequently, its impact, since the complexity of the process depends not only on its structure, but also on the number of output nodes (Fig. 1);
d. the ability of a process to be in multiple states at the same time;
e. disorganization of the process, which means that any process is not ideal, thus it is difficult to apply improvements;
f. permanent changes in external conditions.

Based on these points, the need appears to develop a methodology for measuring the structural information measure of described processes grounded on the information theory [17].

5 Development of a Method for Finding an Extended Fuzzy Set of Fuzzy Rules

This section defines the information measure of the forming partial quality indicators process (see Table 1). To define the processes information measure and an extended fuzzy set of fuzzy rules, it is necessary to take into account the following process features:

- process customers;
- scope of the process;
- resources and tasks submitted to the process input;
- process functions;
- process execution technologies;
- outputs of the process.

The process of obtaining information on partial quality indicators should consist the following operations:

a. transfer of resources and basic materials to perform functions of structural units involved in the process of generating information on partial quality indicators;
b. performing the functions of the structural units involved in the process of forming information on partial quality indicators;
c. control points for checking the results of structural divisions activities;
d. making a decision on the results obtained;
e. elimination of comments.

To define an extended fuzzy set, we describe the process of forming information about quality indicators through a directed graph $G(V, E)$ [18]. Let's define this graph with the following features:

1. Vertices of the graph are the vector $V = (v_1, v_2, ..., v_n)$, where $i = \overline{1, n}$ are the process operations, and the edge is the vector $E = (e_1, e_2, ..., e_m)$, where $j = \overline{1, m}$ is the number of the edge [19].
2. When moving from one operation to another within the scope of the process definition, resources are spent, defined as a vector $S = (s_1, s_2, ..., s_m)$, where $e_j^+ = (v_i, v_{i+1})$ are planned costs, and $e_j^- = (v_i - 1, v)$ are expenses.
3. The amount of expenses depends on the method of comments elimination and the stage of comments detection.
4. Each expense is characterized by its own entropy, therefore, the content of entropy is the volume of negative factors that lead to expenses:

$$H(s_i) = (-k)\log_2\left(1 - \frac{s_i}{SP}\right), \tag{6}$$

where SP are the planned costs; k is the number of operation consumers, on which the comments were noticed; the total entropy of the graph by cost is $H(S) = \sum_{i=1}^{n} H(s_i)$.

To determine the method for constructing a rank scale, we use statistical methods for constructing histograms, namely, finding the number of intervals for a quantitative scale. There are many ways to do this. We will focus on the method used for the equally probable distribution law, since the chosen method for finding entropy (6) is based on the so-called structural entropy (Hartley entropy), the peculiarity of which is the equal probability of occurrence of all given events [20]:

$$k = 4\lg(n), \tag{7}$$

where n is the sample size, and k is the number of partitioning intervals.

To determine expenses, a matrix of the incident $v_n \times s_m$ is preformed, in which v_n are vertices (process operations), s_m are edges (costs/expenses).

Only the negative values of the edges $-s_j$ are involved in the calculation of the partition intervals number (7). After determining the number of intervals, a rank scale is constructed, where 0 is the absence of entropy (no comments), and n is the maximum rank of the scale (the maximum entropy value). The value of n is calculated by (7). An example of the correspondence of the entropy scale and the rank scale at $H(s_1) \approx 11$ and $k(8.04) \approx 14$ is shown in Table 2. The following conditions are applied to the scale construction:

$$n = \begin{cases} 1 \ \mathit{if} \ k < 0, \\ 0 \ \mathit{if} \ k = 0, \\ 2 \ \mathit{if} \ 0.85 \le k < 1 \end{cases} . \tag{8}$$

6 Method for Reducing the Fuzzy Set Dimension and Finding an Updated List of Fuzzy Quality Indicators

To reduce the dimension of the fuzzy set or the ranks of the rank scale, an approach based on the fuzzy clustering of c–means is used [21, 22]. We define the matrix of observations for applying the clustering algorithm in the form $N = \{x_{ij}\}$, where the scale ranks go by indices i ($i = \overline{1, n}$), and attributes of each rank go by indices j ($j = \overline{1, m}$). The feature vector x_j will consist of the following quantitative factors:

1. expenses on eliminating comments without returning to the previous operation and using resources;
2. expenses on eliminating comments without returning to the previous operation, but using resources;
3. expenses on eliminating comments with a return to the previous operation and using resources;
4. the number of quality management system documents used;
5. the number of design and technological documentation used;
6. the number of process operation functions;
7. the number of potential consumers of the process operations.

After determining the observation matrix, a random set of clusters is formed, according to which a fuzzy cluster matrix $M = \{c_{ij}\}, \ i = \overline{1, T}, \ j = \overline{1, m}$ is constructed, where the clusters go by indices i, and by the indices j – the degree of the rank, belonging to a certain cluster of the rank scale. The fuzzy cluster matrix satisfies conditions.

$c_{ij} \in [0, 1], \ \sum_{i=1}^{T} c_{ij} = 1 \ \mathit{if} \ j = \overline{1, m}$, and $0 < \sum_{j=1}^{m} c_{ij} < m \ \mathit{if} \ j = \overline{1, m}$. Quality assessment of the rank scale division into clusters is determined by the degree of belonging [23]:

$$J = \sum_{i=1}^{T} \sum_{j=1}^{m} \left(c_{ij}\right)^{w} d\left(l_i x_j\right), \tag{9}$$

where $d(l_i x_j)$ is the Euclidean distance between the j–th object and the i–th center of the cluster l_i, $w \in (1, \infty)$ is the exponential weight that determines the blurriness of the cluster.

Table 2. Rank scale.

Edges of the incident matrix s_i	Entropy H	Number of partitioning intervals k	Number of operations n
0.2	11.60964	14.14901	14
0.25	10.0	13.28771	13
0.3	8.684828	12.47399	12
0.35	7.572866	11.68336	11
0.4	6.60964	10.89829	10
0.45	5.760015	10.10429	10
0.5	5.0	9.287712	9
0.55	4.312482	8.434074	8
0.6	3.684828	7.526389	7
0.65	3.107442	6.54291	6
0.7	2.572866	5.453505	5
0.75	2.075187	4.212967	4
0.8	1.60964	2.746954	3
0.85	1.172326	0.917497	2
0.9	0.760015	−1.5836	1
0.95	0.370003	−5.73757	1
1.0	0	0	0

The cluster centers make up a matrix $V = \{v_{ik}\}$, whose components are calculated by the formula:

$$v_{ik} = \frac{\sum_{j=1}^{m} \left(c_{ij}\right)^{w} x_{ik}}{\sum_{j=1}^{m} \left(c_{ij}\right)^{w}}, \quad k = \overline{1, n}. \tag{10}$$

Our task is to find a fuzzy clusters matrix, at which j is minimal. In subsequent iterations, elements c_{ij} are calculated as follows:

$$\begin{aligned} c_{ij} &= \frac{1}{\left(d_{ij}\right)^{\frac{2}{w-1}} \sum_{k=1}^{T} \frac{1}{\left(d_{kj}\right)^{\frac{2}{w-1}}}} \quad if \;\; d_{ij} > 0; \\ c_{ij} &= \begin{cases} 1, \; k = i, \\ 0, \; k \neq i \end{cases} \quad if \;\; d_{ij} = 0. \end{aligned} \tag{11}$$

Calculations should be continued until the difference $\|M - M^*\|$ becomes minimal (M^* are matrices in the previous iteration). The proof of this algorithm convergence is presented in [24].

7 Conclusions

The proposed method for constructing a fuzzy term-set allows to solve the problem of determining an updated list of fuzzy 1uality indicators, for their further application in a two-level model for optimizing of product quality assessment. This method is also capable of processing large numerical values of scale division intervals.

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