Dynamics and Stability: From an Ancillary to a Leading Role in the History of AIMETA

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Abstract The role of "Dynamics and Stability" in the history of AIMETA Conferences is discussed. It is emphasized that these subjects, initially away from the interests of the scientific community, have assumed increasing importance over time, culminating in the foundation of the AIMETA Group of Dynamics and Stability (GADeS), which today collects scientific contributions from many members belonging to different scientific sectors. The first timid steps taken by "Dynamics and Stability" in AIMETA are recalled in a historical key, and the causes that first slowed down and then determined their growth are conjectured. With reference to Dynamics and to its classic distinction between linear and nonlinear behaviour, the perturbation method is seen as a key to interpreting nonlinear phenomena. With reference to Stability and Bifurcation, the existence of non-communicating worlds, namely static and dynamic bifurcations, is noted in the world scientific panorama. Once again, the perturbation method can be recognized as the tool that acts as a bridge between the two worlds. Finally, the main topics that have been debated in the GADeS meetings and mini-symposia are briefly reviewed. This paper, in addition to representing a historical synthesis and a cross-section of contemporary research in "Dynamics and Stability" within AIMETA, offers critical considerations on the fragmentation of knowledge, and encourages the development of a unifying vision of the two disciplines.

Keywords Dynamics · Stability · Static and dynamic bifurcations · Center manifold · Perturbation method · Structural mechanics

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1 Introduction

Dynamics and Stability are pillars of both theoretical and applied Mechanics. The first describes the evolution, free or forced, of a system, isolated or interacting with the environment; the latter evaluates the robustness of the equilibrium, or more generally, of a stationary state (periodic or quasi-periodic motion). Both disciplines are declined in multiple sub-disciplines, so diversified from each other, to appear as independent. On the one hand, the linear and nonlinear dynamics, the seismic dynamics, the wind response, the impulsive response, the wave propagation, …; on the other, the static stability (buckling and post-buckling), the dynamic stability of autonomous systems (the Hopf, flip, Neimark-Sacker bifurcations), the parametric excitation, the chaos.

As a matter of principle, there is no mechanical problem whose study does not require the use of concepts and methods of Dynamics and/or Stability. One would therefore expect, looking with a careful eye to the history of AIMETA, to notice significant interest from Italian researchers to these two disciplines. Surprisingly, re-reading the indexes of the Conference Proceedings relating to the period 1971–2011, it turns out that their role was marginal and subordinated to other major issues of interest at the time. Stability was intended as a synonym for static bifurcation (buckling, sometimes post-buckling), almost never placed in dedicated sessions, but more frequently discussed in more general contexts of Structural Mechanics. On the other hand, the dynamic stability of nonconservative systems was almost totally absent. Besides, Dynamics was a topic for a few enthusiasts, often relegated to the last session of the last day of the conference, carelessly followed by colleagues who were waiting for the train with suitcase in hand.

The causes of this apparent lack of interest in fundamental issues, which, on the contrary, should have played a unifying role in the varied panorama of the disciplinary competences of AIMETA members, still appear difficult to explain today. Nonetheless, the authors conjecture here the reasons for the phenomenon, in an attempt to shed light on the roots of our scientific history.

1.1 Ancient History

AIMETA, as is well known, collects skills ranging from General Mechanics to Fluid, Machine, Solid and Structure Mechanics. However, the numerically preponderant role of the two scientific sectors that refer to ICAR/08 (i.e., the disciplinary area of Solids and Structures in the Italian university system), is a fact. All the AIMETA Boards of Directors (CD) have worked hard to spread the "mission" of the Association among the affiliates of the three minority sectors, with results that, unfortunately, have not been comforting, so that, to date, the numerical composition remains "stable", notably unbalanced. The preponderant role of ICAR/08 has therefore markedly addressed the interests of researchers, transferring, in the AIMETA context, the choices, inclinations, beliefs and, sometimes, even prejudices, which pervaded that scientific-disciplinary sector. Dynamics, therefore, which was slow to make its way into ICAR/08, was slow to appear also in AIMETA. Stability, on the other hand, limited to its static aspects, was recognized as an ancient discipline, albeit reserved for a few amateurs and, therefore, was welcomed in the scenario of the great themes of Structural Mechanics, and consequently, in AIMETA. Dynamic stability, on the contrary, which still today (not only in Italy, but in the world) many scholars think that nothing has to do with static stability, was almost completely unknown.

In support of the above assertion, two anecdotes are told. The senior author of this article was lucky enough to attend, as a student, the first edition of the Dynamics of Structures course, which was held in Rome in 1976 by Professor Carlo Gavarini. All the young participants were made aware of the exceptional nature of the event, as there was only one other similar course in Italy, held by Professor Giuseppe Grandori at the Milan Polytechnic. This indicates the fact that the Dynamics of Structures, until then the prerogative of Applied Mechanics and Machines only, entered the context of Structural Mechanics and Engineering late. A second anecdote concerns a discussion, which the senior author himself had at the beginning of the 2000s, with a highly esteemed colleague from Solid Mechanics, who has now passed away, in a selection committee for professorship. The colleague, in examining the titles of an aspiring full professor who had dealt with Dynamics subjects, asked: "But dynamics, is it Structural Mechanics?", thus questioning the relevance of the topic to the core of the scientific-disciplinary sector. This underlines the ancillary role that the same Dynamics played, no more than twenty years ago, in the field of Structural Mechanics.

1.2 Modern History

Recent years (2012–2019) have marked an epochal change regarding the placement of Dynamics in the scientific interests of AIMETA. The authors believe that the establishment of GADeS (AIMETA Group of Dynamics and Stability) represented *the turning point*. The Group was born on a solicitation that Giuseppe Piccardo made to Angelo Luongo, to be a spokesperson at the CD AIMETA, of the need to formalize the growing scientific interest in the themes of Dynamics and Stability with the creation of a new AIMETA Group. The CD, represented by Prof. Paolo Luchini, showed interest and appreciation for the initiative, while expressing some doubts (moreover shared by the promoters themselves) about the success of the operation, especially towards the much desired involvement of all the members of the Association. GADeS was formally founded on October 19, 2012. The first assembly (constituent and scientific) was held in Rome, at the Department of Basic and Applied Sciences for Engineering—Mathematics Section—of the Sapienza University of Rome. 52 sympathizers participated, with significant representation of mechanical engineers and applied mathematicians, in addition to the majority component belonging to structural mechanics, as usual. The first Coordinating Committee was elected, consisting of Profs. Angelo Luongo (coordinator), Sandra Carillo, Walter D'Ambrogio, representing the three souls of the Group. The debut took place at the following AIMETA Conference in Turin (XXI AIMETA Conference, September 17–20, 2013), where, at the mini-symposium organized by GADeS, 46 papers were presented, which occupied all the days of the event. The topics mainly covered the Dynamics and the Control of the Response of Mechanical Systems, the Dynamical Systems and their Stability and Bifurcation. Suddenly, Dynamics had abandoned its minority role in AIMETA to achieve the leading role it deserves. The merit of this was certainly not of the GADeS proponents, who limited themselves to collecting the request of the researchers, but of the many Italian scholars (not only from ICAR/08) who had long shown interest in these disciplinary areas. GADeS, therefore, represented a "home" in which to meet, where everyone could bring their baggage of knowledge, no longer in a personal capacity, but to constitute a recognizable "critical mass". In the following years, the contribution of GADeS to AIMETA remained stable, confirming that the one in Turin was not a singular event. The new coordinating committee, which took the place of the first in 2016, continued the work, urging the development of the scientific subjects that characterize the Group.

Today we can say that Dynamics and Stability are disciplines that contribute significantly to AIMETA's mission of spreading scientific culture in the mechanical field. However, other steps will have to be made to complete the slow transformation processes that have taken place over the past fifty years. In the following, some of these aspects are discussed, focusing on the growth of a desirable unifying vision of the aforementioned sub-disciplines.

2 Dynamics

Dynamics, like many other engineering disciplines, is usually classified into *linear* and *nonlinear* dynamics, as two disjoint worlds. Those who study nonlinear aspects, while knowing basic linear dynamics, turn their attention to the *distinctive features* introduced by nonlinearities, i.e. to all those phenomena that cannot be explained in a linear context. Those who deal with linear problems, on the other hand, often ignore the nonlinear effects, and investigate a multitude of models and application cases, which sometimes reveal unexpected behaviors, and therefore fulfill the need to stimulate scientific curiosity. Thinking today of suggesting the development of a unifying knowledge of the two worlds is an unrealistic undertaking, given the extreme diversification of the themes and, above all, of the applicative interests, which lead to specializations aimed at particular problems. Nonetheless, on these aspects, the "bridging" role played by the *perturbation method*, which we want to comment on, deserves an important consideration.

2.1 The Perturbation Method as a Key to Understanding Nonlinear Phenomena

As is known, the perturbation method, in its various versions (strained parameters, multiple scales, method of averaging, … [1]), reduces a (weakly[1]) nonlinear problem to a succession of linear equations. These are governed by the same operator, and must be solved in sequence, using the information acquired in the previous steps. When applied to a dynamics problem, the (generating) equation that triggers the procedure is constituted by the linear model. The method, therefore, far from representing a mere technical tool, offers an educational and profound key to reading many nonlinear phenomena. These are seen as caused by the occurrence of *resonances* (external, internal, parametric), which arise between the different components of motion, generated by nonlinearities through the creation of multiple frequencies from the basic ones. This vision leads the understanding of nonlinear phenomena (such as the coupling and the transfer of energy from one mode to another, the limitation of the nonlinear response through the creation of limit cycles, …) to the use of a single concept, *the resonance*, which is well known in the linear field. The procedure amply clarifies the role of linear normal modes that contribute to the nonlinear response, and opens the way to the formulation of *reduced models*, with a few degrees of freedom, capable of grasping the essence of phenomena.

On the contrary, it should be noted that, due to the lack of knowledge of perturbation methods, many researchers make recurrent and systematic use of direct integrations of the equations of motion, the results of which often obscure the synthetic understanding of the phenomenon. The lack of a true scientific culture of nonlinear dynamics often leads the researcher to use discrete models with thousands of degrees of freedom, when a few modes (often only two or three) would be sufficient to give qualitatively accurate information and, above all, would be able to reveal the nature of the response. It is worth remembering, in support of this thesis, the animated scientific discussions that developed during the XXIII AIMETA conference in Salerno (2017), around the causes that led to the famous collapse of the Tacoma Narrow Bridge in 1940. The authors of the study, refuting the most accredited interpretation in the literature, namely the occurrence of aeroelastic instability, explained the phenomenon as due to the flexural–torsional modal coupling generated by nonlinearities. However, they did not know how to answer the question of clarifying what was the ratio between the fundamental flexural and torsional frequencies which, if integer (i.e., equal to 2,3, …), is solely responsible for any coupling (from internal resonance), as clearly explained by the perturbation method.

The diffusion in the world scientific community, particularly mechanical, of the perturbation method, as the main tool for the analysis of nonlinear dynamic systems, is due to Ali H. Nayfeh (1933–2017). Author of numerous books and articles on the subject, his work has accompanied the scientific growth of many students around

[1] The smallness of nonlinearity, which limits to consider motions of small but finite amplitude, generally does not represent a (too) strong limitation, but is sufficient to answer most of the engineering problems.

the world. In Italy, the first to early collect his teaching were Angelo Luongo, Giuseppe Rega and Fabrizio Vestroni, who developed some applications, many of which were presented at the AIMETA Conferences. These were followed by other scholars (Stefano Lenci and Walter Lacarbonara among the oldest) and many other young people. Today the Italian school of nonlinear dynamics has an international dimension, worldwide recognized with the appointment of Walter Lacarbonara as Editor-in-Chief of the *Nonlinear Dynamics* journal founded by Ali Hasan Nayfeh.

3 Stability and Bifurcation

Stability and Bifurcation (different concepts, but often confused in the common language) are ancient knowledge, developed in Mechanics but then spread to all physical–mathematical disciplines. In the engineering world, with the singular exception of Systems Theory, the equilibrium bifurcation is almost always understood as a synonym for "Buckling", that is Static Bifurcation. The historical reasons for this lie in the fact that the prevailing attention of researchers has turned to conservative systems (elastic and subject to gravitational forces), whose stability of equilibrium, according to the Lagrange-Dirichlet Theorem, is not influenced by the kinetic energy. The damping, when introduced in these systems, plays a secondary role since, if on the one hand it improves stability transforming it from marginal to asymptotic, on the other it is unable to stabilize unstable equilibria. Therefore, if one ignores the non-conservative external actions (for example the follower forces, or those dependent on the speed), the static approach is correct.

On the other hand, the general theory of stability—Poincarè (1854–1912), Lyapunov (1857–1918)—was developed many years after the pioneering studies on the static stability by Euler (1707–1783). In particular, the fundamental theorem of the *Center Manifold*, which is at the basis of the studies on dynamic bifurcations, has been proved in recent times [2], and it is not yet sufficiently known in the engineering world, remaining for the most part a tool for applied mathematicians.

3.1 Two Non-communicating Worlds

As a result of the evolution of Science in terms of Stability, today there are two distinct worlds on the international scene, apparently not communicating with each other:

- the Elastic Stability community, which draws on the approach of Koiter [3], Budiansky [4], Hutchinson [5], of the English school (Thompson and Hunt [6], Croll and Walker [7], Supple [8]), and which has seen over the years a myriad of contributions, some of which also developed within the AIMETA context;

- the Dynamic Stability community, which operates in the context of the general theory of dynamical systems (Guckenheimer and Holmes [9], Wiggins [10], Troger and Steindl [11]), which has joined AIMETA only in relatively recent years.

The general theory of stability, however, includes the case of static bifurcation, pursuing an economy of thought that can only benefit the rationalization of knowledge. The two theories really differ slightly:

- a bifurcation is static when, as a bifurcation parameter varies, an eigenvalue of the Jacobian matrix, tangent to the equilibrium path, crosses the imaginary axis in zero; a bifurcation is dynamic when a pair of conjugated complex eigenvalues crosses the imaginary axis at points other than zero;
- from a static bifurcation (at least) a new equilibrium branch is born; from a dynamic bifurcation (at least) a family of periodic motions is born.

The same general theory does not attribute the elementary geometric meaning of *branching point* to the bifurcation, suggested for example by the bifurcation diagram of the Euler beam, but the meaning of *instantaneous qualitative modification of the phase portrait* of the system, i.e. of its dynamics. This modification also occurs at the branching point (where the number of equilibrium points changes), but not only at that.

To be able to unify the two worlds, and to look at Stability and Bifurcation as a *unicum*, it is however necessary to abandon the energy criterion of stability and, even more, the static criterion of adjacent equilibrium (very much appreciated by professors and engineering students), and to regard the structure as a dynamical system. An operation that, on the one hand, may appear overabundant compared to the objective, on the other is absolutely necessary when one wants to include non-conservative forces in the model, as in the case of aeroelasticity.

3.2 The Perturbation Method as a "Bridge"

Actually, static and dynamic bifurcations are already shared by a common analytical tool of investigation, which is the perturbation method, although this affinity is often unknown to the authors themselves [12]. The whole theory of post-buckling, aimed at the asymptotic construction of the branched paths, is based on the so-called Method of Static Perturbation [6, 13]. This consists in developing displacement and load in series as a function of a perturbation parameter, equivalent to a curvilinear abscissa along the unknown path. The systematic imposition of compatibility conditions, which remove forces outside the operator's range, allows for the connection between displacement and load. On the other hand, all the perturbation methods developed in nonlinear dynamics (Lindstedt-Poincaré, Multiple Scales, Method of Averaging [1, 14]) can be used in the analysis of dynamic bifurcations. In particular, the multiple scale method appears as the natural extension of the method of static perturbation, where the removal of secular terms plays the same role as static compatibility, since it removes the forces that induce a non-periodic response.

The use of the perturbation method in dynamic bifurcation analysis, however, is not commonly accepted. As mentioned, many researchers use the Center Manifold Theorem to deduce the bifurcation equations of the problem. However, Luongo, Paolone and co-authors [15–18] have shown how the perturbation method provides the same results of the Center Manifold method, in a simpler and more automatic way, even in cases of high degeneracy of the operator (multiple bifurcations, which imply a non-diagonal Jordan canonical form of the Jacobian matrix).

Despite its application complexity, the message transmitted by the Center Manifold Theorem is of incomparable scientific value: close to a bifurcation (i.e. around the critical points in the parameter space), the system evolves on a manifold of the state space of low dimension (usually equal to 1, 2, 3, …), which is tangent to the critical subspace (i.e. spanned by the critical eigenvectors of the Jacobian matrix), of which it retains the dimension. This means that reduced models, with a very low number of degrees of freedom, are able to capture the essence of the bifurcation phenomenon. These models, however, need to be able to capture the curvature of the manifold, and not just to span the critical subspace. For example, the Nonlinear Normal Modes [19, 20] perform this function. The insufficient knowledge of this very important result, however, still leads many researchers today to tackle the problem through brutal numerical computations, using models with many degrees of freedom.

The Center Manifold Method also has a counterpart in static bifurcation, known as the Lyapunov-Schmidt reduction [11], essentially based on the same considerations. This was Koiter's approach in developing his famous doctoral thesis [3], which marked the beginning of the modern Elastic Stability Theory.

4 A Glance at the GADeS Years

As mentioned in Sect. 1, the GADeS group has given a new home to the topics of Dynamics and Stability within AIMETA. In some cases, GADeS has collected subjects already present within the AIMETA conferences, generally placed under the broad heading of Mechanics of Structures and Mechanics of Machines. In many other cases, GADeS contributed to welcoming new topics, widening the audience of the AIMETA conferences. Since its birth GADeS has been the promoter of three Special Issues (SI) in journals of international importance [21–23]. The material contained in these SI covers a broad spectrum of topics in solid mechanics, structural mechanics and control, in both linear and nonlinear fields, in the presence of environmental or artificial excitations; analytical, numerical and experimental methods are presented, using deterministic as well as stochastic approaches. Therefore, references cited mainly refer to developments of some of the research presented during the GADeS mini-symposia and published in these SI. It is worth noting that the most recent SI [23], published in 2021 on Nonlinear Dynamics journal, was initially conceived for presenting a survey of new studies carried out by researchers participating in the

GADeS group, similarly to the previous two SI. However, the breadth of the topics covered in GADeS and their importance at an international level led the editors to enlarge the Italian viewpoint by inviting worldwide renowned scientists in order to give a broader view on recent advances in stability, bifurcations and nonlinear vibrations that may involve different kinds of mechanical systems. In the following, an attempt to show the main subjects presented during the GADeS meetings and mini-symposia will be made, without any claim to be exhaustive.

- Bifurcation and dynamic stability. This subject is common in the research of GADeS participants, with applications to various fields, using reduced order models. By way of example, the dynamic stability of classic case studies, such as the Ziegler's column and the Nicolai paradox, are discussed [24, 25]; the critical aeroelastic behaviour of bridges [26, 27], cables [28, 29], building [30, 31] is analyzed; the stability of parametrically-excited systems is dealt with [32, 33]. Specific studies relating to wind-structure interaction problems seem to be of considerable interest [34–36].
- Buckling. It is one of the historical topics dealt with in AIMETA, especially regarding thin-walled beams, still active at GADeS meetings [37, 38]. The compressive and the tensile buckling in slender beams [39] is also present, along with instability phenomena in double-layered pipes [40], non-classic interactive buckling in paradigmatic discrete systems [41] and flexural–torsional buckling of beam-like structures [42]. Developments in the framework of the Generalized Beam Theory (GBT) are also addressed [43–45].
- Cable dynamics. The subject, known to very few researchers in the seventies of the last century, was presented for the first time by Luongo, Rega, Vestroni at the V AIMETA conference in Palermo (1980), as regards the perturbation analysis of the free nonlinear oscillations, and is admirably summed up in Rega's review papers [46, 47]. This topic continues to be very active within the GADeS group. In addition to the papers on galloping already mentioned [28, 29], new formulations for cable self-damping [48], mechanical models for specific dampers [49], numerical methods using corotational beam elements [50] and modal interactions in beam–cable systems [51] are dealt with.
- Dynamical phenomena in mechanical systems. It is obviously a very vast subject, impossible to summarize in all its aspects. Among the various interests of the researchers participating in the GADeS meetings it is recalled the friction-induced oscillations [52, 53], the dynamical integrity [54, 55], the dynamics of composite/nanocomposite structures [56, 57], the fractional-order system dynamics [58–60], the hysteretic systems [61–63], the linear and nonlinear analysis of structures [64–70].
- Equivalent beam models. This subject has had a notable impact on GADeS meetings under the impulse of the research group of the University of L'Aquila. Starting from equivalent nonlinear models able to reproduce the dynamic behavior of 3D shear-type structures [71, 72], it has moved on to equivalent Timoshenko beam models [73, 74], which are also able to describe micro-structured bodies [75] and

non-symmetrical layouts [76]. This type of modeling can find large application areas in the context of environmental forces [77, 78].

- Experimental dynamics. This subject, characterized by a strong interdisciplinarity, has had a remarkable development in recent years within GADeS. By way of example, it is reported experimental activities in wind-structure interactions [79–81], in monitoring and earthquake engineering [82–84], in vibro-impact dynamics [85], in dynamics of circular cylindrical shells [86, 87].
- Transient dynamics. This is a subject that has had a great increase in recent years, including through GADeS. It covers classic topics such as vibrations induced by moving loads [88, 89] and moving masses [90–92]. Recently the propagation of uncertainties on serviceability assessment of pedestrian bridges has attracted the attention of various researches [93, 94]. Effects caused by non-stationary wind loading seem a promising research field [95, 96].
- Vibration control systems. A large group of GADeS presentations addresses vibration control through various techniques, as also evidenced by the fact that control appeared within several papers cited up to now. Classic systems such as Tuned Mass Dampers and Tuned Liquid Column Dampers are present [97, 98]. At the same time, Nonlinear Energy Sinks (NES) has received great attention [99–101], together with piezoelectric and hysteretic systems [102, 103].

5 Final Remarks

The growing relevance of Dynamics and Stability in the scientific interests of AIMETA researchers, now coordinated by GADeS, was analyzed in a historical key. Some of the topics discussed at the association's conferences were reviewed, recalling the growth process. However, in the opinion of the authors, this requires further development in the direction of the unification of concepts and methods, and consequent overcoming of barriers, aimed at artificially defining sub-disciplines, which stand in the way of a global and not partial approach to the problem. To this end, the following considerations have been developed.

1. Linear and nonlinear dynamics should be considered as two aspects of the same discipline. Nonlinearities induce qualitatively new effects, which cannot be explained with the linear model. However, the perturbation method, by reducing the nonlinear problem to a succession of linear problems, explains these phenomena with the same simple tools of linear analysis. Above all, it transforms nonlinearities into known forcing, acting on the linear system.
2. Static stability is a particular aspect of dynamic stability, in which an eigenvalue of the Jacobian matrix crosses the imaginary axis in zero, rather than in a generic point. The same concept of bifurcation should be considered not as a branch of the equilibrium path, but as a qualitative change of the phase portrait. Even in stability, the perturbation method allows to deal with static and dynamic bifurcations essentially in the same way.

3. The perturbation method is not merely an operational tool, but is a key to interpreting both static and dynamic nonlinear problems. As a fundamental result, it teaches that, at the bifurcation or near a resonance, the response of a static or dynamic system is described by a very small number of degrees of freedom. This complies with the Center Manifold Theory, according to which the evolution develops on a manifold tangent to the critical subspace. This fundamental notion needs to be spread and matured by many researchers, not experts in the field, who use huge numerical models, which threaten to obscure the deep understanding of the phenomena.
4. As evident from the papers derived from the presentations at the GADeS meetings and mini-symposia, the topics of Dynamics and Stability have expanded considerably from the beginning, and are now characterized by a strong interdisciplinarity. GADeS reflects the rich spectrum of topics covered by the modern researches in "Dynamics and Stability", and fits perfectly into the current trends of Nonlinear Dynamics [104, 105]. Reduced-order modeling remains a common denominator present in almost all the works within GADeS.

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