Giuseppe Rega *Editor*

50+ Years of AIMETA

A Journey Through Theoretical and Applied Mechanics in Italy

Springer

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Foreword

Since its foundation, in 1965, by the eight illuminated scientists Bruno Finzi, Giorgio Sestini, Carlo Ferrari, Giulio Supino, Emilio Massa, Giovanni Bianchi, Leo Finzi, and Elio Giangreco, the AIMETA (Italian Association of Theoretical and Applied Mechanics—Associazione Italiana di Meccanica Teorica e Applicata) played an important role in the development of the Italian community of mechanics. At the international level, it is the Italian adhering association to IUTAM (International Union of Theoretical and Applied Mechanics), and thus it was the mean by which Italy participated in the international community, especially at early times, when it was not so easy and not so common to attend international congresses. It is my personal belief that many of the international successes of the Italian community have benefited from the AIMETA environment.

At a domestic level, on the other hand, it has been—and it continues to be—the first stage where young scholars make their first experience of presenting a paper and participating in a scientific event, learning on the field. A very worthy training ground. This happened to me, too, and I am proud of this.

By the Congresses, held every two years (apart from two exceptions, the last one due to the recent pandemic), AIMETA also helped so much in developing knowledge between colleagues, personal relationships, scientific collaborations, scientific discussions, spread of knowledge, and updates on recent developments: in summary, to build up, sustain, and renovate the national community. In this respect, a not secondary role was also played by “Meccanica”, the Journal of AIMETA, that ended years ago its transition to a fully respected and renowned international scientific journal.

AIMETA gave so much to the community, and I felt it should be somehow celebrated. Within this fertile background, the idea to write a book on the history of AIMETA came to me some time ago, when I realized that after 50+ years of life of the Association, it was the time to review his activity and achievements in retrospective and prospective viewpoints, thus complementing and somehow completing what we did for “Meccanica”, that celebrated its 50 years by a Special Issue (Volume 51, issue 12, December 2016). In fact, I believe that knowing our history is a necessary step to continue and somehow to optimize the next evolution. It is also a way to recognize the work of scholars that preceded us and upon which our work is based on.

I had not in mind a barren and aseptic history, but a "live" one, where numbers and data (necessary and welcome from an analytic point of view, or just for curiosity) were accompanied by retrospectives, personal viewpoints, and remembrances, together of course with more technical contributions aimed at drawing the past and shaping the future.

When I presented this idea to the Executive Council of AIMETA (in the first meeting when I was elected as President), it received a very positive welcome, and it was unanimously approved. I sincerely thank the colleagues (Paolo Luchini, Sandra Carillo, Umberto Perego, Fernando Fraternali, Walter D'Ambrogio, and Carlo Massimo Casciola) for supporting it and accepting this "challenge".

The next step was to choose the right person that could take care of this. It was not easy, indeed, because we were perfectly aware of the difficulties that should have been faced, in terms of time, efforts, and patience. We unanimously realized that Giuseppe Rega, past President of AIMETA and past Editor-in-Chief of "Meccanica", was the right person, for his scientific and human qualities and for the respect he has within the whole community.

In some preliminary discussions I had with Giuseppe to illustrate our ideas, he correctly suggested to enlarge a little bit the scope and to not remain only within the history of the Association, but to open a bit to the history of mechanics in Italy. We strongly agreed on that, and then I left to him the "baton".

I am sure that the challenge has been won, and a wonderful work has been made that I hope will remain as a keystone for us and for the next generations of Italian researchers in the field of mechanics.

Sincere thanks are of course due to Giuseppe, for the hard work he did, for practically realizing our somehow vague ideas, and for writing fundamental chapters. I can imagine how demanding it has been to shape the book (for example, deciding to divide the book into two parts) and to involve and coordinate all authors. Authors to which our gratitude is also due, for their hard work and valuable contributions.

Ancona, Italy
October 2021

Stefano Lenci
President of AIMETA

Preface

The AIMETA President Stefano Lenci talked to me of the significance of celebrating the fiftieth anniversary from the foundation of AIMETA with a publication in the year 2018, namely with a time delay of less than three years with respect to the actual anniversary date (AIMETA was established at the very end of 1965). Asking my opinion on the matter, which was favorable in principle yet somehow problematic as to the substance, he subtly inquired about whether I would have been available to take care of the matter. Besides my several other commitments, in Spring 2019, I underwent one more serious surgery to my backbone, with a successive intense rehabilitation going on until the beginning of 2020 when our life started to be heavily affected by the COVID pandemic. This summary is just intended to somehow justify why this volume comes to the fore at the beginning of 2022, i.e. about six years after the anniversary date, this being a circumstance in no way attributable to the President and the Executive Council of AIMETA who took the initiative.

Thinking of the project assigned to me by the Council, I convinced myself of two main elements. (i) Reporting on the history of a collective entity like a scientific association as a single author would definitely make no sense. (ii) Thinking of an exclusively national spotlight for any text written on AIMETA would be discrediting for an association that in the last fifty years has proved capable of obtaining significant international recognition. A third element was immediately evident, as well. (iii) Consistent with the role played by AIMETA in promoting the development of mechanical sciences in Italy, the publication should definitely be the proper occasion for making the richness and variety of the underlying research apparent. In a few words, the project became a bit more ambitious.

The initial idea (Autumn 2020) for a book to be written in English was built around general reports on the research developments that occurred within the five scientific areas of AIMETA (General Mechanics, Fluid Mechanics, Mechanics of Machines, Solid Mechanics, and Structural Mechanics) since about the 70s, prepared by single and well recognized relevant scholars. The idea swiftly showed to be too ambitious, because of requiring authors with knowledge, memory, time availability, and credibility in each of the five areas, which were impossible to identify. And indeed, due to the lack of an adequate historical perspective, dwelling authoritatively on the research carried out over the last fifty years or so would have been extremely pretentious—if not even inelegant—for anyone.

The revised project is the one realized in the present volume, after various thinking and ensuing adjustments. The volume is divided into two parts. The first part, which the writer takes the nearly full responsibility of, consists of a main introductory chapter in which the AIMETA history is attentively retraced by comprehensively looking at the wide basket of its scientific and organizational initiatives, and by assuming it as a proper perspective to look at the evolution of theoretical and applied mechanics in Italy. The weak side of this solution stands in the inevitable part position played by the writer as an exponent of only two (solids and structures) of the five scientific souls of AIMETA, despite his serious efforts to also report on the role played by the other three areas (general mechanics, fluids, and machines) in a possibly objective way. Of course, this could have never been concerned with the substance of the research themes of major interest addressed in those areas over the decades, nor with the underlying academic 'schools', due to an obvious lack of competence. Thus, the report suffers from three main 'voids' in terms of knowledge, references, and the overall summary of scientific aspects. However, the writer has taken the responsibility to dwell on some main research themes developed in the area of solid and structural mechanics and on the relevant evolution, of course presenting his personal view of the matter, though anyway filtered through his long-lasting acquaintance with the academic and scientific environments in the background. The first part of the volume also includes: (i) a detailed tribute to earlier recognized Italian scholars of mechanics who were involved with AIMETA at different levels; (ii) short testimonials by a few retired scholars who were formally committed with AIMETA in various official positions; (iii) a brief updated report on the international journal *Meccanica*. Actually, section (ii) should have duly been more extended. But, unfortunately, the fully understandable discomfort of many important protagonists of AIMETA and Italian mechanics due to a lack of adequate and reliable memory has prevented them from contributing testimonials or memories. Needless to say that those protagonists are anyway in the mind of all aware people of AIMETA.

The second part has a completely different aim. Indeed, it is intended to give an account—though unavoidably incomplete, to a large extent—of what has been going on in terms of advanced research in the five areas of AIMETA in the last few decades. A number of senior, yet still active, authors have been invited to contribute a chapter on a general theme, or a specific topic, which they have been meaningfully worked on in longer or more recent times, or which they are currently active on. The contributions have thus to be intended not as much as original papers, which would actually make not much sense in a volume whose audience will be necessarily varied and not specialized, but rather as scientific surveys, also possibly cross-disciplinary, summaries of achievements of research groups, expressions of research viewpoints and/or criteria, indications of perspectives of development, and so on. As such, collected texts are of a rather different nature, ranging from contributions which overview modeling, methodological, and/or phenomenological aspects in a given (wider or narrower) field, according to a prevailing analytically and/or computationally oriented perspective or to a qualitative one, up to contributions dwelling on a specific research topic. Despite a possible consequent feeling of inherent inconsistency of the contents, to the writer's opinion, this partial inhomogeneity highlights the richness, variety, and full international relevance of Italian research in mechanics, with the last aspect being always evident.

Selecting the colleagues to be invited to contribute to the second part of the volume has been likely the most difficult aspect of my editor's activity, in both scientific and 'personal' terms. I am fully aware that a substantial amount of important research themes remain somehow overlooked in the final outcome of this part of the project, along with the esteemed scholars, and the research groups behind them, who obtained meaningful achievements in about the last ten years or more. In addition to the lack of the younger (say under fifty) generation of scientists (apart from their possible collaborative involvement by a senior author)—generation that will surely have a central role in a future report on the state of Italian mechanics to hopefully appear before another fifty years pass—this is mostly the case with 'not so senior' (say between fifty and sixty) scientists, colleagues, and friends. My sincere apologies go to most of them, well known to the Italian community of mechanics in different areas and internationally recognized.

One more point is concerned with a certain inequal presence of contributions from the different areas, which—besides peculiar situations—also reflects their somehow different presence and centrality within current AIMETA activities, as also pointed out in the writer's introductory chapter of the volume.

Of course, despite the drawbacks related to some imbalance/incompleteness of knowledge and information on the scientific areas of AIMETA and on topics within each area, it is hoped that this book will not only testify to the quantity and quality of the activities handled by AIMETA since its foundation but also provide an idea of what went on and is currently occurring in the Italian research on mechanics. In this regard, it may be of interest to both scholars of the international community of mechanics and to new generations of Italian scientists.

It is definitely necessary to thank a considerable number of people, starting with all the colleagues and friends who have agreed to contribute to this volume, and who have certainly done their best, despite possible personal difficulties related to retirement or, on the contrary, to a very active and busy life. Initially, several of them were reluctant. This can be easily justified from both a general and a specific perspective: (i) for scholars of whatever discipline, it is never comfortable to dwell on research items that occurred in a quite recent period of time; (ii) for scholars of a hard science, as mechanics, this is even more uncomfortable, for their being accustomed neither to a predominantly qualitative treatment of the topics of their interest nor to observe the phenomena from an historical perspective. Aspects, the latter two, which require some revision and lightening, at least according to the opinion of a now-elderly scholar who has done his utmost to overcome the mentioned reluctance.

The support of colleagues from different universities in reconstructing specific aspects of the wide basket of AIMETA activities (international journal, congresses, research groups, awards, and advanced schools) is also gratefully acknowledged. Unfortunately, upon the various (also 'practical') transformations that occurred over

the years in the Italian universities, the old ‘archives’ of AIMETA held at the Polytechnic of Milan (Department of Mechanics and Department of Structural Engineering, where important groups of scientists played a core role in AIMETA activities for a long time) have been ‘substantially’ lost or have become nearly inaccessible. In turn, the AIMETA Officers of, let’s say, the last twenty years (which also included the writer) did not do too much to keep an adequate memory of what was going on (this being a by-product of the mentioned overlooking of historical aspects), despite some efforts to establish and maintain an updated website. Nonetheless, it can be said that, overall, only a few voids of practical information remain in the large amount of recollected data, which are reported in the many Appendices of the volume introductory chapter. They mostly pertain to older times (the 70s and 80s), about which the support received by Professors Leone Corradi and Carlo Cinquini, who made available some material still present at the Istituto Lombardo Accademia di Scienze e Lettere, is definitely acknowledged. Thanks go to Dr. Valeria Settimi, who has provided support and help in several circumstances. Many thanks are also due to Springer, a publisher with which AIMETA has a long-lasting fruitful relationship in connection with its international journal *Meccanica*, which agreed to publish this book. The specific support of Mr. Pierpaolo Riva, Springer Editor for Engineering and Applied Sciences, is gratefully acknowledged.

My final personal thanks go obviously to all members of the AIMETA Executive Council, who conceived the project and offered me the possibility to take care of it. A commitment that has certainly represented a non-trivial part of my overall activity of the last year or so, but has also pleasantly renewed the spirit of service that has guided my involvement with AIMETA and Italian mechanics for a long time, also somehow in emotional terms.

Rome, Italy
October 2021

Giuseppe Rega
Professor Emeritus

Contents

A Short History of AIMETA

AIMETA: A Proper Perspective to Retrace the Evolution of Italian Mechanics, with a Focus on Solids and Structures

Giuseppe Rega

Abstract This contribution aims at providing a survey on the evolution of theoretical and applied mechanics in Italy in about the last fifty years, as observed through the perspective of the Italian Association of Theoretical and Applied Mechanics (AIMETA). Stages of its development are overviewed, by referring to the carefully collected and organized data/information about a variety of related activities, which include the national congresses and the publication of the international journal *Meccanica*, for dwelling on the presence and the contribution of the five scientific areas—general mechanics, solids, structures, fluids, machines—encompassed by AIMETA. Within this general framework, a focus is also provided on the overall evolution of a specific area of mechanical sciences (solids and structures), with also some qualitative considerations.

Keywords Theoretical and applied mechanics in Italy · General mechanics · Solids · Structures · Fluids · Machines · Historical developments

1 Introduction

The Italian Association of Theoretical and Applied Mechanics (AIMETA) was founded in December 1965, with a notarial deed (Appendix 1), by eight renowned Italian scientists (Appendix 2, Table 1) from Schools of Engineering, active in different areas of mechanics. According to its Statute, AIMETA was established: "(i) to promote the development of theoretical and applied mechanics, through the coordination of research efforts and the organization of national congresses and meetings; (ii) to establish relations with similar associations abroad, and with IUTAM in particular; and (iii) to make known the results of Italian research in Italy and abroad, through the publication of a journal in English. The distinguishing features of the new Association were its interdisciplinary approach and the priority accorded to international relations" [1].

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At that time, I was at the beginning of my second year of studies at the Faculty of Engineering of the University of Rome, tackling my first challenging problems in Rational Mechanics. The first AIMETA National Congress I certainly attended was the 5th one, Palermo 1980. Thus, my knowledge of the first decade of AIMETA and beyond is extremely vague, and basically consists of the names (and some faces) of a few important and respected professors of solid and structural mechanics whom I likely encountered elsewhere. Since then, I attended all AIMETA congresses, with the exception of a couple of them (certainly the 20th one, Bologna 2011) due to health reasons.

Thus, I may be not the right person to cover in a comprehensive way the whole AIMETA life, as elapsed in the past 50+ years. Of course, I sincerely apologize for my lack of knowledge of its first glorious decade of activity, and for the ensuing, possibly wrong and incomplete, account of the scientific research conducted in that time interval that I will give in the sequel. However, although missing a huge number of important aspects, I have been living the AIMETA time for about forty years, with variable commitments, but always with dedication, with a feeling of belonging, and with some emotional involvement, too. This is certainly one reason for having been asked to edit this book and write this introductory chapter.

Another reason is likely connected with the circumstance that, over this long time period, I have been active in the area of solids and structural mechanics, thus encompassing two (out of the five) areas of mechanical sciences—the other three being general mechanics, fluid mechanics, and mechanics of machines—into which AIMETA activities are articulated. A subdivision that also reflects, to a certain extent, the organization of engineering studies in Italian universities at an intermediate disciplinary level. In this respect, it is also worth noting that, in the last thirty years or more, the presence of research topics from mechanics of solids and structures, with the involved scientists, has been largely predominant within AIMETA, as discussed forward.

Indeed, starting with the 80s, I have still a relatively good, though variable, memory of Italian research schools in solid and structural mechanics, along with the scientific topics to which they more meaningfully contributed and the involved senior and younger scientists. But I have a definitely poor knowledge of what was going on, along the same lines, in the other three areas.

Thus, on my side, notwithstanding some attempts to also grasp at least a few basic aspects of what was occurring in those companion areas, mostly in more recent times, this is a further meaningful drawback of low acquaintance that strongly limits the appropriateness and significance of what I will be dealing with in the following.

As a result, this survey on the evolution of theoretical and applied mechanics in Italy, as observed through the AIMETA perspective, should be deemed as a combination of two personal viewpoints. The first is concerned with the AIMETA activities in their entirety, as they developed over about the last fifty years. The second reflects the overall evolution of only a part (solids and structures), though important, of mechanical sciences, as perceived by a colleague who has been committed to research in this area for a long time, with a certainly serious attempt of rigorous involvement in science and a spirit of service to the academic community.

2 Up to the End of the 70s: The AIMETA Early Stage

The AIMETA was founded with the "mandate from the Italian National Research Council (CNR) to represent Italy in the International Union of Theoretical and Applied Mechanics (IUTAM)", to which Italy had adhered since 1949, "reviving the ties between Italian researchers and that community, of which Levi Civita had been one of the first promoters" [1].

The AIMETA structure somehow reproduced at the national level the IUTAM organization of mechanical topics into two very large areas (Fluids and Solids), however identifying four main areas of interest (General Mechanics, Mechanics of Solids, Mechanics of Machines, and Mechanics of Fluids), likely also to better reflect the academic and scientific environments behind the eight founders. Indeed, two of them came from theoretical mechanics (Bruno Finzi and Giorgio Sestini), two from solid mechanics (Leo Finzi and Elio Giangreco), two from mechanics of machines (Giovanni Bianchi and Emilio Massa), and two from fluid mechanics (Carlo Ferrari and Giulio Supino). The AIMETA articulation into four areas lasted until the beginning of the 80s, when it was decided to consider the area of solid mechanics as internally articulated into solids and structures (the latter formally introduced as a fifth area starting with the 1986–89 Executive Council), in order to make the evolution of research topics and scientists in the background, occurred over the decade, more apparent. According to the revised Statute, the President of AIMETA (Appendix 2, Table 1) is elected by the Executive Council (Appendix 3), made up of six members elected in turn by the General Assembly of the Association for a four-year term, plus the Past-President.

Less than one year after the establishment of the Association, namely in September 1966, the first double issue of its official journal, *Meccanica*, written in English, was launched under the Editorship of Emilio Massa (Appendix 2, Table 2), with four issues per year being published by a national publishing house. The issue contained a foreword by the first AIMETA President, Bruno Finzi, making reference to Mechanics "in the broadest sense of the word, ….. as extending far into the neighboring realms of Thermodynamics, Electrodynamics, Atomic Physics, … thus being difficult to describe the line of demarcation between Theoretical Mechanics and Mathematical and Theoretical Physics, between Applied Mechanics, Hydraulics, Aerodynamics and Technical Physics" [2].

The main aim of promoting knowledge and information on Italian theoretical, experimental and technical research in the diverse areas of mechanical sciences at the international level was declared explicitly. Accordingly, for a relatively long time, the journal mostly hosted contributions from qualified Italian scientists, despite an international Editorial Board. The first paper by foreign scientists (Bernard Coleman and Morton Gurtin) appeared in the third issue of 1967, and papers co-authored by Italian and foreign scientists were published occasionally, although with an obviously increasing trend over the years.

Browsing through the issues of *Meccanica* until about the end of the 70s, one can get an idea not only of the most frequented research themes but also of the involved

Italian scientists. Most of them were esteemed, and not always easily reachable, academicians and university chair holders, belonging to the generation of Italian scholars of mechanics born before the 2nd, or even the 1st World War (WW), who were protagonists of the scientific and technological reconstruction of Italy in the post-2nd WW time. Irrespective of the involved names, some general aspects lasting for about the whole 70s decade can be pointed out.

1. Research topics and scientists' involvement were relatively well balanced among the four areas of AIMETA, in both qualitative terms, with regard to their national as well as international recognition, and quantitative terms, as regards the number of papers appeared in the journal.
2. The presence of theoretical topics and scholars from the area of General Mechanics was definitely meaningful, greatly varied, and of the highest level.
3. Contributions in two (Solids and Machines), out of three, applied areas of AIMETA were rather varied and articulated, because of encompassing research criteria, and ensuing topics, either more methodologically oriented or more explicitly driven by the application needs of a given technological context. This distinction refers (i) in the solid mechanics area, to contributions of 'Scienza delle Costruzioni'[1] (solid and structural mechanics) versus those from 'Tecnica delle Costruzioni' (structural engineering), and (ii) in the mechanics of machines area, to contributions of 'Meccanica Applicata alle Macchine' (applied mechanics) versus those of 'Costruzione di Macchine' (machine construction). Indeed, mostly in the first part of the considered time interval, both kinds of topics were deemed to be fully consistent with AIMETA interests and activities, also due to them substantially coexisting within the academic organization of scientific disciplines in the Italian schools of engineering. The situation gradually changed over the 70s, also in connection to the newly occurred disciplinary separation of theoretical aspects of applied mechanics from the more technical ones of engineering applications. In the following years, the presence of more finalized research topics, and of communities of scientists pursuing them, within AIMETA activities decreased up to nearly disappearing, or underwent redefinitions also linked with a number of changes going on in science and technology as well as in the background society.
4. Contributions in the third applied area (Fluids) were encompassing topics from both aero/fluid-dynamics, on one hand, and hydraulics, on the other hand. In both fields, they were also distinguished according to being more theoretical-methodological or more technologically-oriented, the latter referring to issues of interest in aeronautics/aerospace/mechanics or in hydraulic constructions. However, the academic/disciplinary frameworks of aero/fluid-dynamics and hydraulics clearly reflected the distinction between the two worlds of industrial

[1] 'Scienza delle Costruzioni' is the classical name of the fundamental discipline encompassing topics of continuum mechanics, elasticity, mathematical-physics, strength of materials, and theory of structures, taught in Italy to students in the Schools of Engineering and Architecture. 'Tecnica delle Costruzioni' is the sequential discipline more technically oriented in the civil engineering realm.

engineering and civil engineering they belong to, respectively. This circumstance somehow helped keeping the two sub-areas within AIMETA also in the decades to come, up to the current days, even though other scientific associations more clearly linked with the two different technological contexts have always exerted a great attraction to them.

5. Last, but not least, it has to be noted that, overall, assigning a contribution to a definite scientific area often reflected much more the academic collocation of the scholars in the background than an actually marked difference as to the way a certain topic was addressed in terms of methodology and/or phenomenology. This was indeed a generally valuable outcome of the meaningful permeability between neighboring scientific areas, typical of former times, this being a feature which, unfortunately, has been lost in the following decades, governed by a drive towards specialization always running the risk to entail a narrower vision of the world. Luckily, in about the last decade, interest to scientific cross-disciplinarity is meaningfully increasing, again.

Until beginning of the 70s, AIMETA put apparently all his efforts in giving visibility to its international journal. Indeed, its second important scientific initiative (Appendix 2, Table 3), the establishment of a National Congress, was postponed until 1971, when its first edition was held in Udine, hosted by the International Centre of Mechanical Sciences (CISM). Over the first decade, the congress was handled by a single Committee, including a President and a Secretary. Since the sixth edition (Genova 1982), the Organizing Committee taking care of all practical aspects of the congress was paralleled by a Scientific Committee (Appendix 4, Table 5) taking care of all research aspects, as well as by a Honorary Committee, at least in some early editions. In all congresses, a few renowned foreign or Italian Key Lecturers were invited (Appendix 4, Table 6).

The 1st National Congress hosted 75 presentations, of which 12 in General Mechanics, 27 in Solid Mechanics, 15 in Mechanics of Machines, and 21 in Mechanics of Fluids (Appendix 4, Table 7). General scientific reports and summaries of presentations were published in a Special Issue of *Meccanica* [3], and represent a good starting point to dwell on scientific themes of main interest in the four areas.

- Dionigi Galletto reported on 'Some recent results and developments in theoretical mechanics and mathematical physics', providing a thoughtful and comprehensive overview of the most recent international advancements in mechanics of rigid bodies, continuum mechanics, mathematical theory of simple materials and mixtures of materials, plasma physics and magneto-fluid-dynamics, and relativity. The list clearly highlights the breadth of scientific interests gathered around the area of Mechanics, as per the aforementioned interpretation by Bruno Finzi.
- Michele Capurso reported on Mechanics of Solids, grouping presentations in three main sections, namely, elasticity theory, stability theory, and plasticity theory, and providing a list of international references, too.
- Andrea Capello reported on Mechanics of Machines, collecting presentations in a number of sectors, namely: vibrations; lubrication; fluidics; automatic control

theory; mechanisms; mechanics of solids and behavior of materials; methods and means of experimental investigation; automatic design and drawing.

- As regards Mechanics of Fluids, only summaries of presentations were reported.

The 2nd National Congress, Napoli 1974, is worth to be mentioned because of the decision of the AIMETA Council "to collect in a singular dedicated issue of its journal the papers which would fall within the fields of computational mechanics and applications of functional analysis to mechanics, …… two topics which, as also witnessed by a number of IUTAM Symposia, rank among the latest and more interesting essential new developments of mechanical sciences" [4]. The focus on computational mechanics via the large-size electronic calculators typical of the beginning of the 70s, towards which an explosion of interest had been developing in all areas of applied mechanics, is definitely to be underlined. Another issue of *Meccanica* [5] published the text of a round table on the stability of continuous systems, also held at the 2nd National Congress in Napoli.

Since 1974, the AIMETA National Congress was held each even year, with a sequence later shifted forward to odd years (Appendix 2, Table 3) in order to accommodate the organization in Italy of the 2nd European Solid Mechanics Conference of EUROMECH, Genova 1994.

As regards the 4th National Congress, Firenze 1978, abstracts of presentations given within the four AIMETA areas appeared in *Meccanica* [6] without a relevant general report. Thereafter, no more summaries of papers presented at National Congresses were ever published, giving up an editorial choice that, apparently, was swiftly realized to make no sense in an international journal.

Browsing through the two pillars of AIMETA, *Meccanica* issues [7] and Congress Proceedings, one can clearly observe also the substantial change of the core generation of known Italian scientists in mechanics occurred in between the 60s and the 70s, with the progressive affirmation and the scientific characterization of new research schools spreading all over the peninsula. Their prestige and role would have become even more apparent in the following decade, also in connection with a modified organization of the Italian university system in terms of formal institutions and recruitment procedures, which entailed a wider and more democratic exchange of information and knowledge also among young scientists.

3 Italian Schools of Solid and Structural Mechanics Since the Late 60s and Far Beyond

Recognized schools of solid mechanics with a core of well-identified advanced knowledge in a certain field already existed in Italy since a long time. However, looking backwards from the viewpoint of a young researcher facing at those times the nearly first steps of his scientific and academic career, they became fully apparent between the 70s and the 80s. In this respect, some main scientific fields of widespread interest for the community of solid and structural mechanicians are to be mentioned, along with the schools that obtained more meaningful achievements.

The theory of elasticity had its core center at the University of Pisa, with Piero Villaggio's elegant and often solitary studies on a great variety of relevant topics and the capability to use it "as a chisel to create models from the complexity of reality" [8]. A though partial account of his many contributions can be found in two fundamental books. The first deals with the characterization and solution of general boundary value problem of linear elastostatics, including mean value properties and inequalities, strong formulations, energetic a priori estimates, pointwise bounds and special other topics [9]. The second offers a critical collection of various mechanical theories for modeling the behavior of structures, and is a somehow natural progression from the earlier work on the elastic solid [10]. But Villaggio also made significant contributions in many other fields, including plasticity, fracture, contact and impact, stability, optimization, masonry constructions, and history of science, with a wide culture also allowing him later to comprehensively and critically revisit decades of development and progress in solid and structural mechanics [11].

With his extraordinary capability to extract the best from students, in a really Socratic way, and despite a severe and even sometimes provocative personal attitude, Villaggio raised a first group of outstanding young scientists, who "immensely benefited from his contacts with two outstanding research groups, ….. the group of Pisan mathematicians around Guido Stampacchia and his theory of variational inequalities, …….. and the group of Clifford Truesdell and other protagonists of the revival of continuum mechanics that took place in the second half of the past century" [12]. In the years and decades to come, other brilliant scientists greatly benefited from Villaggio's ability "to awake a person's intellectual interest on a subject" even away from the specific one he was dealing with, also in connection with an "uncanny ability to spot promising new research trends". And indeed, his students were always free to make their choices, several of them later following with success quite different scientific paths, yet always keeping Villaggio's rigorous style and attitude in tackling scientific problems.

Among Villaggio's earlier disciples, at least two are mentioned for having become worldwide known scholars in continuum mechanics, finite elasticity, and mechanics of materials and structures. Gianpietro Del Piero was a rational scholar, and a teacher with a rigorous sense of justice, active in unilateral problems, structured deformations of continua [13], linear viscoelasticity, and unified modelling of material behavior, also including, in most recent times, fracture, plasticity, damage and creation of microstructure, based on incremental energy minimization. In turn, Paolo Podio Guidugli [14], still active, is recognized as the most aristocratic intellectual in the field of rational continuum physics, where he has been working, among other, on rods, plates, and shells, phase interfaces in solids, deformable ferromagnets, carbon nanotubes and graphene, continuum and statistical thermodynamics, conceptual issues in molecular dynamics, also forming a significant number of followers.

Since about the early 60s, theory of plasticity was another major research field in structural mechanics with a national program supported by the CNR, and with Italian scientists giving meaningful contributions to both theoretical advancements and their computational implementation in engineering problems. In between the University of Firenze and the University of Roma, Giulio Ceradini fruitfully worked on limit analysis of structures as a linear programming problem (in collaboration with his authentic disciple, Carlo Gavarini), on a maximum principle in the incremental theory [15], up to the formulation of the first theorem of dynamic shake-down. His results were published in Italian journals, owed to a still restricted vision of the national, and in particular Roman, scientific environment, being summarized in an international journal only much later [16]. Yet, they paved the way to a remarkable series of results of international level, later obtained by the Italian school of plasticity involving important research groups from several other universities, also collaborating with each other.

At the Polytechnic of Milano, for a long time, Giulio Maier was extremely active on a variety of related themes, along with collaborators, suitably balancing between theoretical results and computationally oriented formulations for the elastic–plastic problem based on finite element modelling. He obtained outstanding results on the links between plasticity and mathematical programming theories [17], dynamic shakedown of hardening structures with kinematic theorem [18], incremental plastic analysis in the presence of physical instabilizing effects [19], extremum theorems for holonomic elastic–plastic problems [20], overall stability of structural systems with individual softening components, boundary elements and their symmetric Galerkin version [21, 22]. Maier established fruitful early connections with the top-level international groups of William Prager, Daniel Drucker and Paul Symonds at Brown University, Providence, but also with scholars from University of Illinois at Urbana-Champaign, University of Minnesota, Faculté Polytechnique de Mons, and University of Cambridge. However, along with young collaborators, he has contributed significantly also to optimum design and structural optimization, parameter identification in elastoplasticity based on mathematical programming and Kalman filter techniques, statistically loaded structures, crack-propagation analysis, cohesive crack models, and structural dynamics.

Editor of *Meccanica* (1982–85) and President of AIMETA (1986–89), Maier was awarded a huge number of prestigious Italian and international prizes, which highlighted his large recognition in different scientific fields. Based also on many other links, he constantly feeded excellent scholars working on a variety of themes, ranging from finite elements, heterogeneous materials, and fracture processes, up to inverse problems, structural diagnosis, and micro-electro-mechanical-systems. All of this has qualified Maier and his varied research group, for decades, as the center of reference for advanced research in structural mechanics in Italy, as also internationally witnessed by his still being a Member-at-Large of the IUTAM General Assembly (see Appendix 9 forward).

The Italian school of plasticity was represented significantly in other academic institutions, too, in a context of transversal intellectual commonality and also personal friendships. At the University of Bologna, Michele Capurso contributed meaningful results on the extremum characterization of incremental elastic–plastic solutions [23], on rigid-plastic dynamic responses, and on upper bounds on values attained by history-dependent quantities after shakedown of elastic–plastic structures under cyclic loads [24], also in the presence of fractured materials [25]. Sadly, he passed away prematurely, at the very height of his scientific maturity. In turn, at the University of Palermo, Castrenze Polizzotto, who also nourished younger scholars and is still active at a ripe old age, obtained valuable results on a variety of topics in plasticity. They include shakedown within dynamics [26] and damage mechanics, related bounding techniques of plastic deformations [27], steady state response of structures under cyclic thermo-mechanical loads, strain gradient plasticity theory [28], elastic–plastic, limit and shakedown analysis by the symmetric BEM [22].

Stability, structural dynamics, and engineering applications of elasticity were worthily addressed at the University of Genova in Riccardo Baldacci's school. In the years to come, his two main disciples, Edoardo Benvenuto and Alfredo Corsanego, would have become a scholar of international reference in the history of structural mechanics [29], and a scientist of national reference as to the seismic risk and vulnerability assessment of buildings and territorial systems, respectively. Both paid a main care to the conservation and safety of Italy's architectural and monumental heritage, according to a more conceptual or more practically-oriented perspective. Actually, the early Italian center for earthquake engineering was Giuseppe Grandori's school at the Polytechnic of Milano, with his disciple Vincenzo Petrini later playing an important role, along with Carlo Gavarini from the University of Roma La Sapienza, in the methodological and organizational definition of technical procedures for effectively handling seismic and post-seismic emergencies.

In the even more applied context of structural engineering, two early meaningful schools grown up in the academic environment of solid mechanics are to be mentioned: Franco Levi's school on reinforced concrete structures at the Polytechnic of Torino, and the school on steel structures established at the Polytechnic of Milano by Leo Finzi, who was one of the eight founders of AIMETA.

Finally, at the University of Napoli, just after the establishment of AIMETA, there was the academic separation of the engineering-oriented group around another AIMETA founder, Elio Giangreco, widely active on theory of structures, from the group of Vincenzo Franciosi, active on more fundamental topics of solid and structural mechanics. In this more pertinent context, the school of the latter has to be mentioned, too, for the variety of its interests and for the capability to train a group of smart young scientists, who would have later contributed significantly to both theoretical and more technical aspects.

Scientific results obtained within all these schools were always presented at AIMETA national congresses and were often also published in *Meccanica*, especially in its early years when almost only the pertinent schools' 'fathers' were involved.

4 80s, 90s and Beyond: The Time of AIMETA Consolidation

Over the 70s, AIMETA congresses were attended by a meaningful and increasing, yet still relatively limited, number of presenting scholars (Appendix 4, Table 7), coming mostly—though obviously not only—from the communities around the many different and well-recognized protagonists of the long post-2nd WW revival of mechanics in Italy. Roughly speaking, the following two decades may be considered the time of the definitive affirmation of AIMETA congresses as the reference biennial event for the exchange of scientific knowledge within a community which was undergoing an important quantitative growth and a meaningful generation change, with young professors born at about the 2nd WW time or just after it coming significantly into play. In this respect, a non-trivial role was also played by the reorganization of university institutions and recruitment procedures occurred at the beginning of the 80s, and by the explosion of scientific mobility at both international and national level.

While in the 70s contributions were relatively well balanced among the four areas of AIMETA in numerical terms, over the 80s changes in the composition of congresses' attendees started to occur. Participation from the solid mechanics area strongly increased, with also a differentiation between contributions more clearly focused on fundamental aspects of mechanics of solids and materials, and contributions paying attention also, or mostly, to phenomenological aspects of the structural response. Since Genova 1982, this differentiation reflected in the formal grouping of the two kinds of contributions under distinct headings ('mechanics of solids' and 'mechanics of structures') in both the congresses' programs and the relevant proceedings. This distinction was sometimes questionable from the scientific viewpoint. Anyway, the increased attendance from mechanics of solids and structures, further enhanced in the decades to come, entailed an increase from four to five in the nominal number of areas' representatives in the AIMETA Executive Council (Appendix 3), and in other formal initiatives (e.g., the number of AIMETA Junior Prizes, see Appendix 7 forward) later undertaken by the AIMETA governing boards.

On the other hand, starting with the 80s, attendance of scientists from the other AIMETA areas slightly decreased (certainly in percentage), although with variable and oscillating trends over the decades. From about the 90s, this occurred in particular as regards general mechanics, also somehow in connection with a progressive reduction in the presence and role of the related disciplines (typically, Rational Mechanics) in the teaching curricula of Italian schools of engineering. This trend further increased around the turn of the millennium, with the transition in the organization of the relevant studies from the early, successful and worldwide recognized, 5-year system to the 3 + 2 system, which substantially deprived the mechanical disciplines of their most important theoretical fundamentals, irrespective of some possible (though anyway doubtful) merits occurring in other engineering areas. Scholars of theoretical mechanics increasingly considered their topics better centered in scientific events organized by associations like the National Group for Mathematical Physics (GNFM) or the Italian Society of Applied and Industrial Mathematics (SIMAI).

A certain reduction of centrality of AIMETA congresses with respect to topics in the areas of mechanics of machines and mechanics of fluids was perceived in the relevant communities, too, although to a definitely minor extent. As to the former, after a non-trivial increase of attendance occurred in the first decade of the new millennium, a somehow uncertain attitude towards AIMETA of scientists active in more technologically-oriented fields, like machine construction and design, has to be mentioned, even though a meaningful core of applied mechanicians always feel themselves fully at home within AIMETA. Actually, this evolution should also be related to a general widening of scientific perspectives and topics of interest (e.g., micro/nanomechanics and biomechanics) occurred mostly in the new millennium, and also reflected in a modified organization of the AIMETA congress with a progressive transition to a symposia-based scheme (Appendix 4, Table 8).

As regards fluid mechanics, the 80s and 90s marked a kind of bifurcation between excellent scientists from mechanical university environments constantly considering themselves, and their topics, well centered within AIMETA activities/events, and scientists from aeronautical environments more explicitly attracted by other conference series, e.g. the one organized by the Italian Association of Aeronautics and Astronautics (AIDAA). Anyway, fluid topics and people from qualified hydraulic environments of civil engineering have always been well represented in AIMETA.

All of the above issues are concerned with the presence of the different scientific areas in the AIMETA congresses. Somehow surprisingly, the situation was quite different as regards the AIMETA journal. Indeed, therein, over both the 80s and the 90s, quantitative contributions from Italian scholars in solid and structural mechanics were only comparable to, if not even lower than, those from each one of the other three areas. In particular, the 'new' generation of mid-age scientists in the former area often preferred to submit their works to other international journals, with either a longer and better established tradition or a more clear focus in a given, and often 'novel', scientific field. An undue and obviously undeclared, yet underlying, need of verifying the quality of personal and Italian research at an 'actually' international level certainly played a role. However, other circumstances were also important, namely *Meccanica* being a journal (i) published by an Italian company (until 1990), (ii) with a too general scope (possibly not at the highest level, according to some perception) in mechanics, and (iii) with a board of solely Italian Associate Editors (until March 1991). Why this mostly occurred in the area of solid and structural mechanics and lasted in the 90s, too, when the number of contributions to *Meccanica* by highly qualified foreign scientists just from that area started to meaningfully increase, is a matter left to the thoughts of science historians.

In the 90s, the limitations for the journal success associated with the above three points were addressed in an editorial by Giuliano Augusti. He observed that the "*Meccanica* birth-mark ……. of publishing in English the most important Italian contributions to Mechanics", even with the non-acceptance of a paper "unless written by an Italian author or developed with some connection with the world of Italian Mechanics", had been overcome only a few years before [30]. Indeed, the search for "an Editor willing (and able) to solicit papers from non-Italian scientists", and the transition to an actually international publisher (Kluwer Academic) guaranteeing a wider circulation occurred in 1991. Over the 90s, papers by qualified non-Italian authors were around half the total number, or more.

As to the very broad scope of the Journal, already considered "a rare occasion for the productive encounter, exchange and cross-fertilization of ideas in a time of exasperated specialization in the publishing world" [1], Augusti claimed that in a time of "so many specialized journals on the market, a general mechanics journal has its scientific reasons and its viable space". Later, the matter was addressed by other Editors of *Meccanica*. They pointed out that the focus of the journal is on "sharing the common methodological framework of all scientists working in whatever field of mechanics, and on highlighting phenomenology and application aspects occurring in modern mechanical problems" [31], while also noting that, due to the variety of papers, "balancing theoretical and more application-oriented works is an important item for a journal which likely, in the past, gave major emphasis to the former" [32]. Overall, the choice to maintain the general and interdisciplinary character of the journal was confirmed, however strengthening measures to favorably account for the ever increasing, and likely irreversible, trend towards specialization. Among them, special issues built around a clearly identified topic, and often devoted to novel research lines, with also the involvement of foreign guest editors of high-quality, are to be mentioned (Appendix 5). Indeed, since about the end of the 90s, they have increasingly appeared in *Meccanica*, up to becoming a substantially regular and appealing ingredient of the journal in the last decade, as it has occurred for other high-quality journals.

After all, the trend towards explicitly identifying specific topics of interest for the community and grouping expert scientists around them—which is at one time a cause and an effect of specialization—had become apparent also in other AIMETA activities. Indeed, since about mid-80s, a third pillar was established in the basket of AIMETA activities, i.e. the so-called AIMETA Groups steadily devoted to a given subfield of mechanics and aimed at connecting the involved scientists through specifically dedicated conferences, with also features of transversality among the different areas (Appendix 6). The early Groups (Computational Mechanics, Theory of Machines and Mechanisms, Stochastic Mechanics, Mechanics of Materials and Structures, Tribology) were progressively paralleled/replaced by new/renamed ones (Mechanics of Materials, Biomechanics, Continuum Mechanics, Turbulence, Kinematics and Dynamics of Multibody Systems, Dynamics and Stability), with also further articulations and changes introduced to better characterize the fields of interest. Data and information on some main initiatives undertaken by the currently active Groups are listed in Appendix 6 (Tables 10, 11, 12, 13, 14, 15, and 16).

Focusing on the area of solid and structural mechanics, it is worth compiling a gross list of core fundamental fields steadily pursued in the 80s and 90s, and extending to the beginning of the new millennium, too. Non-exhaustive lists of academic institutions where the related topics have been best practiced, with also international recognition, are also mentioned, however with no names of involved scientists being provided, due to the lack of an adequate historical perspective and a matter of opportunity.

- Computational mechanics (Pavia, Polytechnic of Milano, Padova, Cosenza, Brescia, …….)
- Continuum mechanics, finite elasticity and plasticity (Pisa, Roma Tor Vergata, Ferrara, Udine, Bologna, Parma, Trento, Napoli Federico II, Polytechnic of Milano, Palermo, Bari, …….)
- Dynamics, stability, bifurcation, chaos, and their control (Genova, Firenze, L'Aquila, Roma La Sapienza, Polytechnic of Marche, Palermo, Messina, …..)
- Inverse problems and identification (Polytechnic of Milano, Roma La Sapienza, Udine, …..)
- Mechanics of materials, damage and fracture (Polytechnic of Torino, Polytechnic of Milano, Genova, Bologna, Padova, Trento, Napoli Federico II, Cassino, ………..)
- Modern and historical structures: modeling, analysis, response (all universities/polytechnics)
- Stochastic mechanics, probability, wind and earthquake engineering (Pavia, Palermo, Messina, Genova, Roma La Sapienza, L'Aquila, Napoli Federico II, Firenze, …….)

As to *Meccanica*, it is also worth mentioning its somehow peculiar nature of a journal well founded in the Italian tradition of mechanics, on one hand, and opened to a wide vision of the evolution of science, on the other hand, by shortly dwelling on two related items. The first item is concerned with the attention worthily paid also to some historical aspects of the evolution of mechanics, mostly (though not only) in the national context. This is witnessed by a number of articles appeared in both earlier and recent times on the Italian contribution to mechanics [33], or about general and specific achievements of Italian scientists of earlier centuries in both the mathematical and the engineering environment [34–40]. A general care to the evolution of mechanics in the international context has also to be noticed (see, e.g., [11, 41, 42]). The second item is consistent with the wide scope of the journal in general cultural terms, and refers to the publication of papers, written by well-known mechanicians, dwelling on criteria, features and trends of research, with also critical and warning considerations [43–45].

5 The New Millennium: Widening of Scientific Perspectives, Further Generation Change, and New AIMETA Initiatives

Due to the great acceleration of technological transformations in the XIX and, mostly, XX century, changes in whatever realm of life (cultural, economic, social) have been increasingly characterized by changes of paradigm under the sign of innovation.

This has also happened in science where, however, remarkable features of continuity can also be recognized. In particular, at about the turn of the millennium, a number of meaningful technological changes entailed a non-trivial redefinition of the fields of scientific interest in applied mechanics, making trends already existing in the previous few decades fully apparent and, mostly, ubiquitous. At the same time, a new generation of scientists came to the fore.

Of course, classical scientific fields did persist, along with the relevant themes. However, their characters of unity and internal coherence began to be overlooked in favor of the explicit identification of specific sub-themes, according to a trend ever increasing towards particularization and specialization. At the same time, another, and seemingly opposite, trend became apparent: the need to overcome traditional boundaries between scientific areas and to hybridize the relevant themes, by virtue of an increasingly recognized transversality of methods and technological scales, and cross-correlation between theoretical/physical contexts and the associated phenomena. This being a circumstance already experienced in the realm of physical sciences, and transferred to the engineering realm with a physiological time delay.

As regards AIMETA congresses, since about beginning of the new millennium papers accepted for presentation in a given area, out of the five constituent ones, were grouped a posteriori around a main characterizing theme/subject. Alternatively, although in the same spirit, the selection of well identified scientific fields and of a number of expert scholars suitable to meaningfully and possibly comprehensively deal with them, led to the a priori organization of Special Sessions and/or Minisymposia in the programs of congresses (Appendix 4, Table 8). In this regard, attention was also paid to their possible transversality with respect to AIMETA areas, mostly in more recent times, as per the cross-disciplinary nature and scope of the Association.

In turn, *Meccanica* published more and more studies in interdisciplinary fields at the border between different areas of mechanics, such as fluid–solid interaction, acoustic–structural coupling, and thermomechanics, or between mechanics and other mathematical and engineering sciences, including control, advanced materials, dynamical systems, computation, electromechanics, and biomechanics [46]. New challenges concerned with the interaction between mechanics and chemical reactions, and biological signal transmissions were considered of interest for the journal, too [47].

In the international perspective, the list of 'novel' fields of interest to the community, with the underlying research themes, could be built by looking at the articulation of the AIMETA activities, as made explicit through the congresses, the journal organization and publications, and the groups. Indeed, browsing through the lists of (i) key lectures delivered at national congresses, (ii) relevant subgroups of papers, special sessions and minisymposia, (iii) special issues of *Meccanica*, and (iv) formally established AIMETA groups, as reported in Appendices 4–6, helps getting an overall view of the scientific topics of interest in a given area and in a certain period of time. Moreover, looking at the numbers of involved people gives an idea of the most frequented of those topics. Overall, the relative qualitative and quantitative evolutions over the last two decades can be better monitored than for previous decades, this being in the writer's opinion the maximum effort that can be made in order to get a comprehensive and comparative view of what has been going on with the matter up to now.

Focusing again on the area of solid and structural mechanics, the generation of scholars born during the 2nd WW or just after it is now smoothly coming to its term, upon having been active for about 40 years or more. The new generation, which started replacing it since about the beginning of the new millennium, is obviously more prepared to catch the new signs of the times. The core research fields listed in Sect. 4 were still practiced with continuity, yet introducing proper relevant redefinitions and complementing them with fully 'novel' research topics, either more specialized (even when dealing with fundamentals) in accordance with the objectives and purposes of an underlying scientific/technological environment, or more transversal to different areas of mechanics and beyond them. A partially updated rough list of novel/redefined macro-fields can be given as follows:

- Advanced computational mechanics, also including parallelizable algorithms
- Biomechanics
- Exploiting nonlinearity in mechanics: analysis, geometry, computation, engineering design
- Modeling, analysis and phenomenology at different space and time scales, with emphasis on micro/nano-systems and multifunctional structures
- Multifield complex and/or architected materials, mechanical metamaterials
- Multiphysics problems
- New/hybridized topics beyond mechanics, including artificial neural network, additive manufacturing, renewable energy systems, probabilistic data-driven models, machine learning-based methods, uncertainty quantification issues, …………
- Nonlinear dynamics, bifurcation, chaos, synchronization, wave propagation, localization, control
- Nonlinear identification, structural health monitoring, energy harvesting
- Reduced order modelling.

Here, no academic institutions are mentioned. This is also due to the definitive affirmation of a couple of newly established research paradigms. On one hand, the increased collaboration between people from groups based at different (and often also international) institutions, made possible by the improvements of tools for virtual meetings, and then brought to its extreme consequences in the Covid-pandemic time. On the other hand, the enhanced cross-disciplinary nature of the 'novel' fields, with collaborations being established between people belonging to completely distinct academic groups. All of this has somehow reduced the possibility to unequivocally identify schools of reference geographically localized, while enhancing the role played by single scientists in the framework of trans-institutional and/or cross-disciplinary collaborations.

In the mood for a better dissemination, characterization, and possible reward of scientific research, changes occurred also in the two main activities of AIMETA, since about the beginning of the new millennium, along with some later new initiatives. Upon moving to the Springer publishing company (since 2002), whose professionalism was crucial to the definitive success of the journal, *Meccanica* gradually increased the number of issues per year up to twelve, in order to face the meaningful growth of good papers authored by foreign scientists from both advanced and 'emerging' countries in all over the world. The number of special issues devoted to specific research topics and possibly edited also by solely foreign internationally recognized scholars markedly grew, too, up to making them nearly periodic. Currently, nearly two thirds of the Associate Editors of the journal are renowned foreign scientists from all over the world, with full responsibility of managing, in their field of expertise, the review process of each submission, on invitation of the Editor in Chief. Papers by foreign scientists have become the vast majority. AIMETA 'fathers' and some following generations of Italian scholars of mechanics would likely complain the fully pursued internationalization of the journal, which certainly deprives it of its Italian 'identity', even though some senior and cultured foreign scholars are still able to catch it. This is the case of, e.g., the renowned ones having relationships with the Italian academy, through scientific collaboration with AIMETA or other Italian mechanicians, who were invited to celebrate the 50th Anniversary of *Meccanica* with papers published in a meaningful, dedicated special issue [48]. Moreover, it can likely be caught, behind the standard international appearance, also in some lasting features of the journal, as the mentioned care to historical aspects of mechanical sciences and to 'societal' effects of scientific research. The above is of course a general price to pay to globalization, whose effects in terms of journal visibility in the worldwide community of theoretical and applied mechanicians, which include a possibly high journal ranking, have become definitely important, irrespective of the inflated value certainly given to this kind of classifications.

In turn, as already mentioned, scientific programs of AIMETA congresses paid increasing attention to the apparent specification of research fields, up to being organized nearly only as a collection of large minisymposia, possibly, but not necessarily, reflecting researches being conducted in the framework of AIMETA Groups. One more relevant aspect to be mentioned is the frequently resumed discussion, in the new millennium, about the possible transformation of AIMETA congresses in international events, as per, e.g., the German GAMM (Gesellschaft für Angewandte Mathematik und Mechanik) scheme. However, while participants were asked to write contributions for the congress proceedings in English, it was finally agreed that the community of Italian mechanics is already large enough and qualified to guarantee a biennial meeting of the underlying communities with a satisfactory exchange of advanced knowledge and information, also by virtue of their anyway active involvement in other, and even possibly too many, international events. Mostly, AIMETA congresses represent the ideal place to meet periodically varied people from the rich Italian community of mechanics, and to become acquainted with new generations of scientists.

Two important new initiatives undertaken in the new millennium should also be noted. The first is the establishment (2009) of AIMETA Junior Prizes awarded by a dedicated committee any two years, on the occasion of the national congress, to the best young researcher in each one of the five constituent areas of the association (Appendix 7). This new initiative has been widely appreciated by young scientists, with the submission of a good number of excellent nominations. This occurred notwithstanding a traditional allergy of Italian people towards individual recognitions, differently from other countries, which is likely a somehow improper byproduct of the affirmation of even too many individual entities at all (i.e., personal and institutional) levels in the long history of our country. Along the same line, Awards for best PhD Theses have been established by several AIMETA Groups (Appendix 6).

The second initiative is concerned with the organization of a summer school for PhD students and post-doc researchers on a variable topic, which, after some earlier attempts, has been eventually realized via a formal agreement with the International Centre for Mechanical Sciences, through a CISM-AIMETA Advanced School being held in Udine every year (Appendix 8).

Links of AIMETA with international associations in the area of mechanics are also to be noted, starting with the meaningful presence of Italian mechanics in the IUTAM activities. Notwithstanding the chronical low care paid by the Italian governments to the needs of scientific research, Italy is one of the few countries (along with Canada, France, Germany, Japan, Russia and UK) having since long times four representatives in the IUTAM General Assembly (with only China and USA having more) (Appendix 9). Three of them are designated by the AIMETA and one by the National Research Council. Italian scholars are also meaningfully present within IUTAM Committees (Congress Committee, Fluid and Solid Symposia Panels), and as invited organizers/lecturers at both the International Congress of Theoretical and Applied Mechanics (ICTAM)—which is the biggest scientific event in the area of mechanical sciences held every four years—, and in the IUTAM Symposia held every year on different subjects. As a general recognition of the overall quality of Italian mechanics, the 25th ICTAM (originally planned in Milano for 2020) has been held successfully online in 2021, due to the pandemic issue, under the organization of the solid and structural mechanics group of the Polytechnic, and the meaningful support of AIMETA and the whole Italian community.

Links with other associations include the European Mechanics Society (EUROMECH), via a long lasting affiliation (since 1995), the International Association of Computational Mechanics (IACM), the European Community on Computational Methods in Applied Sciences (ECCOMAS), which the AIMETA Group of Computational Mechanics is the Italian member of, and more recently with the Chinese Society of Theoretical and Applied Mechanics (CSTAM).

6 Conclusions, with an Overlook to the Future

In this paper, the evolution of theoretical and applied mechanics in Italy has been overviewed, as observed from the perspective of its national association AIMETA, after fifty plus years from its foundation. While being articulated in five constituent scientific areas (general mechanics, solids, structures, fluids, machines) since about its foundation, the various AIMETA activities have been developed over the decades within a substantially unitary framework, this being definitely a worth feature as regards the organization of mechanical sciences in Italy. Qualitative and quantitative evolution of the Association and of its two long-lasting activities—the international journal *Meccanica* and the National Congress –, besides the most recent ones, has been retraced in detail, by also referring to the seemingly whole set of carefully reconstructed data and information collected in the Appendices. It has been possible to get an overview of the main research themes addressed within the five component areas over fifty plus years, and of some relevant involved scientists, with also a focus on what has been going on in the specific area of solid and structural mechanics where the writer has been active for more than forty years. Overall, the fundamental role played by AIMETA in monitoring the evolution of theoretical and applied mechanics in Italy, and in somehow orientering and fostering the research of the underlying communities has clearly emerged.

AIMETA is obviously expected to be the Association of reference for mechanical sciences in Italy also in the decades to come. However, since about the turn of the millennium, the role of mechanics within the whole basket of mathematical and engineering sciences has non-trivially evolved. In his foreword appeared in the first issue of *Meccanica* (1971), the first AIMETA President, Bruno Finzi, referred to Mechanics as the "most ample science, multiform, in unceasing, bewildering development; the 'Paradise of the mathematical sciences', as Leonardo said, and the cornerstone of every physical science too, which informs our present-day civilization to such an extent that it has well been said that we are living in the Age of Mechanics" [2]. Now, the situation is definitely different.

Within the current context of mathematical and engineering sciences, mechanical ones can be considered to have reached a plateau, if not even a turning point, of their glorious and long-lasting parabola of growth, though still having in front of them rich perspectives in terms of further renovation and, mostly, of hybridization and cross-fertilization with other sciences. Of course, this will somehow affect also the identity, the scope and the activities of the associations of reference at both the international level (IUTAM) and the national ones (AIMETA and equivalent foreign associations). To what extent, in which directions, and in how much time this will occur is left to the vision and the practice of the follow-up generations of scholars of Mechanics.

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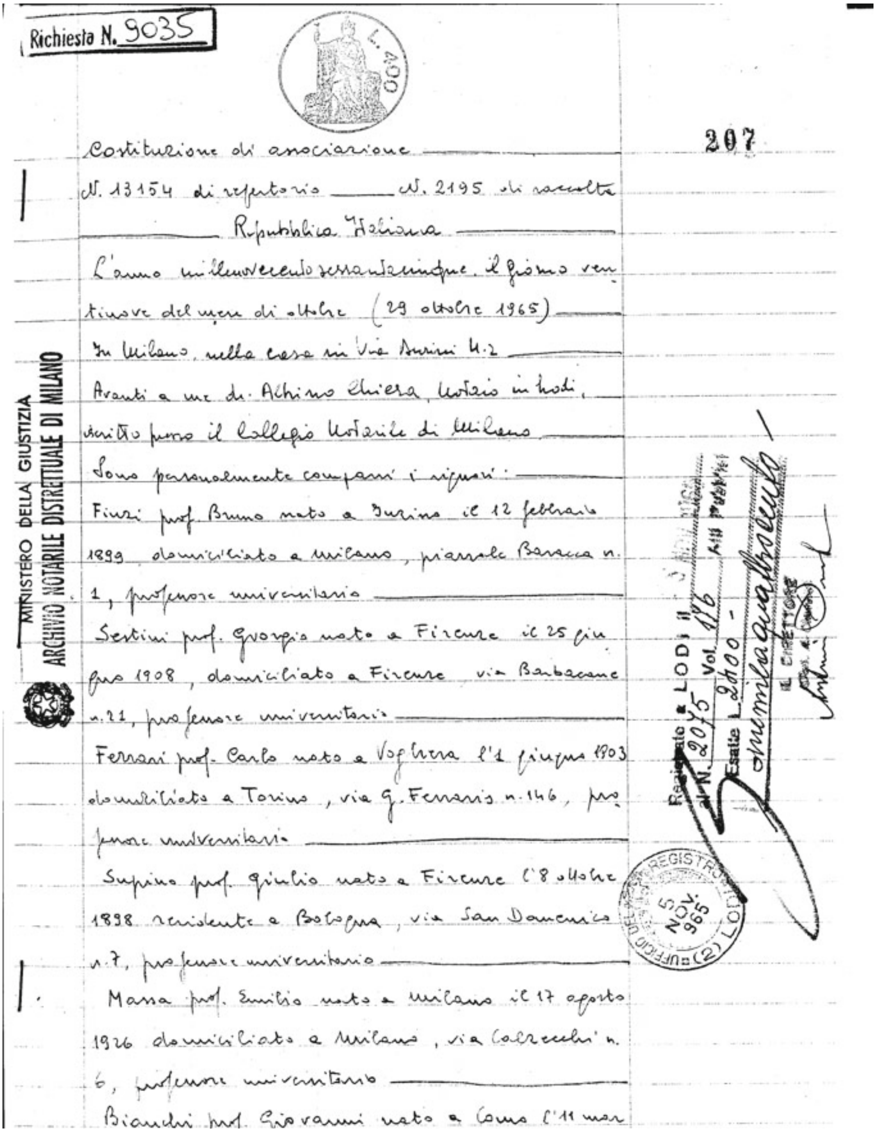
Appendix 1: Early Notarial Deed of Constitution



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N. 13154 di repertorio — N. 2195 di raccolta

Repubblica Italiana

L'anno millenovecentosessantacinque, il giorno ventinove del mese di ottobre (29 ottobre 1965)

In Milano, nella casa in Via Aurini n. 2

Avanti a me dr. Albino Chiesa, Notaio in Lodi, iscritto presso il Collegio Notarile di Milano

Sono personalmente comparsi i signori:

Finzi prof. Bruno nato a Torino il 12 febbraio 1899 domiciliato a Milano, piazzale Baracca n. 1, professore universitario

Sestini prof. Giorgio nato a Firenze il 25 giugno 1908, domiciliato a Firenze via Barbacane n. 21, professore universitario

Ferrari prof. Carlo nato a Voghera l'1 giugno 1903 domiciliato a Torino, via G. Ferraris n. 146, professore universitario

Supino prof. Giulio nato a Firenze l'8 ottobre 1898 residente a Bologna, via San Domenico n. 7, professore universitario

Massa prof. Emilio nato a Milano il 17 agosto 1926 domiciliato a Milano, via Colzecchi n. 6, professore universitario

Bianchi prof. Giovanni nato a Como l'11 mar

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IL DIRETTORE

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re 1924 domiciliato a Milano, via Ampere n. 47, professore universitario

Finzi prof. Leo nato a Milano il 24 settembre 1924 domiciliato a Milano, viale Giustiniano n. 10, professore universitario

Giangreco prof. Elio nato a Paceco il 6 ottobre 1924 domiciliato a Napoli via Tasso n. 480, professore universitario

Persone aventi cittadinanza italiana e della cui identità sono io notaio certo, che rinunziano – d'accordo fra loro e col mio consenso – all'assistenza dei testimoni al presente atto col quale dichiarano di volere costituire, come col presente atto costituiscono una associazione denominata: "Associazione Italiana di Meccanica Teorica ed Applicata" (A.I.M.E.T.A)"; con sede legale in Milano, Piazza Leonardo da Vinci n. 32, retta dallo Statuto che i comparenti qui mi esibiscono e che da me letto ai comparenti, viene allegato sotto A al presente atto quale sua parte integrante e sostanziale

Il Signor Finzi prof. Bruno viene dai comparenti unanimemente designato a rappresentare legalmente la Associazione fino alla elezione delle cariche sociali previste dallo

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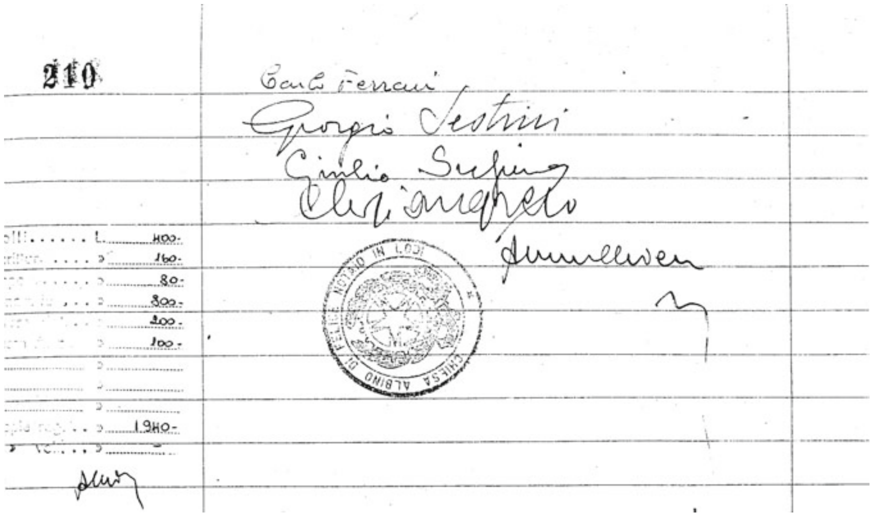
Statuto elezione che avrà luogo nella prima
assemblea dei soci che verrà convocata a cura
del predetto Signore.
La prima Assemblea dei soci provvederà pure
a determinare l'ammontare del contributo an-
nuo.
Il nominando Presidente viene fin d'ora autoriz-
zato a compiere le pratiche necessarie per il conse-
guimento del riconoscimento dell'Associazione e
per l'acquisto della personalità giuridica e viene
facoltizzato ad apportare allo Statuto le modifi-
che richieste dalle competenti autorità.
Ri-
chiesto io notaio ho ricevuto il presente atto
che ho letto, in uno con l'annesso allegato,
ai sovracomparsi i quali lo approvano e con
me lo sottoscrivono.
Consta
quest'atto di un foglio scritto da me e da ma-
no fida per due facciate intere e righe ven-
tuno di questa terza
Bruno Finzi
Leo Finzi
Giulio Krall
Giovanni Bianchi



Appendix 2: AIMETA Main People

See Tables 1, 2, 3.

Table 1 AIMETA Founders and Presidents

AIMETA Founders (1965)	AIMETA Presidents	
Name	Years	President
Giovanni Bianchi	1966–1969	Bruno Finzi
Carlo Ferrari	1970–1973	Giulio Supino
Bruno Finzi	1974–1977	Carlo Ferrari
Leo Finzi	1978–1981	Giorgio Sestini
Elio Giangreco	1982–1985	Giovanni Bianchi
Emilio Massa	1986–1989	Giulio Maier
Giorgio Sestini	1990–1993	Carlo Cercignani
Giulio Supino	1994–1997	Enrico Marchi
	1998–2001	Gianfranco Capriz
	2002–2005	Angelo Morro
	2006–2009	Giuseppe Rega
	2010–2013	Carlo Cinquini
	2014–2017	Paolo Luchini
	2018–2021	Stefano Lenci
	2022–2025	Walter D'Ambrogio

Table 2 Editors of *Meccanica*

Years	Editor
1966–1981	Emilio Massa and Giovanni Bianchi
1982–1985	Giulio Maier
1986–1989	Carlo Cercignani
1990–1997	Giuliano Augusti
1998–2003	Giuseppe Rega
2004–2011	Vincenzo Parenti Castelli
2012–2014	Alberto Carpinteri
2015–2020	Luigi Gambarotta
2021-	Anna Pandolfi

Table 3 Congress Chairs

Congress	Congress Committee
I Udine 1971	Luigi Sobrero
II Napoli 1974	Rodolfo Monti, Luigi G. Napolitano
III Cagliari 1976	Giuseppe Aymerich
IV Firenze 1978	Giuliano Augusti
V Palermo 1980	Nicola Alberti

Congress	Organizing Committee	Scientific Committee
VI Genova 1982	Giovanni Bianchi	Riccardo Baldacci
VII Trieste 1984	Fulvio Di Marino	Giuseppe Grioli
VIII Torino 1986	Bruno Piombo	Ario Romiti, Dionigi Galletto
IX Bari 1988	Antonio Trentadue	Alfredo Sollazzo
X Pisa 1990	Roberto Bassani	Marino Marini
XI Trento 1992	Roberto Contro	Luigi Salvadori
XII Napoli 1995	Luciano Nunziante	?
XIII Siena 1997	Paolo Toni	Piero Villaggio
XIV Como 1999	Alberto Fontana	Roberto Contro
XV Taormina 2001	Francesco Petrone	Giuliano Augusti
XVI Ferrara 2003	Claudio Alessandri	Giannantonio Sacchi Landriani
XVII Firenze 2005	Claudio Borri	Mario Primicerio
XVIII Brescia 2007	Angelo Carini	Renzo Piva
XIX Ancona 2009	Stefano Lenci	Paolo Podio Guidugli
XX Bologna 2011	Francesco Ubertini	Antonio Tralli
XXI Torino 2013	Giuseppe Lacidogna	Alberto Carpinteri
XXII Genova 2015	Luigi Gambarotta	Angelo Morro
XXIII Salerno 2017	Fernando Fraternali	Antonio Tralli
XXIV Roma 2019	Achille Paolone, Antonio Carcaterra	Giorgio Graziani
XXV Palermo 2022	Mario Di Paola, Fabrizio Micari	Giuseppe Rega

Appendix 3: AIMETA Executive Council

See Table 4.

Table 4 Executive Council

Year	Member	Affiliation	Role	Area
1966–1969	Bruno Finzi	Polytechnic of Milano	President	General Mechanics
	Carlo Ferrari	Polytechnic of Torino		Fluids
	Leo Finzi	Polytechnic of Milano		Solids and Structures
	Elio Giangreco	University of Napoli		Solids and Structures
	Emilio Massa	Polytechnic of Milano		Machines
	Giorgio Sestini	University of Firenze		General Mechanics
	Giulio Supino	University of Bologna		Fluids
	Giovanni Bianchi	Polytechnic of Milano	Secretary	Machines
1970–1973	Giulio Supino	University of Bologna	President	Fluids
	Carlo Ferrari	Polytechnic of Torino		Fluids
	Bruno Finzi	Polytechnic of Milano		General Mechanics
	Leo Finzi (?)	Polytechnic of Milano		Solids and Structures
	Elio Giangreco (?)	University of Napoli		Solids and Structures
	Giorgio Sestini	University of Firenze		General Mechanics
	?	?		?
	Giovanni Bianchi	Polytechnic of Milano	Secretary	Machines
1974–1977	Carlo Ferrari	Polytechnic of Torino	President	Fluids
	Carlo Cattaneo	University of Roma La Sapienza		General Mechanics
	Mario Como	University of Napoli		Solids and Structures
	Lucio Lazzarino	University of Pisa		Machines
	Giulio Maier	Polytechnic of Milano		Solids and Structures
	Giorgio Sestini	University of Firenze		General Mechanics
	Giulio Supino	University of Bologna		Fluids
	Giovanni Bianchi	Polytechnic of Milano	Secretary	Machines
1978–1981	Giorgio Sestini	University of Firenze	President	General Mechanics
	Giuliano Augusti	University of Firenze		Solids and Structures
	Carlo Cercignani	Polytechnic of Milano		General Mechanics
	Carlo Ferrari	Polytechnic of Torino		Fluids
	Ettore Funaioli	University of Bologna		Machines
	Giulio Maier	Polytechnic of Milano		Solids and Structures
	Luigi G. Napolitano	University of Napoli		Fluids

(continued)

Table 4 (continued)

Year	Member	Affiliation	Role	Area
	Giovanni Bianchi	Polytechnic of Milano	Secretary	Machines
1982–1985	Giovanni Bianchi	Polytechnic of Milano	President	Machines
	Enrico Marchi	University of Genova	Vice-President	Fluids
	Carlo Cercignani	Polytechnic of Milano	Secretary	General Mechanics
	Luciano De Socio	University of Roma La Sapienza	Treasurer	Fluids
	since September 1983 replaced by			
	Giuseppe Grioli	University of Padova		General Mechanics
	Giulio Maier	Polytechnic of Milano		Solids and Structures
	since September 1983 replaced by			
	Piero Villaggio	University of Pisa	Treasurer	Solids and Structures
	Silvio Nocilla	Polytechnic of Torino		General Mechanics
	Giorgio Sestini	University of Firenze		General Mechanics
1986–1989	Giulio Maier	Polytechnic of Milano	President	Solids and Structures
	Enrico Marchi	University of Genova	Vice-President	Fluids
	Pietro Caparrini	University of Firenze	Treasurer	Machines
	since September 1986 replaced by			
	Tristano Manacorda	University of Pisa	Treasurer	General Mechanics
	Giovanni Bianchi	Polytechnic of Milano		Machines
	Carlo Cercignani	Polytechnic of Milano		General Mechanics
	Giuseppe Grioli	University of Padova		General Mechanics
	Piero Villaggio	University of Pisa		Solids and Structures
	Leone Corradi	Polytechnic of Milano	Secretary	Solids and Structures
1990–1993	Carlo Cercignani	Polytechnic of Milano	President	General Mechanics
	Piero Villaggio	University of Pisa	Vice-President	Solids and Structures
	Leone Corradi	Polytechnic of Milano	Secretary	Solids and Structures
	Roberto Bassani	University of Pisa	Treasurer	Machines
	Giulio Maier	Polytechnic of Milano		Solids and Structures
	Renzo Piva	University of Roma La Sapienza		Fluids
	Castrenze Polizzotto	University of Palermo		Solids and Structures
1994–1997	Enrico Marchi	University of Genova	President	Fluids
	Piero Villaggio	University of Pisa	Vice-President	Solids and Structures
	Angelo Morro	University of Genova	Secretary	General Mechanics
	Roberto Contro	Polytechnic of Milano	Treasurer	Solids and Structures
	Roberto Bassani	University of Pisa		Machines
	Carlo Cercignani	Polytechnic of Milano		General Mechanics

(continued)

Table 4 (continued)

Year	Member	Affiliation	Role	Area
	Renzo Piva	University of Roma La Sapienza		Fluids
1998–2001	Gianfranco Capriz	University of Pisa	President	General Mechanics
	Roberto Contro	Polytechnic of Milano	Vice-President	Solids and Structures
	Angelo Morro	University of Genova	Secretary	General Mechanics
	Maurizio Pandolfi	Polytechnic of Torino	Treasurer	Fluids
	Giuliano Augusti	University of Roma La Sapienza		Solids and Structures
	Roberto Bassani	University of Pisa		Machines
	Enrico Marchi	University of Genova		Fluids
2002–2005	Angelo Morro	University of Genova	President	General Mechanics
	Giuliano Augusti	University of Roma La Sapienza	Vice-President	Solids and Structures
	Carlo Cinquini	University of Pavia	Secretary	Solids and Structures
	Maurizio Pandolfi	Polytechnic of Torino	Treasurer	Fluids
	Roberto Bassani	University of Pisa		Machines
	Gianfranco Capriz	University of Pisa		General Mechanics
	Mario Di Paola	University of Palermo		Solids and Structures
2006–2009	Giuseppe Rega	University of Roma La Sapienza	President	Solids and Structures
	Maurizio Pandolfi	Polytechnic of Torino	Vice-President	Fluids
	Angelo Morro	University of Genova	Secretary	General Mechanics
	Maria Lampis	Polytechnic of Milano	Treasurer	General Mechanics
	since March 2007 replaced by			
	Antonio Fasano	University of Firenze	Treasurer	General Mechanics
	Massimo Guiggiani	University of Pisa		Machines
	Mario Di Paola	University of Palermo		Solids and Structures
	Luigi Gambarotta	University of Genova		Solids and Structures
2010–2013	Carlo Cinquini	University of Pavia	President	Solids and Structures
	Paolo Luchini	University of Salerno	Vice-President	Fluids
	Angelo Morro	University of Genova	Secretary	General Mechanics
	Massimo Guiggiani	University of Pisa	Treasurer	Machines
	Guido Borino	University of Palermo		Solids and Structures
	Luigi Gambarotta	University of Genova		Solids and Structures
	Giuseppe Rega	University of Roma La Sapienza		Solids and Structures
2014–2017	Paolo Luchini	University of Salerno	President	Fluids
	Walter D'Ambrogio	University of L'Aquila	Vice-President	Machines
	Guido Borino	University of Palermo	Secretary	Solids and Structures
	Carlo Cinquini	University of Pavia	Treasurer	Solids and Structures

(continued)

Table 4 (continued)

Year	Member	Affiliation	Role	Area
	Sandra Carillo	Sapienza University of Roma		General Mechanics
	Stefano Lenci	Polytechnic University of Marche		Solids and Structures
	Elio Sacco	University of Cassino and Southern Lazio		Solids and Structures
2018–2021	Stefano Lenci	Polytechnic University of Marche	President	Solids and Structures
	Paolo Luchini	University of Salerno	Vice-President	Fluids
	Sandra Carillo	Sapienza University of Roma	Secretary	General Mechanics
	Umberto Perego	Polytechnic of Milano	Treasurer	Solids and Structures
	Carlo M. Casciola	Sapienza University of Roma		Fluids
	Walter D'Ambrogio	University of L'Aquila		Machines
	Fernando Fraternali	University of Salerno		Solids and Structures
2022–2025	Walter D'Ambrogio	University of L'Aquila	President	Machines
	Stefano Lenci	Polytechnic University of Marche	Vice-President	Solids and Structures
	Sandra Carillo	Sapienza University of Roma	Secretary	General Mechanics
	Umberto Perego	Polytechnic of Milano	Treasurer	Solids and Structures
	Carlo M. Casciola	Sapienza University of Roma		Fluids
	Fernando Fraternali	University of Salerno		Solids and Structures
	Elio Sacco	University of Napoli Federico II		Solids and Structures

Appendix 4: AIMETA Congresses

See Tables 5, 6, 7, and 8.

Table 5 Scientific Committee

Member	Affiliation	Area
I Udine 1971		
Luigi Sobrero (Chair)	University of Trieste	General Mechanics
Elio Giangreco	University of Napoli	Solids and Structures
Luigi G. Napolitano	University of Napoli	Fluids
Ario Romiti	Polytechnic of Torino	Machines
Giovanni Bianchi	Polytechnic of Milano	Machines
II Napoli 1974		
Luigi G. Napolitano (Chair)	University of Napoli	Fluids
Giovanni Bianchi	Polytechnic of Milano	Machines
Giulio Ceradini	University of Roma La Sapienza	Solids and Structures
Giovanni Jarre	Polytechnic of Torino	Fluids
Giulio Mattei	University of Siena	General Mechanics
Antonio Grimaldi	University of Napoli	Solids and Structures
III Cagliari 1976		
Giuseppe Aymerich (Chair)	University of Cagliari	General Mechanics
Angelo Berio	University of Cagliari	Solids and Structures
Giovanni Bianchi	Polytechnic of Milano	Machines
Vittorio Cantoni	Polytechnic of Torino	General Mechanics
Costantino Fassò	University of Cagliari	Fluids
IV Firenze 1978		
Giuliano Augusti (Chair)	University of Firenze	Solids and Structures
?	…	…
V Palermo 1980		
Nicola Alberti (Chair)	University of Palermo	Machines
Giuliano Augusti	University of Firenze	Solids and Structures
Guglielmo Benfratello	University of Palermo	Fluids
Giovanni Bianchi	Polytechnic of Milano	Machines
Carlo Ugo Galletti	University of Genova	Machines
Antonio Greco	University of Palermo	General Mechanics
Rodolfo Monti	University of Napoli	Fluids
Castrenze Polizzotto	University of Palermo	Solids and Structures
Salvatore Rionero	University of Napoli	General Mechanics
VI Genova 1982		
Riccardo Baldacci (Chair)	University of Genova	Solids and Structures
Aldo Belleni Morante	University of Firenze	General Mechanics
Edoardo Benvenuto	University of Genova	Solids and Structures
Andrea Capello	Polytechnic of Milano	Machines
Carlo Cercignani	Polytechnic of Milano	General Mechanics

(continued)

Table 5 (continued)

Member	Affiliation	Area
Enrico Marchi	University of Genova	Fluids
Umberto Meneghetti	University of Bologna	Machines
Rinaldo Michelini	University of Genova	Machines
Luigi Napolitano	University of Napoli	Fluids
Castrenze Polizzotto	University of Palermo	Solids and Structures
Edoardo Storchi	University of Genova	General Mechanics
VII Trieste 1984		
Giuseppe Grioli (Chair)	University of Padova	General Mechanics
Antonino Antonini	University of Trieste	Machines
Raffaele Cola	University of Padova	Fluids
Rinaldo Ghigliazza	University of Genova	Machines
Maurizio Pandolfi	Polytechnic of Torino	Fluids
Giannantonio Sacchi	Polytechnic of Milano	Solids and Structures
Alfredo Sollazzo	Polytechnic of Bari	Solids and Structures
Enzo Tonti	University of Trieste	General Mechanics
VIII Torino 1986		
Ario Romiti (Chair)	Polytechnic of Torino	Machines
Dionigi Galletto (Co-Chair)	University of Torino	General Mechanics
Giuseppe Bernasconi	Polytechnic of Milano	Machines
Elisa Udeschini Brinis	Polytechnic of Milano	General Mechanics
Giulio Ceradini	University of Roma La Sapienza	Solids and Structures
Vincenzo Franciosi	University of Napoli	Solids and Structures
Ettore Funaioli	University of Bologna	Machines
Tristano Manacorda	University of Pisa	General Mechanics
Angelo Morro	University of Genova	General Mechanics
Silvio Nocilla	Polytechnic of Torino	Fluids
Giannantonio Pezzoli	Polytechnic of Torino	Fluids
Castrenze Polizzotto	University of Palermo	Solids and Structures
IX Bari 1988		
Alfredo Sollazzo (Chair)	Polytechnic of Bari	Solids and Structures
Sergio Benenti	University of Torino	General Mechanics
Alfredo Corsanego	University of Genova	Solids and Structures
Franco Maceri	University of Roma Tor Vergata	Solids and Structures
Michele Maiellaro	University of Bari	General Mechanics
Michele Napolitano	Polytechnic of Bari	Fluids
Luciano Pirodda	University of Cagliari	Machines
Guido Ruggieri	Polytechnic of Milano	Machines

(continued)

Table 5 (continued)

Member	Affiliation	Area
Giambattista Scarpi	University of Bologna	Fluids
X Pisa 1990		
Marino Marini (Chair)	University of Pisa	Machines
Pasquale M. Calderale (Co-Chair)	Polytechnic of Torino	Machines
Gianfranco Capriz	TECSIEL - University of Pisa	General Mechanics
Luigi Cedolin	Polytechnic of Milano	Solids and Structures
Giulio Ceradini	University of Roma La Sapienza	Solids and Structures
Giovanni G. Lisini	University of Firenze	Machines
Giulio Mattei	University of Pisa	General Mechanics
Renzo Piva	University of Roma La Sapienza	Fluids
Mario Primicerio	University of Firenze	General Mechanics
Giovanni Romano	University of Napoli	Solids and Structures
Giovanni Seminara	University of Genova	Fluids
Furio Vatta	Polytechnic of Torino	Machines
XI Trento 1992		
Luigi Salvadori (Chair)	University of Trento	General Mechanics
Giovanni Bianchi	Polytechnic of Milano	Machines
Carlo Cinquini	University of Pavia	Solids and Structures
Umberto Meneghetti	University of Bologna	Machines
Mario Pitteri	University of Padova	General Mechanics
Giuseppe Rega	University of L'Aquila	Solids and Structures
Filippo Sabetta	University of Roma La Sapienza	Fluids
Gianbattista Scarpi	University of Bologna	Fluids
XII Napoli 1995		
Paolo Blondeaux	University of Genova	Fluids
Leone Corradi	Polytechnic of Milano	Solids and Structures
Gianpietro Del Piero	University of Ferrara	Solids and Structures
Antonio Fasano	University of Firenze	General Mechanics
Vincenzo Parenti Castelli	University of Ferrara	Machines
Amilcare Pozzi	University of Napoli Federico II	Fluids
Salvatore Rionero	University of Napoli Federico II	General Mechanics
Bernhard Schrefler	University of Padova	Solids and Structures
Aldo Sestieri	University of Roma La Sapienza	Machines
Fabrizio Vestroni	University of Roma La Sapienza	Solids and Structures
XIII Siena 1997		
Piero Villaggio (Chair)	University of Pisa	Solids and Structures
Renzo Piva	University of Roma La Sapienza	Fluids

(continued)

Table 5 (continued)

Member	Affiliation	Area
Giuliano Augusti	University of Roma La Sapienza	Solids and Structures
Giovanni Frosali	Polytechnic University of Marche	General Mechanics
Giovanni G. Lisini	University of Firenze	Machines
Carlo Marchioro	University of Roma La Sapienza	General Mechanics
Luciano Nunziante	University of Napoli Federico II	Solids and Structures
Giuseppe Rega	University of L'Aquila	Solids and Structures
Antonio Tralli	University of Ferrara	Solids and Structures
Furio Vatta	Polytechnic of Torino	Machines
XIV Como 1999		
Roberto Contro (Chair)	Polytechnic of Milano	Solids and Structures
Piero Bassanini	University of Rome La Sapienza	Fluids
Guido Belforte	Polytechnic of Torino	Machines
Andrea Carpinteri	Polytechnic of Torino	Solids and Structures
Vittorio Cossalter	University of Padova	Machines
Maria Lampis	Polytechnic of Milano	General Mechanics
Paolo Luchini	Polytechnic of Milano	Fluids
Paolo Podio Guidugli	University of Roma Tor Vergata	Solids and Structures
Giovanni Romano	University of Napoli Federico II	Solids and Structures
Filippo Sabetta	University of Roma La Sapienza	Fluids
XV Taormina 2001		
Giuliano Augusti (Chair)	University of Roma La Sapienza	Solids and Structures
Angelo M. Anile	University of Catania	General Mechanics
Guido Buresti	University of Pisa	Fluids
Leone Corradi	Polytechnic of Milano	Solids and Structures
Sergio Della Valle	University of Napoli Federico II	Machines
Mario Di Paola	University of Palermo	Solids and Structures
Giuseppe Oliveto	University of Catania	Solids and Structures
Maurizio Pandolfi	Polytechnic of Torino	Fluids
Bruno Pizzigoni	Polytechnic of Milano	Machines
XVI Ferrara 2003		
Giannantonio Sacchi Landriani (Chair)	Polytechnic of Milano	Solids and Structures
Guido Buresti	University of Pisa	Fluids
Roberto Contro	Polytechnic of Milano	Solids and Structures
Vincenzo D'Agostino	University of Salerno	Machines
Gianpietro Del Piero	University of Ferrara	Solids and Structures
Carlo Ugo Galletti	University of Genova	Machines

(continued)

Table 5 (continued)

Member	Affiliation	Area
Giorgio Graziani	University of Roma La Sapienza	Fluids
Giuseppe Muscolino	University of Messina	Solids and Structures
Franco Pastrone	University of Torino	General Mechanics
Umberto Perego	Polytechnic of Milano	Solids and Structures
Furio Vatta	Polytechnic of Torino	Machines
XVII Firenze 2005		
Mario Primicerio (Chair)	University of Firenze	General Mechanics
Gianni Bartoli	University of Firenze	Solids and Structures
Davide Bigoni	University of Trento	Solids and Structures
Guido Borino	University of Palermo	Solids and Structures
Ennio Carnevale	University of Firenze	Machines
Alberto Corigliano	Polytechnic of Milano	Solids and Structures
Massimiliano Lucchesi	University of Firenze	Solids and Structures
Paolo Luchini	University of Salerno	Fluids
Aleramo Lucifredi	University of Genova	Machines
Angelo Luongo	University of L'Aquila	Solids and Structures
Ettore Pennestrì	University of Roma Tor Vergata	Machines
Terenziano Raparelli	University of L'Aquila	Machines
Paolo Rissone	University of Firenze	Machines
Giampiero Spiga	University of Parma	General Mechanics
XVIII Brescia 2007		
Renzo Piva (Chair)	University of Roma La Sapienza	Fluids
Roberto Bassani	University of Pisa	Machines
Stefano Bennati	University of Pisa	Solids and Structures
Paolo Blondeaux	University of Genova	Fluids
Alberto Carpinteri	Polytechnic of Torino	Solids and Structures
Raffaele Casciaro	University of Calabria	Solids and Structures
Carlo Cinquini	University of Pavia	Solids and Structures
Mauro Fabrizio	University of Bologna	General Mechanics
Antonio Fasano	University of Firenze	General Mechanics
Francesco Genna	University of Brescia	Solids and Structures
Giovanni Mimmi	University of Pavia	Machines
Giorgio Novati	Polytechnic of Milano	Solids and Structures
Edzeario Prati	University of Parma	Machines
Elio Sacco	University of Cassino	Solids and Structures
Roberto Verzicco	Polytechnic of Bari	Fluids
XIX Ancona 2009		
Paolo Podio Guidugli (Chair)	University of Roma Tor Vergata	Solids and Structures

(continued)

Table 5 (continued)

Member	Affiliation	Area
Alessandro Bottaro	University of Genova	Fluids
Carlo M. Casciola	University of Roma La Sapienza	Fluids
Vincenzo Ciampi	University of Roma La Sapienza	Solids and Structures
Claudia Comi	Polytechnic of Milano	Solids and Structures
Fabrizio Davì	Polytechnic University of Marche	Solids and Structures
Luca Deseri	University of Trento	Solids and Structures
Giovanni Falsone	University of Messina	Solids and Structures
Giovanni Frosali	University of Firenze	General Mechanics
Luigi Garibaldi	Polytechnic of Torino	Machines
Giovanni Legnani	University of Brescia	Machines
Tommaso Ruggeri	University of Bologna	General Mechanics
Giovanni Santucci	University of Roma La Sapienza	Machines
Marco Savoia	University of Bologna	Solids and Structures
Giovanni Solari	University of Genova	Solids and Structures
XX Bologna 2011		
Antonio Tralli (Chair)	University of Ferrara	Solids and Structures
Benedetto Allotta	University of Firenze	Machines
Paolo Bisegna	University of Roma Tor Vergata	Solids and Structures
Maurizio Brocchini	Polytechnic University of Marche	Fluids
Giorgio Busoni	University of Firenze	General Mechanics
Giorgio Graziani	University of Roma La Sapienza	Fluids
Donatella Marini	University of Pavia	General Mechanics
Gianpiero Mastinu	Polytechnic of Milano	Machines
Achille Paolone	University of Roma La Sapienza	Solids and Structures
Luigi Preziosi	Polytechnic of Torino	General Mechanics
Santi Rizzo	University of Palermo	Solids and Structures
Luciano Rosati	University of Napoli	Solids and Structures
Michele Russo	University of Napoli	Machines
Antonio Taliercio	Polytechnic of Milano	Solids and Structures
Paolo Vannucci	University of Versailles	Solids and Structures
XXI Torino 2013		
Alberto Carpinteri (Chair)	Polytechnic of Torino	Solids and Structures
Nicola Bellomo	Polytechnic of Torino	General Mechanics
Francesco Benedettini	University of L'Aquila	Solids and Structures
Leonardo Bertini	University of Pisa	Machines
Salvatore Caddemi	University of Catania	Solids and Structures
Massimo Callegari	Polytechnic University of Marche	Machines
Claudio Canuto	Polytechnic of Torino	General Mechanics
Paolo Fuschi	University of Reggio Calabria	Solids and Structures
Raimondo Luciano	University of Cassino	Solids and Structures

(continued)

Table 5 (continued)

Member	Affiliation	Area
Franco Pastrone	University of Torino	General Mechanics
Gianni Pedrizzetti	University of Trieste	Fluids
Federico Perotti	Polytechnic of Milano	Solids and Structures
Terenziano Raparelli	Polytechnic of Torino	Machines
Gianni Royer Carfagni	University of Parma	Solids and Structures
Roberto Verzicco	University of Roma Tor Vergata	Fluids
Giorgio Zavarise	University of Salento	Solids and Structures
XXII Genova 2015		
Angelo Morro (Chair)	University of Genova	General Mechanics
Michele Ciarletta	University of Salerno	General Mechanics
Enrico Ciulli	University of Pisa	Machines
Domenico De Tommasi	Polytechnic of Bari	Solids and Structures
Claudio Giorgi	University of Brescia	General Mechanics
Giacomo Mantriota	Polytechnic of Bari	Machines
Antonino Morassi	University of Udine	Solids and Structures
Anna Pandolfi	Polytechnic of Milano	Solids and Structures
Antonina Pirrotta	University of Palermo	Solids and Structures
Egidio Rizzi	University of Bergamo	Solids and Structures
Maria Vittoria Salvetti	University of Pisa	Fluids
Giovanni Seminara	University of Genova	Fluids
Giovanni Solari	University of Genova	Solids and Structures
Patrizia Trovalusci	University of Roma La Sapienza	Solids and Structures
Mauro Velardocchia	Polytechnic of Torino	Machines
XXIII Salerno 2017		
Antonio Tralli (Chair)	University of Ferrara	Solids and Structures
Marco Carricato	University of Bologna	Machines
Antonella Cecchi	University of Venezia IUAV	Solids and Structures
Andrea Collina	Polytechnic of Milano	Machines
Mauro Fabrizio	University of Bologna	General Mechanics
Attilio Frangi	Polytechnic of Milano	Solids and Structures
Giuseppe Giambanco	University of Palermo	Solids and Structures
Nicola Ivan Giannoccaro	University of Salento	Machines
Renato Masiani	University of Roma La Sapienza	Solids and Structures
Roberta Massabò	University of Genova	Solids and Structures
Angelo Morro	University of Genova	General Mechanics
Giuseppe Muscolino	University of Messina	Solids and Structures
Roberto Paroni	University of Sassari	Solids and Structures
Eugenio Pugliese Carratelli	University of Salerno	Fluids
Alessandro Talamelli	University of Bologna	Fluids

(continued)

Table 5 (continued)

Member	Affiliation	Area
XXIV Roma 2019		
Giorgio Graziani (Chair)	University of Roma La Sapienza	Fluids
Lorenzo Bardella	University of Brescia	Solids and Structures
Davide Bigoni	University of Trento	Solids and Structures
Francesco Braghin	Polytechnic of Milano	Machines
Mario Di Paola	University of Palermo	Solids and Structures
Luciano Feo	University of Salerno	Solids and Structures
Domenico Guida	University of Salerno	Machines
Maria Grazia Naso	University of Brescia	General Mechanics
Nicola Rizzi	University of Roma Tre	Solids and Structures
Giuseppe Pascazio	Polytechnic of Bari	Fluids
Francesco Pellicano	University of Modena and Reggio Emilia	Machines
Alessandro Reali	University of Pavia	Solids and Structures
Vincenzo Tibullo	University of Salerno	General Mechanics
Patrizia Trovalusci	University of Roma La Sapienza	Solids and Structures
Antonio Viviani	University of Campania L. Vanvitelli	Fluids
XXV Palermo 2022		
Giuseppe Rega (Chair)	University of Roma La Sapienza	Solids and Structures
Vincenzo Armenio	University of Trieste	Fluids
Alessandro Bottaro	University of Genova	Fluids
Antonella Cecchi	University of Venezia IUAV	Solids and Structures
Claudia Comi	Polytechnic of Milano	Solids and Structures
Mauro Fabrizio	University of Bologna	General Mechanics
Luigi Gambarotta	University of Genova	Solids and Structures
Fabrizio Greco	University of Calabria	Solids and Structures
Marco Paggi	IMT School Lucca	Solids and Structures
Antonina Pirrotta	University of Palermo	Solids and Structures
Ferruccio Resta	Polytechnic of Milano	Machines
Alessandro Rivola	University of Bologna	Machines
Maurizio Romeo	University of Genova	General Mechanics
Elio Sacco	University of Napoli	Solids and Structures
Rosario Sinatra	University of Catania	Machines

Table 6 Keynote Lectures

Congress	Area	Author	Affiliation	Title
I Udine 1971	Fluids/Solids	R.E D. Bishop	London University	A survey of strength calculations for ship hulls
	General Mechanics	D. Galletto	University of Torino	Some recent results and developments in general mechanics and mathematical physics
	Solids	M. Capurso	University of Bologna	Mechanics of solids: General report
	Machines	A. Capello	Polytechnic of Milano	Mechanics of machines: General report
II Napoli 1974	?	?	?	?
III Cagliari 1976	General Mechanics	G. Grioli	University of Padova	On the mechanics of oriented tridimensional continua
	Machines	E. Funaioli	University of Bologna	?
	Fluids	A. Pezzoli	Polytechnic of Torino	?
IV Firenze 1978	General Mechanics	F. Brezzi	University of Pavia	?
	?	?	?	?
	?	?	?	?
V Palermo 1980	General Mechanics	C. Cercignani	Polytechnic of Milano	Kinetic theory of gases and thermomechanics of continua
	Solids and Structures	R. Baldacci	University of Genova	Phenomenological and structural plasticity
	Machines	S. Stecco	University of Firenze	Vibration phenomena in turbomachines ensuing from fluid-blade interaction
	Fluids	M. Ippolito	University of Napoli	Interaction effects on viscous properties of solid–liquid dispersed systems. Non-Newtonian behaviour and fluidynamic resistances
VI Genova 1982	Solids and Structures	W. T. Koiter	Delft University	?
	General Mechanics	L. Salvadori	University of Trento	?
	Fluids	R. Monti	University of Napoli	?
	Solids and Structures	G. Maier	Polytechnic of Milano	?
	Machines	A. Romiti	Polytechnic of Torino	?

(continued)

Table 6 (continued)

Congress	Area	Author	Affiliation	Title
VII Trieste 1984	General Mechanics	H. Ziegler	Polytechnic of Zurich	Thermodynamics
	General Mechanics	D. Galletto	University of Torino	Classical mechanics and cosmology
	Machines	L. Lazzarino	University of Pisa	Mechanics of machines: Contribution to the development of modern engineering
	Fluids	G. Moretti	Polytechnic Institute of New York	Numerical fluid mechanics
	Solids and Structures	G. Ceradini	University of Roma La Sapienza	Development and structural implications of the theory of plasticity
VIII Torino 1986	General Mechanics	P. Cicala	Polytechnic of Torino	Comments on recent developments of asymptotics
	Machines	J. Wittenburg	University of Karlsruhe	Multibody dynamics. A rapidly developing field of applied mechanics
	Machines/Structures	G.R. Tomlinson	Heriot-Watt University, Edinburgh	Hilbert transform procedures for detecting and quantifying non-linearity in modal testing
	Fluids	M.J. Werle	United Technologies Research Center, West Hartford	R. Thomas Davis. His contributions to numerical simulation of viscous flows—Part I, historical perspective
IX Bari 1988	General Mechanics	C. Truesdell	John Hopkins University	Some reflections upon theoretical mechanics in the past fifty years
	Solids and Structures	J. Lemaitre	University of Paris 6	Mechanics and micromechanics of damage
	Fluids	R. Piva	University of Roma La Sapienza	Boundary integral equations in fluidynamics
	Industrial	L. Guerriero	Italian Space Agency	Italian contributions to space activity
X Pisa 1990	General Mechanics	T. Manacorda	University of Pisa	Origin and development of the concept of wave
	Solids and Structures	V. Tvergaard	Technical University of Denmark, Lyngby	Mechanical modelling of ductile fracture
	Fluids	J.C.R. Hunt	University of Cambridge	How fluid mechanics can help solve problems in the environment
	Machines	W. Schiehlen	University of Stuttgart	Recent developments in multibody system dynamics
XI Trento 1992	General Mechanics	G.I. Barenblatt	Russian Academy of Sciences, Moscow	Intermediate asymptotics, scaling laws and renormalization group in continuum mechanics
	General Mechanics	L. Salvadori	University of Trento	?

(continued)

Table 6 (continued)

Congress	Area	Author	Affiliation	Title
	Solids and Structures	J.B. Martin	University of Cape Town	Piecewise smooth dissipation and yield functions in plasticity
	Fluids	K. Kirchgassner	University of Stuttgart	Structure of permanent waves in density-stratified media
XII Napoli 1995	General Mechanics	I. Müller	Technische Universitat Berlin	Instructive instabilities in non-linear elasticity: Biaxially loaded membrane, and rubber balloons
	Solids and Structures	N. Olhoff	Aalborg University	On optimum design of structures and materials
	Fluids	A. Hirschberg	Eindhoven University of Technology	Aeroacoustics of musical instruments
	Machines	A. Sestieri	University of Roma La Sapienza	Circumventing space sampling limitations in mechanical vibrations
XIII Siena 1997	General Mechanics	D.G. Crighton	DAMTP Cambridge University	Recent developments in structural acoustics
	Solids and Structures	Z. Mróz	IFTR Polish Academy of Sciences, Warsaw	Models of damage, slip and wear at material interface
	Fluids	G. Seminara	University of Genova	Stability and morphodynamics
	Machines	D. Dowson	University of Leeds	Modelling of elastohydrodynamic lubrication of real solids by real lubricants
XIV Como 1999	General Mechanics	H. Zorski	Polish Academy of Sciences, Warsaw	Locally rigid model of the peptide chain
	Solids and Structures	S.C. Cowin	City University of New York	Structural change in living tissues
	Fluids	P. Luchini	Polytechnic of Milano	New concepts for fluid-dynamics stability: Algebraic growth and added calculation of receptivity
	Machines	F. Pfeiffer	Technical University of Munich	Unilateral problems in dynamics
XV Taormina 2001	General Mechanics/Solids	M. Šilhaví	Mathematical Institute of AVČR, Prague	Dissipation postulates in finite-deformation plasticity
	Solids and Structures	P.J. Prendergast	Trinity College Dublin & TU Delft	Mechanical aspects of function and adaptation in the skeleton
XVI Ferrara 2003	General Mechanics/Solids	D.R. Owen	Carnegie Mellon University, Pittsburgh	Decompositions and identification relations as guides in the formulation of multiscale continuum field theories
	Solids and Structures	S.C. Cowin	The City College, New York	Bones have ears

(continued)

Table 6 (continued)

Congress	Area	Author	Affiliation	Title
	Fluids	A. Bottaro	Université Paul Sabatier, Toulouse	Initial stages of the transtion to turbulence in near-wall flow
	Machines	A.Z. Szeri	University of Delaware, Newark	On some inconsistencies in the application of the Reynolds lubrication theory
XVII Firenze 2005	Solids and Structures	N. Makris	University of Patras	Dimensional response analysis of yielding structures under near source ground motions
	Machines	A. Kahraman	Ohio State University	On the relationship between gear dynamics and surface wear
XVIII Brescia 2007	General Mechanics	C. Cercignani	Polytechnic of Milano	Fluid dynamics in MEMS and NEMS: A recent application of kinetic theory
	Solids and Structures	A.H. Nayfeh	Virginia Polytechnic Institute	Nonlinear phenomena in MEMS and NEMS
	Solids and Structures	G. Geymonat	Ecole Polytechnique Palaiseau	Some problems suggested by multi-materials and functionally graded materials
	Fluids	A. Prosperetti	Johns Hopkins University	The average stress in fluid-particle flows
	Industrial	B. Murari	STMicroelectronics	Breaking innovations: The role of lateral thinking
	Industrial	G. Audisio	Pirelli Tyre System	New frontiers of automotive innovation: The key factors
XIX Ancona 2009	Solids and Structures	J.J. Marigo	École Polytechnique Palaiseau	From initiation of cracks to fatigue: Some fundamental contribution of the variational approach to fracture
	Solids and Structures	G. Stepan	Budapest Univeristy of Technology	Balancing and vision - or The dynamics of poise
	Solids and Structures	G. Del Piero	University of Ferrara	The variational approach to fracture and to other inelastic phenomena
	Fluids	G. Pedrizzetti	University of Trieste	Cardiac fluid mechanics: From theory to clinical applications
	Industrial	A. Mencarini	Indesit	Innovation process in household appliances segment: Issues and opportunities
	Industrial	G. Rivetti	Cantieri Navali Marchigiani	Management and projects of the nautical supply chain
XX Bologna 2011	Solids and Structures	G. Romano	University of Napoli Federico II	On the geometric approach to non-linear continuum mechanics
	Solids and Structures	G. Sacchi Landriani	Polytechnic of Milano	Solid mechanical problems in the Risorgimento generation

(continued)

Table 6 (continued)

Congress	Area	Author	Affiliation	Title
	Machines	T. Bewley	University of California San Diego	New approaches for observation and forecasting of contaminant release plumes via coordination of swarms of sensor vehicles
	Industrial	D. Barana	Ducati Motor Holding	The development of high specific power engines
	Industrial	L. Marmorini	Ferrari GeS	Energy recovery in F1 cars: A necessity to win
XXI Torino 2013	General Mechanics	F. Brezzi	University of Pavia	Virtual element methods in structural mechanics
	Solids and Structures	V. Tvergaard	Technical University of Denmark, Lyngby	Ductile fracture at different levels of hydrostatic tension
	Solids and Structures	P. Podio Guidugli	University of Roma Tor Vergata	On the validation of theories of thin elastic structures
	Fluids	F. Charru	CNRS-INP-UPS – Université de Toulouse	Sand ripples and dunes
	Industrial	S. Re Fiorentin	FIAT Research Centre	Multidisciplinary optimization as a key step forward in the automotive design
XXII Genova 2015	Solids and Structures	C. Comi	Polytechnic of Milano	On chemo-mechanical degradation phenomena in concrete
	Solids and Structures	M. Di Paola	University of Palermo	Fractional calculus in mechanics and dynamics
	Fluids	L. Brandt	KTH Stockolm	Numerical simulations of particle suspensions
	Machines	M. Guiggiani	University of Pisa	Transient vehicle dynamics
	Industrial	M. Debenedetti	Fincantieri	Research and innovation in shipbuilding: The approach of Fincantieri
	Industrial	G. Metta	Italian Institute of Technology	iCub: A research platform for robotics & AI
XXIII Salerno 2017	General Mechanics	C. Giorgi	University of Brescia	Global analysis of asymptotic behavior for infinite dimensional dynamic systems
	Solids and Structures	J.N. Reddy	Texas A&M University	Recent developments in shell finite elements and non-local theories for composite structures
	Solids and Structures	F. Vestroni	Sapienza University of Roma	Resonance phenomena in hysteretic systems
	Machines	V. Parenti Castelli	University of Bologna	Frontiers of machine mechanics
	Industrial	J. Pasfall	Fiberline Composites A/S	Perspective on market, barriers and examples of recent projects realized with GFRP

(continued)

Table 6 (continued)

Congress	Area	Author	Affiliation	Title
XXIV Roma 2019	Solids and Structures	C. Daraio	California Institute of Technology	Mechanics of robotic matter
	Solids and Structures	A. Pandolfi	Polytechnic of Milano	OTM: Combining optimal transportation theory and meshless discretization for the simulation of general solid and fluid flows
	Fluids	R. Di Leonardo	Sapienza University of Roma	The statistical and fluid mechanics of swimming bacteria
	Machines	M. Ruzzene	Georgia Institute of Technology	Metastructures for wave and vibration control: Internal resonances, edge states and quasi-periodicity
	Archeology	P. Carafa	Sapienza University of Roma	Construction techniques in the Rome of the Palatine
XXV Palermo 2022	Solids and Structures	R. Massabò	University of Genova	to be announced
	Solids and Structures	S. Reese	Aachen University	to be announced
	Fluids	D. Ohl	University of Twente	to be announced
	Machines	G. Diana	Polytechnic of Milano	to be announced

Table 7 Regular papers

Congress	General Mechanics	Solids and Structures	Fluids	Machines	Special Sessions	Mini-Symposia	Total
I Udine 1971	12	27	15	21			75
II Napoli 1974	25	46	25	39			135
III Cagliari 1976	18	35	29	15			97
IV Firenze 1978	23	33	14	21			91
V Palermo 1980	30	25	8	30			93
VI Genova 1982	32	67	29	55			183
VII Trieste 1984	23	57	20	58			158
VIII Torino 1986	22	37	32	51			142
IX Bari 1988	26	71	26	49			172
X Pisa 1990	14	63	29	61			167
XI Trento 1992	22	86	23	50			181
XII Napoli 1995	18	146	44	54			262
XIII Siena 1997	22	117	27	59			225
XIV Como 1999	16	120	38	61			235

(continued)

Table 7 (continued)

Congress	General Mechanics	Solids and Structures	Fluids	Machines	Special Sessions	Mini-Symposia	Total
XV Taormina 2001	12	84	20	56	66	55	293
XVI Ferrara 2003	12	73	23	70	152		330
XVII Firenze 2005	13	138	23	88		28	290
XVIII Brescia 2007	14	167	50	87	31		349
XIX Ancona 2009	12	162	29	57	99		359
XX Bologna 2011	20	148	27	67	150		412
XXI Torino 2013	18	75	27	27		167	314
XXII Genova 2015	8	30	30	16		259	343
XXIII Salerno 2017	9	28	17	22		303	379
XXIV Roma 2019	3	19	21	10		332	385
XXV Palermo 2022	?	?	?	?	?	?	?

Table 8 Special Sessions and Mini-Symposia, with specific organizers (if any)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
XV Taormina 2001	SS	Non-convex energies—G. Del Piero	9
		Fracture mechanics—A. Carpinteri	19
		Damage in composite materials—A. Corigliano	17
		Computational mechanics—U. Perego	21
	MS	Mechanics of tissues and implants—R. Contro, P.J. Prendergast	27
		Multifield theories in mechanics of materials—G. Capriz, P.M. Mariano	9
		Interaction problems in structural mechanics—W.S. Hall, G. Oliveto	19
XVI Ferrara 2003	SS	Dynamics of mechanical systems, linear and nonlinear dynamics, control and structural response—A. Sestieri, G. Muscolino	42
		Microstructures in elasticity and plasticity—G. Del Piero	15
		Fracture problems and interface problems in composite materials—A. Corigliano	21
		Inverse problems in the mechanics of structural materials—G. Maier, F. Vestroni	26
		Unusual thin structures—P. Podio Guidugli, A. Di Carlo	7
		Computational mechanics—U. Perego	41
XVII Firenze 2005	Structures	Finite elements	10
		Analysis and identification of damage and fracture	11
		Stability	5
		Composites	8
		Masonry	11
		FRP	5

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Dynamics and vibrations	4
		Dynamics	6
		Stochastic	6
	Solids	Dynamics and stability	6
		Nanostructures	4
		Constitutive laws	7
		Fracture mechanics	6
		Analytical solutions	7
		Finite elements and solids not resistant to traction	6
		Damage analysis and identification	5
	Fluids	Computational fluid dynamics	4
		Turbulence	7
		Biofluid-dynamics	5
	Machines	Gears	6
		Contact	6
		Multibody	4
		Robotics	11
		Vehicles	16
		Cams and lubrication	7
		Biomechanics and magnetic suspension	6
		Dynamics	6
		Simulation	9
		Analysis and control of vibrations	5
	MS	Aerodynamics of separate flows and squat bodies	6
		Stochastic mechanics in structural engineering applications	10
		Nanotechnologies: building up structures at the nano and meso-scales	12
XVIII Brescia 2007	General Mechanics	Numerical methods in dynamics	4
		Non-linear dynamics	5
	Structures	Instability of structures and solids	5
		Beam theory	6
		Structural analysis	12
		FRP	5
		Structural safety	4
		Structural dynamics	5
		Structural dynamics: beams and cables	4
		Structural dynamics: models	4
		Structural dynamics: identification	6
		Structural dynamics: moving loads	3
		Structural optimization	5

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Structural identification	4
		Structural diagnostics	5
		Seismic engineering	4
		Masonry	10
	Solids	Phase transformations and shape memory materials	5
		Biomechanics	6
		Plasticity, limit analysis, shakedown	6
		Rocks, soil, snow	6
		Elasticity	9
		Interfaces	5
		Multi-phase problems	4
		Saint-Venant solid	4
		Finite elements	4
		Boundary elements	10
		Models for rubber and polymeric materials	4
		Fracture mechanics	13
		Masonry	4
		Homogenization	6
	Fluids	Biofluid-dynamics	4
		Combustion and aeroacoustics	6
		Turbulence	5
		Vorticity	4
		Numerical simulations in turbulence	4
		Gas-dynamics	5
		Fluid mechanics: stability and fluid dynamics	4
		Aerodynamics of separate flows and squat bodies	3
		Hydraulics	9
		Applications of hydraulics and fluid-dynamics	6
	Machines	Articulated systems and cams	4
		Belt drives and machine dynamics	5
		Gears	6
		Rotor dynamics	7
		Biomechanics	7
		Robotics	17
		Mechanisms	5

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Tribology	10
		Railway vehicles	6
		Dynamics and fluids	4
		Vehicle dynamics	12
		MEMS	5
	SS	Functionally graded materials (FGM)	5
		Fluid-dynamics of real gases	4
		Miniaturized mini-robotic systems	6
		Joint session AIAS-AIMETA: Mechanics of composite materials	16
XIX Ancona 2009	Structures	Membranes, plates	6
		Finite elements, boundary elements	17
		Identification, control, optimization	6
		Elasto-plastic analysis, limit analysis, real cases	6
		Monodimensional continua	6
		FRP	9
		Instability and collapse	12
		Structural dynamics: models	9
		Masonry	11
		Non-linear dynamics	12
		Composites, laminates, FGM	11
	Solids	Interfaces	6
		Coupled and multiphase problems	6
		Plasticity and damage	17
		De St. Venant solid	3
		Models for rubber and polymeric materials	3
		Elasticity	12
		Fracture	5
		Homogenization	5
	Fluids	Complex fluids and heat transfer	5
		Free surface waves	5
		Numerical simulation techniques	5
		Vorticity and aerodynamics	3
		Turbulence	5
		Non-conventional numerical methods	6
	Machines	Vibrations	6
		Mechanisms	5
		Vehicles	5
		Biomechanics	6

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Drives and actuators	9
		Components	11
		Dynamics	3
		Robotics	12
	SS	Dynamical methods of experimental investigation—A. Morassi, F. Benedettini	26
		Biomechanics and biomaterials—L. Deseri	20
		Mechanics of materials and systems at micro and nano scales—C. Comi, L. Deseri	17
		Innovation and research as a support to industrial competitiveness—M. Callegari, P. Sermellini	15
		Variational models for fracture—M. Angelillo	10
		Control of flows—R. Donelli, P. Luchini	6
		The analytical approach to the two-dimensional dynamics of vorticity. Is it just an obsolete fact?—G. Riccardi	5
XX Bologna 2011	SS	Multiphase flows—A. Soldati, C.M. Casciola	6
		Inverse problems in mechanics of solids and structures—R. Fedele, A. Morassi	17
		Towards the assessment of quality and reliability of large-eddy simulations—M.V. Salvetti	8
		Micro- or nano-mechanics—A. Corigliano, N. Pugno	29
		Recent developments in the mechanics of masonry structures—L. Gambarotta, E. Sacco	34
		Biomechanics of the eye: experiments, theoretical and numerical modelling—M. Angelillo, A. Pandolfi, T. Rossi	11
		Mathematical contributions to the study of thin structures—R. Paroni, E. Zappale	17
		Models and methods for the nonlinear analysis of slender structures—R. Casciaro, F. Ubertini	11
		Structural joints, physical discontinuities and material interfaces: modeling, experimental and numerics—L. Contrafatto, F. Fraternali, N. Valoroso, G. Ventura	12
		Computational mechanics—M. Cuomo, F. Auricchio, F. Ubertini	5
XXI Torino 2013	MS	Fracture and structural integrity—G. Ferro, D. Firrao, M. Paggi, A. Spagnoli	10
		Advances in mechanics of materials—L. Deseri, R. Massabò, O. Vena, A. Corigliano	49
		Advanced methods for computational mechanics: beyond classical finite elements—F. Auricchio, J. Kiendl, A. Reali	11
		Computational biomechanics: applications to cardiovascular problems—F. Auricchio, M. Conti, S. Morganti, A. Reali	6
		Dynamics and control of the response of mechanical systems—W. D'Ambrogio, S. Casciati	23

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Dynamical systems, stability and bifurcation—A. Luongo, F. Tubino	23
		Smart and biological materials: mathematical models and applications—S. Carillo, M. Ciarletta	5
		Advanced beam models for homogeneous and non-homogeneous structures—A. Pagani, M. Petrolo, P.S. Valvo, E. Zappino	14
		Computational methods for shell structures—C. Chinosi, M. Cinefra, L. Della Croce, F. Tornabene	5
		Interdisciplinary problems in the physics of porous media: models, numerics and experiments for biomechanics to hydrogeology—A. Grillo	12
		Super-hydrophobic surfaces and heterogeneous nucleation processes—C.M. Casciola, G. Carbone	6
		Innovative computational methodologies in mechanics—C. Canuto	3
XXII Genova 2015	MS	GIMC: Recent advances in computational mechanics—S. Marfia, A. Pandolfi, A. Reali, G. Zavarise	30
		GADES: Dynamics and stability of mechanical systems—A. Luongo, S. Carillo, W. D'Ambrogio	59
		Advances in biomechanics: from basic research to applications—P. Bisegna, V. Parenti Castelli, G. Pedrizzetti	20
		Cellular mechanobiology and morphogenesis of living matter—D. Ambrosi, P. Ciarletta, L. Preziosi	12
		Masonry modeling: from theory to numerical and simplified approaches—D. Addessi, G. Milani, E. Sacco	31
		Mobile robotics—L. Bruzzone, G. Quaglia, G. Reina	9
		MEMS and NEMS: Models and analysis of micro- and nano-electro-mechanical systems—A. Corigliano, S. Lenci, A. Mariani	19
		GMA: Mechanics and materials 2015—L. Bardella, R. Massabò, P. Vena	37
		GMA Specialist session: Active soft materials—G. Noselli, A. Lucantonio	10
		GMA Specialist session: Non-local modeling of materials—A. Bacigalupo, F. Dal Corso, A. Piccolroaz	20
		GMA Specialist session: Materials for tissue engineering—F. Barberis, A. Lagazzo	5
		GMA Specialist session: Damage characterization in composite materials—V. Lopresto, A. Langella	7
XXIII Salerno 2017	MS	Theoretical and applied biomechanics for cardiovascular problems—M. Conti, M. Marano, G. Vairo, M. Zingales	12
		GADES: Dynamics and stability of mechanical systems—M.G. Naso, F. Pellicano, G. Piccardo	46
		Variational methods and applications in solid mechanics—G. Cricrì, E. Zappale	12
		Innovative lattice structures and materials—A. Favata, L. Feo, F. Fraternali, A. Micheletti, R.E. Skelton	24
		Mechanical behavior of masonry: modeling and numerical procedures—D. Addessi, E. Sacco	21

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Mechanical behavior of masonry: analysis of shell structures—F. Marmo, G. Milani, L. Rosati	17
		GIMC: New approaches in computational mechanics—S. Marfia, A. Pandolfi, A. Reali	22
		Fluid-structure interaction: methods and applications—F. Auteri, M.D. de Tullio, F. Giannetti	11
		Fracture: interface models and "phase-field" approaches—R. Alessi, F. Freddi, G. Lancioni, E. Sacco	31
		GMA: Mechanics and materials—L. Bardella, M. Paggi, P. Vena	16
		GMA: Soft active materials—G. Noselli, A. Lucantonio	9
		GMA: Mechanics of "green" composites: mechanical characterization and related technological aspects—F. Fabbrocino, P. Russo, F. Colangelo	16
		GMA: Recent advances in mechanical modeling of composite materials and periodic structures- A. Bacigalupo, F. Dal Corso, M.L. De Bellis, A. Piccolroaz	25
		GMA: Mechanics and reliability of piezoelectric materials—P.S. Valvo, M. Paggi	7
		Advanced and physically oriented numerical methods for continuous mechanics simulations—G. Coppola, M.D. de Tullio, F. Capuano	6
		GMS: Stochastic and probability computation approaches in mechanics—A. Pirrotta, L. Rosati, S. Sessa	11
		Coatings for tribological applications: modeling and characterization—G. Carbone, M. Di Donato, G. Favaro	7
		Extreme material mechanics: graphene, composites, metamaterials and biological /bioinspired materials—F. Bosia, M. Fraldi, N.M. Pugno	10
XXIV Roma 2019	MS	Interface models and phase-field approaches for fracture and damage mechanics—R. Alessi, M. Brunetti, F. Freddi, G. Lancioni, E. Sacco	22
		Composites in civil engineering—F. Ascione, V. Carvelli, P. Colombi, R.S. Olivito, G. Vairo	18
		Mechanics and materials (GMA) —L. Bardella, G. Noselli, M. Paggi	57
		Modelling and analysis of small-scale structures—R. Barretta, M. Fraldi, F. Marotti de Sciarra	10
		Theoretical and applied biomechanics (GBMA)—P. Bisegna, V. Parenti Castelli, G. Pedrizzetti, M.D. De Tullio, M. Marino, N. Sancisi, G. Vairo	28
		Shell and spatial structures—S. Gabriele, F. Marmo, A. Micheletti, V. Varano	12
		Vehicle dynamics—B. Lenzo, M. Guiggiani	13
		Novel approaches in computational mechanics (GIMC)—S. Marfia, A. Pandolfi, A. Reali	28
		Mechanics and geometry—P. Nardinocchi, R. Paroni	11
		Dynamics and stability of mechanical systems (GADeS)—F. Pellicano, G. Piccardo, M. Romeo	46
		Stochastic mechanics and probability in engineering—A. Pirrotta, A. Di Matteo, F.P. Pinnola	18

(continued)

Table 8 (continued)

Congress	Special Sessions (SS) and Mini-Symposia (MS)		Papers
		Recent advances and challenges in structural mechanics and engineering—L. Rosati, S. Sessa, N. Vaiana	18
		Hydrothermal ageing of natural fibre polymer composites—P. Russo, F. Nanni, F. Fabbrocino	6
		Masonry constructions: from material to structures, modelling and analysis approaches—E. Sacco, D. Addessi, F. Marmo, G. Milani	32
		Theoretical, numerical and physical modelling in geomechanics—C. Tamagnini, C. Jommi, A. Amorosi	13
XXV Palermo 2022	MS	Novel approaches in computational mechanics—S. Marfia, G. Garcea, S. de Miranda	?
		Theoretical and applied biomechanics GBMA—P. Bisegna, V. Parenti Castelli, G. Pedrizzetti, M.D. De Tullio, M. Marino, N. Sancisi, G. Vairo	?
		Masonry modelling and analysis: from material to structures—D. Addessi, G. Castellazzi, F. Clementi, G. Milani	?
		Dynamical systems and applications in civil and mechanical structures—F. D’Annibale, M. Ferretti, M. Romeo	?
		Control and experimental dynamics—F. Pellicano, G. Piccardo, D. Zulli	?
		Mechanical modelling of metamaterials and periodic structures—A. Bacigalupo, F. Dal Corso, M.L. De Bellis, A. Piccolroaz	?
		Novel stochastic dynamics methodologies & signal processing techniques for civil engineering applications—V. Gusella, A. Pirrotta	?
		Open issues on procedures and methodologies for the vibration-based monitoring and dynamic identification of historic constructions—M. Betti, G. Boscato, N. Cavalagli, A. Cecchi, F. Clementi	?
		Modeling and analysis of nanocomposites and small-scale structures—F.P. Pinnola, M.S. Vaccaro	?
		Reaction-diffusion-drift equations and gradient flows in mechanics and continuum physics—F. Davì, M. Paggi, A. Gizzi	?
		New frontiers in multibody systems vibration analysis—A. Cammarata, P.D. Maddio, F. Garesci, M. Cammalleri	?
		Mechanics of renewable energy systems—C. Baniotopoulos, C. Borri, C.L. Bottasso, L. Cappietti, E. Marino	?
		Advances in mathematical modeling and experimental techniques for quantification and prediction of fluid dynamic noise—V. Armenio, R. Camussi, M. Felli, M. Gennaretti	?
		Advanced process mechanics—G. Buffa, L. Filice, A. Ghiotti	?

Appendix 5: *Meccanica*: Special Issues

Looking at the list of Special Issues published in a journal allows to follow the evolution of both the editorial means preferentially used to disseminate scientific knowledge/results and the topics of major interest by a given community of scientists. Indeed, in the last few decades, publication of a number of Special Issues per year has become a general editorial policy to attract scientists' interest to a given journal, also in connection with the explosion of both journals and conferences, as well as the right occasion to draw new research lines and/or advancements in well-established areas.

This is apparent also in the case of *Meccanica* (Table 9). In about the first half of its more than fifty years life, very few Special Issues were published and nearly only for celebration and anniversary purposes (such as the retirement, birthday or memory of recognized scholars). Since around the turn of the millennium, the trend has modified radically, with Special Issues being published nearly regularly, and independent of solely episodical celebration purposes, also in connection with the greater visibility secured by an important international publisher. A circumstance which also entails the editorial involvement of qualified international scientists active in different fields, sometimes in collaboration with Italian ones. Anyway, the also independent presence of the latter witnesses the vitality of the Italian school of mechanics, already apparent in the earlier publication stages of *Meccanica*, which persists, and indeed increases, in accordance with the high position it generally occupies in qualified international rankings, independent of possible opinions about their overwhelming significance and pervasiveness. Indeed, in a considerable number of meaningful cases, new research lines (concerned, among others, with micro/nano-scale levels, multiphysics, nonlinear and control aspects, novel materials, more clearly identified technological applications) are being effectively documented, along with important advancements in well-established areas. This occurs despite some proliferation of associated wording (recent advances/progress, new trends, etc.) which may prevent the reader from clearly catching the stage of development of the considered research area along its overall parabola.

See Table 9.

Table 9 Special Issues of *Meccanica*

Title	Guest Editors	Vol (issue)	Year	No. papers
Recent advances in computational mechanics and innovative materials (*for the 75th birthday of J.N. Reddy*)	G.H. Paulino, E. Sacco	56(6)	2021	18
Modelling and numerical/experimental investigation of dynamical phenomena in mechanical systems	J. Awrejcewicz, M. Amabili, A. Nabarette	56(4)		16
Recent advances in nonlinear dynamics and vibrations	P. Perlikowski, J. Warminski, S. Lenci	55(12)	2020	18
Computational models for 'complex' materials and structures, beyond the finite elements	P. Trovalusci, F. Cui	55(4)		20
Recent advances in modeling and simulations of multiphase flows	F. Picano, O. Tammisola, L. Brandt	55(2)		9
Mechanics of extreme materials	F. Bosia, M. Fraldi, N.M. Pugno	54(13)	2019	10
Stochastics and probability in engineering mechanics	C. Bucher, A. Pirrotta, C. Proppe, L. Rosati	54(9)		13
Progress in mechanics of soils and general granular flows	P. Giovine, P.M. Mariano, G. Mortara, K. Soga	54(4–5)		11
New trends in mechanics of masonry	E. Sacco, D. Addessi, K. Sab	53(7)	2018	20
Novel computational approaches to old and new problems in mechanics	S. Marfia, A. Pandolfi, A. Reali	53(6)		17
Recent advances on the mechanics of materials	L. Bardella, M. Paggi, P. Vena	53(3)		10
Active behavior in soft matter and mechanobiology	A. De Simone, G. Noselli, A. Lucantonio, P. Ciarletta	52(14)	2017	16
New trends in dynamics and stability	S. Carillo, W. D'Ambrogio	52(13)		18
Advances in biomechanics: from foundations to applications	P. Bisegna, V. Parenti Castelli, G. Pedrizzetti	52(3)		19
50th Anniversary of Meccanica	L. Gambarotta	51(12)	2016	22
Nonlinear dynamics, identification and monitoring of structures (*in memory of Francesco Benedettini*)	A. Luongo, G. Rega	51(11)		22
Recent progress and novel applications of parallel mechanisms	A. Müller, V. Parenti Castelli, T. Huang	51(7)		9

(continued)

Table 9 (continued)

Title	Guest Editors	Vol (issue)	Year	No. papers
Computational micromechanics of materials	J. Segurado, T. Sadowski, J. Llorca, S. Schmauder	51(2)		17
Soft mechatronics	G. Berselli, X. Tan, R. Vertechy	50(11)	2015	15
Advances in the mechanics of composite and sandwich structures (*for the 65th birthday of Marco Di Sciuva*)	S. Abrate, M. Gherlone, R. Massabò	50(10)		10
Advances in dynamics, stability and control of mechanical systems	A. Luongo	50(3)		21
Experimental solid mechanics (*for the 65th birthday of Emmanuel E. Gdoutos*)	A.N. Kounadis	50(2)		24
Multi-scale and multi-physics modelling for complex materials	T. Sadowski, P. Trovalusci, B. Schrefler, R. de Borst	49(11)	2014	12
New trends in fluid and solid mechanical models	M. Fabrizio	49(9)		22
Nonlinear dynamics and control of composites for smart engineering design	S. Lenci, J. Warminski	49(8)		18
Micro- or nano-mechanics	A. Corigliano, N.M. Pugno	48(8)	2013	20
Asperity contacts and lubrication aspects	E. Ciulli, F. Franek	46(3)	2011	13
Fundamental issues and new trends in parallel mechanisms and manipulators	C. Gosselin, V. Parenti Castelli, F. Pierrot	46(1)		21
Simulation, optimization and identification	J.L. Zapico Valle, M.P. González Martínez	45(5)	2010	9
Recent advances in experimental and theoretical analysis of stress and strain	D. Amodio, S. Lenci	43(2)	2008	14
Advanced problems in mechanics	M. Boltežar, M. Wiercigroch, D. Indeitsev	41(3)	2006	9
Mechanics from nano to macroscopic level	G. Capriz, P.M. Mariano	40(4–6)	2005	11
Dynamical systems: theory and applications	J. Awrejcewicz	38(6)	2003	14
(*for the 70th birthday of Piero Villaggio*)	G. Augusti, S. Bennati	38(5)		10
Control and condition monitoring of engineering systems	M. Wiercigroch, A.A. Rodger	38(2)		8
Nonlinear dynamics of mechanical systems	M. Wiercigroch, E. Kreuzer, T. Kapitaniak	38(1)		12

(continued)

Table 9 (continued)

Title	Guest Editors	Vol (issue)	Year	No. papers
Mechanics of tissues and tissue implants	P.J. Prendergast, R. Contro	37(4–5)	2002	15
Stochastic dynamics of non-linear mechanical systems	M. Di Paola	37(1–2)		13
Topics in tribology	R. Bassani	36(6)	2001	14
Boundary element methods in soil-structure interaction	W.S. Hall, G. Oliveto	36(4)		10
(*for the 70th birthday of Giulio Maier*)	L. Corradi, G. Novati, U. Perego	36(1)		9
(*in memory of Carlo Ferrari*)	S. Nocilla	33(5)	1998	7
Nonlinear and random dynamics	G. Solari	33(3)		11
Control and diagnostics in automotive applications	A. Gambarotta	32(5)	1997	9
Transform methods in solid mechanics	H. Grundmann, A. Liolios	32(3)		10
Thermodynamics of continua (*for the 70th birthday of Gianfranco Capriz*)	G. Augusti	31(5)	1996	13
Bifurcation and chaos in solid and structural dynamics	G. Rega, F. Pfeiffer	31(3)		8
Hydrometeorology	L. Ubertini	31(1)		9
Microstructures and phase transitions in solids (*for the 70th birthday of J.L. Ericksen*)	C. Davini, M. Pitteri	30(5)	1995	17
Dynamics and geometry of vortical structures	P. Orlandi, E. Hopfinger	29(4)	1994	15
(*in memory of Giorgio Sestini*)	A. Fasano, M. Primicerio	28(2)	1993	10
Masonry construction: structural mechanics and other aspects (*for the retirement of Jacques Heyman*)	C.R. Calladine	27(3)	1992	8
Progress of the structural analysis problem since Castigliano *(for the death centenary of Alberto Castigliano)*	P. Cicala, F. Levi, U. Rossetti	19(1)	1984	11
Presentations from the 1st AIMETA National Congress	D. Galletto, M. Capurso, A. Capello	7(1)	1972	3+ summ.
(*for the retirement of Bruno Finzi*)	E. Massa, G. Supino	5(1)	1970	7

Appendix 6: AIMETA Groups (current)

See Tables 10, 11, 12, 13, 14, 15, and 16.

Table 10 Italian Group of Computational Mechanics (GIMC). Since 1984, founder A. Cannarozzi

Executive committee	S. Marfia	University of Roma Tre
	S. de Miranda	University of Bologna
	G. Garcea	University of Calabria
Meetings	2018	XXII Italian Congress of Computational Mechanics—IX GMA Meeting, University of Ferrara
	2016	XXI Italian Congress of Computational Mechanics—VIII GMA Meeting, IMT School for Advanced Studies, Lucca
	2014	XX Italian Congress of Computational Mechanics—VII GMA Meeting, University of Cassino
	2012	XIX Italian Congress of Computational Mechanics, Rossano
	2010	XVIII Congress of Computational Mechanics, Siracusa
	2008	XVII Congress of Computational Mechanics, Sassari
	2006	XVI Congress of Computational Mechanics, Bologna
	2004	XV Congress of Computational Mechanics, Genova
	2002	XIV Congress of Computational Mechanics—III Joint Conference with Ibero-Latin Association of Computational Methods in Engineering, Giulianova
	2001	I CSMA-GIMC Joint Workshop, Cefalù
	2000	XIII Congress of Computational Mechanics, Brescia
	1999	XII Congress of Computational Mechanics, Napoli
	1998	XI Congress of Computational Mechanics, Trento
	1996	X Congress of Computational Mechanics—I Joint Conference with Ibero-Latin Association of Computational Methods in Engineering, Padova
	1995	IX Congress of Computational Mechanics, Catania
	1994	VIII Congress of Computational Mechanics, Torino
	1993	VII Congress of Computational Mechanics, Trieste
	1991	VI Congress of Computational Mechanics, Brescia
	1990	V Congress of Computational Mechanics, Cosenza
	1989	IV Congress of Computational Mechanics, Padova
	1988	III Congress of Computational Mechanics, Palermo
	1987	II Congress of Computational Mechanics, Roma
	1986	I Congress of Computational Mechanics, Milano

(continued)

Table 10 (continued)

Awards for best Ph.D. thesis	2019	S. Meduri
	2019	V. Diana
	2018	E. Gaburro
	2018	D. Magisano
	2017	F. Fambri
	2017	P. Di Re
	2016	N. Nodargi
	2016	A. Montanino
	2015	D. Grazioli
	2015	W. Boschieri
	2014	G. Scalet
	2013	R. Dimitri

Table 11 AIMETA Group of Stochastic Mechanics (GAMS). Since 1987, founder F. Casciati

Executive committee	A. Pirrotta	University of Palermo
Meetings	2016	Stochastic Mechanics'16, Capri, Napoli
	2012	Stochastic Mechanics'12, Ustica, Palermo
	2008	Stochastic Mechanics'08, Cefalù, Palermo
	2004	Stochastic Mechanics'04, Pantelleria, Trapani
	1998	Stochastic Mechanics'98, Lampedusa, Agrigento
	1993	Stochastic Mechanics'93, Taormina, Messina

Table 12 AIMETA Group of Tribology (GAIT). Since 1988, founder R. Bassani

Executive committee	A. Ruggiero	University of Salerno
	L. Mattei	University of Pisa
	F. Colombo	Polytechnic of Torino
Meetings	2019	7th ECOTRIB, Vienna, Austria
	2017	6th ECOTRIB, Ljubljana, Slovenia
	2015	5th ECOTRIB, Lugano, Switzerland
	2013	4th ECOTRIB, within the 5th World Tribology Congress (WTC), Torino
	2011	3rd ECOTRIB, Vienna, Austria
	2009	2nd ECOTRIB, Pisa
	2007	1st ECOTRIB-European Conference on Tribology, Ljubljana, Slovenia

(continued)

Table 12 (continued)

	2006	5th AITC, Parma
	2004	4th AITC, Roma
	2002	3rd AITC, Vietri sul Mare
	2001	2nd AITC, within the World Tribology Congress (WTC), Vienna, Austria
	2000	1st AITC-AIMETA International Tribology Conference, L'Aquila
	1998	5th AIMETA Tribology Conference, Varenna
	1996	4th AIMETA Tribology Conference, Santa Margherita Ligure
	1994	3rd AIMETA Tribology Conference, Capri
	1993	2nd AIMETA Tribology Conference, Firenze
	1991	1st AIMETA Tribology Conference, Parma

Table 13 Mechanics of Materials (GMA). Since 1994, founder G. Del Piero

Executive committee	L. Bardella	University of Brescia
	M. Paggi	IMT School for Advanced Studies, Lucca
	G. Noselli	SISSA—International School for Advanced Studies, Trieste
Meetings (recent)	2019	GMA 2019—within AIMETA 2019, Sapienza University of Roma
	2018	GMA 2018—within GIMC-GMA 2018, University of Ferrara
	2017	GMA 2017—within AIMETA 2017, University of Salerno
	2016	GMA 2016—within GIMC-GMA 2016, IMT School for Advanced Studies, Lucca
	2015	GMA 2015—within AIMETA 2015, University of Genova
	2014	GMA 2014—within GIMC-GMA 2014, University of Cassino
	2013	GMA 2013—within AIMETA 2013, Polytechnic of Torino
	2012	GMA 2012—Fondazione Campus of Lucca
	2011	GMA 2011—University of Udine
	2010	GMA 2010—University of Palermo
	2009	GMA 09—Polytechnic of Milano
	2008	GMA 08—University of Genova
	2007	GMA 07—University of Trento
Awards for best Ph.D. thesis	2021	D. Agostinelli
	2020	F. Recrosi
	2019	A.S. Barjoui
	2018	P. Lenarda
	2017	E. Cattarinuzzi

Table 14 AIMETA BioMechanics Group (GBMA). Since 1995, founders A. Vallatta, R. Contro

Executive committee	P. Bisegna	University of Roma Tor Vergata
	V. Parenti Castelli	University of Bologna
	G. Pedrizzetti	University of Trieste
Meetings	2019	GBMA Minisymposium "Theoretical and Applied Biomechanics"- within AIMETA 2019, Sapienza University of Roma
	2017	Minisymposium "Theoretical and Applied Biomechanics for Cardiovascular Problems"—within AIMETA 2017, University of Salerno
	2015	Minisymposium "Progresses in Biomechanics: From Fundamentals to Applications"—within AIMETA 2015, University of Genova
Awards for best Ph.D. thesis	2020	F. Regazzoni
	2019	S. Palumbo

Table 15 Kinematics and Dynamics of Multibody Systems (CDSM). Since 2003, founder E. Pennestrì

Executive committee	F. Cheli	Polytechnic of Milano
	D. Guida	University of Salerno
	P. Masarati	Polytechnic of Milano
	C.M. Pappalardo	University of Salerno
	E. Pennestrì	University of Roma Tor Vergata
	A. Tasora	University of Parma
	P. P. Valentini	University of Roma Tor Vergata
	F. Cheli	Polytechnic of Milano
Meetings	2021	3rd International Multibody System Dynamics Workshop & Summer School (online)
	2019	2nd International Multibody Summer School, Parma
	2016	1st Italian Multibody Summer School, Parma

Table 16 AIMETA Group of Dynamics and Stability (GADeS). Since 2012, founder A. Luongo

Executive committee	G. Piccardo	University of Genova
	F. Pellicano	University of Modena and Reggio Emilia
	M. Romeo	University of Genova
Meetings	2019	GADeS 2019—within AIMETA 2019, Sapienza University of Roma
	2018	GADeS 2018—University of Cagliari
	2017	GADeS 2017—within AIMETA 2017, University of Salerno
	2016	GADeS 2016—University of Brescia
	2015	GADeS 2015—within AIMETA 2015, University of Genova
	2014	GADeS 2014—University of Firenze
	2013	GADeS 2013—within AIMETA 2013, Polytechnic of Torino
	2012	GADeS 2012—Sapienza University of Roma
Awards for best Ph.D. thesis	2018	A. Di Matteo
	2016	V. Settimi

Appendix 7: AIMETA Junior Prizes

See Table 17.

Table 17 AIMETA Junior Prizes

Year	Committee	Area	Awardees	Area
2009	M. Pandolfi	President-AIMETA Representative	L. Fusi	General Mechanics
	F. Pastrone	General Mechanics	S. Camarri	Fluids
	E. Virga	General Mechanics	M. Gabiccini	Machines
	P. Luchini	Fluids	L. Bardella	Solids
	A. Sestieri	Machines	S. Vidoli	Structures
	F. Vatta	Machines		
	A. Corigliano	Solids and Structures		
	G. Del Piero	Solids and Structures		
	L. Gambarotta	Solids and Structures		
	G. Muscolino	Solids and Structures		
	E. Sacco	Solids and Structures		
2011	N. Bellomo	General Mechanics	G. Napoli	General Mechanics
	M. Fabrizio	General Mechanics	F. Picano	Fluids

(continued)

Table 17 (continued)

Year	Committee	Area	Awardees	Area
	R. Piva	Fluids	M. Carricato	Machines
	E. Pennestrì	Machines	G. Tomassetti	Solids
	C.U. Galletti	Machines	L. De Lorenzis	Structures
	S. Bennati	Solids and Structures		
	D. Bigoni	Solids and Structures		
	D. Bruno	Solids and Structures		
	A. Carpinteri	Solids and Structures		
	A. Luongo	Solids and Structures		
2013	G. Rega	President-AIMETA Representative	L. Vergori	General Mechanics
	G. Caricato	General Mechanics	M. Dona	Fluids
	G. Spiga	General Mechanics	A. Artoni	Machines
	D. Boffi	General Mechanics	F. Dal Corso	Solids and Structures
	F. Bassi	Fluids	A. Reali	Solids and Structures
	M.V. Salvetti	Fluids		
	F. Sorge	Machines		
	P. Fanghella	Machines		
	P. Bisegna	Solids and Structures		
	A. Morassi	Solids and Structures		
	L. Rosati	Solids and Structures		
	F. Ubertini	Solids and Structures		
	M. Cuomo	Solids and Structures		
2015	S. Lenci	President-AIMETA Representative	I. Bochicchio	General Mechanics
	M. Fabrizio	General Mechanics	S. Cherubini	Fluids
	G. Frosali	General Mechanics	F. Farroni	Machines
	G. Buresti	Fluids	M. Bruggi	Solids
	G. Iuso	Fluids	G. Noselli	Structures
	M. Carricato	Machines		
	B. Allotta	Machines		
	C. Comi	Solids and Structures		
	F. Davì	Solids and Structures		
	G. Giambanco	Solids and Structures		
	A. Paolone	Solids and Structures		
2017	C. Cinquini	President-AIMETA Representative	A. Giacomello	Fluids
	D. Andreucci	General Mechanics	M. Scaraggi	Machines
	M. G. Naso	General Mechanics	A. Amendola	Solids and Structures

(continued)

Table 17 (continued)

Year	Committee	Area	Awardees	Area
	M. Quadrio	Fluids	D. Misseroni	Solids and Structures
	A. Soldati	Fluids		
	E. Ciulli	Machines		
	E. Sabbioni	Machines		
	S. Caddemi	Solids and Structures		
	G. Royer Carfagni	Solids and Structures		
	R. Massabò	Solids and Structures		
	P. Trovalusci	Solids and Structures		
2019	C.M. Casciola	President-AIMETA Representative	I. Carlomagno	General Mechanics
	S. Carillo	General Mechanics	V. Citro	Fluids
	M. Romeo	General Mechanics	C. Putignano	Machines
	R. Verzicco	Fluids	M. Cremonesi	Solids and Structures
	E. Campana	Fluids	M. Marino	Solids and Structures
	A. Collina	Machines		
	A. Trivella	Machines		
	F. Greco	Solids and Structures		
	V. Gusella	Solids and Structures		
	A. Corigliano	Solids and Structures		
	G. Borino	Solids and Structures		

Appendix 8: CISM-AIMETA Advanced Schools

See Table 18.

Table 18 CISM-AIMETA Advanced Schools

Year	Title/Coordinators	Lecturers	Affiliations
2014	Shell-like Structures: Advanced Theories and Applications H. Altenbach V.A. Eremeyev	H. Altenbach	Otto-von-Guericke University, Magdeburg, Germany
		V. A. Eremeyev	Rzeszów University of Technology, Poland
		G. Mikhasev	Belarusian State University, Minsk, Belarus
		P. Podio-Guidugli	Accademia Nazionale dei Lincei, Rome, Italy
		K. Sab	Université Paris-Est, France

(continued)

Table 18 (continued)

Year	Title/Coordinators	Lecturers	Affiliations
		K. Wisniewski	IFTR, Polish Academy of Sciences, Poland
2015	The Art of Modeling Mechanical Systems F. Pfeiffer H. Bremer	H. Bremer	Johannes Kepler Universität Linz, Austria
		F. Pfeiffer	Technical University of Munich, Germany
		M. Raous	Lab. de Mécanique et d'Acoustique, Marseille, France
		A. Shabana	University of Illinois at Chicago, USA
		S. Shaw	Michigan State University, East Lansing, USA
		P. Wriggers	Leibniz Universität Hannover, Germany
2016	Global Nonlinear Dynamics for Engineering Design and System Safety G. Rega S. Lenci	P. B. Goncalves	PUC-Rio, Rio de Janeiro, Brazil
		S. Lenci	Polytechnic University of Marche, Italy
		G. Rega	Sapienza University of Roma, Italy
		J.-Q. Sun	University of California, Merced, USA
		M. Thompson	University of Cambridge, UK
		M. Younis	KAUST, Tuval, Saudi Arabia and Binghamton University, NY, USA
2017	Dynamic Stability and Bifurcation in Nonconservative Mechanics D. Bigoni O. Kirillov	D. Bigoni	University of Trento, Italy
		O. Doaré	ENSTA Paris Tech, France
		E. Hemingway	University of California, Berkeley, USA
		O. Kirillov	Russian Academy of Sciences, Moscow, Russia
		A. Metrikine	Delft University of Technology, The Netherlands
		A. Ruina	Cornell University, Ithaca, USA
2018	Cell Mechanobiology: Theory and Experiments on the Mechanics of Life A. De Simone V. Deshpande	C. Bouten	TU Eindhoven, The Netherlands
		V. Deshpande	University of Cambridge, UK
		A. De Simone	SISSA, Trieste, Italy
		D. E. Discher	University of Pennsylvania, USA

(continued)

Table 18 (continued)

Year	Title/Coordinators	Lecturers	Affiliations
		J.M. Garcia-Aznar	University of Zaragoza, Spain
		R. M. McMeeking	University of California at Santa Barbara, USA
		P. Recho	LIPhy, Grenoble, France
		U. Schwarz	University of Heidelberg, Germany
2019	Anisotropic Particles in Viscous and Turbulent Flows C. Marchioli G. Verhille	J. Butler	University of Florida, Gainesville, USA
		E. Guazzelli	Université de Paris, France
		C. Marchioli	University of Udine, Italy
		F. Picano	University of Padova, Italy
		A. Pumir	ENS de Lyon, France
		G. Verhille	Aix-Marseille University, CNRS, France
		G. Voth	Wesleyan University, Middletown, USA
2020 + 2	Exploiting the Use of Strong Nonlinearity in Dynamics and Acoustics O. Gendelman A.F. Vakakis	B. Cochelin	LMA, CNRS, France
		O. Gendelman	Technion, Haifa, Israel
		G. Kerschen	University of Liege, Belgium
		M. Krack	University of Stuttgart, Germany
		G. Rega	Sapienza University of Roma, Italy
		A. Vakakis	University of Illinois at Urbana-Champaign, USA
		F. Vestroni	Sapienza University of Roma, Italy

Appendix 9: IUTAM General Assembly

See Table 19.

Table 19 Italian representatives

Name	Area	Years	Total
Bianchi G	Machines	1976–2001	26
Bigoni D	Solids/Structures	2015–2021	7
Bottaro A	Fluids	2015–2021	7
Campana E F	Fluids	2015–2018	4
Cercignani C	Fluids	1984–2006	23
Colonnetti O	General Mechanics	1948–1967	20
Crocco L	Fluids	1948–1970	23
De Bernardis E	General Mechanics	2019–2021	3
Ferrari C	Fluids	1970–1975	6
Finzi B	General Mechanics	1966–1969	4
Galletto D	General Mechanics	1976–1993	18
Maier G	Solids/Structures	1976–1983	30
		2000–2021[a]	
Morro A	General Mechanics	2007–2014	8
Napolitano L G	Fluids	1976–1983	8
Podio Guidugli P	Solids/Structures	1994–2014	21
Polizzotto C	Solids/Structures	1987–1997	11
Rega G	Solids/Structures	2009–2021	13
Schrefler B A	Solids/Structures	2012–2014	3
Sestini G	General Mechanics	1973–1975	3
Sobrero L	General Mechanics	1971–1977	7
Supino G	Fluids	1970–1972	3
Vatta F	Machines	1995–2008	14
Villaggio P	Solids/Structures	1984–1986	3

[a] Member-at-Large since 2012

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Recognized Italian Scholars of Mechanics

Giuseppe Rega

Abstract The history of AIMETA has been made up by the scholars of mechanics who have engaged in it, putting their scientific personality and achievements into play, and contributing in a non-trivial way to its recognition as the reference Italian association in the field of mechanical sciences. This contribution is a tribute paid to earlier recognized Italian scholars of mechanics who were involved in a somehow meaningful way with AIMETA in about the last fifty years. The reported list, based on the commemorations published in the AIMETA journal *Meccanica*, does not claim to be comprehensive of all scholars who would have deserved to be included, nor is intended to present the 'best' scholars of Italian mechanics in the considered interval of time.

Keywords 20th century Italian scholars of mechanics · General mechanics · Solids · Structures · Fluids · Machines

1 Introduction

According to the historian of sciences Steven Shapin, quoted in the 'Selected Biographies of Mechanicians' in Maugin's recent book on the twentieth century history of continuum mechanics [1], "the very idea of paying homage to the great scientists of the past is problematic …………, and notions of an impersonal scientific method, which have gained classical dominance over ideas of scientific genius, make the personalities of scientists irrelevant". This should prevent since the beginning from trying to draw up a list of recognized Italian scholars, even more so if the reference period concerns the last half century, or a little more.

However, the history of a scientific society is made up of the scholars who have engaged in it, putting their scientific personality and achievements into play, and contributing in a non-trivial way to the recognition of the same as the reference association in a given field of science. This is just the case of AIMETA, whose long

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history reflects the scientific contribution given in about the last fifty years by many recognized Italian scholars of mechanics. Thus, the list below has to be considered as a tribute paid by the relevant Italian community to scientists who were involved with the association in a somehow meaningful way.

In this perspective, and in accordance with the preliminary choice not to include living people in order not to touch their personality and ego due to inevitable selections, the list is rigorously based on the commemorations published over the years in the AIMETA journal *Meccanica*, sometimes integrated with information available on Wikipedia, with only few exceptions relating to scholars anyway strongly involved with the association. As such, the list certainly does not claim to be comprehensive of all Italian scholars who would have deserved to be included, nor it aims at presenting the listed ones as the somehow 'best' Italian scholars of mechanics in the considered interval of time. This being obviously a circumstance for which the writer sincerely apologizes.

For each of the scholars listed below with the respective area of reference, a brief scientific profile is provided in the sequel. In compiling them, texts of the commemorations published in *Meccanica* or elsewhere (in a few cases) have been suitably rearranged. However, for the sake of an easier presentation, sentences/paragraphs taken from commemorations are not explicitly highlighted in quotation marks. Anyway, for each profile, relevant bibliografic references are given

Bruno Finzi (1899–1974)	General Mechanics
Giulio Supino (1898–1978)	Fluids
Giuseppe Colombo (1920–1984)	Machines
Riccardo F. Baldacci (1917–1986)	Solids and Structures
Pietro Caparrini (1923–1986)	Machines
Luigi Crocco (1909–1986)	Fluids
Michele Capurso (1935–1987)	Solids and Structures
Cataldo Agostinelli (1894–1988)	General Mechanics
Gianni Jarre (1924–1988)	Fluids
Vincenzo Franciosi (1925–1989)	Solids and Structures
Dario Graffi (1905–1990)	General Mechanics
Luigi Gerardo Napolitano (1928–1991)	Fluids
Giorgio Sestini (1908–1991)	General Mechanics
Placido Cicala (1910–1996)	Solids and Structures
Ennio De Giorgi (1928–1996)	General Mechanics
Carlo Ferrari (1903–1996)	Fluids
Gaetano Fichera (1922–1996)	General Mechanics
Edoardo Benvenuto (1940–1998)	Solids and Structures
Lucio Lazzarino (1913–1998)	Machines
Emilio Massa (1926–1998)	Machines
Leo Finzi (1924–2002)	Solids and Structures

(continued)

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Giovanni Bianchi (1924–2003)	Machines
Giulio Ceradini (1918–2005)	Solids and Structures
Alfredo Corsanego (1936–2008)	Solids and Structures
Carlo Cercignani (1939–2010)	General Mechanics/Fluids
Piero Villaggio (1932–2014)	Solids and Structures
Francesco Benedettini (1956–2015)	Solids and Structures
Giuseppe Grioli (1912–2015)	General Mechanics
Raffaele Casciaro (1943–2020)	Solids and Structures
Gianpietro Del Piero (1940–2020)	Solids and Structures
Giovanni Solari (1953–2020)	Solids and Structures

2 Scientific Profiles

Bruno Finzi (1899–1974) [2]

Chair of Rational Mechanics at the University of Milano (1931) and then at the Polytechnic of Milano (1946).

Contributed general theorems and specific solutions to the theory of elasticity and plasticity, as well as to hydro and aerodynamics, including the general integral of the equilibrium equations of continua. Interested also in wave groups and particle showers, presented in variational form many general issues of mechanics, electricity and magnetism, relativity.

One of the founders and first President of AIMETA, was member of the Accademia Nazionale dei Lincei, which awarded him the Feltrinelli Prize for Mechanics and its applications (1958), of the Istituto Lombardo di Scienze e Lettere, the Accademia delle Scienze di Torino, the Accademia delle Scienze di Bologna, the Accademia Nazionale di Modena. Was in the editorial committees of several scientific journals, including *Il Nuovo Cimento*, the *Bulletin of the Unione Matematica Italiana*, the *Annali di Matematica*, the *International Journal of Engineering Science*.

Rector of the Polytechnic of Milan (1967–69).

Giulio Supino (1898–1978) [3]

Chair of Hydraulics at the University of Bologna (since 1946), upon teaching various courses before the second world war and abandoning the university due to fascist racial laws.

Contributed theoretical results (theorems on the dynamics of viscous fluids, demonstration of Saint Venant principle in convex fields, extension of the method of the volume of stemming in hydrographic systems), always with a strong flair for their technical applications (dam and acqueduct projects).

Dean of the Faculty of Engineering, was one of the founders of AIMETA, which was also President of, and of the Unione Matematica Italiana.

Member of the Accademia Nazionale dei Lincei, the Accademia delle Scienze di Bologna, the Accademia delle Scienze di Torino, the Accademia dei Georgofili di Firenze, received the Honorary Degree from the Technische Hochschule of Munich.

Took part in a number of public works, including the Supreme Council for Public Works, the interministerial Commission for the regularization of the Arno river in Florence, and was President of the Technical and Scientific Committee for the preservation of Venice.

Giuseppe Colombo (1920–1984) [4]

Chair of Applied Mechanics (1955), and then of Space Engineering, at the University of Padova.

Upon adding modern developments of celestial mechanics and the completely new subject of spacecraft dynamics to his first field of research in theoretical mechanics, from 1960 was research consultant at the Harvard Smithsonian Astrophysical Observatory at Cambridge, MA. From 1970 collaborated with the Jet Propulsion Laboratory of Pasadena, CA, in the study of NASA projects and research on the structure of the solar system, and with various Italian and European Space Administrations.

Member of the Accademia Nazionale dei Lincei, the Accademia dei XL, the Accademia Pontificia delle Scienze, the American Academy of Arts and Sciencies, and the Academies of Torino, Venezia e Padova, was also recipient of prestigious awards: the Feltrinelli Prize for Astronomy (1971), the Prize of the City of Columbus (Ohio) for Space Technology (1976), and the NASA Gold Medal for exceptional scientific achievements (1983).

Some of his results earned him the particular recognition of scientific circles all over the world: the discovery of the coupling between rotational and orbital movements of Mercury; the plotting of the orbit of the Earth-Venus-Mercury mission, which made it possible to pass close to Mercury three times; the design of the satellite connected to the shuttle or space station by cables hundred kilometer long (later known as ‘tethered system’) to study the properties of the upper atmosphere; gravitational phenomena and the propagation of disturbances in the earth’s magnetic field.

Riccardo F. Baldacci (1917–1986) [5]

Chair of Scienza delle Costruzioni[1] at the University of Genova (1955).

More relevant contributions concern a rigorous revision and improvement of basic concepts and usual treatments of the engineering application of elasticity, of the technical theory of stability, and of other fields of structural mechanics, starting from nonlinear mechanics, by means of advanced mathematical methods. A large

[1] Scienza delle Costruzioni’ (Solid and Structural Mechanics) is the classical name of the fundamental discipline encompassing topics of continuum mechanics, elasticity, mathematical-physics, strength of materials, and theory of structures, taught in Italy to students in the Schools of Engineering and Architecture.

culture allowed him to reach important and sometimes fundamental results, and to exert a decisive influence on the development of the Italian researches on strength of materials and theory of structures. Succeeded in joining a deep knowledge of theoretical mechanics with a wide practice of structural engineering.

His treatise Scienza delle Costruzioni has been a necessary reference for students and scholars in this subject, for decades.

Dean of the Faculty of Engineering (1973–79), was also a member of the Accademia delle Scienze di Genova and the Accademia dei XL.

Pietro Caparrini (1923–1986) [6]

Chairs of Agricultural Engineering with Design Applications at the University of Catania (1965), Agricultural Mechanics and then Applied Mechanics at the University of Calabria (1974), Mechanics of Machines and Machines at the University of Firenze (1979).

Contributed research in agricultural mechanization, and technical and economic aspects of driving and operating machines, also from the point of view of technology and plant organization.

Established scientific institutes and departments in different universities and was active at various academic levels.

Luigi Crocco (1909–1986) [7]

Professor of Aviation Engines at the University of Roma (1939); Chair of Jet Propulsion at Princeton University (1949); Professor at École Centrale, Paris (1970).

Research in the area of aerodynamics and gasdynamics, with fundamental contributions to the explanation and solution of high frequency combustion instabilities; NASA called him to solve these problems in the Saturn rocket which brought man on the Moon in 1969.

Member of the Accademia Nazionale dei Lincei, the Accademia delle Scienze di Torino, the International Academy of Astronautics, Paris, the National Academy of Engineering, Washington, the Académie Nationale de l'Air et de l'Éspace, Toulouse, and in the Honorary Board of *Meccanica.*

Was awarded the following prizes: Pendray Award (1965); Wilde Award (1969); Columbus International Prize and Gold Medal (1973); Premio Panetti of the Accademia delle Scienze di Torino (1985); Nibelungen Ring, Berlin (1985). Was also awarded an Honorary Degree in Mechanical Engineering by the Politechnic of Milano (1985), with a volume gathering papers in his honor by well-known scientists from all over the world.

A European 'aristocrat" in the best sense of the term, was gently mannered and cultured, with a keen intellect and a special ability to make clear the complex phenomena which he worked with.

Michele Capurso (1935–1987) [8]

Professor of Scienza delle Costruzioni at the University of Bologna (1970).

Was active in a broad spectrum of areas, including theoretical and structural plasticity, analysis of plates and shells expecially in reinforced concrete, theory of thin-walled beams, structural applications of mathematical programming, computer-aided design and analysis of civil engineering structures. Gave particularly fruitful contributions on the extremum characterization of incremental elastic–plastic solutions, rigid-plastic dynamic responses, and upper bounds on values attained by history-dependent quantities after shakedown of elastic–plastic solids; all of this being widely praised far beyond his school and the national solid and structural mechanics community.

His dedicated, passionate and inspiring work as a teacher witnessed a special taste and skill for clear presentations and an effective combination of scientific rigour and engineering evaluation, with the capability to make his activities as researcher, professor and engineer harmoniously complementary to each other and synergetic.

Cataldo Agostinelli (1894–1988) [9]

Chair of Advanced Mechanics at the University of Torino (1943).

One of the last outstanding representatives of the classical Italian mathematical-physics school, that included remarkable scholars in Turin, contributed to analytical mechanics, with particular regard to the integration by separation of variables of the Hamilton–Jacobi equation and to the definition of the Painlevé corresponding dynamical systems.

Made noteworth research into the motion of an electrically charged corpuscle in the presence of a magnetic dipole, and into the equilibrium configurations of a homogeneous fluid mass attracted by several distant centres according to Newton's law of gravitation. This allowed him to prove that for the above-mentioned fluid mass there are ellipsoidal equilibrium configurations with unvariable axes in terms of magnitude and direction. Also worked on the three-body problem, on special motions in the dynamics of rigid bodies, and on electromagnetism.

Pioneered the field of magneto-hydrodynamics in Italy, performing important studies on the propagation of magneto-hydrodynamic waves, symmetric motions with respect to an axis, vortical motions and motions governed by Helmholtz's theory on vortexes, ellipsoidal equilibrium figures, adiabatic equilibrium of rotating and gravitating fluid masses, radiating equilibrium of gaseous masses, stability of stationary magneto-hydrodynamical motions, shock waves, and plasma dynamics.

Coauthored treatises on Rational Mechanics, Analytical Mechanics, and Magneto-hydrodynamics, as well as a remarkable textbook on the Foundations of Mathematical Physics.

Member of the Accademia Nazionale dei Lincei, the Accademia delle Scienze di Torino, which was also President of, the Istituto Lombardo di Scienze e Lettere, the Accademia di Scienze, Lettere ed Arti di Modena, and the Accademia delle Scienze di Bologna.

Gianni Jarre (1924–1988) [10]

Chair of Aerodynamics (1958), and then of Gasdynamics, at the Polytechnic of Torino.

Obtained important results in different fields of investigations, including turbomachinery, heat transfer, applied mechanics, aerothermodynamics and others.

Always paid attention to matching research and teaching in order to understand and make understandable to others actual complex phenomenologies, with emphasis on the essential features properly cleaned up from factors of secondary relevance.

Member of the Accademia delle Scienze di Torino, of the Advisory Board of INSEAN (experimental nautical tank), Roma, and of the Scientific Committee of the Studi di Tecnica Navale (CETENA), Genova, was Director of the Centro Studi Dinamica dei Fluidi of CNR (National Research Council), and President of the Scientific Committee of the Centro Ricerche sulla Propulsione ed Energetica (CNPM), Milano.

Vincenzo Franciosi (1925–1989) [11]

Chair of Scienza delle Costruzioni at the University of Napoli (1956).

Contributed meaningful research on plastic limit analysis of structures, the generalized Betti's theorem, instability, analysis of bridges, and masonry structures.

Wrote a monumental treatise of Scienza delle Costruzioni, among several other books, and was an exceptional teacher, combining rigour and clarity, and educating a meaningful number of excellent young scientists and then university professors.

Dario Graffi (1905–1990) [12]

Chair of Rational Mechanics (1938), and then Emeritus, at the University of Bologna, was a member of the Institute of Advanced Studies in Princeton, and Visiting Professor at the Sorbonne.

Gave important contributions in mechanics, thermodynamics and electromagnetism, with papers on hereditary phenomena, published between 1928 and 1935, placing him among the precursors in the field. Wide interests ranging from reciprocity theorems in elasticity and electromagnetism to the mechanics of variable masses, from the thermomechanics of continua to the free and guided electromagnetic wave propagation in heterogeneous media, from the mechanics of nonlinear vibrations to the properties of adiabatic invariants.

Dean of the Faculty of Sciences (1960–65), was a member of the National Committee for Mathematical Sciences of CNR (1964–68), and Treasurer and then Secretary (1952–1967) of the Unione Matematica Italiana.

Member of the Accademia dei Lincei, the Accademia delle Scienze di Bologna (which was also President of), the Accademia delle Scienze di Torino, the Accademia di Scienze, Lettere ed Arti di Modena, the Istituto Lombardo di Scienze e Lettere, the Accademia Ligure di Scienze e Lettere, the Accademia dei Concordi di Rovigo, and the Accademia delle Scienze di Ferrara.

Was awarded the Prize Presidente della Repubblica by the Accademia dei Lincei (1965), the Gold Medal of the Benemeriti della Scuola, della Cultura e dell'Arte (1964), and the Honorary Degree in Electronic Engineering by the University of Bologna.

Luigi Gerardo Napolitano (1928–1991) [13]

Chair of Aerodynamics at the University of Napoli (1960); also professor at the University of California, Berkeley (1965), at the Sorbonne (Paris, 1967), at the École National de Méchanique et Aérotechnique (Poitiers, 1974).

A reference scientist for defining and promoting research in the space community, in Italy and overseas. A champion of experimentation in the zero gravity environment of space, studied surface driven flows in space (which would later grow into the fields of microgravitational science and microgravitational fluid dynamics), and demonstrated the real existence of the Maragoni effect in the famous SL-1 experiment. For his lifelong contributions to science, *Aviation Week and Space Technology* lauded him as the most representative personality in aerospace for 1985. Also made fundamental contributions to the study of surface phases and flow regimes, with his work best symbolized by the Order of Magnitude Analysis (OMA), which is recognized worldwide as Napolitano's method.

Director of the Department of Fluid Mechanics at CISM (International Centre of Mechanical Sciences), Udine (1970–74), was member of several committees of CNR, and of the European Space Agency (ESA).

President of the IAF (for two terms); President of the Fluid Dynamics Panel of the AGARD; founder and President of the European Low Gravity Association; Editor of *Acta Astronautica*, and of *Aerotecnica Missili e Spazio*; founder and Editor of *Earth Oriented Applications of Space Technology*, and of *Microgravity Quarterly*. Was also President of CIRA (Italian Centre for Aerospace Research), of the MARS Center, and of the SPACE CAMP, and member of the board of Directors of the Italian Space Agency (ASI), of the Accademia Nazionale dei Lincei, and of the International Academy of Astronautics. Was also a catalyst behind the Columbus Project, and devoted time and energy to the promotion of science and technology for industry in Italy.

A non-conformist teacher, a perfectionist hating compromise, and a dedicated, though tough and demanding, professor.

Giorgio Sestini (1908–1991) [14, 15]

Chair of Rational Mechanics (1956), and then Emeritus, at the University of Firenze.

Gave original contributions to scientific research in the fields of mechanics of particles and systems, of hydrodynamics, and especially of heat conduction and diffusion. In particular, brought phase-change problems to the attention of European mathematicians and obtained significant results in the field of Stefan-like problems, opening a fruitful research line on various kinds of free boundary problems encountered in applications.

Member of the Accademia Nazionale dei Lincei, the Accademia delle Scienze, Arti e Lettere di Modena, and the Accademia di Scienze e Lettere di Firenze.

One of the founders of AIMETA, as well as its President (1978–82), was Italian representative in the General Assembly of IUTAM (1973–75), VicePresident of the National Committee for Mathematical Sciences of CNR (1968–1972), and Rector of the University of Firenze (1970–1973), where established the Faculty of Engineering.

Wonderful teacher, created a serene atmosphere among his students, lavishing his best energies in passionate lectures, and in training younger co-workers and obtaining the best from them, on the basis of the genuine respect inspired by his honesty and frankness. Man with a unique mixture of irony, humour and wisdom, never pretended to be a philosopher, liking to be considered as a practical man with absolute dedication and professionalism, in compliance with a few basic moral principles.

Placido Cicala (1910–1996) [16]

Chair of Aeronautical Constructions (1945) and then Scienza delle Costruzioni (1957) at the Polytechnic of Torino.

Another brilliant disciple of Modesto Panetti who brought the Torino School of Applied Mechanics up to the world highest levels, with a truly ingenious and creative mind and a surprising versatility which, under a shy and unpretentious demeanour, led him into the field of structural mechanics, well beyond the boundaries of his original themes.

Made original and often pioneering contributions in aerodynamics of wing profiles, in particular on: non-stationary motion; calculus of variations, with a novel approach allowing engineers to search for extremal conditions without recurring to methods of functional analysis; shell theory, with particular reference to reinforced shells; influence of imperfections on buckling loads and jump instabilities; asymptotic approaches; non-homogeneous and elastoplastic materials.

Dealing with these subjects by mastering mathematical tools, however only used based on their actual difficulties, and aiming to make physical phenomena as evident as possible, his troublesome approach was subtly, yet keenly, within the range of both the mathematicians' fire, who could find it not rigorous enough, and the engineers' fire, who often objected it being too little engineering-minded. An approach coherently followed also in lecturing to students.

Visiting Professor at Purdue University, and lecturer at Stanford, Yale and the University of Illinois, was a member of the Accademia delle Scienze di Torino (1952) and of the Accademia Nazionale dei Lincei (1972), and obtained numerous awards and honours.

Ennio De Giorgi (1928–1996) [17]

Chair of Mathematical Analysis at the Scuola Normale Superiore of Pisa (1960).

A precocious talent in mathematics, was one of the greatest mathematicians of the twentieth century. Developed an extensive sequence of impressive results: a theorem on regularity of extremals in the calculus of variations for multiple integrals; a counter-example in the Cauchy problem for differential equations; the regularity properties of minimal surfaces; a new theory on perimeters; the theory of G-convergence and Γ-convergence; the variational theory of functionals simultaneously defined on volumes and surfaces; the evolution theory of minimal surfaces depending on a parameter; a generalisation of gravitational theory.

Also worked in logic, enlarging the class into which collecting concepts according to wider notions of quality and relation based on an algebra having the same rigour as that of sets, and reconsidering the minimum number of axioms of a theory.

Was member of the Accademia Nazionale dei Lincei, the Pontificia Accademia delle Scienze, the Académie des Sciences, and the U.S. National Academy of Sciences, among others.

Received numerous prestigious awards, including the Caccioppoli Prize of the Unione Matematica Italiana (1960), the Prize Presidente della Repubblica of the Accademia dei Lincei (1973), and the Wolf Prize (1990).

A man of great humanity and extraordinary civil vocation, interested in politics, though with a proper detachment, and active in defending human rights.

Carlo Ferrari (1903–1996) [18, 19]

Chair of Aerodynamics (1932) and Mechanics Applied to Machines (1948) at the Polytechnic of Torino, and then Emeritus. Also lectured at Brown University, Providence (1961–62), and New York University (1965–66).

Scientifically active in many sectors of mechanics, from fluid mechanics to biomechanics, with major involvement in the most diverse fields of aerodynamics, and research ranging from the study of air flows and pressures around airplanes and their components to the fundamental problem of turbulence, with results which made him famous all over the world.

Taking over the Laboratory of Aeronautical Engineering founded by Modesto Panetti, established therein a large wind tunnel where a wide variety of tests on models of airplanes, propellers, wing surfaces, cars, hulls, and buildings were carried out. A project realized during the great change occurred in aeronautics in the post-war period with the creation of jet engines.

His fundamental treatise Transonic Aerodynamic, coauthored with the mathematician Francesco Tricomi, and encompassing physical bases of the phenomenon, its mathematical modelization, and different approximations and solution methods, was translated into English, with worldwide visibility. Also important were didactic contributions in fields of theoretical mechanics and fluid dynamics, appeared in the Encyclopedic Dictionary of Physics (1961) and Italian encyclopedias.

One of the founders of AIMETA, was Italian representative in the IUTAM General Assembly (1970–75), member of the Accademia Nazionale dei Lincei, the Accademia delle Scienze di Torino (which was also President of), honorary member of ASME (American Society of Mechanical Engineers), and member of several Italian and foreign scientific academies and institutions. Awarded with the Prize Presidente della Repubblica by the Accademia dei Lincei (1950), and the Torino Prize by the Association of Engineers and Architects of Torino (1965), was also an unrivalled teacher, with a large group of clever disciples.

Gaetano Fichera (1922–1996) [20]

Chairs of Mathematical Analysis (1956) and then Higher Analysis (1959) at the University of Roma.

Obtained important results in both pure mathematics and applications to mathematical physics.

The former were concerned with: mixed boundary value problems of elliptic equations; generalized potential of a simple layer; second order elliptic–parabolic equations; well posed problems; weak solutions; semi-continuity of quasi-regular integrals of the calculus of variations; two-sided approximation of the eigenvalues of a certain type of positive operators and computation of their multiplicity; uniform approximation of a complex function; extension and generalization of the theory for potentials of simple and double layer; specification of the necessary and sufficient conditions for the passage to the limit under integral sign for an arbitrary set; analytic functions of several complex variables; solution of the Dirichlet problem for a holomorphic function in a bounded domain with a connected boundary, without the strong conditions assumed by Francesco Severi in a former study; construction of a general abstract axiomatic theory of differential forms; convergence proof of an approximating method in numerical analysis and explicit bounds for the error.

Results in applied mathematics and mathematical physics were essentially concerned with the existence, uniqueness and regularity of solutions in linear elastostatics, in particular the mixed boundary problem and Signorini's problem. Other studies regarded the energy approach to the Saint Venant's problem, and the theory of materials with memory, where obtained useful information on the analytical structure of the memory kernel. Some mathematical research concerned electrology and biology, and interesting works were devoted to the contribution of Italian mathematicians to functional analysis and theory of elasticity.

Member of the Accademia Nazionale dei Lincei, which awarded him the Premio Feltrinelli for Mathematics (1976), was also member of the Accademia Nazionale dei XL, of the Russian Academy of Sciences, and of other institutions.

Edoardo Benvenuto (1940–1998) [21]

Professor of Scienza delle Costruzioni at the University of Genova (1975).

Early works mostly concerned with central themes of elasticity and inelasticity, stability, and structural dynamics, through approaches exploring methodological ways alternative to the most followed ones.

Research into history of structural mechanics, with the Springer book An Introduction to the History of Structural Mechanics considered by Clifford Truesdell as "one of the finest I have ever read", gave him a great fame among scholars of mechanics. In contrast with the common separation between the essentially deductive role of mechanics and its documentary and critical history, his great innovation consisted of linking the retrospective analysis of the development of technical-scientific thought with the social and economic changes and the transformation of ideologies and philosophical thought. In this way, meaningfully contributing to the mitigation of the XX century contraposition between sciences of nature and human science.

Teaching for more than 20 years in the Faculty of Architecture, which he was also Dean of (1980–1997), dedicated most part of his studies to the analysis of the relations between mechanics and architecture, a human activity which he was deeply interested in, because of looking upon it as a millenary testing ground for thinking of statics of bodies and strength of materials. Refused, as conceptually inconsistent, certain usually stated separations between technique and inspiration, as well as science and art of construction, maintaining that looking at Brunelleschi, Michelangelo, Wren and many others only as great artists took away the merit of their brilliant aptitude for engineering. With his historical attention, helped revisiting the long trail that preceeded present mechanics, reconstructing the itineraries along which some great fundamental themes gradually shifted from ontology towards mathematics, and drawing the perception of how much these itineraries mean in terms of hypotheses, intuitions, confutations, debates.

His profound knowledge of history also entailed an innovative teaching of structural mechanics in the school of Architecture, under the convinction that transmitting contents of mechanics via a critical approach in which also the historical moments of their evolution are discussed arouse the attention of architecture students towards the synthesis between empirical intuition and rational analysis. All of this qualifying him as a meeting point for dialectic comparison of neutral and directed disciplines.

Made important contributions in many fields of thought, with meaningful studies of epistemology. A man of great lucidity and rare imagination, and a scientist attentive to the problems of our time and society.

Lucio Lazzarino (1913–1998) [22]

Professor of Aeronautical Construction (1944) and Construction of Machines (1960) at the University of Pisa, and then Emeritus.

Worked as aircraft designer at FIAT CMASA, where was involved in the design of several airplanes, then moved to the academy and made fundamental studies on propellers, flight mechanics and, mostly, shell type structures in light alloy, which was a topic at its first stage of development.

Teaching also other disciplines, gave an outstanding contribution to the education and training of hundreds of engineering students, with an approach innovative, tied to reality, and always aimed at improving and maintaining a fruitful relationship between academy and industry.

Was Dean of the Faculty of Engineering for 23 years.

Emilio Massa (1926–1998) [23]

Chair of Mechanics Applied to Machines at the Polytechnic of Milano (1964).

Research characterized by a broad and unusual variety of interests strongly linked with reality, moving from concrete problems in engineering however addressed through systematic theoretical analyses. They range from spatial kinematics of rigid bodies to the dynamics of machines as a system of rigid elements, to the nonlinear vibrations of systems with concentrated parameters, to vibrations of continua in the presence of thermal effects, to the mechanics of fluids in problems of lubrication. When becoming strongly involved with major administrative responsibilities, also in national committees, supervised younger collaborators' researches on wind-induced vibrations of electric cables, dynamics of automobiles, and behavior of ring seals in non-conventional motors.

One of the founders of AIMETA and first Editor of *Meccanica* (1966–1982), was Dean of the School of Engineering (1980–1987) and Rector of the Polytechnic of Milano (1987–1994), an institution to which he dedicated a life of commitments, by also implementing a regional structure in Lombardy and promoting a strong cooperation with industry.

Leo Finzi (1924–2002) [24]

Chair of Scienza delle Costruzioni at the Polytechnic of Milano (1960).

In the first years of academic career, obtained original results in continuum mechanics of elasticity and elastoplasticity, in rigid-plastic dynamics, in shell and structural stability theories. Later, gradually moved to scientific subjects more directly connected with engineering applications, such as methods of analysis of tall buildings and of joints in steel frame structures. In the field of steel structures, became an internationally acknowledged leader and a member and chairman of various Italian and European committees for elaboration of technical codes and research promotion, orientation and coordination. His originality and expertise in structural engineering led him to outstanding achievements as designer of large, unusual, and challenging structures.

Along his productive life, advocated and practised a synergetic interaction between scientific research and applications to real-life problems.

One of the founders of AIMETA, was member of the Accademia Nazionale dei Lincei, and of the Istituto Lombardo di Scienze e Lettere; Fellow of the American Society of Civil Engineering (ASCE); Honorary Member of the International Association for Bridge and Structural Engineering (IABSE).

Giovanni Bianchi (1924–2003) [25]

Chair of Mechanics Applied to Machines (1970), and then Emeritus, at the Polytechnic of Milano.

Research activity in linear and nonlinear mechanics, mechanisms, articulated systems, robotics and system dynamics.

Was a founder of institutions and a maker of people and scientists. Contributed to the founding of IFToMM (International Federation for the Promotion of Mechanism and Machine Science) with passion, tenacity and creativity. Acting as General Secretary and President of IFToMM for many years, is remembered for his innate ability to propose ideas and promote culture. In that context, in conjunction with Adam Morecki, directed the Robotics Committee, and proposed the Romansky Symposia, a reference event for scientists, academics, designers and researchers since 1972.

One of the founders of AIMETA, as well as its President (1982–85), was General Secretary of CISM in Udine (1977–2000), ensuring its international prestige, and developing activities in innovative sectors such as robotics and biomechanics. Was also Italian representative in the General Assembly of IUTAM.

Regarded knowledge and action as a means of communicating with others, creating study and research centers, indicating possible guidelines, offering new approaches to know how in a theoretical and concrete perspective. Besides scientific merits, was a man of kindness, humility and depth.

Giulio Ceradini (1918–2005) [26]

Professor of Scienza delle Costruzioni (since 1967), and then Emeritus, at the University of Roma.

Upon early research activity on fatigue, stress concentration, in-situ testing, elastic instability, viscosity, thin shells, and curved bridges, focused on plasticity problems, giving some early, fundamental, theoretical contributions to the limit analysis of structures as a linear programming problem; to the incremental theory, via the formulation and implementation of a maximum principle; to elastoplastic dynamics, with a suitable numerical approach; to plastic shake-down, with the first dynamic theorem equivalent of the Bleich-Melan theorem. Overall paving the way to a remarkable series of results of international level, obtained by the Italian school of plasticity also involving research groups from other universities.

A reserved scientist and an effective teacher, was a true gentleman, shy and somehow detached from the logic of academic power. Always left full freedom of choice of scientific themes to a meaningful number of disciples, and then professors, who were however strongly influenced by his style as a master and a non-invasive reference scholar, endowed with a variety of scientific interests and paying attention to both rigorous formulation of structural mechanics problems and their engineering implications.

Alfredo Corsanego (1936–2008) [27]

Professor of Scienza delle Costruzioni at the University of Genova (1975).

In the first half of his career, was meaningfully active on classical problems of elasticity, dynamics and stability of solids and structures, elastic systems in unilateral contact. Then, upon disastrous seismic events in Italy, research interests shifted away from classical topics of structural mechanics to the emerging problems of vulnerability and seismic risks of buildings. This was a sign of his deep-rooted yearning to contribute with all his scientific skill to the practical needs of the society, which would be later crowned with a strong commitment to the academic community.

Since 1980, worked on interpretative models for the nonlinear dynamics of soil and soil-structure interaction, and on seismic design of new structures and strengthening of existing structures. Seismic vulnerability was the fundamental field of interest, where contributed innovative methodologies for buildings, territorial systems, local seismic hazard, and for forecasting post-seismic scenarios, always working in a pioneering multidisciplinary setting.

Played a determinant role in the issue of safety and conservation of Italy's architectural and monumental heritage, taking important part in the debate on Italian and European seismic standards, and developing Recommendations issued by the National Committee for Protecting the Cultural Heritage against Seismic Risk.

President of the Scientific Council of the Seismic Risk Research Institute of CNR, in the last decade of his life coordinated the Committee of Civil Engineering and Architecture of the National University Council (CUN), actively participating to the ongoing discussion on the great transformation of universities.

A cultured scientist, a true engineer, a noble, fair and far-sighted man, ready to consider the reasons and needs of those he was working with, tolerant with those who disagreed, and always careful to safeguard the institutions and the dignity of individuals.

Carlo Cercignani (1939–2010) [28, 29]

Professor of Rational Mechanics at the Polytechnic of Milano (1975).

One of the leading mathematical physicists, active in PDEs, numerical analysis, semigroup theory, Monte Carlo methods, spectral theory, Riemann-Hilbert problems, Fourier analysis, functional analysis, and other topics.

Gave fundamental contributions on kinetic theory and, in particular, the Boltzmann equation, where was a reference scientist from a mathematical, physical, historical and, more generally, cultural point of view. Upon developing a variational method for the integro-differential formula of the linearized Boltzmann equation, obtained meaningful results on boundary conditions, models with discrete velocities, relativistic gases, polyatomic gases, the H-Theorem and shock waves, the kinetic modeling of granular media, evaporation–condensation processes, the long time behavior of the spatially homogeneous Boltzmann equation, its derivation from microscopic models, the search for self-similar solutions and the study of solutions with infinite energy, theorems of existence and uniqueness for the Boltzmann and Enskog equations.

Improved turbulence models for numerical simulation of the filtered Navier–Stokes equations, with numerical applications to turbulent jets, convection problems, turbulent flows in a plane channel, homogeneous isotropic turbulence, turbulence associated with combustion processes. Studied instability in rarefied gases, with main results pertaining to the long time behaviour and the formation of coherent structures and attractors.

In the last decade of his activity, combining the kinetic methods of the Boltzmann equation with the continuum Reynolds equation, obtained interesting results also in the modeling and analysis of MEMS.

Was always present at important international symposia on Rarefied Gas Dynamics and the Oberwolfach workshops in Many-Particle Systems.

Italian representative in the IUTAM General Assembly for 23 years, was also a member of the IUTAM Bureau (2000–03). At the national level, devoted his attention to C.I.M.E. Summer Schools for several years, convinced of the need of an institution to foster contacts between young Italian mathematicians and most active international scholars.

Authored and co-authored appreciated books on kinetic theory and gas dynamics, including The Boltzmann Equation and its Applications (Springer, 1988)—a treatise which became the standard reference in kinetic theory—and the monograph Ludwig Boltzmann, The man who trusted atoms (Oxford University Press, 1988), which highlights his fascination for the vision of Boltzmann in terms of both scientific theories and philosophical thought.

Awarded with several prizes and honors, including the Gold Medal for Mathematics of the Accademia dei XL (1982); the Prize Città di Cagliari for Applied Mathematics (1992); the Honorary Degree of the Université Pierre et Marie Curie, Paris VI (1992); the Humboldt Prize (1994); the Italian State Medal for Science and Culture (1999). Was a member of the Accademia Nazionale dei Lincei, the Académie des Sciences (Paris), the Istituto Lombardo di Scienze e Lettere.

A worldwide esteemed scholar, a man of vast culture also beyond science, a rich and friendly human personality, a great example of enthusiasm, tenacity, commitment to scientific research, courage in a premature serious illness.

Piero Villaggio (1932–2014) [30–32]

Professor of Scienza delle Costruzioni at the University of Pisa (1966), and also Professor of Fluid Mechanics at the Scuola Normale Superiore. Visiting Professor at the Johns Hopkins University, Baltimore, the Herriot-Watt University, Edinburgh, and the University of Minnesota, Minneapolis.

One of the most honored and recognized personalities in the field of continuum mechanics, continued the long tradition of excellence established by Italian elasticians, while being an engineer with a much higher knowledge of mathematics and physics than a traditional engineer. His extreme variety of scientific interests ranged from engineering problems where the elastic model was the key tool to suggest a correct and effective design, to purely mathematical contributions. Studied important problems in elasticity, plasticity, fracture, stability (also of thermoelastic media and fluid mixtures), structural optimization. In many works, including his elastic theory of Coulomb friction, dealt with fundamental questions cleverly using an elementary classical linear elasticity framework to interpret and explain eminently nonlinear phenomena. Made extensive studies on variational inequalities: variational formulation of thermodynamic processes, a great number of unilateral elastic problems including the search for numerical methods of solutions, buckling under unilateral constraints, a general variational approach to the theory of structures, contact in elastic bodies ranging from onedimensional up to the threedimensional class of Signorini problems. General optimization was addressed on both theoretical grounds and application to specific cases, including the optimal distribution of loads in elastic solids, optimal reinforcements, shape-optimization. Also worked on more traditional problems of equilibrium, stability and optimization of plates and shells, the Saint–Venant principle, planar linear elastic problems, complex variable method in classical elasticity, energetic bounds in linear and nonlinear elasticity, linear elastic fracture mechanics, inclusions in elastic media, incompressible elasticity, aeroelasticity and aerodynamic stability, stability of finite element methods, non-linear hypoelastic materials. Collaborated with eminent colleagues around the world.

Notwithstanding the extreme variety of his scientific interests, "Piero was not an interdisciplinarian, because for him there were no disciplines; in a period of increasing specialization he remained a Natural Philosopher" [31]. Was always very attentive to the changing trends in scientific research, about which wrote critical and penetrating essays.

Published three books, including the treatise Mathematical Models for Elastic Structures (Cambridge University Press, 1997) containing a summa of analytical models in linear elasticity, and a critical review of selected works by Johann I. Bernoulli (1667–1748).

A seemingly severe and provocative man, was actually generous, honest, elegant and witty, with a profound classical culture, an extraordinary fascination, and a Socratic capability to extract the best from students or colleagues.

Extremely reluctant to honors, with difficult times for people attempting to celebrate him, was anyway a member of the Accademia Nazionale dei Lincei, Associate Editor of many international scientific journals, and Vice-President of AIMETA.

Francesco Benedettini (1956–2015) [33]

Professor of Structural Dynamics at the University of L'Aquila (2000), where helped establishing and was responsible of the Experimental Laboratory of Nonlinear Dynamics (later expanded to include In-situ Dynamic Testing) since 1992.

Contributed meaningful research on nonlinear vibrations of systems and structures, using the combination of advanced techniques, analytical, computational, geometrical, and experimental, needed to carefully detect and reliably characterize a variety of nonlinear and complex dynamic phenomena possibly occurring in different engineering areas. Worked on finite amplitude oscillations of elastic monodimensional systems with initial curvature (suspended cables and arches), first obtaining high-order multiple time scale solutions of reduced order models in different external and internal resonance conditions, and then dealing with bifurcation and chaos phenomena with advanced numerical and geometrical techniques. A pioneering experimental activity allowed him to cross-validate theoretically observed phenomena via sophisticated techniques of reconstruction of the nonlinear response of a variety of flexible physical systems.

In the last 15 years of activity, shifted his main research interests towards more applicative problems in the area of structural mechanics and engineering, focusing on the identification and monitoring of structures, where contributed to experimental modal analysis in operational conditions, damage identification and model updating, monitoring dynamical properties of a great variety of bridges and other important structures subjected to ambient vibrations. Chaired the International Operational Modal Analysis Conference (IOMAC).

Constantly complemented theoretical and numerical investigations with the advanced use and the innovative implementation of experimental techniques suitable to analyze the response of systems at both the small-scale of a specialized nonlinear dynamics lab and the large-scale of real structures in civil engineering.

An open-minded, determined, and tireless scholar, full of brilliant initiatives and human richness and availability towards students and colleagues, was a true gentleman in both the behavior and the soul depth.

Giuseppe Grioli (1912–2015) [34]

Professor of Rational Mechanics (1949), and then Emeritus, at the University of Padova.

Follower of Antonio Signorini, was a specialist of mathematical problems in asymmetric elasticity and media with couple stresses, with international prestige in most celebrated environments of continuum mechanics. Was also significantly active in the dynamics of rigid bodies, dealing with problems of celestial mechanics where his abstract approach to the search for families of exact solutions somehow complemented the more operational one of his assistant and collaborator Giuseppe Colombo.

Dean of the Faculty of Science (1968–75), was member of the Accademia dei Lincei, the Accademie delle Scienze di Torino e di Palermo, the Accademia Patavina di Scienze, Lettere ed Arti, the Accademia Peloritana dei Pericolanti, and the Istituto Veneto di Scienze, Lettere ed Arti, as well as President of the National Group of Mathematical Physics of the National Research Council for many years. Was also member of the AIMETA Council.

An old-fashioned gentleman of calm authoritativeness and great poise, with many devoted disciples and collaborators.

Raffaele Casciaro (1943–2020) [35]

Professor of Scienza delle Costruzioni at the University of Calabria (1988).

Since the early 1970s, scientific interests focused on computational mechanics, an area where he was one of the pioneers and founders, with several academic activities undertaken over the years. Developed structural models and numerical methods for simulating the mechanical behavior of structures, always paying attention to practical problems. Contributed meaningful research to a variety of topics: limit and shakedown analysis, structural dynamics, nonlinear analysis of slender structures prone to buckle, modeling of geotechnical problems, solution algorithms for nonlinear problems, nonlinear structural models and their finite element formulations, multigrid methods, modeling of masonry and reinforced concrete structures.

Dealing with nonlinear problems through incremental methods, significantly strengthened the arc-length formulation for the analysis of slender structures via a mixed description in stress and displacement outperforming standard displacement descriptions in terms of robustness and efficiency. Also generalized the Riks path-following analysis to a strategy suitable for solving the optimization problem deriving from the static shakedown theorem and for handling the unstable static response of softening materials in masonry walls.

A milestone of his research activity was the finite element implementation of Koiter's theory of elastic stability, resulting into an innovative tool able to efficiently estimate the initial post-critical behavior of elastic slender structures taking the imperfection sensitivity into account also in case of almost coincident buckling loads.

Was an original and fascinating teacher, capable to communicate complex and advanced topics in a simple and synthetic way, focusing on general aspects of solid and structural mechanics. With modern and unconventional lectures and a great charisma, trained a generation of engineers. His early (late 70s) course of Theory of Structures was way ahead of time as regards treating topics with a clear computational mechanics approach for the analysis of real-life structures discretized with the finite element method. He loved spending hours working side by side with young researchers, and quickly transferring knowledge to them.

Intelligence, brilliant intuition, and a profound generosity were his best qualities. His ability to explore in-depth new topics in a few minutes was impressive, along with the capability to eliminate redundant details from a complex problem and to focus on its main logical scheme, often being able to detect weaknesses or strengths via few theoretical concepts. Was also a sophisticated intellectual with a broad culture including interests in history, music and literature.

Gianpietro Del Piero (1940–2020) [36, 37]

Professor of Scienza delle Costruzioni at the University of Udine (1979) and the University of Ferrara (1991), was also visiting professor at prestigious foreign universities.

Contributions to continuum mechanics, and mostly finite elasticity, place him among the outstanding scientists of contemporary mechanics.

Was involved in research on the applicative nature of materials, in particular W. Noll's 'new theory' for material behavior and numerical techniques for solving boundary problems. Contributed significantly to the theory of no-tension materials and materials with microstructure, with results giving rise to the theory of structured deformations. Investigated the symmetries of the elastic tensor, the notions of state and free energy in linear viscoelasticity, the elasticity tensor in transversally isotropic materials. Worked on a unified model of material behavior, based on an energetic approach, and on the mechanisms of rupture and dissipation in a wide class of materials.

In the new millennium, research interests turned to the thermodynamic foundations of linear viscoelasticity, the notion of body in continuous mechanics, a theoretical–experimental study on the behavior of expanded and cellular polymers, a study of local minima in finite elasticity, and to adhesion problems. Worked on the numerical calculation of the fracture load and the determination of the fracture pattern in nonlinear elastic solids.

In the last few years, expanded his interests to the research of a common basis to the phenomena of fracture, plasticity, damage and creation of microstructure, treating them in a unified manner via the incremental energy minimization. Obtained meaningful theoretical advances, revisiting the foundations of the mechanics of generalized continua, and proposing an axiomatic theory independent of the concepts of motion and inertia and able to provide a simple and unitary formulation for many classes of generalized continua.

On most of the above topics, collaborated with top-level foreign scholars and advised a huge number of young Italian scientists, later become university professors. Indeed, was also an exceptional teacher, able to transmit complex concepts with great clarity, logic, rigor and formal order, mostly at high-level education. Combining interests to traditional and advanced issues, "he was the perfect specimen of last century's Italian professor of Scienza delle Costruzioni, the accomplished product of a noble school" [37].

A man of outstanding scientific and moral rigor, "adamant that he was not going to settle for anything less" than his very high standards [37], and with an assiduous quest for depth of knowledge, an exceptional culture, a great sense of justice, a sincere humanity and a substantial modesty.

Giovanni Solari (1953–2020) [38, 39]

Professor of Structural Engineering at the University of Genoa (1990).

A giant in the field of wind engineering, where he made insightful research on modeling wind loads on structures, with special regard to the closed form solution of the alongwind and 3D response of structures, the equivalent wind spectrum technique, the proper orthogonal decomposition and double modal transformation, the wind-induced fatigue, the extreme wind speed statistics and the thunderstorm outflows. Many of these contributions had a relevant impact on engineering practice, structural design, and codification sector. Also for these reasons, in 2017 was awarded with an ERC Advanced Grant, THUNDERR, which has produced some of the most cutting edge research in the field and has matriculated a very talented cadre of students from the newly installed Ph.D. program at the University of Genova in "Wind Science and Engineering".

In 1999, the Wind Engineering community entrusted him to chair the panel appointed to create a new framework and organization of the International Association of Wind Engineering (IAWE), and then elected him as the first IAWE President.

Received numerous scientific awards of great international prestige, including the Jack Cermak Medal and the Robert H Scanlan Medal of ASCE (American Society of Civil Engineers), the Alan Davenport Medal of IAWE, the Otto H.G. Flachsbart Medal of Windtechnologische Gesellschaft e.V. Germany-Austria-Switzerland, the Senior Special Award of EASD (European Association for Structural Dynamics). Also received an Honorary Degree from the Technical University of Civil Engineering of Bucharest (Romania) and was Honorary, Guest and Adjunct Professor in several foreign universities.

Was an honest, committed, driven, unassuming scholar, and a real gentleman, as well as a "maestro" for many young people, who had the privilege of collaborating with him in his research activities.

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Mathematics and Engineering: A Fruitful Interaction

Gianfranco Capriz

Abstract Memories of a long life of scientific research offer an unusual point of view on the interactions between mathematics and engineering.

Keywords Mathematics and engineering · Borders and contacts · Automatic computation · Friuli's geography and history

The title could be understood as an encouragement to write an enormous book listing decisive events or, vice versa, declaim short but not interest-free invocations. But the present occasion is calling for felicities not for the boring preaching or the giving advice. And I could be considered, taking a long look, as an immigrant, though born in 'Gemona del Friuli' and so 'friulano' by birth. But Gemona is not far from the Italian state border and to its north there is a number of valleys of difficult access from the west but more easily accessed from the east. Topographically Friuli is a complex region escaping easy and general statements (just as the title topic of the present essay).

Besides all know how difficult it is to decide and declare without struggle and accept with serenity a border. In principle, on the meaning and consequences of the border of a body (possibly facing a variety of even foreign environments) both mathematicians and engineers must take a stand on the question if it makes sense to deem them as comparable with the consequences of internal contacts along an imagined internal frontier, an imaginary 'Euler cut'. Even, more specifically, if those consequences could be taken as depending only on strictly local geometric properties such as the existence of a tangent plane.

Ennio De Giorgi himself expressed embarrassment when we asked him about the possible ampler validity of his definition of 'reduced' frontier. Besides one may

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express doubts about the attribution of qualities naively and brusquely to a two-dimensional object as mathematicians suggest and wonder, rather, contrariwise, how one 'sees' the frontier from a near internal point, possibly with a perception declining steadily with the distance, as, possibly, an engineer would prefer to do. Finally, but not less importantly, there is the ambience within which the body is intended to be placed or even a container inside which the body may be constrained. An empty space (corpora et inane as a poet [and a mathematician, perhaps] would prefer considering, the choice as accessory, anyway) or whichever ambience a contract imposes to an engineer? A mathematician seems, sometimes, to arrogate the more convenient choice from a formal, functional stand. An engineer suffers, as nobody seems even to have started to classify ambiences and some, which the engineer is forced to confront, are the most diverse, sometimes unfortunate, even dangerous in the long run.

To pass on a larger view it is inevitable to recall that we all lived, from the beginnings of the past 50 years, in a period during which the political and military world circumstances have conditioned the choices (even the life) almost of every one of us. There was no alternative than to answer to the apocalyptic menaces and, in all great nations, a frenetic research activity relevant for our topic was launched: the interaction between mathematics and engineering was not only encouraged, it was even imposed, and financed generously also for the universities.

Even if one avoids mention of products of direct military interest, there is a sector (even that inevitably concerned, though) with revolutionary general consequences: automatic computers. They were also initially thought of to forecast exact trajectories of artillery shells but they invaded almost all activities, tacitly. There is no doubt that the whole spread of human geniality, from the most abstract logic for operating systems to the finest machining for output and printers, was involved.

An essential role was played by a strict collaboration between mathematicians and engineers. Personally, I traversed day by day that period of gestation even if at the margins and what I now mention is colored by my memories. That being premised, there is no doubt that the dominant personality in the field was a great genius, miscast then, Alan Turing. He had already described, in principle [1], a machine capable of self-control to carry out mathematical operations. But then, during the war, Turing worked hidden in the absolutely secret environment of military decryption. It was known later, much later, that, in parallel with the polish scientist Marian Rejewski, he had managed to decrypt the code ENIGMA used by the German Army. In that success a machine, called BOMBA, appropriately designed, had a decisive role. Soon after the end of the war, the superintendent of the mathematics division of the British National Physical Laboratory (NPL) offered Turing a job to design and, hopefully, build a large computer (denominated ACE, automatic computing engine), to match US projects. In a matter of months, Turing provided a document with detailed plan, times to specify details, time scales of activities required, costs to cover them. The project was approved, adding the need for an initial smaller version called DEUCE. Importantly, a member of the NPL Advisory Council was Sir (later Lord) George Nelson, president of a large conglomerate of companies English Electric Co Ltd. He intervened to provide the necessary staff not available at NPL (1949) so that

DEUCE was working within 2 years. ACE required still some years to exist (1958). But Turing was a homosexual, which was a crime in Britain then. Recognized as such in 1954, he could choose to be castrated or go to prison. He committed suicide during the chemical castration process.

As it happened, I was employed to work in the central research laboratories of the English Electric from 1956 and for 6 years, using for my researches one of the two DEUCE installed there. The conglomerate provided no end of problems, particularly of concrete mechanical engineering, which became my field of competence.

In the same years, other scientists, who were engaged into defense during the war, claimed the need of aligning investments in the civil Britain manufacture industry, after the war, to the level reached during the war itself. This was the so-called *white heat scientific revolution.* P. Blankett, a labour party adviser, was one of the most active promoters. Other successful persons contributed, as C. P. Snow, who was at that times in the English Electric board of directors. Several initiatives emerged; an important one was the creation of Nelson Research Laboratories, dedicated to fundamental research in the fields that industrial group was interested in.

It is time, however, that I come back to some personal issues.

31 years old—it was 1956—since some tentative of starting in Rome a serious academic career failed, I decided to move towards industry. Curious circumstances pushed me to apply for a research position in General Electrics. Nelson Research Laboratories offered me a contract (with an initially modest salary) for the mathematical section. There, the activity was essentially focused on DEUCE computers. At that times, the user was almost in direct contact with the inner hardware, in the absence—so to say—of an appropriate software. It was about to program numerical calculations able to furnish in acceptable time approximate solutions to rather complex problems. Learning several specific things was necessary. Eventually, I was able to collaborate with J. H. Wilkinson, who received the Turing Award in 1970 for his research in numerical analysis. On my background I had a mathematical-physics culture, which was essential in modeling engineering problems. I was in constant contact with the design offices in the group, the NPL at Reading, already quoted above, and also with the National Engineering Laboratory, opened near Glasgow, for data on new alloys, which were appropriate for high-temperature environments. I'll mention only some special cases for the pertinent prominence of unusual mathematical questions asked by engineers: (i) the shear in heating environment, which involves fractional derivatives, (ii) the whole rheology with the corresponding imaginative proposals, (iii) the effects of condensation drops on a supersonic turbine, (iv) thermal stresses on selenium discs, even those doped, for current rectified in electric locomotives. I'll dwell a bit more on a vibration problem concerning a rotating beam on lubricated bearings because it requires a mathematical formulations from which peculiar problems emerge: the boundary conditions for the beam involve per se evolution equations! As it moves, the beam floats over the lubricant and such a floating obeys to the balance equations describing the viscous fluid motion. The liquid film undergoes cavitation if the bearing clearance has no cylindrical symmetry. Consequently, the boundary conditions are themselves evolution equations: the pin can bounce within the bearing so that the breaking effects due to viscosity can or

cannot prevail on the rotational energy transfer with the consequence of exciting vibrations. The whole system sizing requires care to avoid disastrous consequences, which historically occurred in realized machines. In this case, engineers imposed to mathematicians a new class of problems, still open as far as I know.

I already quoted above the vastness of the interactions between engineers and mathematicians for military purposes, during XX Century's first half, dwelling on the first computers because they have eventually had a universal influence. At the same time there were other chapters that were temporarily very important and engaging, but their value was only specific (see, e.g., [2]).

In those years, the English Electric was involved in the construction of power plants based on fossil fuel or nuclear power. Its laboratories, specifically the Nelson Research Laboratories, were engaged in the analysis of pertinent problems and DEUCE was largely used. I must also mention another project imposed to Great Britain by the existing politic-military situation. The Soviet bombers Tupolev 22 could haul an atomic bomb through the North Sea to keep East England strategic installations down. And, therefore, the Great Britain needed to have in its possession a so high speed interceptor able to destroy the possible threat, after a sudden alarm, tackling it on the North Sea, before the bomber could reach the British airspace. English Electric was able to offer a solution with an extraordinary airplane able to reach Mach 2 and corresponding rate of climb to altitude, with the power of two jet engines fitted one over the other. In our laboratory we named it *P1*. The more appropriate official name was *Lightning* (see, also, [3]).

In 1962 I returned to Pisa, called back by Alessandro Faedo to assure continuity for the pioneering work carried out up to then by Marcello Conversi, who followed a suggestion by Enrico Fermi, to design and make servicing an electronic computer called CEP. I quote these personal events because they give me the chance to insist once again on how much fruitful the interactions between mathematicians and engineers can be. In Pisa I found two strongly involved actors: Alfonso Caracciolo di Forino and Giovanni Battista Gerace; the first interested and competent in logic and theory, even if always conscious of the needs of practicality, the other a disciplined engineer, with specific competence in the more advanced electronics. Two perfect examples of the interactions this essay is dedicated to.

Some years later I received an offer to become president of a new company, named Tecsiel and created within the financial corporation Finsiel by its president, Carlo Santacroce—a former student of the Scuola Normale Superiore and Faedo's friend—to manage what at that times was called an *intermediate software*. That company produced a heterogeneous net software, namely a net involving computers with different origins (on this there was in Pisa the leading expert, Luciano Lenzini, who was professor in the engineering faculty).

After that experience I came back to the Department of Mathematics in the University of Pisa still doing research on materials with (say) active microstructure, along a view unifying different specific models. Later I started to study the possibility of constructing continuum mechanics avoiding the axiom, rarely rendered explicit, of the exclusive and perennial existence of each material element. That axiom is even too convenient for a mathematician because it allows one-to-one correspondence between reference and current macroscopic shapes of a continuous body. Already Galileo Galilei mentioned it referring, for a concrete example, to the bending of beams made by carpenters in the Venetian arsenal. At times that axiom is nefarious for an engineer; for example, if rearrangements of matter due to temperature variations are non negligible. From thoughts about that problem, the invention of a new conceptual scheme, the one of so-called *ephemeral materials*, emerged, with a new line of studies (see [4], and references therein).

So, in my professional life, I floated on the perilous sea which links and divides the shore of engineering from that of mathematics. Perilous, I wrote, and, appropriately, not long ago, I was elected member of the Accademia dei Pericolanti of Messina. The, now, colleagues, who voted me in, have long meditated on whether the stretch of sea, they are used to admire, be more perilous to cross for a mathematician who pretends to act as an engineer or for an engineer who wants to argue as a mathematician.

Perhaps this peculiarity was among the factors which led me to the presidency of AIMETA (Italian Association of Theoretical and Applied Mechanics) from 1998 to 2001. In that period, beyond standard duties, I essentially worked to protect *Meccanica*, the AIMETA journal, from influences that could have potentially reduced its character of a true research journal.

My memory jumps and comes back, for the moment, to Friuli, mentioned in beginning this text, and to the topography of its northern border, which is determined by ‘Alpi Carniche’ and ‘Alpi Giulie’. As an old man, I adopt here names that have stayed in my mind from my childhood, when I wandered in those places. I know that nowadays, officially, we need to use those names selected by southern Slav, who came after Istrians and Romans. That Alps (as the branch that separates Styria from Slovenia) are permeable because the pertinent heights and asperities are modest. Consequently, various populations were able to overcome them easily, above all descending along the valley traced by the river ‘Isonzo’. Then, as a name, ‘Forum Iulii’ moved west and named the region: Friuli, precisely. The city, a border city, referred to itself briefly as ‘civitas’, i.e., ‘Cividale’.

Barbarians of various tribes overcame long ago that Alps, not only Huns led by Attila, to escape whom, some Venetians went on the islands scattered in front of the Adriatic coast. The first Slavic invaders arrived from the far north of, say, Slavonia from the surroundings in the bay that St. Petersburg now overlooks. They probably followed the amber route that extended from the Baltic coasts and, much appreciated by the Romans, was the first imperial market in Friuli and among the Venetians.

Those proto-Slavs camped in the widening of the Isonzo valley which now has Saga as its center around A.D. 500. Then, around 700 the cruel Longobards, to whom the genocide of those found there was no stranger, came. Some proto-Slavs, among those I have already mentioned, were able to escape climbing through an extremely inaccessible valley, which then opens to the west, arriving at what is now called the ‘Tanamea pass’. A small river, active only when the snow melts, now called Mea, flows from there towards the west to the sources of a stream—named nowadays in Italian as ‘Torre’, in Friulian ‘Tor’, in their language ‘Ter’—flowing south, at a certain point crossing a second precipitous and impassable narrow passage now called ‘Crosis’. Shortly after the source of ‘Torre’, the valley widens and offers the possibility of growing something edible, so that they built a village still called 'Ter' (nowadays 'Pradielis', in both Friulian and Italian). They have managed to survive hardship for a thousand years and speak their language, studied in 1800 by a Polish scholar, only a few now (see [5] for further details).

At the end of 1600, one of them managed to climb a pass between the mountain ranges of 'Musi' and 'Quarnan', up to descend to 'Gemona'. He said his name was Crapiz; he married a Gemonese. An error, perhaps, in the parish register moved the letter *r* as early as 1700. Currently, the closed valley is called by them 'Terski Dolina' ('Alta Valle del Torre' in Italian). There is still a family in Pradielis called Crapiz.

I went into details, partly irrelevant, also to justify being still alive at 96: I derive, at least in half, from a people who have survived a thousand years of hardship.

On closing this essay I must add a remark on the tool used by both communities (those of mathematicians and engineers) to diffuse the theorems they proved or the structures they planned and made real: the scientific journals. Great is the courage needed to propose a new one and the fatigue required to keep one alive which is already on the shelves, waiting to be read.

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Memoirs of a Few Senior AIMETA Officers

Giuliano Augusti, Leone Corradi Dell'Acqua, Carlo Cinquini, and Giuseppe Rega

Abstract This chapter collects short memories written by some retired Italian scholars of structural mechanics, already engaged with AIMETA in various official positions, as a dutiful representative tribute to all scholars and scientists who have contributed to building AIMETA's role as a showcase of Italian Theoretical and Applied Mechanics.

Keywords AIMETA · Structural mechanics · Memories

1 Introduction

The Italian Association of Theoretical and Applied Mechanics, AIMETA, was established in 1965, built in over half a century, and brought to its current status of a representative society of Italian Mechanics recognized in the world, thanks to the strong dedication and intense effort of a large number of scholars and scientists.

Of course, many AIMETA 'constructors' from the previous generations are no more with us. Nonetheless, a volume that aims at somehow retracing the history of AIMETA could not fail to provide testimonials of some of its most recent protagonists. Actually, the list of contributions should have been more extended, and include people from the different areas of AIMETA. However, the lack of adequate memory has unfortunately prevented some protagonists from providing a reliable account of what happened in their time of greatest involvement with the Association. It goes

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without saying that they are esteemed colleagues well-known to the community of the most aware AIMETA scientists, and also to the youngest.

Thus, in the sequel of this chapter, short memories written by only a few retired scholars of structural mechanics, already formally engaged with AIMETA in various official positions, are reported. Grateful thanks go to them for having kindly accepted to be present in some way in this anniversary celebration of over 50 years of AIMETA.

2 Some Recollections of Florence and AIMETA[1]

My first recollection of my involvement with the University of Firenze and AIMETA is very vivid: Udine 1972. During the National Congress of Earthquake Engineering, Giorgio Sestini, then Rector and President of AIMETA, took me under his arms and told me that he was enthusiastic about the interest of a promising young scientist like me to participate in the founding and building of a Faculty of Engineering in his University.

At the time I had the Ph.D. from the University of Cambridge but no position in Italy: thus, I was flabbergast of Sestini's approach and accepted immediately. I have never regretted my decision: the years I have spent in Florence (approximately 15) have been active and productive, although difficult and physically tiring because I had to commute weekly from my home in Naples.

My first commitment was to start the course of Civil Engineering, that was in the University's Statute (together with Mechanical and Electronic Engineering) but had not been activated because of some opposition from the Faculty of Architecture. Actually, this was immediately overcome also because of my friendly relationship with another Neapolitan, Salvatore di Pasquale, Professor of "Scienza delle Costruzioni" in that Faculty: indeed, I was his guest in Piazza Brunelleschi until the Faculty of Engineering moved to its premises in Via di Santa Marta, where it still is (now as "School").

Thus, I started to teach in the new Faculty, and the same time started looking for adequate teachers in Structural Engineering for the years to come. I got the first collaboration from who was to become a very close friend of mine: Franco Angotti, who rapidly got a Chair himself, was several times Dean of Engineering and had other important commitments in the University.

As for the new Professors, my first choice was a truly "Florentine", who had graduated in Bologna where he was an Assistant Professor, but was happy to settle in Florence: Andrea Chiarugi (who unfortunately is no more with us). He got the appointment for teaching "Tecnica delle Costruzioni" (with some difficulties because of the powerful "sponsors" of somebody else) and gave a very significant contribution to the development of Engineering in Florence: I recall specifically his theoretical and experimental researches on Brunelleschi's Dome.

[1] Written by Giuliano Augusti.

In 1975 the first three students graduated in Civil Engineering in our Faculty: they were Paolo Spinelli, Andrea Vignoli and Antonio Borri, now three Full Professors, two in Firenze, one in Perugia. Their Graduation Theses were the start of researches in numerical techniques and in earthquake engineering, and soon in wind engineering, in which Florence was leader in Italy. The CRIACIV (InterUniversity Research Center in Building Aerodynamics and Wind Engineering) and the National Association for Wind Engineering (ANIV) were thus born, both located in Florence, where the First National Conference in Wind Engineering IN-VENTO took place in 1990.

Another important step for the new Faculty was my engagement in 1976 for the formal test and licensing ("collaudo statico") of the new Bridge-Viaduct over the river Arno, named "all'Indiano". Following a decision of mine, all tests were conducted in due time by staff and "volunteers" of our Faculty, and were a very important experience for us all.

In the meantime, I was engaged in AIMETA in several positions (member of the Board in 1978–1981; Vice-President in 2002–2005; Chair of the Organizing Committee of the 1978 Congress in Florence and of the Scientific Committee of the 2001 Congress in Taormina; ...). But by far my most important engagement for AIMETA was the Editorship of its international journal *Meccanica* from 1990 to 1997: I am proud to recall that during this time the printer was successfully moved from a "local" one (Bologna's Pitagora Editrice) to a widely known international one, that still publishes *Meccanica* with everybody's full satisfaction.

But this is now present time and no more "recollections" !!!

3 AIMETA Role in Shaping the Italian Research in Structural Mechanics[2]

In the first decades of the second half of the last century scientific research underwent a significant growing in Italy, both in universities and in a few external research centers, such as CNR. However, when computers were not available (as was the case at the time), communication between researchers from different institutions was not as easy as it is today and this situation was felt as a bridle slowing down progress.

Among AIMETA's purposes was to bridge this gap in the field of mechanics, considered in all its aspects, from theoretical or rational mechanics to engineering applications. The operation no doubt was successful. Main instruments were the journal *Meccanica* and the national conferences, held every two years, which immediately gathered a numerous audience. They gave the opportunity of reading papers and listening to presentations covering all the different fields of mechanics, considerably broadening the spectrum of knowledge of researchers and giving rise to interdisciplinary cooperation in a number of cases.

Results, however, were not limited to making people aware of the research going on in Italy on the different aspects of mechanics. They also included a form of

[2] Written by Leone Corradi Dell'Acqua.

geographical synergy, in that researchers from different institutions in Italy, which previously were only in marginal contact, realized that they were sharing similar interests and started cooperating with each other. Today within AIMETA there are quite a few thematic groups, formally recognized, on specific subjects, such as computational mechanics or biomechanics, which meet periodically and represent moments of exchange of experience among researchers of all parts of Italy.

The groups above were established in comparative recent times, but already in the late sixties of the last century some spontaneous steps were undertaken. In particular, at one of the first AIMETA conferences, researchers working on different aspects of structural plasticity, a topic that gained significant interest in the years following the second world war, recognized the opportunity of sharing their experience and results and decided to organize a kind of informal forum to this purpose.

Giulio Ceradini, at the time professor in Florence, agreed to be the reference person for this spontaneous "plasticity group" and made available a room in his university to this end. People from different parts of Italy, from assistant professors to the youngest members of the research teams (the majority), reached Florence by train or car, sharing gas expenses, usually on Saturdays, to join the meetings, which were extremely informal: no written papers, only presentations of the research going on, results, problems, doubts and, above all, thorough discussion, which lasted till the departure of the last train.

From the neighboring university of Pisa also came professor Piero Villaggio, who became the second senior member of the group. Villaggio was not particularly interested in plasticity, but extremely fond of research and possessed a sincere love for research people. He had a strong mathematical background and was extremely rigorous in his activity. His capabilities brought an important contribution to the discussion, not much on plasticity in itself, though Villaggio was not completely extraneous to the subject, rather on methodology, in that all the steps of the research procedure, sometimes somewhat daring due to the naivety of the youngest, were carefully examined and weaknesses were pointed out with extreme care, even with fussiness, but always with an extremely friendly attitude.

Professor Villaggio pupils also joined the meetings and contributed by presenting their work, mostly dealing with advanced topics in elasticity and solid mechanics. This helped participants to realize that there were subjects worth studying other than plasticity, a topic, moreover, that was no longer as fashionable as a few years before. People began dedicating their attention to other interests and in the last meetings of the group plasticity had only a marginal presence, the main roles being played by different topics, such as computational mechanics or structural dynamics.

The plasticity group did not go on for long. In the early seventies professor Ceradini moved to Rome and meetings lost their natural and well established location. On the other hand, the enthusiasm and the freshness of the beginning was inevitably fading out. Researchers, even the youngest now more experienced, pursued their activities in their own institutions, but the structural and solid mechanics community was well established now, with professors Ceradini and Villaggio maintaining their reference role. The plasticity group ceased its activity, but it was not a fugacious experiment, as witnessed by the fact that most of the structural mechanics professors at the end of the twentieth century attended this group, sometimes finding in it their first stimuli.

The plasticity group was not considered as an official branch of AIMETA. Nevertheless, the society played a key role on his birth, in that it is through its activities that contacts were established and the benefits of mutual cultural exchanges recognized. In this sense, one can say that AIMETA strongly contributed in shaping the Italian structural mechanics community as it was till the turn of the centuries. Nowadays all the surviving members of the group are retired, but it does not seem too daring to say that some kind of indirect influence still remains.

4 A Few Personal Memories[3]

AIMETA has represented for me, over many years of University life, a constant point of referral.

From the beginning, National Congresses were very significant and meaningful: you got to know and be known within academic and scientific circles, you could find expert and experienced professors, you had a chance to meet and be introduced to prestigious names. In those meetings, you also got to know and befriend young scientists of your own age range, creating relationships that flourished over the years, developing into friendships that represent a wealth of connections still valuable today.

Not to mention that a paper presented to the National Congress (having to go through Scientific Committee's vetting) was considered as a published paper. Even when scientific value of the Congress Proceedings has waned, due to the ever increasing importance of publishing on journals, the very specific value of meeting and getting to know colleagues stayed, even for those who work in different areas of the discipline.

In AIMETA you always have the chance for an interesting overview on Mechanics world and its principal actors.

Considering the very positive experiences always, I was honoured to receive the proposal to be part of the Executive Committee; when later on friends and colleagues proposed my name for Presidency, I have been extremely grateful and gratified. Having free time from administrative and management activities within University, it was a pleasure to devote my time working for the Association.

On top of handling administrative complexities (to be fair, not so heavy but increasingly demanding due to changing of rules) I have had the pleasure to take part in the preparation of biannual Congresses and in promoting and following up on the different Group initiatives. These last have been growing in volume, numbers and importance in the last years, as focus of knowledge sharing and confrontation on the most specific and diverse topics.

Fulfilling the role of President and being part of the Executive Committee has been an extremely rewarding experience as well when it came to develop and grow

[3] Written by Carlo Cinquini.

the presence of AIMETA among international organizations, culminating in Italy being assigned the task of organizing the 25th International Congress of Theoretical and Applied Mechanics of IUTAM, sixty years after the 10th one held in Stresa, 1960.

The Congress, initially planned for August 2020 in Milan, was postponed to August 2021 and then held fully virtually, for the obvious and well known pandemic reasons.

Tribology Within AIMETA

Roberto Bassani and Enrico Ciulli

Abstract Tribology deals with every problem related to friction, wear and lubrication. Tribological studies range from theoretical to applied ones. Tribology subjects were included in the Congresses of the Italian Association of Theoretical and Applied Mechanics (AIMETA) since the first edition in 1971. The Tribology Group of AIMETA (GAIT) was founded in 1988. GAIT played an important role for the Tribology Commission of the Italian National Organization of Standardization. It organized five national Tribology Conferences from 1991 to 1998 and five International Tribology Conferences from 2000 to 2006. After the foundation of the Italian Tribology Association (AIT) in 2005, GAIT collaborated with AIT for the organization of the biennial European Conferences on Tribology (ECOTRIB) held every odd year and for the biennial Workshops "Tribologia e industria" held every even year. Virtual seminars have been also organized by GAIT since 2021. The strong activity of GAIT and AIT has taken to the awarding of the Tribology Gold Medal for the first time to an Italian scientist in 2006 and to the assignation of the World Tribology Congress 2013 to Italy.

Keywords Tribology · AIMETA Tribology Group · Tribology Conferences · Tribology Gold Medal

1 Introduction

Tribology is an interdisciplinary science dealing with all problems related to friction, wear and lubrication. It involves many aspects of mechanical engineering, physics, mathematics, chemistry, materials science, biomechanics, manufacturing and computer science, ranging from theoretical to applied studies and covering from nano/micro to macroscopic aspects.

The name Tribology officially appeared in 1966 [1], but its applications go back to the ancient times.

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Tribology studies concern materials, lubricants, additives, surface treatments and coatings, and manufacturing, with applications to several components of machines, as bearings and gears, and all kinds of dry and lubricated pairs, as cardiovascular devices and prostheses. The vastness of the tribological subjects is also evidenced by the birth of specific disciplines during the years, as for instance Nanotribology, Industrial Tribology, Biotribology and Ecotribology. Green Tribology was also introduced regarding with all tribological studies more directly related to sustainability. Many tribological researches aim to friction and wear reduction, which is connected to material and energy saving and reduced emissions and therefore to sustainability.

Tribology presentations were included starting from the first Congresses of the Italian Association of Theoretical and Applied Mechanics (Associazione Italiana di Meccanica Teorica ed Applicata, AIMETA) since the seventies (see for instance [2, 3]). Specific scientific sessions totally dedicated to Tribology were included in the AIMETA Congresses since the eighties (see for instance [4–18]). In 1988 the Tribology Group of AIMETA (Gruppo AIMETA di Tribologia, GAIT) was founded. In 2005 AIMETA became a founding member of the Italian Tribology Association (AIT) which main aims are to increase the cooperation between Universities and Industries, to contribute to the development of industrial technologies related to Tribology, to promote Tribology with cultural and scientific activities, as seminars, conferences and courses, and to promote international relations and exchanges. GAIT immediately started to collaborate with AIT. Thanks to the strong activity of GAIT and AIT members, Tribology had in Italy a big development that was internationally acknowledged by two extraordinary events: the Tribology Gold Medal was awarded for the first and still unique time to an Italian scientist in 2006, and the World Tribology Congress (WTC) 2013 was assigned to Italy, Fig. 1. The Tribology Gold Medal is the world's highest honor in Tribology and is awarded each year since 1972 for some outstanding and supreme achievement in the field of Tribology. The 2006 Tribology Gold Medal was awarded to Professor Bassani. The WTC is the largest

Fig. 1 The Tribology Gold Medal and Tribology Gold Laureates at WTC 2013. From the left: Talke, Ioannides, Spikes, Bassani, Frene, Jost, Goryacheva, Bartz, Xue

Tribology Congress organized every four years. It was held in Torino in September 2013 and was attended by more than 1,300 participants from five continents.

A summary of the main activities of the Tribology Group of AIMETA is reported in the following.

2 The AIMETA Tribology Group

After its foundation in 1988, the Tribology Group of AIMETA contributed to increase the cooperation between Universities and Industries. The group was founded again according to new rules in 2013. The logos of the original and new group are shown in Fig. 2.

GAIT played a fundamental role for the applications for research funds, such as the grants delivered by the Italian Ministry or Scientific and Technological Research. Members of GAIT chaired and contributed to the works of the Tribology Commission of UNI (Italian National Organization of Standardization), founded in 1989 by the Group. Some standards related to Tribology were produced [19–22].

Since the nineties, under the auspices of AIMETA, GAIT started the organization of a series of Conferences totally dedicated to Tribology that usually held with biennial frequency. The first AIMETA Tribology Conference was held in Parma in October 1991, the second one in Firenze (September 1993), the third one in Capri (September 1994) the fourth one in Santa Margherita Ligure (October 1996) and the fifth one in Varenna (October 1998). The number of participants increased in every edition of the Conference till reaching more than 30 presentations. Sample references are [23–28]. The covers of the proceedings of the five Conferences are shown in Fig. 3.

Since the year 2000, the Conference became international, with the acronym AITC (AIMETA International Tribology Conference) and was held every two years in September. The 1st AITC was held in L'Aquila in 2000 with more than 90 presentations. The 2nd AITC was a special edition entitled "Tribology: Science and application" and was held as a part of the World Tribology Congress, WTC 2001, in Vienna.

Fig. 2 Logo of the AIMETA Tribology Group: original (left) and new (right)

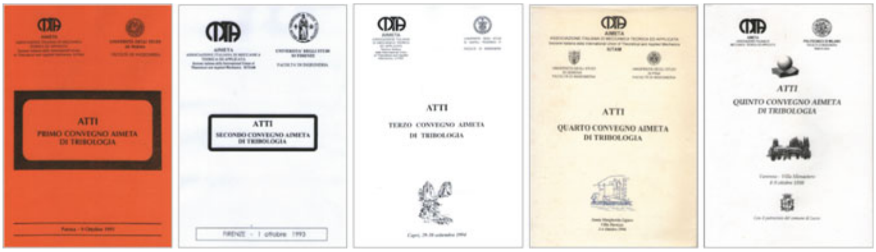

Fig. 3 Covers of the proceedings of the first five AIMETA Tribology Conferences

The following Conferences were the 3rd AITC in Vietri sul Mare (2002) and the 4th AITC in Roma (2004). The 5th International Conference on Tribology was held in Parma in 2006 after the birth of the Italian Tribology Association (AIT). The acronym AITC-AIT was used for that Conference. Almost a hundred participants from all over the world generally attended the AITCs with an average of 40 papers presented by Italian researchers. Some graphics of the five International Conferences are shown in Fig. 4.

Italian contributions presented at AITCs covered a great variety of subjects, such as friction and wear (erosion, scuffing, adhesion, indentation, alloys and steel, ceramics and composite materials, elastomers, coatings), Tribology in transportation (wheel-rail and tire-road contacts, brakes, clutches, passive magnetic levitation), lubricants and lubrication (hydrodynamic, elastohydrodynamic, hydrostatic and aerostatic lubrication), biotribology, micro- and nanotribology, high performance friction and rolling bearings and in internal combustion engines pairs [29–34].

In the frame of an international agreement among the Austrian Tribology Society, the Italian Tribology Association, the Slovenian Society for Tribology and Swiss Tribology, the European Conference on Tribology (ECOTRIB) was established where some Conferences organized by the four national societies merged. Therefore AITC merged inside ECOTRIB starting from the 6th AITC that coincided with the 1st ECOTRIB held in Ljubljana, Slovenia, in 2007 (Fig. 5). Since then, ECOTRIB is a biennial Conference held in the odd years usually in June. The editions held till now are: 2nd ECOTRIB in Pisa (Italy, 2009), 3rd ECOTRIB in Vienna (Austria, 2011), 4th ECOTRIB in Torino (Italy, inside the 5th World Tribology Congress WTC

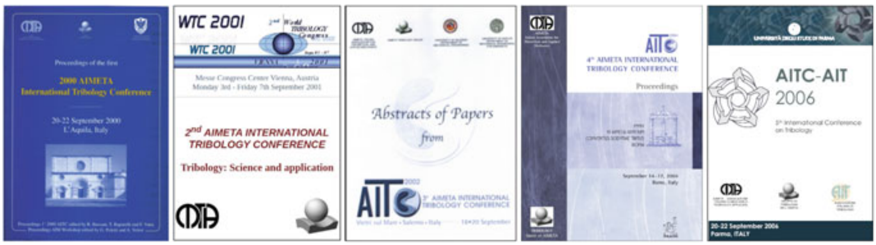

Fig. 4 Graphics of the five AIMETA International Tribology Conferences

Fig. 5 CD with the proceedings of the 1st European Conference on Tribology

2013), 5th ECOTRIB in Lugano (Switzerland, 2015), 6th ECOTRIB in Ljubljana (Slovenia, 2017) and 7th ECOTRIB in Vienna (Austria, 2019). The 8th ECOTRIB has been moved to 2023 due to the pandemic situation.

Starting from 2008 GAIT also collaborates with AIT to the organization of the national Workshop "Tribologia e industria" held every even year: 2008 Navacchio (Pisa), 2010 Bari, 2012 Milano, 2014 Modena, 2016 Salerno, and 2018 Torino. The 2020 edition, originally organized in Pisa, was held online. The heading of the leaflet of the 2nd Workshop is shown in Fig. 6.

In addition to the above activities in collaboration with AIT, GAIT started to organize virtual seminars in 2021. The first two seminars were held online in April and October 2021.

3 Conclusions

Tribology, dealing with every problem related to friction, wear and lubrication, has been always included among the subjects of AIMETA. The Tribology Group of AIMETA, together with AIT since its foundation, has played an important role in the tribological activities in Italy also by contributing to increase the cooperation between Universities and Industries. Several national and international Conferences have been organized. The activities have had international acknowledgment with the awarding of the Tribology Gold Medal for the first time to an Italian scientist in 2006 and with the assignation of the World Tribology Congress 2013 to Italy.

Fig. 6 Heading of the leaflet of the 2nd Workshop "Tribology and industry"

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New Challenges for *Meccanica* After 55 Years

Anna Pandolfi

Abstract The long journey of AIMETA has been sided for 55 years by Meccanica. In compliance with the ideals of the founders, the journal has been a distinguished reference for the Italian Mechanics community. In the last 20 years, Meccanica has turned progressively to become an international window for Eastern counties. The journal is facing new challenges, related to the uncontrolled run for publishing and citing, but the strong scientific picture envisioned by the founders remains the guidance for submission and acceptation.

Keywords Meccanica · AIMETA · Metrics · Science

In 1966, one year after the founding of the Italian Association of Theoretical and Applied Mechanics, the official Journal of the association, Meccanica, was launched as a forum to collect the contributions to the several branches of Mechanics [1]. From an initial prevailing Italian authorship, the journal has become more international, seeing a progressive growth in popularity especially in Eastern emerging countries. Today, internationality characterizes not only the contributing authors, but also the Associate Editor Board. In the next paragraphs, after a brief introduction of the recently renewed Editorial board, a compendium of the current trends of Meccanica is presented, according to the statistics of submissions and acceptance during the short span of time between January 2021 and September 2021. The discussion does not involve the relatively large number of manuscripts published in 2021, so that the 2020 submissions are excluded. Some comments on the observed trends and on influence of the impact numbers on the average quality of the published research conclude this overview.

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1 Editorial Board

Luigi Gambarotta has served as Editor in Chief of Meccanica since 2015 to the end of 2020. His dedication to the Journal, in terms of time and passion, has pushed the notoriety of Meccanica and increased its reputation among the Italian and International scientific community. After five years, he decided to resign from the position and I have been appointed to replace him. The move had not indifferent repercussion on the Editorial Team and Associate Editor Board of the journal. The role of Managing Editor is now taken by Maurizio Quadrio, from the Department of Aerospace Sciences and Technology of the Politecnico di Milano, and the role of Assistant Editor by Rosalba Ferrari, from the Department of Engineering and Applied Sciences of the University of Bergamo.

The current Associate Editor Board of Meccanica includes fourteen foreign members, most of which have been appointed by the previous Editors in Chief since the beginning of the new century:

- Katia Bertoldi, Harvard University, USA
- Luca Brandt, Royal Institute of Technology, Sweden
- Eliot Fried, Okinawa Institute of Science and Technology, Japan
- Paulo Batista Gonçalves, Pontifical Catholic University, Rio de Janeiro, Brazil
- Wei Hong, Southern University of Science and Technology, P.R. China
- Djimedo Kondo, Sorbonne University, France
- Andreas Müller, Johannes Kepler University, Austria
- Chérif Nouar, Université de Lorraine, France
- Zhongxiao Peng, University of New South Wales, Sydney, Australia
- Franck Plouraboué, Université de Toulouse, France
- Tomasz Sadowski, Lublin University of Technology, Poland
- Stephanos Theodossiades, Loughborough University, United Kingdom
- Zaihua Wang, Nanjing University of Aeronautics and Astronautics, P.R. China
- Mohammad I. Younis, King Abdullah University of Science & Technology, Saudi Arabia

and ten Italian members:

- Maurizio Brocchini, Università Politecnica delle Marche, Italy
- Claudia Comi, Politecnico di Milano, Italy
- Giorgio Dalpiaz, University of Ferrara, Italy
- Antonio DeSimone, Scuola Internazionale Superiore di Studi Avanzati, Italy
- Eugenio Dragoni, Università degli Studi di Modena e Reggio Emilia, Italy
- Alessandro Iafrati, Consiglio Nazionale delle Ricerche, Italy
- Antonina Pirrotta, Università di Palermo, Italy
- Luigi Preziosi, Politecnico di Torino, Italy
- Elio Sacco, University of Naples Federico II, Italy
- Epifanio G. Virga, Università degli Studi di Pavia, Italy.

The precious contribution of the Associate Editors is fundamental to the journal. They are distinguished scientists, with strong expertise in various fields of Mechanics, and represent the task force for the selection of the proper reviewers for each submission. Furthermore, their presence in the Editorial Board of Meccanica increases the respectability and the appeal of the Journal especially to young scientists that look at them as reference. We foresee, indeed, the possibility that the innovative research fields object of the current studies of the Editorial Board members (bio-fluid-dynamics, liquid crystals, metamaterials, bio-robotics, to mention a few) can find a proper collocation in Meccanica.

2 First Submission

Meccanica is an international journal. In the first nine months of 2021, out of 532 submissions, only 3% of the submitted manuscripts are from Italy, while 75% of the submissions are from Asia, and 60% specifically from China, Iran, and India, see Fig. 1. The chart confirms the general trend observed during the last decade in many international Journals, and it is the natural consequence of the heavy pressure to publish coming from the eastern countries.

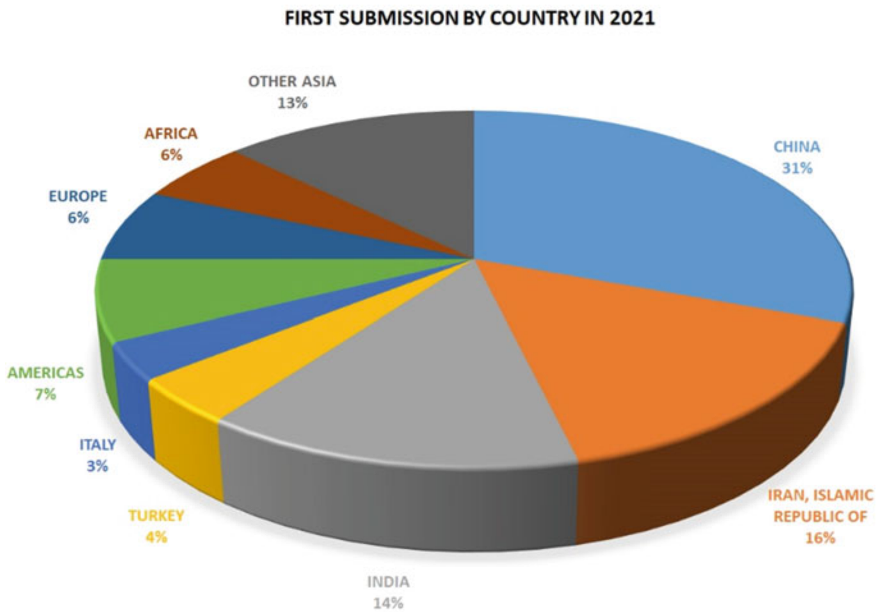

Fig. 1 Distribution of the corresponding author country for the 532 manuscripts submitted to Meccanica in the first nine months of 2021

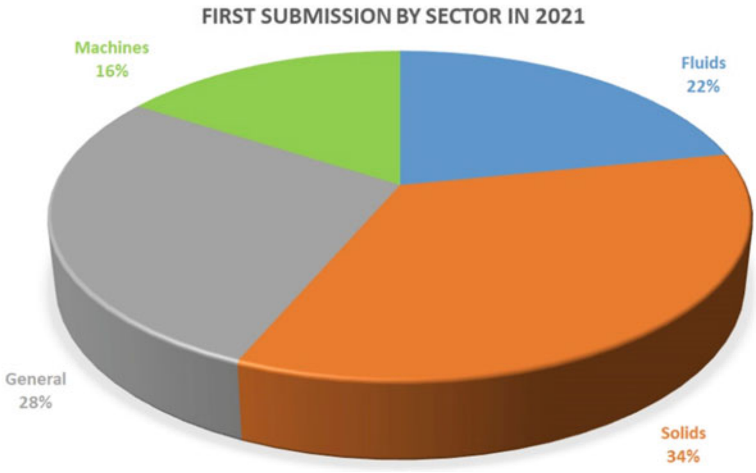

Fig. 2 Distribution of the submission section (Solids, Fluids, Machines and General) for the 532 manuscripts submitted to Meccanica in the first nine months of 2021

Internationality affects the traditional classification of submissions in Solids, Fluids, Machines and General Mechanics. The four sectors, still carrying some significance for AIMETA and for Italian authors, are not a clear indication for foreign submission. Figure 2 shows the subdivision in the four sectors according to the 532 first submissions in 2021.

The distinction between machines and solids does not hold for international contributions. Very difficult is for authors to select the proper sector for problems that involve fluid–structure or fluid–solid interactions. Papers dealing with coupled physics are not properly located between Solid or General sectors. The difficulty is certainly related to the variety of modern topics associated to mechanics, including computational mechanics, biomechanics, metamaterials. As matter of fact, the selection operated by the authors must be often ignored in the first evaluation phase.

3 Contents and First Editorial Decision

A high rate of submissions is unfortunately related to a low average quality of the manuscripts. Many manuscripts discuss a specific application, without presenting innovative ideas or original aspects of research related to mechanics. If they do not fit the scope of Meccanica, but their overall quality is still acceptable, they are transferred to other journals. Regrettably, many manuscripts are affected by other serious issues such as plagiarism or self-plagiarism, that can reach levels up to 50% of the content, or insufficient quality of the editorial presentation, or absence of

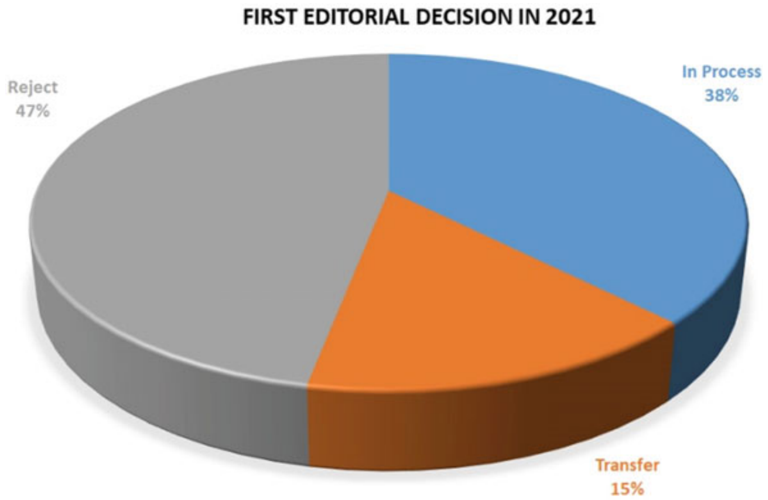

Fig. 3 Distribution of the first editorial decision for the 532 manuscripts submitted to Meccanica in the first nine months of 2021

any scientific novelty. These submissions are rejected without undergoing a review process.

Traditionally, Meccanica has been a rigorous journal, with rather low acceptance rates. According to the editorial of Luigi Gambarotta [2], in 2015 the ratio between published papers and submissions was 35%. The tradition of severity has been confirmed also in the first period of 2021. Figure 3 shows the results of the first screening operated by the Editorial team on the new submissions. Only 38% of the submitted papers have initiated a review process, while 15% have been transferred.

In this brief compendium we explicitly omit the data on the published papers in 2021, for two reasons: two more issues are missing to complete the volume 56, and most of the published papers in the first months of 2021 were not initially managed by the present Editorial Team.

The overall impression that the Editorial Team has matured in this brief time is that Italian authors have a considerable respect of Meccanica. Thus, Italian contributions are in general characterized by high scientific quality and accurate editorial presentation, and we wish that this impression can persist in the future.

4 The Battle of Quality Versus Numbers

In the last decade, Meccanica has been experiencing the increment of the number of submissions. This trend, shared with all scientific journals, is motivated by two main reasons, one respectable, one deplorable. The respectable motivation is the increase of number of people involved in research, due also to the emergence in the scientific community of highly populated countries such as China and India. The deplorable motivation is the pressure induced by the more and more recurrent use of variety of metrics of scientific productivity, including citations, Hirsch index, views, and downloads. Quantitative indicators, originally conceived by publishers to monitor the performance of a journal, have become a too easy way to replace the laborious qualitative evaluation of the research with the automatized collection of numbers. Quantity of production is sometimes misinterpreted as synonymous of quality for career advance and promotions [3] and in selective calls for grants or awards. The increasing reliance on metrics to evaluate scholarly publications has produced radically new forms of academic fraud and misconduct [4].

The pressure to increase numbers pushes for publishing an article also when the research is not self-contained, assuming the form of “salami slicing” and lowering the average quality of the papers submitted to journals: simultaneous or concurrent submissions of a manuscript describing the same research to more than one journal, production of self-similar papers with incremental differences and lack of originality, presence of non-contributing authors, inclusion of additional authors after the first round of review, unacceptable count of self-citations, citation rings, requests from the supposedly anonymous reviewers to cite their own works, to mention the most frequent.

Meccanica is not exempt: submissions revealing misconducts (mostly self-plagiarism) are received in a regular basis and unethical reviewers are not isolated phenomena. The journal, however, fully complies with the ethical policy of the publisher, and the Editorial Board in the last years has taken a strong point of view to discourage such behaviors.

The consequences of a strong position may be unfavorable to the immediate visibility of the journal: a rigorous journal is scientifically respected but often it stands in lower positions in the journal ranking. But in the long run, it is going to be the winning choice. Today, Meccanica is behaving well, having a respectable impact factor (2.258 in 2021), albeit in the classification of it is surpassed by other journals that are less critical with submissions.

To the young authors and readers of Meccanica, the suggestion is to keep strong in good research, developing an open mind and curiosity about scientific questions without falling in the vortex of the imperative “impact or perish”. Correct behaviors will increase the credibility of the Journal and reflect the ideals of the members of the AIMETA.

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A Glance at Italian Research in Mechanics

Italian Mechanics: Overviews, Viewpoints, Perspectives

Giuseppe Rega

Abstract A summary of the contributions by senior yet still active authors appearing in this volume is presented, with the aim to provide an overview of the Italian research in Mechanics in the last few decades, with regard to both 'classical' and 'novel' topics, and a glimpse of ongoing developments and open perspectives. The contributions are discussed according to a sequence that attempts to outline a kind of scientific path from a glorious past to a challenging future.

Keywords Italian mechanics · Overviews · Research viewpoints · Perspectives

1 Introduction

The second part of this volume, which celebrates AIMETA's 50+ years of activity, aims to present an at least partial overview of the Italian research in Mechanics in the last few decades, while providing a glimpse of ongoing developments and open perspectives, with regard to both 'classical' topics and 'novel' ones.

To pursue this objective, a number of senior, yet still active, authors have been invited to contribute a chapter, also possibly in collaboration with younger authors, on a general research theme or a specific topic, which they have been working on in longer or more recent times with internationally acknowledged results, or which they are currently active on. Of course, within a scenario of highly varied and worldwide recognized scientific activity, selecting the colleagues to be invited to contribute has been a highly demanding task for the writer, as Editor of this volume, in both scientific and personal terms.

Considering the interdisciplinary nature of a volume whose audience will be varied and with different specializations, it would not have made much sense to include fully original contributions, profitably readable only by scholars expert in a particular field. Accordingly, contributions in the following chapters mostly provide reviews of achievements of research groups, expressions of research viewpoints

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and/or criteria, indications of perspectives of development, and so on. As such, the collected texts are of a quite different nature, because of ranging from contributions which overview modeling, methodological and/or phenomenological aspects in a given (wider or narrower) field according to a prevailing analytically- and/or computationally-oriented perspective or a qualitative/phenomenological one, up to contributions dwelling on a specific research topic. Despite a consequent feeling of possible inherent inconsistency of contents, to the writer's opinion this partial inhomogeneity highlights the richness, variety and full international relevance of the Italian research in Mechanics, this last aspect being apparent in all papers.

In the sequel, contributions reported in the following chapters according to a sequence that attempts to outline a kind of scientific path from a glorious past to a challenging future, are overviewed.

2 The Panorama of Covered Topics

An albeit partial overview of Italian Mechanics could not lack an initial contribution highlighting in a somehow exemplary way the consolidated and long lasting tradition of *historical* studies on the origins of applied mathematics and mechanics, within the framework of the close relationships and fruitful exchanges of ideas occurred between advanced protagonists of *European mechanics*. This is the theme of the contribution by Sacchi Landriani and Russo [1] on the scientific roles of Eugenio Beltrami's and Barré de Saint–Venant's in the nineteenth century, which are also framed in the evolution of political and cultural relations between Italy and France.

The following three contributions present surveys of research topics addressed within the AIMETA environment over the last few decades in three classical wide areas, namely Kinematics of Machines, Masonry Structures, and Dynamics and Stability.

Given the impossibility to provide a generally agreed framework for dealing with *kinematics of machines*, Belfiore and Pennestrì [2] focus on the considerable amount of relevant research published in the AIMETA international journal Meccanica by Italian and foreign scientists. The authors shortly dwell on the early advancements of kinematics of machines, and on the historical evolution of the discipline, at both international and Italian level, that led to the identification of six different kinematic-related categories: mechanism topology; kinematics analysis of planar and spatial mechanisms; kinematic synthesis; cams, gears and transmissions; computational kinematics and optimization methods in mechanism; compliant mechanisms. Topics addressed in the papers are grouped accordingly, providing a significant set of bibliographic references.

Masonry mechanics is a scientific area in which the impact of Italian research has been dominant at the international level, in connection with the long tradition of studies originated by the worldwide recognized importance of the Italian monumental and artistic heritage, and by the dangerous coexistence of a wide patrimony of masonry constructions and of important seismic actions damaging them.

All of this makes a correct modeling of masonry structures mandatory. In this framework, Sacco et al. [3] dwell in detail on the significant impact of Italian contributions in the field over the past five decades, as reflected by the AIMETA activities (both conferences and Meccanica journal). No-tension models, limit analysis-based models, phenomenological models, multiscale and homogenization approaches, and block-based models are discussed in a quite comprehensive and often also interrelated perspective.

Dynamics and stability is the subject of Luongo and Piccardo's [4] contribution. Also these authors focus on a wide field of mechanics having nontrivial connections with branches of applied mathematics, too, according to an AIMETA-driven perspective. Indeed, they first retrace the evolution of the role played by the two disciplines in the panorama of Italian Mechanics, dwelling on a quite peculiar feature of the Italian academic environment, which entailed their full incorporation in the basket of AIMETA activities only quite recently, notwithstanding earlier nontrivial achievements by a few relevant scholars. Then, they focus on the important methodological aspect of the unifying role that perturbation techniques can play in addressing problems of nonlinear dynamics and stability, which is however still overlooked and not yet fully exploited. In this respect, the contribution can be considered the paradigmatic expression of a research viewpoint, as typical of position papers. Finally, restricting to the GADeS (AIMETA Group of Dynamics and Stability) environment, a glance is given to the broad spectrum of considered topics in solid mechanics, structural mechanics and control, in both linear and nonlinear regime, via analytical, numerical and experimental methods, using deterministic as well as stochastic approaches.

A substantial number of following contributions is devoted to a variety of well-established research fields/themes in solid or fluid mechanics that, although anyway encompassing a wide number of important topics, are either somewhat less broad than the previous ones or surveyed according to the possibly more oriented perspective of an active Italian research group in the background. Consistent with their long lasting habit of scientists nourished in the strongly founded Italian school of continuum mechanics, some authors deal with the considered theme via mathematical treatments that could be deemed even too much rigorous for the circumstance of this anniversary volume. Some other authors prefer to give a more general overview of problems and phenomena in the background of the addressed theme, and of the models and methods to tackle and unveil them. In all cases, however, by suitably framing the considered research issues in a proper historical perspective.

Polizzotto, Fuschi and Pisano [5] deal with *nonlocal elasticity* theories, whose roots are traced back to the 60s and 70s. Upon scrutinizing the inconsistencies encountered within the Eringen's purely nonlocal model and the remedies required to overcome shortcomings and paradoxical situations known from the literature, the authors overview a family of strain-difference based nonlocal elasticity theories they have been working on in the last two decades, showing how they provide effective methods to address boundary-value problems. Applications to plates by nonlocal finite elements and size effects analysis of beams in bending are presented as illustrative examples of the obtained results.

Davini [6] deals with the theory of inhomogeneities in solids, whose interest is also motivated by the attempt to provide a microscopic basis to *anelasticity theories*. First, the author revisits the divisive continuum theory of defects grown in the years 1950s–70s, and dwells on the elastic invariants and Ericksen's theory of X-ray observations of crystals. Then, he focuses on an overlooked, yet far-sighted, earlier paper by C. Eckart, aiming to indicate how the classical theory of elasticity can be extended to include anelastic phenomena if properly removing two of its cornerstones. Drawing attention to a contribution that anticipated instances found, for instance, in the nowadays research on growth in which many young Italian researchers are involved, Davini reminds them of the need to look with open eyes also to the past when traveling along seemingly new paths.

Within the framework of the Eshelbian mechanics, Bigoni et al. [7] deal with *configurational forces* in elastic structures, which are related to the possibility of changing the structure configuration, thus inducing a variation in its potential energy. The authors theoretically demonstrated, experimentally validated and implemented these forces in the realisation of novel reconfigurable elastic devices, showing their strong effects on the structure behaviour. Applications to soft devices (elastica arm scale, dripping of an elastic rod, torsional actuator), implications on stability (penetrating blade), and connections with limbless locomotion (snaking rod) are reviewed. The results inspire new interpretations and pave the way to the territory of *configurational structural mechanics*, with innovative applications of reconfigurable mechanisms in fields ranging from nanomedicine to aerospace.

Materials with memory is a field of major scientific interest in continuum physics, which goes back to the second half of the nineteenth century with fundamental contributions by a number of giants of science, up to being framed in the general scheme of the thermodynamic theory of materials in the 1960s. Yet, the subject is open to many important improvements, as discussed by Giorgi and Morro [8] in their contribution. The authors follow some historical developments of viscoelasticity within the theory of materials with memory, and show how a new approach to dissipative and hysteretic phenomena via the use of the second law of thermodynamics as a restriction on the constitutive equations allows a decisively more general setting of materials models. In this setting, they establish a far reaching scheme of hysteretic properties.

Viscoelasticity is also the topic of the contribution by Di Paola and Pirrotta [9] according to the modern perspective of *fractional calculus*, which is very popular in the engineering community because of its capability to predict the viscoelastic response of various materials in both time and frequency domain. Upon shortly dwelling on the inconsistencies of classical elementary models and on fractional derivative based models to describe damping, the authors focus on continuous viscoelastic beams with a fractional derivative element, which naturally appears in the stress–strain equations if creep and relaxation functions of power-law type are considered in the Boltzmann superposition integral, as also confirmed by experimental studies. Results of the research group at the University of Palermo are summarized, discussing also variable-order fractional operators for viscoelastic materials and applications in biomechanics. While the integration issue of fractional operators is totally overcome, a wide experimental campaign is needed for a proper description of the parameters in the relevant equations.

Fracture and damage is another long lasting subject of investigation within the international community, with the formulation of accurate mathematical models, but challenging tasks still existing as regards the prediction of damage evolution in a structure up to failure. The Italian community gathered around two AIMETA Groups (Computational Mechanics and, later, Mechanics of Materials) has been very active in the field, with a huge amount of innovative contributions. Comi and Perego [10] retrace some main stages of the research evolution from the only apparently partial point of view of the activity on computational modeling carried out in the last 30 years at the Politechnic of Milan. Topics overviewed include damage models for concrete, strain localization and regularization strategies for overcoming ill-posedness of the boundary value problem, continuum and discrete approaches in the finite element modeling of crack propagation, interface models for delamination of composites. The authors also dwell on new challenges entailed by the need of a refined design accounting for, e.g., possible defect growth at different scales and extreme loading conditions in multiphysics contexts.

Within the large area of system identification, Vestroni and Morassi [11] deal with *dynamic structural identification* and *damage detection*, pursued by suitably complementing mathematical models with the results of experimental investigations, in the framework of inverse problems. After carrying out the dynamic structural characterization, localization and quantification of damage turn out to be an important sub-problem, also in view of the health monitoring of structures. The authors refer to their own results and to a selection of Italian contributions, framing them in the international literature. A general overview is provided, focusing on some topical points: the interpretative models for describing the structural behaviour, the evaluation of optimal estimators of model parameters, the availability and type of experimental data, uniqueness issues of the related inverse problems, and tools to govern their peculiar difficulties in damage identification. Practical use of structural identification methods still requires extensive study, to be pursued also via a multidisciplinary approach.

Within the general theme of structural identification, D'Ambrogio and Fregolent [12] overview their own results on the specific topic of *experimental dynamic substructuring*. Both substructure coupling and substructure decoupling are considered. The former consists of the identification of the dynamic behavior of an assembled structural system starting from the dynamic behavior of component subsystems. The latter can be defined as the identification of the dynamic model of a structural subsystem embedded in a structural system known from experiments (assembled system) and connected to the remaining part of the system (residual subsystem) through a set of coupling degrees-of-freedom (DoFs). Decoupling is needed for subsystems that cannot be measured separately, but only when coupled to neighboring substructures. Issues concerned with coupling DoFs, configuration dependent interfaces, and interface optimization are addressed, along with perspectives of developments which include coupling with localized nonlinearities and joint identification.

In a cross-disciplinary contribution on *bluff-body aerodynamics*, Buresti and Piccardo [13] dwell on research challenges posed by *wind engineering* to civil structures, by reviewing a few important topics and recent developments. Vortex shedding response and galloping are considered, along with their possible interaction, which is a complex and open issue. Then, attention is paid to the interference between bluff bodies and to its effects on aerodynamic loads and possible aeroelastic phenomena, pointing out the still unsatisfactory level of relevant information. The problem of the prediction of the loads acting on bodies subjected to accelerating flows, as those occurring in thunderstorms, is also tackled, dwelling on the procedures to evaluate acceleration-induced forces and on their potential importance in wind engineering. Final comments refer to further general challenges. For aerodynamicists, the need to attain an increased understanding of the involved physical phenomena by a proper synergy between numerical simulations and wind tunnel tests; for wind engineers, the great variability of shape and arrangements of civil structures, and the need to exploit novel techniques of data analysis founded on AI-based schemes and the uncertainty quantification issue.

Blondeaux and Vittori [14] dwell on the periodic *sedimentary patterns* generated by *tidal currents* flowing over a cohesionless sea bottom, and characterized by spatial and temporal scales ranging from centimetres to kilometres and from minutes to centuries. The theoretical models used to investigate the mechanisms leading to the appearance of different tidal bedforms are based on different approaches. The authors consider separately the formation of ripples and dunes, sand waves, long bed waves and sand banks, overviewing some main results obtained with the stability analysis in the last few decades. Linear stability furnishes reliable predictions of the wavelength and orientation of a large number of periodic bedforms, while in a few cases a weakly nonlinear stability analysis can be used to estimate the equilibrium amplitude of the bedforms. However, in many cases, the equilibrium bottom configuration can be predicted only by accounting for strongly nonlinear effects and by using a numerical approach.

Despite a declared bias for the topics in which they have been involved directly, Luchini and Quadrio [15] dwell in general to the meaningful contribution given by Italian Mechanics to advanced research in *fluid flow turbulence*, which is probably the last unsolved mystery of classical physics. The authors describe progress in the comprehension of wall-bounded turbulence and the reduction of turbulent skin-friction drag, in the four aspects of (i) shape of the mean velocity profile and applicability limits and precise values of the coefficients of the classical logarithmic law, (ii) statistics of turbulent fluctuations and their multidimensional characterization in space–time, (iii) passive (static) surface modifications that may offer some reduction in skin-friction drag, and (iv) active (moving-wall) modifications that may provide a larger reduction at the expense of greater complications.

Gallo and Casciola [16] deal with *multiphase fluid flows*, overviewing the rich theory that, starting with the basic aspects of statistical mechanics and density functional theory, touches upon the subject of nonhomogenous fluids and their dynamics. The authors discuss a recently devised mesoscopic approach to deal with the *nucleation dynamics* of a daughter phase in a metastable mother fluid, with application to bubble/droplet nucleation in metastable liquids/vapors. Despite the intention of not going into too many mathematical details, the physical phenomena are viewed according to a mainly theoretical perspective, providing a quite rigorous summary of the techniques to be used and of recently obtained results. By numerical solution of the stochastic system of partial differential equations the model is shown able to deal with both homogenous and heterogenous nucleation over surfaces of different wettability and geometry, and to couple the nucleation process of the new phase with the large-scale flow dynamics. The approach bridges the gap between the atomistic scale where nucleation takes place and the macroscopic dynamics, reaching a wide range of length and time scales in a problem where thermal fluctuations are crucial ingredients.

Homogeneization is a powerful modeling theory to address problems of heterogeneous media of different nature, as also highlighted in this volume by two contributions dealing with its use in fluid and solid mechanics.

Bottaro [17] employs multiscale homogenization to derive boundary conditions which model the effect of microscopically *patterned surfaces* on the macroscale flow of a *fluid*. The procedure revolves around the separation of spatial scales, and comprises two steps. Auxiliary problems in a representative volume element capturing fine-scale details are solved, to identify appropriate tensorial coefficients which are used in effective conditions of the macroscale problem at a smooth virtual boundary, thus avoiding the need of numerically solving fine-grained details near the textured wall. Summarizing some main recent results, the approach is applied to treat the motion of a fluid near a porous medium, to assess the macroscopic behavior of a liquid near regularly patterned superhydrophobic surfaces, to describe the effect of wall riblets on the skin friction drag for a turbulent boundary layer flow. Homogenized boundary conditions are deemed advantageous to describe a microstructured wall, e.g. as an initial approximation in an inverse design approach aimed at identifying those surface patterns more likely to yield a desired effect.

Gambarotta, Bacigalupo and Lepidi [18] focus on *periodic architected materials* such as lattice-like and rigid blocky materials, in which the periodicity of the microstructure determines considerable scale effects, implying boundary layer phenomena and dispersive propagation of elastic waves. Although these materials may be accurately modeled through discrete Lagrangian systems, the need to derive synthetic descriptions of the mechanical properties and to reduce the computational burdens motivates the formulation of non-conventional non-local homogenization techniques able to accurately describe the static and dynamic response. Authors' most recent theoretical contributions in the field are overviewed. Upon generally dwelling on architected and mechanical metamaterials and on standard continualization methods, the focus is on enhanced *non-local homogenization* schemes, and on innovative surrogate optimization techniques for the spectral design of a new generation of metafilters.

Micro Electro Mechanical Systems (MEMS) is a 'new' branch of Engineering that over the last thirty years had an impressive development in terms of research, potentialities and diffusion. MEMS are now widespread as micro sensors and/or micro actuators, and can be found in many objects of common use. Summarizing the activity carried out in the *MEMS modelling and design* group at the Politechnic of Milan along the last 20 years, Corigliano et al. [19] give an overview of the importance of Mechanics in the study, design and fabrication of MEMS. Inertial and piezoelectrically actuated MEMS are first described. Then, key issues concerning microsystems reliability are discussed, such as fracture, fatigue and consequences of impacts due to accidental drop, along with uncertainty-related issues at the device scale. Future research perspectives are concerned with smart MEMS, in which sensing and actuation are accompanied by a pre-treatment of acquired data, also via machine learning approaches; with bio-MEMS, with the high level of reliability necessary for application to humans; with meta-MEMS, exploiting ideas coming from the study of metamaterials; with MEMS designed ad hoc, by using additive manufacturing techniques at the micro-scale.

Indeed, despite its widespread adoption among many key industrial sectors over the recent years, the potentiality of *Additive Manufacturing* (AM) is still far away from being fully exploited. Besides technological aspects, AM is still a very active and multidisciplinary field of research, which includes material science, metrology, computational mechanics, mathematics, chemistry, biology, medicine, and other disciplines. Auricchio and many younger co-authors [20] overview recent progresses on AM research, with a special focus on the work carried out in the last decade by the Computational Mechanics and Advanced Materials team at the University of Pavia. They dwell in phenomenological terms on computational issues in thermomechanical models, structural and topology optimization, 4D printing, advanced simulation technologies for design, support for surgery, bio-printing, and new materials.

The last three papers deal with *biomechanical* problems, either in general terms or focusing specifically on the theoretical description and understanding that Mechanics can provide as regards movement in humans or animals, and their control.

Making reference to the AIMETA background, Bisegna, Parenti Castelli and Pedrizzetti [21] present a wide review of advanced studies in *biomechanics*, as exponentially developed mostly in about the last two decades. The field is definitely cross-disciplinary even from a multiple perspective, because of particularizing to and possibly exploiting knowledge and expertise from different areas of mechanics, i.e. solids, fluids and machines, for effectively dealing with a great variety of problems in the science and practice of life. The breadth of the theme has programmatically given rise to the longest research contribution of this anniversary volume, articulated in three main subsections devoted to tissue/solid biomechanics, biological fluid mechanics, and articular joints up to rehabilitation. While somehow reflecting the articulation of research knowledge in the above mentioned three 'souls' of Mechanics, the chapter aims at looking at biomechanics problems according to a somehow unitary perspective, which is also the fruitful outcome of the worth cooperation established among the three authors in their joint leadership of the AIMETA Group of Biomechanics, along with other colleagues.

De Simone and Teresi [22] dwell on the problem of shape control, also known as *morphing problem*. First, they review how ideas about controlling shape have emerged in the last few decades in the literature on the mechanics of materials where, upon being initially formulated to study ductile metals and shape memory alloys (hard materials), they have been successfully applied to liquid–crystal elastomers (soft materials), and eventually to biological matter to describe stress-driven growth. In all cases, the central mechanical notion is the multiplicative decomposition of the gradient of the visible deformation into elastic and inelastic parts. Then, the authors focus on how the same ideas can be applied to the *mechanical modeling of muscles*, showing that the notion of multiplicative decomposition is useful to understand the different functions of a muscle as motor or brake, and the key difference in the way skeletal, cardiac, and smooth muscles solve the push problem of soft tissues. Perspectives of development are outlined in terms of nonlinear elasto-plasticity with large, time evolving distortions, characterization of remodeling actions via coarse grained models of the phenomena occurring at smaller scale, and general studies of stress-free morphing in both biological matter and engineered structures.

Fish swimming is a subject of interest in fluid mechanics, at the border with other disciplines in the field of environmental sciences, whose main complexity is given by the interaction between the fish body and the unbounded fluid domain, otherwise at rest. Graziani and Piva [23] deal with the problem within the context of the relevant literature, and summarize the more significant results obtained recently by their research group. Claiming that *self-propulsion* is the proper unified procedure to evaluate the performance in terms of optimal locomotion speed, expended energy, and cost of transport, for both undulatory and oscillatory swimming, the authors present a simple theoretical approach to study free swimming and the most suitable simulation technique to obtain significant results. The proposed impulse approach succeeds in capturing the most subtle aspects of the problem, highlighting the roles of the added mass and of the vorticity release contributions to determine, beyond the fish locomotion, the recoil motions which are of major importance for the evaluation of swimming performance.

3 Personal Remarks, Apologies, and Worth Aspects

Of course, due to the inevitable selection made, a substantial amount of important research topics addressed by the Italian Mechanics community are missing, or are not adequately treated, in the presented overview, together with the role of esteemed and recognized international scholars, and the research groups behind them, who have achieved significant results in the past decade or more.

Actually, a few invited senior colleagues were unable to contribute, due to personal reasons (job commitments and, unluckily, also health issues) or even to a certain *hýbris* sometimes associated with the brilliance and depth of thought of aristocratic intellectuals. This was the case with a qualified review paper on one more ‘new’ and trendy topic, prepared and submitted for publication. But the contribution was unfortunately much longer than the average of all the others, moreover often dedicated to thematic areas of mechanics even considerably wider. The authors were kindly asked to shorten the contribution in some way, although they were still allowed a number of pages not trivially higher than the average. Unfortunately, they refused to do it and withdrew the contribution, preferring to send it to a journal, as the writer had suggested for the long version. In addition to neglecting the basic rules of democracy and resulting in a total number of book pages higher than that agreed with the publisher, allowing the contribution to appear in long form would have been ethically incorrect in thematic/cultural terms, due to the related implicit recognition of a greater relevance of that theme compared with the others present in the book. Something which would have been unfair to all authors who diligently, albeit often regrettably, accepted a reduction in length.

In any case, sincere apologies are due from the writer, as the book editor, for the at least partial incompleteness of the panorama of themes dealt with by the Italian community of scholars of Mechanics, present in this volume.

Yet, worth aspects of the resulting collection are to be noted, too. They include, on one hand, the presentation of a variety of models and methods with the needed mathematical rigor, albeit in necessarily abridged forms. On the other hand, the qualitative description of physical phenomena in ways understandable even by not specialists, according to the intended transversal audience of the book. Authors acquainted with ‘hard’ (i.e., analytical/quantitative) treatments of their topics, who forced themselves to dwell with them according to a somehow ‘softer’ perspective, are gratefully acknowledged for the efforts done. These are also consistent with the need for a scientific and cultural cross-fertilization between different research areas which is in the AIMETA scope, as per the rationale of its national congresses and its international journal. It is also worth mentioning the panorama of ongoing research and of challenging problems in both ‘classical’ and ‘novel’ topics, which emerges from the basket of presented contributions, along with the sketch of perspectives for future developments. An issue that is certainly in the interests of both scholars of the international community of Mechanics and of new generations of Italian scientists.

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Eugenio Beltrami à l'Académie des Sciences and Adhemar de Saint-Venant all'Accademia dei Lincei

Giannantonio Sacchi Landriani and Alessandro Russo

Abstract The term "long nineteenth century" to indicate the historical period from 1789 to 1914 (i.e. a dilated XIX century) is due to the English historian Eric Hobsbawn for its historical and cultural complexity. This paper concerns two political and civil cultural protagonists of the "long century": Eugenio Beltrami (1835–1900) and Adhémar-Jean-Claude Barré de Saint-Venant (1797–1886).

Keywords History of mechanics · Nineteenth century

1 Saint-Venant Linceo

1.1 Accademia Pontificia dei Nuovi Lincei in Rome

In 1847 the ancient Accademia Pontificia dei Lincei in Rome was refounded by Pope Pius IX. Times are turbulent. In 1848 there are important revolutions and in 1849 the Roman Republic is founded with the Pope's exile in Gaeta. France supports the Pope and gives him power. The Pope's gratitude towards France will be attenuated due to the alliance of Napoleon III with the king of Sardinia for the war against Austria in 1859.

It is the beginning of the unification of Italy. On the base of the Plombières Agreement, and as a consequence of the Battle of Solferino, Austria cedes Lombardy and Savoy becomes French. The Pope excommunicates Cavour, the architect of the unification of Italy. Relations between the Church and France are again spoiled but not catastrophic because of the pro-Catholic policy of the French Emperor. In the period up to 1870 the political equilibrium is precarious and de Saint-Venant is elected member of the Accademia dei Nuovi Lincei.

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It is with a letter of March 6, 1866 that the president of the Accademia dei Nuovi Lincei sends to the Particular Secretariat of the Pope (the *Camerlengato di Santa Romana Chiesa*) the report of the meeting of the Accademia where, by election, new members are proposed. Among them appears the name of Adhémar Claude Barré de Saint-Venant "emeritus chief engineer in the corps of engineers of the Ponts et Chaussées of the French Empire, known because of very acclaimed publications concerning either rational or industrial mechanics".[1] The 9th of the same month is the date indicated on the document which ensures the approval of the Pope.

We find only weak traces of an active participation of Saint-Venant in the work of the Accademia, which moreover was going through very difficult years because of the precariousness of the State of the Church.

A letter, which we had in our hands, had been sent (1869) to Saint-Venant by the perpetual secretary (P. Volpicelli) of the Accademia. It concerned the promise of presentation of certain memoirs at a future meeting of the Accademia and the assurance of the return to Saint-Venant of duplicates of publications. The letter is addressed to the Count (Saint-Venant had received the title from the Pope in the same year) and member of the Académie des Sciences (he had been received the nomination in the previous year, when he was 71 years old).

In the de Saint-Venant's archives at the École Polytecnique of Paris it can be found the reply of de Saint-Venant, moved for having been nominated Academic member of the Church of Rome.

1.2 The Istituto Lombardo Accademia di Scienze e Lettere

In the eighteenth century Lombardy had experienced an important period from the cultural point of view under the reigns of Maria Teresa and her son Joseph II, enlightened Habsburg kings. The "Milano dei Lumi" (Enlightened Milan) had seen eminent men of culture such as Cesare Beccaria and the Verri brothers, experts in economics and law, Ruggero Giuseppe Boscovich and Alessandro Volta, in astronomy and physics. Many of them were in correspondence with Diderot and d'Alembert, authors of the Encyclopédie.

At the end of the 18th century, Napoleon Bonaparte ensured the continuity of the Enlightenment culture and set up the Istituto Lombardo Accademia di Scienze e Lettere in Milan, with Alessandro Volta as president [1]. In 1814, with the fall of Napoleon, the restoration gave rise to the second Austrian domination which exercised a police censorship on cultural activities [2].

In 1848 with the revolt in Milan it opens a period that will be called Risorgimentale and will re-emerge, as members of the Istituto Lombardo, scholars of mechanical and mathematical science with a new notoriety. Gabrio Piola (1791–

[1] The Vatican motivation reads: "*Adhémar-Jean-Claude Barré de Saint-Venant ingegner capo emerito nel corpo degli ingeneri di Ponts et Chaussées de l'Impero francese, conosciuto a causa delle molto applaudite pubblicazioni concernenti la meccanica sia razionale sia industriale*.".

1897) astronomer and mathematician, Francesco Brioschi (1824–1897) animator of research in mechanics of solids and fluids. He was responsible for the appointment to the Istituto Lombardo of Augustin-Louis Cauchy (1855) and Rudolf Clebsch (1869) and Italians such as Enrico Betti, Eugenio Beltrami, Luigi Cremona.

Lagrange (1736–1813) was born in Turin where he lived until 1766. His pupil Cauchy, also a fervent legitimist, abandoned Paris in 1830 and lived in Turin until 1833. In this period he had met in Milan Gabrio Piola (legitimist).

Brioschi had been a pupil of Piola and it can be argued that Beltrami is among Brioschi's pupils the one who studied and criticized most effectively, and with the greatest precision, several aspects of the theory of elasticity due to the unquestionable master de Saint-Venant.

Clebsch will be in Milan in 1868 where he will meet Beltrami, helping him to get acquainted with the work of de Saint-Venant.[2] After the unification of Italy (1861), he had contacted Francesco Brioschi who had initiated him to the studies of Geometry and then of Mechanics with considerable success. It is understood that his political attitude was liberal and that the environment in which he worked was definitely so. A circumstance not without interest in regards to his difficult relations with de Saint-Venant.

1.3 The Reale Accademia dei Lincei

The Accademia Pontificia dei Nuovi Lincei after 1870 resumed its activity with new directives and liberal staff. It changed the name to *Reale Accademia dei Lincei* and the most authoritative members of the Istituto Lombardo became members, including Eugenio Beltrami who will be its president in 1898. From 1870[3] the political climate of the Accademia dei Lincei had changed, which must have upset de Saint-Venant.

Legitimist Saint-Venant could not be comfortable in an Accademia dei Lincei which was no longer Pontificia and which was dominated by Liberals, whose minds were seized by the building of the new nation and who saw still in the Pope an opponent of the unity of Italy.

On the occasion of the death of de Saint-Venant, in the Atti della Reale Accademia dei Lincei appeared a small announcement, very dry, certainly not corresponding to the magnitude of the deceased. It's very hard to find in the archives of the École Polytechnique evidence, after 1870, of any correspondence between our men of science and the Accademia, and we couldn't find any reply to Beltrami, who was, among the Italian scholars, one the finest connoisseurs of the work of de Saint-Venant. One can understand the difficulty of a direct relationship if one thinks of the weight of the difference in political and cultural position, at least on a psychological level. In this regard, we can even cite a French example, that is, the absence of documents regarding the relations between de Saint-Venant and Eiffel.

[2] Saint-Venant declares himself a pupil of Lagrange and Cauchy.

[3] 1870 is a key year for Rome, no more papal because of the Franco-Prussian war and the Commune.

It is nevertheless important to point out that in 1889, the year of the Tower, the Academy of Sciences awarded the price of Mechanics Montyon to Gustave Eiffel for all his metal constructions. Were part of the jury Maurice Lévy and Boussinesq, two of the closest students of Saint-Venant. And again, on the death of Beltrami (a member of the Institute) occurred in 1900, Maurice Lévy wrote on the Comptes Rendu de l'Académie des Sciences, of which he was president, a very detailed and very laudatory obituary. All of this leads us to imagine that the reluctance of de Saint-Venant were more in his outward manifestations than in his mind, since the closest students of him openly expressed judgments and praises that he omitted.

1.4 Beltrami and de Saint-Venant

It is appropriate to linger on some observations that Beltrami made to de Saint-Venant.

On the Reports of the Istituto Lombardo appeared (1885) a memoir entitled *Sulle condizioni di resistenza dei corpi elastici.* The problem of knowing the causes of breakage of a material had been posed for a long time (Galileo, Coulomb, Clebsch, de Saint-Venant). Beltrami quotes Clebsch to report the opinion of de Saint-Venant which identifies the cause of rupture in the maximum expansion. That is, a material is able to withstand an expansion whose value does not exceed the characteristic limit. De Saint-Venant however, in the case of triaxial loading, neglects the influence of the transverse Poisson contraction.

Beltrami raises criticisms underlining that the proposed criterion would have physically unacceptable consequences and proposes that a specific elastic characteristic limit energy be considered for each material. He sends a copy with a dedication of his proposed correction to de Saint-Venant but he doesn't answer.

Beltrami, however, must be considered the first to have identified energy as a cause of rupture. After a few years, Huber and von Mises criteria will appear, in the wake of the Beltrami criterion, which are still useful today in solid mechanics.

It's worth mentioning the *post scriptum* of the paper, in which the author declared his scientific debt to the *dotto ingegnere* Alberto Castigliano (1847–1884) who had died at a young age the year before. The *post scriptum* shows the Beltrami's great respect for Castigliano and considers his theorem as a milestone of the Elastic Structures theory. "*Looking through all [the] applications, it is easy to see that little has been added to this branch of the theory of structures since Castigliano wrote his famous book*" as Stephen Timoshenko wrote. In the de Saint-Venant's documents at the École Polytechnique de Paris there is a copy of the Beltrami's paper with a dedication to de Saint-Venant.

A second interesting proposition by Beltrami, and perhaps the one that made him more famous in the domain of the mechanics of continuous media, lies in the proof that the specific conditions of small strains, which de Saint-Venant had shown to be necessary conditions, are also sufficient. It should be emphasized that the Beltrami memoir about this question *Sull'interpretazione meccanica delle formole di Maxwell* dates back to February 1886, and de Saint-Venant had died in the previous January. It can therefore be assumed that de Saint-Venant knew Beltrami's proposition.

Beltrami entered into the still opened debate on the elasticity theory with authority, assuming a courageous critical position, especially in international sessions, towards the most accredited experts and sharing completely the thesis, not yet universally accepted, defended by Lamé. The proof of sufficiency of the internal strain compatibility conditions for small strains, appeared as a note in the paper mentioned above. Beltrami claims that the formulae, with which Maxwell defines a stress state, represented in an elastic medium by a potential function, do not have a general validity and in the case of isotropy they are only a potential linear function of the place. To support his position Beltrami has resorted to sufficient internal congruence strain conditions. He argues that Saint-Venant's necessary conditions are also sufficient, and he proves it in a note.

In modern mathematical language, the results of de Saint-Venant and Beltrami can be stated in the following way, quoting [3].

> Let $\boldsymbol{v}$ a smooth vector field defined on Ω, then the corresponding strain field $\nabla_s(\boldsymbol{v}) = \frac{1}{2}(\nabla \boldsymbol{v}^T + \nabla \boldsymbol{v})$ is a symmetric matrix field. The characterization of the smooth symmetric matrix fields $\boldsymbol{E}$ that are strain fields, i.e. that can be written as $\boldsymbol{E} = \nabla_s(\boldsymbol{v})$ for some $\boldsymbol{v}$, goes back to the second half of the Nineteenth Century. Indeed, it was discovered by A. J. C. B. de Saint-Venant (1864) the following result:
>
> **Theorem 1.1** (Saint-Venant's necessary compatibility theorem). *The strain field $\boldsymbol{E}$ corresponding to a class $C^\infty(\Omega; \boldsymbol{R}^3)$ displacement vector field $\boldsymbol{v}$ satisfies the compatibility equations*
>
> $$\boldsymbol{CURL\,CURL\,E} = \boldsymbol{0}.$$
>
> The components of the matrix $\boldsymbol{CURL\,E}$ are:
>
> $$(\boldsymbol{CURL\,E})_{ij} = \epsilon_{ipk}\, E_{jk,p}$$
>
> where the commas stand for partial differentiations with respect to x and ϵ_{ipk} denotes the alternator:
>
> $$\epsilon_{ipk} = \begin{cases} +1 & \text{if } (i,p,k) \text{ is an even permutation of } (1,2,3); \\ -1 & \text{if } (i,p,k) \text{ is an odd permutation of } (1,2,3); \\ 0 & \text{if } (i,p,k) \text{ is not a permutation of } (1,2,3). \end{cases}$$
>
> The first rigorous proof of sufficiency was given by E. Beltrami (1886) in the following form.
>
> **Theorem 1.2** (Beltrami's sufficiency compatibility theorem). *If Ω is a simply connected domain and if a symmetric matrix field $\boldsymbol{E} \in C^\infty(\Omega; M^3_{\text{sym}})$ satisfies the compatibility equations above, then there exists a vector field $\boldsymbol{v} \in C^\infty(\Omega; \boldsymbol{R}^3)$ satisfying the strain-displacement relations:*
>
> $$\nabla_s(\boldsymbol{v}) = \frac{1}{2}(\nabla \boldsymbol{v}^T + \nabla \boldsymbol{v}).$$

We gladly quote a third contribution from Beltrami to mechanics, although it appeared relatively late (that is to say in 1892 and therefore such that it could not be considered in direct relation to Saint-Venant) and which provides insight into a problem that has, so to speak, crossed the nineteenth century.

Beltrami quotes the translation in French (written by de Saint-Venant), from the text of Clebsch, about the coefficients of elasticity, and he uses the expressions proposed by Lamé. This shows that Beltrami had accepted, in agreement with Lamé, Green's proposals. Proposals on which de Saint-Venant had never taken a definitive position.

The result obtained by Beltrami consists in the formulation of elastic equilibrium equations in terms of stresses.

2 Beltrami Académicien

Vito Volterra defined the year 1858 as *annus mirabilis* in a conference held in Paris in 1900, because it can be considered as the symbolic year of the actual affirmation of the Italian mathematics in Europe.

Indeed Brioschi organized and made a journey in Europe with Enrico Betti, and the young Felice Casorati. They went to Göttingen, where they got acquainted with Riemann (1826–1866), Dirichlet (1805–1859) and Clebsch (1833–1872), then they went to Berlin where they met Kronecker (1813–1891) and Weierstrass (1815–1897), and to Paris where they got acquainted with Hermite (1822–1901) and Bertrand (1822–1900), and, probably, with Lamé (1795–1870). This event was pregnant also for the development of solid mechanics (especially for the Elasticity Theory), because, as we will better see, Brioschi's scholars gave the most significant contributions to this branch of mechanics. We must remember that Brioschi, Betti and Casorati left Milan that was still under the Austrian Government, and the cultural euphory diffused by the coming of the unity of Italy vivified the atmosphere.[4]

It is generally accepted, that the origin of the modern solid mechanics is to be found in the publication of Louis Navier (1782–1836) *Mémoires sur les lois de l'équilibre et du movement des solides élastiques* (Mémoires de l'Institut Nationale, 1821) [4] regarding the constitution of matter. A generic solid was conceived as a group of simple masses connected with springs. The model, usually called *molecular*, enabled Navier to write, in terms of displacement, the indefinite equation of elastic equilibrium. In the case of isotropy of the material the equations are characterized by a single constant G', that represented the deformability of the ideal strings modeling the elastic solid.

In 1828 appeared the paper *Sur l'équilibre et le movement d'un système de points matérieles sollecités par des forces d'attraction ou de répoulsion naturelle* (Exercises de mathématique, 3, Paris, 1828) of Augustin-Louis Cauchy (1789–1857), that will leave an indelible mark in all the modern solid mechanics [5]. The definition of stress, its representation by three vectors in a Cartesian orthogonal system, its law of variation with respect to the reference system will characterize the notion of stress tensor: the milestone of continuum mechanics, as Clifford Truesdell wrote [6].

[4] In the classical work of Isaac Todhunter and Karl Pearson *A History of Elasticity and of the Strength of Materials* more than twenty-three pages are dedicated to Gabrio Piola with a critical and detailed descriptions of six papers published in *Opuscoli* (1833) and in *Memorie della Società Italiana di Scienze moderne di Modena*.

The year of the Unification of Italy (1861) constituted an appeal to the renewal of the scientific research, whose protagonist was in our field Francesco Brioschi, former a well-known mathematician. He left the University of Pavia, of which he was Rector, to found in Milan in 1862 the *Istituto Tecnico Superiore* (today *Politecnico*). He became shortly Senator of the Kingdom of Italy, President of the *Istituto Lombardo*, President of the *Accademia Nazionale dei Lincei*, Academician of the *XL*, Editor of the *Annali di Matematica*.

The productive cooperation between the French and German Schools came to an arrest in 1870 because of the Franco-Prussian war. The weakening of France due to the defeat and the tragic events of the *Commune de Paris*, allowed the conquest of Rome and the Unification of Italy.

The new Italy succeeded in proposing itself neutral and became the ideal forum of the two leading scientific communities beyond the Alps.

Beltrami became an appreciated member of the *Académie des Sciences de Paris*; Cremona had contacts with Eiffel and Culmann, prominent engineers.

The querelle about the continuum concepts of Navier and Cauchy, continued for almost 50 years, and constituted a scientific event in some respect if we think that personalities such as de Saint-Venant and Poisson (1781–1870) had been reticent in agreeing with Green's thesis supported by Lamé. In 1829 Poisson had defined, for an isotropic elastic solid, the relationship among the normal elasticity modulus E, the tangential elasticity modulus G, and the coefficient of transversal contraction ν [7].

At that time there was among the scholars an agreement in accepting that the isotropic elastic solid could be described with just one constant. It seemed acceptable, by the law of Poisson and by the uniqueness of the constant, a value of 0.25 for ν, and a rate between G and E of 0.4. The fundamental input to the debate on the elastic constants defining the isotropic solid given by Green (1793–1841) in 1837 restored substantially the macroscopic model of Cauchy. De Saint-Venant, however, was still reticent for a long time on the molecular model of Navier. The different experiences, in which also Kirchhoff was engaged (1859), didn't confirm the wrong rate 0.25 of ν.[5]

Eugenio Beltrami agreed with the thesis of Green; in particular, referring to the English author who was interested in the definition of the elastic potential energy concept, published different notes on the argument.

The Riemann and Beltrami's long permanence in Pisa gave to Enrico Betti the opportunity of studying differential geometry, bound to become an effective instrument in the analysis of non-Euclidean manifolds.[6]

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[5] De Saint-Venant himself, in one of the long notes to the French translation of the Clebsch's Treatise, though accepting the Lamé's and Green formulation, believes that for a isotropic compact material, the elastic constants can be reduced to one only.

[6] The engineers mastered quickly the graphical methods and these methods became effective and often irreplaceable instruments, handled with certainty but with no interest towards the advanced higher Geometry that had produced them. And this point for Cremona, who conceived geometry as a training for the spirit, should have been surely disappointing.

Eugenio Beltrami[7] gave important and original contributions to the elasticity theory, but also to the differential geometry, that had found in Riemann the most important inspirer during the time in Pisa with Betti.[8] The paper *Sulla condizione di resistenza dei corpi elastici*, published in *Rendiconti dell'Istituto Lombardo* in 1885 assumes as reference configuration for the strain energy a prestressed state.

3 Different Opinions of Beltrami and Morera About Elastic Equilibrium Equations [8]

Beltrami published (*Osservazioni sulla Nota Precedente, Rendiconti Acc. dei Lincei, Vol. 1, Serie V,* 1° *Sem., pp. 141–143, Roma 1892*) the indefinite equations of the elastic equilibrium in terms of stress: therefore a dual formulation of the Navier equations modified by Lamé. The note, published as a remark of a Morera's paper presented by Beltrami, proposed the general solution of the equilibrium indefinite equations through three scalar arbitrary functions to express the stress components. Consider that Lamé and Clapeyron had already written the Navier's equations starting from the Cauchy's formulation in 1833.

In the *Osservazioni* E. Beltrami points out that Morera's solution corresponds to a particular case of the equilibrium equations. One month later, Morera replies that instead it is possible to solve the problem in its full generality. Morera's note is again presented by Beltrami himself, so we can deduce that Beltrami agreed with Morera's conclusions.

Morera's statement is formally right, but it is limited to the integration of the equilibrium equations. On the other hand Beltrami's goal is to write the elastic equilibrium equations in terms of stress, so that it is justified the focus on all six strain components α, β, γ, λ, μ, ν.

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[7] Eugenio Beltrami (1835–1900), was student of Mathematics of the *Collegio Ghisleri di Pavia*, when, in 1855, he was expelled because he had showed sentiments of Italian character. After 1859 he introduced himself to Brioschi, who groomed him for the scientific research, though he hadn't a degree (he will never get it). Mathematician, Brioschi's student, he began his Professor career at the University of Bologna in 1862 and continued it in Pisa, Pavia and, finally, in Rome. He was Senator of the Kingdom and President of the *Accademia dei Lincei*. In 1892 he delivered the commemoration speech of Betti (*Rend. Circolo Matem. di Palermo*). He gave the commemoration speech of Brioschi and a speech honoring Kelvin at the *Accademia dei Lincei*. He succeeded to Clausius at the *Académie Francaise*. He participated in the life of the Istituto Lombardo with several memoirs. Somigliana pronounced his necrology. Maurice Levy published Beltrami's commemoration in the *Comptes Rendus de l'Académie des Sciences*. Among the most important memoirs written by Beltrami we can cite: *Sulle equazioni generali dell'elasticità. Annali di Matematica pura ed applicata*, Milano 1881; *Sulla teoria del potenziale, R. Ist. Lomb.*, 1883; *Sulla rappresentazione di forze newtoniane per mezzo di forze elastiche. R. Ist. Lomb.* 1884; *Sull'uso delle coordinate curvilinee nelle teorie del potenziale e dell'elasticità. Regia Accademia delle Scienze di Bologna, serie IV, tomo VI,* 1885; *Intorno al mezzo elastico di Green nota II. R. Ist. Lomb*, 1891.

[8] The Beltrami's cap, that is the pseudosphere of negative constant Gauss curvature, is hold by the Mathematics Department of Pavia University.

3.1 A Modern Interpretation of the Discussion

In modern terms, if we write the equilibrium equations as

$$\operatorname{div} \boldsymbol{S} = 0, \quad \boldsymbol{S} = \boldsymbol{S}^T, \tag{1}$$

then Beltrami's solution corresponds to representing $\boldsymbol{S}$ as

$$\boldsymbol{S} = \boldsymbol{CURL}\ \boldsymbol{CURL}\ \boldsymbol{A} \tag{2}$$

where $\boldsymbol{A}$ is an arbitrary symmetric tensor. Morera's solution corresponds to take $\boldsymbol{A}$ with zero entries on the diagonal, i.e.

$$\boldsymbol{A} = \begin{bmatrix} 0 & \omega_1 & \omega_2 \\ \omega_1 & 0 & \omega_3 \\ \omega_2 & \omega_3 & 0 \end{bmatrix}, \tag{3}$$

while Maxwell's solution (corresponding to the case $\lambda = \mu = \nu = 0$) corresponds to taking $\boldsymbol{A}$ as a diagonal tensor:

$$\boldsymbol{A} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}. \tag{4}$$

The proof that Morera's solution has the same generality of Beltrami's is easy, while the same result for Maxwell's solution has been shown only in 1979 by R. Rostamian.

3.2 Some Remarks Concerning Beltrami's Formulation

We report here some interesting remarks concerning Beltrami's formulation.

Remark 1 It is easy to switch to the two-dimensional case supposing that it is allowed to connect plane stress state to plane strain state. Indeed, if we assume instead of the general Beltrami's indefinite equilibrium expression (Eq. 16 in [8]) one of its particular forms, for instance with indices l, m equal to 3, considering that the equivalent Ricci's tensor takes the form $\epsilon_{i3r} = \boldsymbol{a}_i \times \boldsymbol{a}_3 \cdot \boldsymbol{a}_r$, we have

$$\sigma_{ij} = \epsilon_{i3r}\, \epsilon_{j3s}\, B_{33,rs}. \tag{5}$$

Assuming for i, j, r, s only the values 1 and 2, i.e. simulating that l, m have assumed the value 3 without going into index operation, we obtain the well-known Airy's form (see [9]) which involves, if we take into account the elastic isotropic constitutive law and the compatibility, the biharmonic equation found by Maxwell in 1868 [10]:

$$\Delta_2 \Delta_2 B_{33} = 0. \tag{6}$$

Remark 2 A case of two-dimensional problem is related to the bending of a thin plate so that the displacement can be represented by the vector (see [11, 12])

$$\mathbf{V} = \mathbf{S} + w\mathbf{N} \tag{7}$$

where $\mathbf{S}$ is the (vector) component of the displacement in the plane $x_1 - x_2$ of the plate and w is component of $\mathbf{V}$ in the normal direction $\mathbf{N}$. If the displacement is large compared to the thickness of the structure, it is necessary to consider the strain as defined by Green:

$$\varepsilon_{ik} = \frac{1}{2}(S_{i/k} + S_{k/i} + w_{/i} w_{/k}). \tag{8}$$

In this case the continuity condition of strain is given by the non zero component of the Riemann tensor:

$$\bar{R} = -\frac{1}{2}\epsilon^{ir}\epsilon^{ks} b_{is} b_{kr} - \frac{1}{2}\epsilon^{ir}\epsilon^{ks} S_{n/ik}\, S^{n}_{/rs} \tag{9}$$

where $b_{ik} = w_{/ik}$ are the components of the curvature tensor of the deformed structure and ϵ^{ir} are the controvariant components of the two-dimensional Ricci skew tensor in general coordinates. It is possible to write, neglecting the second order term $\frac{1}{2}\epsilon^{ir}\epsilon^{ks} S_{n/ik}\, S^{n}_{/rs}$, the elastic equilibrium equation as

$$\chi /^{mn}_{mn} = -EK \tag{10}$$

where χ is the Airy's function and K is the Gauss total curvature of the deformed surface. We remark that the left-hand-side of (10) corresponds to the left-hand-side of (6). If K is not zero, the deformed surface is not developable and we remark, considering $\sigma_{ik} = \epsilon_{ir}\epsilon_{ks}\chi^{/rs}$, that the membrane stresses can be present also when there are no planar forces acting on the boundary.

When it is possible to develop the deformed surface, K is equal to zero, and (10) becomes (6), so the problem of defining the stress state in the middle surface is equivalent to the case of a plane structure loaded on the boundary. In this case the bending state and the membrane state of stresses are independent. Note that this case corresponds to the Lagrange-Germain problem regarding the flexural plate (see [11]).

Remark 3 A remark that it is worth to point out is the *duality* between Beltrami's and Navier's equations (see [4]):

$$(\lambda + \mu) S_{j,ij} + \mu S_{i,jj} = 0 \tag{11}$$

from which we get

$$S_{i,jji} = \Delta_2 S_{i,i} = 0 \tag{12}$$

where λ and μ are Lamè's constants.

We notice that Navier's equations highlight the nature of the displacements as potential of the strain.

4 Conclusion

It is necessary to recall some remarks which are well-known to scholars of structural mechanics. We point out that Beltrami's presentation is very elegant and rigorous at the same time (see also [13, 14]). However, Beltrami's equations are less used than Navier's because they contain second order partial derivatives which are less suited for numerical approximation. On the other hand, in the approximation of Navier equations much more attention has to be paid to satisfy the stress boundary conditions.

It can also be noted that some aspects of the modern writing of Mathematics have evolved towards an increasingly synthetic representation over the centuries, "long" or "short" they were.

The considerations above represent a small part of the cultural heritage of *Meccanica*, that since its foundation in 1966 has always been aware of an extraordinary rich historical past.

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Kinematics of Machines: Contributions from *Meccanica*

Nicola Pio Belfiore and Ettore Pennestrì

Abstract Specific studies on Kinematics date back to the XVIII Century. Hence, it is rather complicate to provide any generally agreed framework where a pertinent paper can be classified. This task has been herein bravely attempted for the papers published in *Meccanica*. Particularly, six kinematic-related categories have been identified and 84 papers have been classified accordingly. A further division was made between two equally numerous groups of contributions from either Italian or Foreign Authors.

Keywords Kinematics · Mechanisms topology · Cams and transmissions · Mechanism design and analysis · Compliant mechanisms

1 Introduction

The constrained transmission of motion between machine parts is a basic design goal of main interest in mechanical engineering.

Motion is connected to the transmission of power and the consideration of forces involved is necessary. However, the dynamical problems are conceptually divided into two parts, one kinematic and the other kinetic. Traditionally, the kinematic part deals with the basic geometry laws regulating the motion of bodies, without considering their outward forms and acting forces. The kinetic part relies upon the application of the general mechanics laws to a mechanical system defined within the kinematic part. For this reason, since the time of Ampère, it forms a distinct scientific branch.

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For the purposes of this review, the *kinematics of machine* is herein interpreted as that broad branch of Mechanics gathering the wealth of knowledge to be used for the development of analysis and synthesis procedures. On this regard, geometry provides a powerful framework in studying mechanisms, and it provides criteria that lead often to actual designs. The quote from Phillips' book depicts well this view [101]:

> …mechanism is presented here as the geometric essence of machinery. The study of mechanisms is important because the geometry of mechanical motion is often the crux of a real machine's design.

This paper presents a critical classification of the contributions that the papers published in the *Meccanica* journal within the period 1970–2020 bestowed to the realm of Kinematics. The investigation has been founded on a series of queries launched on the SCOPUS database. *Meccanica* is the reference International Journal of the Italian Theoretical and Applied Mechanics Association (AIMETA): from Volume 1, which dates back to 1966, to the present days (November 2021), a volume of 3,820 papers has been published there. Among them, it is very difficult to extract all the contributions affecting Kinematics and therefore the Authors of the present review are aware of the possible existence of some contributions that might be inadvertently excluded (Authors apologize if this actually happened). A general list of papers has been obtained and then manually organized into different sections representative of well known research areas of kinematics: Mechanism topology; Kinematic analysis of planar and spatial mechanisms; Kinematic synthesis; Cams, gears and transmissions; Computational kinematics and optimization methods in mechanisms; Compliant mechanisms.

Just to offer some glimpses of the overall historic scenario at European level, the earliest phases of the kinematics of machines (second half of the XIX and beginning of XX Century) have been characterized by French (Monge, Lanz and Betancourt, Labulaye), English (Willis, Sylvester, Roberts, Kempe), German (Burmester, Reuleaux, Redtenbacher) and Russian (for example, Chebyshev) investigators. However, the Italians Tessari, Borgnis, Cavalli, Allievi and Cesàro provided significant contributions. The first half of the XX Century the contributions of German scientists (Krause, Beyer, Alt, Kraus, Rauh, Hain) mark main advancements of kinematics of machines. This is witnessed not only by the number of publications with relevant scientific advancements, but also by the quality of textbooks. The so-called German school of kinematics laid the foundations of many branches of kinematics. However, starting from the 1950, significant research advancements in the USA, Russia, Europe and China have been recorded. In Italy, University of Genova and Polytechnic of Milan pioneered the use of computer for the solution of kinematic problems. In particular, the role of Galletti and Lucifredi in the spread of digital computer use in kinematics must be acknowledged. Their work, titled Computer-Aided Automatic Design [17], outcome of series of lectures at CISM of Udine, marks the beginning of an era and witnesses the advancement of the University of Genova researchers in the development of software tools with user interface specialized for kinematics. Their general CAD system, with possibilities of a man-machine interaction, included/envisaged different modules to assist the designer in the different tasks such as: Number and type synthesis, Functional dimensions, Structural dimensions, Reliability analysis, Automatic drawings and CNC programming.

The following Sections explore the different topics for which Kinematics played a fundamental role.

2 Mechanisms Topology

At the beginning of the Sixties, starting from some concepts proposed by Auslander and Trent [8, 125], the idea of using graph theory to model the topological characteristics of a mechanism was introduced in Mechanism Science to represent the connectivity relationships between the links. A few years later, the correspondence between graphs and kinematic chains was introduced by Crossley [35], while Freudenstein and Dobrjanskyj addressed the first group of research lines. During the following years, mechanism topology, not necessarily related to Graph Theory, was very popular among kinematicians who were directly involved in some emerging topics such as

- Structural Synthesis [10, 14, 71, 79], also for parallel kinematic chains [73], and Analysis [41, 94], detection of isomorphism [3], classification [19], methods of identification of the degrees of freedom of mechanisms or kinematic chains [4], Type Synthesis [52, 81] and Number Synthesis [34]
- Automatic representation of kinematic chains and mechanisms in planar forms [11]
- Topological methods of Kinematic Analysis and synthesis [1]
- Topological methods of Static force and Dynamic analysis [12, 13]
- Assur's Groups based Methods [55].

A recent review [97] has been also dedicated to the developments of graph-based algorithms for the analysis of mechanisms and gear trains.

Considering "Mechanism and Machine Theory" to be the preferred journal for the investigators involved in mechanisms topology, at the date of publication of the present article, some different queries have been launched on Scopus database. By summing up all the pertinent papers collected during the ranges 1972–1990, 1991–2000, 2000–2010, and 2010 to present, the concept of graph has been increasingly used in Mechanism Science, with an overall increasing number of papers from 23 (2% of the papers), to 38 (4.5%), to 56 (5%) and to 85 (3.4%), respectively.

2.1 Contributions from Meccanica

- From Italian authors:
  - Position analysis by means of Assur's groups: Galletti [55].

- From Foreign authors:
  - Structural analysis of planar mechanisms: Durango et al. [44];
  - Asymmetry in kinematic structures: Simas et al. [119].

3 Kinematic Analysis of Planar and Spatial Mechanisms

The kinematic analysis of mechanisms is a vast well-known topic that has been extensively reviewed and investigated in tens of textbooks, spanning from Italian to International Literature: to only name a few, Di Benedetto and Pennestrì [37] provided an accurate account of the methods of kinematic analysis in Italy, while in the international scenario Shigley [118] and Dukkipati [43], offered an insight on plane and spatial mechanisms, respectively.

3.1 Contributions from Meccanica

- From Italian authors:
  - Sensitivity analysis of plane mechanisms: Lucifredi [82]; Five-bar mechanisms: Cambiaghi et al. [21]; Multiloop kinematic chains: Cambiaghi et al. [22]; Multiloop spatial mechanisms: Garziera et al. [58]; Stewart platform: Innocenti et al. [72]; Kinematics of robot arms: Fanghella [46]; Pin wheel joints: Bagnoli et al. [9]; Parallel mechanisms for simulation of human joints: Sancisi et al. [112]; Methods of kinematic and dynamic analysis: Di Gregorio [38], Applications of parallel mechanisms: Parenti-Castelli et al. [89].
- From Foreign authors:
  - Method based on quaternions: Rico-Martinez et al. [107]; Spatial mechanisms: Racila et al. [104]; Kinematic models for biomechanics: Klopčar et al. [77]; Methods of kinematic and dynamic analysis: Attia [6, 7]; Methods in kinematic analysis: Casey [23]; Kinematic analysis of parallel mechanisms and manipulators: Gallardo-Alvarado et al. [53, 54]; Tale et al. [123], Company et al. [31], Rodriguez-Leal et al. [109], Chen et al. [29]; Mobility analysis of mechanisms: De Bustos et al. [47]; Spherical parallel mechanisms: Sun et al. [122]; Kinematic models of spherical platforms: Hayes et al. [66], Higher kinematic pairs: Müller [88]; Redundant manipulators: Jin et al. [74], Kim et al. [76], Patkó et al. [95].

4 Kinematic Synthesis

Most part of theoretical foundation of kinematic synthesis have been known since decades. The Italian kinematician Allievi, since 1895 [2] had developed methods for 3rd and 4th order stationary curvature path generators, while a few years earlier Burmester [18] had founded the theory that nowadays bears his name. However, the complexity of calculus that had to be involved in kinematic synthesis gave rise to a variety of graphical or hand-calculated methods that offered some satisfying results, although with many restrictions due to the limited available resources. The epic change to a new Era begun in 1954, when Freudenstein started working on an IBM 650, on which an interpretative language called Bell-One offered him not more than a thousand words for programming. Nevertheless F^2 (as Ferdinad Freudenstein was affectionately called by his students, with his consent [113]) was able to implement a method of kinematic synthesis of planar four-bar function generator up to seven precision points [50]. Later on, the members of the F^2 *family tree* together with the so-called *inspired people* [102], as well as many other distinguished scientists have been developing a myriad of techniques to synthesize function generators, path generators and rigid body guidance mechanisms until nowadays.

One of Freudenstein landmark papers was on higher-order path curvature analysis, where the concept of Generalized Burmester points has been introduced. The first numerical solution for the computation of such points was published in Meccanica [98]. Recently, the first full polynomial solution of the problem, and extension of the concept to line and circle envelopes, have been deduced by Cera and Pennestrì [25–27].

4.1 Contributions from Meccanica

- From Italian authors:
  - Stephenson linkage: Riva [108]; Burmester theory: Pennestrì et al. [98]; Flapping mechanism: Callegari et al. [20], Negrello et al. [91]; Parallel mechanism: Mazzotti et al. [84], Di Gregorio et al. [39]; Exoskeletons mechanisms: Conti et al. [32].
- From Foreign authors:
  - Function generators: Hain [64]; Inversion cells: Dijksman [40]; Synthesis 1-dof linkages: Noriega et al. [92]; Parallel mechanisms: Huda et al. [70]; Methods based on nodal coordinates: García-Marina et al. [57]; Path generators: Singh et al. [120].

5 Cams, Gears and Transmissions

The transmission of power between two or more shafts is quite an old topic in mechanical engineering. One of the most famous examples in this field is represented by the "South Pointing Chariot", that appeared in China in around 2,600 B.C. [42]. This ingenious and complex differential gear train was designed in such a way that a pointer positioned on a vehicle was always pointing to the geographic South, no matter the amount of vehicle turns on the path. The analysis and synthesis of gear planetary systems received special attention in literature because they were difficult, although required, tasks. Freudenstein [99, 100], Tsai [30], their students, and many others provided deep insight into the topics, also making use of graph theory. Finally, a special type of transmission is the well-known cam-follower system, which is able to synthesise a large variety of functions, depending on the application. An extensive study of cams and cam followers has been made by Norton in 1993 [93].

5.1 Contributions from Meccanica

- From Italian authors:
  - Transmission joints: Romiti [110], Davoli [36]; Homokinetic joints: Bellomo [15]; Geared robotic wrists: Pennestrì [96], Uyguroğlu et al. [126, 127]; Planetary gear systems: Ricci [106]; Spatial cam mechanisms: Ramahi et al. [105]; Timing systems in ICE: Sequenzia et al. [116]; Noise and efficiency of gears: Höhn [67]; Spatial cycloidal gears: Figliolini et al. [48]; Power-flow and efficiency of gear trains: Wang et al. [129]; Full toroidal CVT: Milazzo et al. [86].
- From Foreign authors:
  - Dynamic loads in gears: Nadolski [90]; Epicyclic-type automatic transmissions: Esmail [45]; Twin cams design: Tiermas, [124]; Complex gear systems: Cervantes-Sánchez et al. [28]; Dual four-bar linkages transmission: Kim et al. [75].

6 Computational Kinematics and Optimization Methods in Mechanisms

The classical problems of *function generation*, *path generation* and *rigid-body guidance* have been approached by using either *exact* or *approximate* methods. *Exact* methods aim to solve exactly one or more equations obtained from specific conditions on the mechanisms, such as loop equations, constraints or design requirements. Inequalities represent typically an additional difficulty for these methods. On the other hand, due to the absence of rigid constraint on error sources, *approximate* methods are more versatile than the *exact* ones. Back in History, before the arrival of the computers, the approximate methods were chiefly executed by graphic procedures [130], while nowadays they rely on the definition of design *variables*, an objective function to be minimized, one set of inequalities equations and another one of equality equations, both representing design, geometric, kinematic or even dynamic conditions the synthesised mechanism will obey to. The least-square approach has been suggested since 1944 [80], while in 1955 Freudenstein proposed an approximate approach to synthesise a plane four-bar function generator [51].

6.1 Contributions from Meccanica

- From Italian authors:
  - Knuckle lever linkage: Maggiore et al. [83]; Function generators: Guy et al. [63], Santoro [114]; Spatial function generators: Cossalter et al. [33]; Robot workspace: Ceccarelli [24]; Parallel manipulators: Gosselin et al. [62]; Kinematics of human joints: Sancisi et al. [111]; Underactuated robotic hand: Sarac et al. [115].
- From Foreign authors:
  - Parallel mechanisms: Gogu [61], Laliberté et al. [78]; Function generators: Shariati [117], Mirmahdi et al. [87]; Kinematical analysis of overconstrained and underconstrained mechanisms: Arponen et al. [5]; Path generators: Stan et al. [121]; Bio-inspired mechanisms: Hassanalian et al. [65]; Synthesis of planar mechanisms: García-Marina et al. [56].

7 Compliant Mechanisms

Mechanisms with elastic links and joints have been used for centuries. They have been widely employed in many applications spanning from the bow and arrows to the vehicles suspension systems. The basic idea consists in allowing energy to be stored in the mechanisms for further use or dissipation, giving rise to configurations significant changes, thanks to either local or distributed deformations. This kind of mechanisms have been revisited and classified in the Nineties [85]. The new approach gave rise to a great interest in the field of Kinematics, especially because of new design methods. One of the first methodology to design compliant mechanisms is based on the adoption of the pseudo-rigid body mechanism PRBM [69] and flexure joints [68]. Later, the design of compliant mechanisms, specially with distributed compliance, involved the use of topology optimization techniques. More recently selective compliance has been associated to conjugate surfaces in a flexure, giving rise to Conjugate Surface flexure hinges (CSFH) [128]. Up to date (October 25th, 2021), 1,271 documents can be found from a query of articles whose title includes the words “compliant mechanisms”: 56 of them include the words “pseudo rigid”, in the title also, while 203 include the words “topology optimization”.

7.1 Contributions from Meccanica

- From Italian authors:
  - Flexible planar linkages: Giovagnoni [59, 60]; Intrinsically compliant electromechanical systems: Berselli et al. [16]; Bistable mechanisms: Follador et al. [49].
- From Foreign authors:
  - Compliant parallel manipulator: Quennouelle et al. [103].

8 Conclusions

The boundaries between Kinematics and Design have been well understood for centuries. However, in the last decades the importance of kinematics increased together with complexity of the systems and their requirements. This is particularly true for high-speed, differential, compliant or multibody systems. This trend is well portrayed by the contributions published in *Meccanica*. On this regard, the International Journal *Meccanica* has been playing an important role for the development of Mechanics, specially in Italy. The present paper has reviewed the contributions that this Journal offered in the fields of Kinematics and gathered together, mentioned and briefly commented 84 pertinent papers out of 3,820. The *kinematic* papers have been classified according to an arbitrary framework that has been set up by relying on the Authors personal experience. *Meccanica* deservedly attracted not only Italian, but also Foreign Authors. As a matter of fact, a half of the papers published in the field of Kinematics originates from Foreign Authors. Finally, great care has been dedicated in order not to leave any contributing paper aside from the lot. The reader is therefore asked to consider as an involuntary occurrence, any possible inconvenient omission, due to the high number of published articles in more than fifty years.

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The AIMETA Contribution to the Development of Masonry Mechanics

Daniela Addessi, Antonio M. D'Altri, Elio Sacco, and Antonio Tralli

Abstract In this chapter, the contribution of the AIMETA community to the development of masonry mechanics over the past five decades is illustrated. A review of the contributions to this field within the AIMETA conferences, as well as in the journal Meccanica, is presented by discussing no-tension models, limit analysis-based models, phenomenological models, multiscale and homogenization approaches, and block-based models. The significant impact of the AIMETA contribution on masonry mechanics is highlighted.

Keywords Masonry · No-tension material · Limit analysis · Phenomenological models · Multiscale models and homogenization

1 Introduction

The impact of the research activity developed in Italy on the topic of masonry and, in particular, masonry mechanics is definitely relevant in the world. Indeed, it can be stated that Italian researchers are undoubtedly the most productive throughout the world concerning masonry modeling, developing theoretical, computational and experimental studies, also dealing with issues related to the design of masonry structures and the reinforcement of existing structures, often proposing innovative reinforcement interventions.

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Table 1 Papers indexed in Scopus concerning research activity on masonry (September 2021)

Keywords\Country	Italy	France	UK	Germany	Spain	Portugal	USA	China
"Masonry"	5649	615 (66)	1739 (263)	860 (46)	1025 (113)	1089 (234)	3420 (201)	2737 (47)
"Masonry" and "Mechanics"	435	53 (7)	94 (18)	49 (1)	54 (5)	70 (17)	180 (23)	199 (6)
"Masonry" and "No-tension"	216	2 (1)	8 (5)	–	1 (0)	4 (3)	9 (5)	1 (0)

The quantity of Italian papers treating masonry mechanics issues that can be found in well-consolidated international database is impressive. In Table 1, the numbers of papers available in the database Scopus are reported referring to the most productive research countries, where those reported in brackets were written in collaboration with Italian authors.

It clearly appears that the Italian research on masonry modeling is much more widespread than in other countries, even taking into account that the number of researchers and research funds are considerably lower than in other countries. The motivation for such great interest on masonry relies on multiple reasons. Romans were expert masonry builders (e.g. Vitruvio) and many monuments and important constructions have been built in Italy during the centuries; the Italian monumental and artistic heritage is perhaps the largest and most valuable worldwide; Italy is typically subjected to severe seismic actions that damage existing masonry buildings. Thus, the importance of the accurate modeling of masonry structures is consequence of the old tradition and history of the Italian country.

It is towards the mid-1970s that the interest in the mechanics of masonry structures began to assume a growing importance in Italy, in correspondence with the beginning of the biennial AIMETA conferences. The reasons can be found both in the publication about ten years before of a seminal article by Heyman [28] focused on the calculation of the failure of masonry arches, and the disastrous earthquakes of Friuli (in 1976) first and then Irpinia (in 1980) which revealed the great vulnerability of Italian masonry buildings and monumental heritage.

Although research originated within the AIMETA community on masonry mechanics led to a remarkable presence of Italian contributions in the international environment (see e.g. Table 1), here the attention is focused only on contributions (occasionally preliminary) presented at AIMETA conferences or in the journal Meccanica. Indeed, a huge number of papers have been presented in the last forty years at the AIMETA conferences or published in the journal Meccanica concerning many aspects and modeling approaches of masonry mechanics. Particularly, 71 papers dealing with masonry mechanics have been published in the journal Meccanica. This can be interpreted in two ways: the international community recognizes the great Italian interest in the mechanics of masonry; the Italian Association of Theoretical and Applied Mechanics (AIMETA) encourages the research activity on the masonry and its diffusion in the world.

This chapter presents a review of the contributions within the AIMETA activities on masonry mechanics. The proceedings of the AIMETA conferences from the beginning of the 1970' have been examined and, due to the large number of contributions, the completeness of the references cannot be guaranteed. The chapter is organized in sections, containing typical modeling approaches developed during the last half century. Section 2 is devoted to the review of no-tension models. Section 3 discusses limit analysis-based approaches. Section 4 reviews phenomenological models specifically developed for masonry. Section 5 is addressed to the review of multiscale and homogenization approaches for masonry. Finally, Section 6 discusses block-based models.

2 No-Tension Models

The masonry material, mainly in historic constructions, is generally characterized by a very poor tensile strength accompanied by a good compressive strength. Moreover, even reduced dynamic actions and environmental effects, as slow penetration of humidity into the pores, can produce the development of micro-cracks in the masonry further reducing the material tensile strength. Relying on these considerations, it appears well-grounded to completely neglect the tensile strength and assume the masonry as an elastic no-tension material. The following hypotheses, initially established by Heyman [28], are classically introduced:

i. masonry is incapable of withstanding tension;
ii. masonry has infinite compressive strength;
iii. sliding cannot occur.

The no-tension masonry modeling can be considered a typical Italian approach for studying the response of masonry structures. In fact, several theoretical and computational studies have been developed during the last forty years, as highlighted by the many papers published and presented as part of the AIMETA activities.

In this framework, Di Pasquale [9] addressed the problem of masonry structures composed by rigid blocks joined by elastic no-tension material. A procedure based on the introduction of suitable eigenstrains to account for the presence of fractures was proposed. The paper can be considered as a first step toward the formalization of the no-tension model. Then, Di Pasquale [10] presented the two-dimensional model of the unilateral elastic isotropic material, introducing the non-positive definiteness constraint for the stress. The diffuse fracture pattern is described considering distributed eigenstrains and invoking the normality rule. Closed form and finite element solutions for a rectangular masonry panel were presented. Baratta and Toscano [4] illustrated the no-tension model substantially in the form still in use today. Strain is split in an elastic and a fracture part, stress is constrained to be semi-negative defined and normality rule between stress and fracture is introduced. Then, the complementary energy approach is developed and the uniqueness of the solution, if it exists, in terms of stress field is discussed.

The no-tension model was further discussed and studied in deep by: Di Pasquale [11], who provided analytical and numerical solutions for masonry elements proofing the indeterminacy of the fracture width, Baratta [5], who developed several numerical methods for approaching the no-tension problem via complementary energy, Romano and Sacco [18], who discussed the basis of the model defining a variational condition on the loading that ensures the existence of a solution.

The equations governing the constitutive law of the isotropic elastic no-tension material can be summarized as:

- strain splitting

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\delta} \tag{1}$$

  where $\boldsymbol{\varepsilon}$ is the total strain tensor, while $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{\delta}$ represent the elastic and inelastic (fracturing) strain tensors, respectively;
- stress admissibility

$$\boldsymbol{\sigma}\mathbf{n} \cdot \mathbf{n} \leq 0 \quad \forall \mathbf{n} : \|\mathbf{n}\| = 1 \tag{2}$$

  where $\boldsymbol{\sigma}$ denotes the stress tensor and $\mathbf{n}$ is the normal vector describing the typical direction in a point;
- stress–strain relationship

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}^e \tag{3}$$

  where $\mathbf{C}$ is the fourth-order isotropic elastic tensor;
- fracture strain property

$$\boldsymbol{\delta}\mathbf{n} \cdot \mathbf{n} \geq 0 \quad \forall \mathbf{n} : \|\mathbf{n}\| = 1 \tag{4}$$

- normality rule

$$(\boldsymbol{\sigma} - \boldsymbol{\tau}) \cdot \boldsymbol{\delta} \geq 0 \quad \forall \boldsymbol{\tau} : \boldsymbol{\tau}\mathbf{n} \cdot \mathbf{n} \leq 0 \tag{5}$$

The no-tension material model is, hence, characterized by some interesting and specific features. In fact, a convex strain energy function exists, so that the constitutive law is reversible and no-energy dissipation occurs during any loading history. Thus, the material model can be considered as nonlinear hyperelastic. A very exhaustive analysis of the no-tension material model was presented by Del Piero [8], rigorously deriving the constitutive equation of the masonry-like material and proving necessary and sufficient conditions for the existence of a strain energy function. Necessary conditions for equilibrium in the absence of tensile strength were also derived. The material model was reviewed in Di Pasquale [12], where masonry is thought as a special fluid–solid transition material. The governing equations of the elastic no-tension material modeling were, then, illustrated in Angelillo [2].

The no-tension model aroused great interest in the development of effective numerical methods for solving 2D and 3D equilibrium problems of masonry-like solids. Numerical procedures were usually based on suitable finite element displacement-based formulations, but others (and very original) approaches were also proposed. Lucchesi et al. [16] implemented the no-tension constitutive law into a displacement-based finite element code evaluating the tangent matrix for solving the nonlinear problem. A finite element approach based on the closest point algorithm for incremental formulation of the no-tension model was proposed in Fuschi et al. [14].

The incremental boundary element method was adopted for studying the response of no-tension masonry elements by Alessandri [1], where the nonlinear analysis is performed implementing an incremental algorithm and considering the no-tension material as a very special elasto-plastic one. The complementary energy approach, initially proposed by Baratta [5], was reviewed and implemented in Grimaldi et al. [15] into a finite element procedure introducing the equilibrium and non positiveness constraints on the stress field for solving problems on 1D and 2D no-tension structural elements. A stress approach was proposed in Fraternali [13], where the equilibrium of the stress field is implicitly satisfied introducing the Airy function and discretized over the finite element mesh by a piecewise linear approximation. This approach, named as lumped stress method, allows to define concentrated stresses arising along the interfaces of the mesh, which can also be regarded as trusses of an ideal reticular structure.

The no-tension model was, then, extended to the case of finite strain formulation in Cuomo and Fagone [7], proving that symmetric part of the gradient of the velocity tensor can be additively decomposed in an elastic and an inelastic part. The model was implemented in a standard isoparametric finite element model using a linearization of the time derivative of the right Cauchy-Green tensor, the velocity of deformation tensor and the Lie derivative of the Kirchhoff stress tensor.

A further enhancement of the classical normal, elastic, no-tension model was proposed by Angelillo et al. [3], where this was extended to associate path-dependent plasticity, allowing the onset of fracture and irreversible crushing of the material, and highlighting the progress of pseudo-rigid kinematics. The elastoplastic problem was decomposed into a sequence of nonlinear elastic variational problems, solved by searching for the functional minimum. An alternative numerical approach for the analysis of no-tension masonry-like solids was presented by Bruggi and Taliercio [6], who reformulated the problem within the framework of topology optimization. Thus, the equilibrium of a 2D no-tension structure was investigated through an equivalent no-tension orthotropic material, obtained by prescribing negligible stiffness in the relevant direction, such that the potential energy of the solid is minimized. Finally, a generalization of the constitutive equation of masonry-like materials was carried

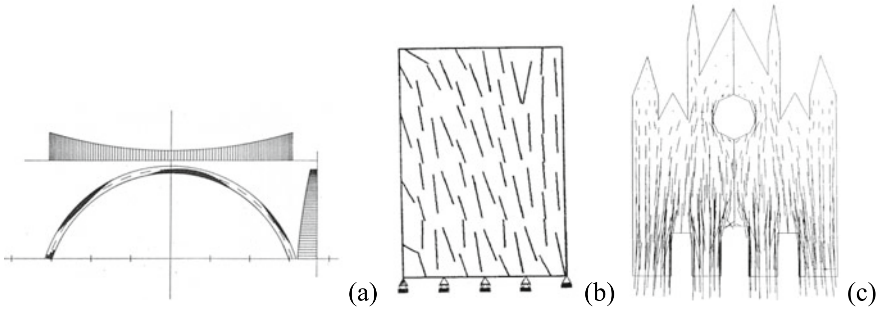

Fig. 1 Examples of no-tension continuum models: pictures from (**a**) Romano and Sacco [18], (**b**) Baratta [5], (**c**) Di Pasquale [12]

out by Lucchesi et al. [17], introducing a limit for the tangential component of the stress, that is assumed to be proportional to the normal component.

Figure 1 illustrates three pictures showing the solutions of interesting applications of the no-tension model. Figure 1a, from Romano and Sacco [18], illustrates the inelastic strain field, representing the diffuse cracking path for the round arch of the Pont-Saint Martin on Lys subjected to constant vertical load and an increasing horizontal load simulating the seismic effect. Increasing the value of the horizontal load, four zones are significantly cracked leading to the formation of four hinges and, then, the activation of the collapse mechanism. Figure 1b, from Baratta [5], shows the (compressive) principal stresses solution of the problem of a masonry panel subjected to a vertical and shear loading, determined adopting a complementary energy approach within the finite element method. Figure 1c, from Di Pasquale [12], illustrates the principal stress field arising in the Siena Cathedral façade, erected in the fourteenth century, subjected to its own weight. It is interesting to remark that the stress field for the no-tension model is significantly different from that occurring for the linear elastic model.

3 Limit Analysis-Based Models

The pioneering contribution by Heyman [28], who assumed a rigid no-tension model for masonry, represents a milestone in the limit analysis of masonry structures. The equations governing rigid no-tension models can be deduced from the no-tension constitutive laws by assuming null elastic strain, i.e.:

$$\boldsymbol{\varepsilon}^e = 0 \quad \rightarrow \quad \boldsymbol{\varepsilon} = \boldsymbol{\delta} \quad \rightarrow \quad \boldsymbol{\varepsilon}\mathbf{n} \cdot \mathbf{n} \geq 0 \tag{6}$$

i.e. strains can never be contractions. Accordingly, the normality rule becomes:

$$\boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon} \leq 0 \tag{7}$$

i.e. at any material point strain can develop along a given direction only if the compression acting along the same direction vanishes. Consequently, there is no internal energy dissipation in these models.

The interest in demonstrating the applicability of limit analysis theorems to masonry structures, introduced by Heyman [28], was soon highlighted within the AIMETA community, see for example the contributions by Como [24], Briccoli Bati et al. [21], and Como [25]. More exhaustively, Del Piero [26] showed that the classical static and kinematic theorems of limit analysis, proven for perfectly-plastic materials, actually hold for a larger class of materials including also masonry-like. This is based on two hypotheses peculiar to both no-tension and elastic–plastic materials. Indeed, for both classes of materials a constraint on the stresses is enforced which translates into the existence of a convex set of admissible stresses and an orthogonality property holds which links admissible stresses and inelastic strains. An application of the Del Piero's theorems to the case of arches, allowing the strains to be concentrated at discrete points, was presented by Lucchesi et al. [31]. A computational model for the limit analysis of three-dimensional masonry structures was developed by Livesley [29], based on the static theorem of limit analysis (Fig. 2a).

Other developments considered also frictional energy dissipation within rigid no-tension models, i.e. adopting non-associated flow rules, see for example [30].

Limit analysis-based models found a remarkable development in the AIMETA community in the last decade, thanks to the use of advanced numerical methods also based on machine learning.

In the framework of the static theorem of limit analysis, Angelillo et al. [19] adopted the Airy stress formulation to construct the singular stress fields for the problem of equilibrium of no-tension structures, while Berardi et al. [20] implemented a genetic algorithm within an adaptive refinement framework to find a safe thrust surface of a masonry vault within a design domain, i.e. by minimizing the average value of the principal tensile stresses in the structure. Using a discrete thrust network instead of a thrust surface, Marmo et al. [32] reformulated and extended to

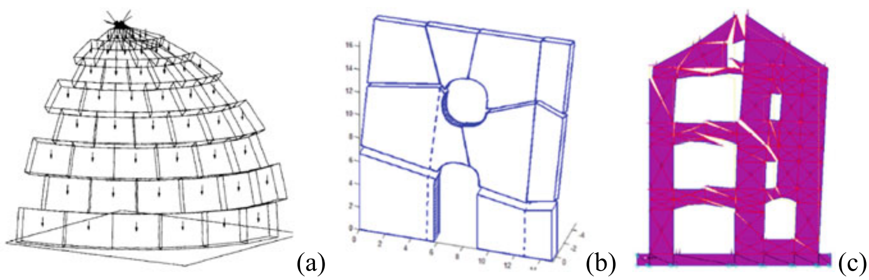

Fig. 2 Examples of limit analysis-based numerical models: pictures from **a** [29], **b** [23], and **c** [27]

seismic loads the thrust network analysis, i.e. a numerical approach which models masonry structures as a discrete network of forces in equilibrium with external loads.

In the framework of the kinematic theorem of limit analysis, Chiozzi et al. [22, 23] presented a NURBS-based adaptive method for the kinematic limit analysis of masonry structures, in the context of homogenized upper bound limit analysis which takes also into account the main characteristics of masonry material (e.g. material strength). In this approach, given a certain loading condition on a masonry structure, a genetic algorithm is used to adaptively adjust the collapse mechanism in order to find that related to the minimum collapse multiplier (Fig. 2b). In the context of a purely rigid no-tension material, Fortunato et al. [27] proposed an approach in the framework of free discontinuity methods. In this latter, the load bearing capacity and collapse mechanism of the structure are obtained through a fully variational approach, by minimizing a kinetic functional that admits the collapse crack pattern as a variable. Accordingly, the collapse multiplier and collapse mechanism of the structure are obtained without any a priori assumption on the crack pattern (Fig. 2c).

4 Phenomenological Models

Phenomenological models consider masonry material as a homogeneous continuum medium, without distinction between blocks and mortar layers (Fig. 3). These models, which rely on continuum nonlinear constitutive laws and can, somehow, approximate the overall mechanical response of masonry in an incremental framework, have the advantage that the mesh discretization does not have to describe the main heterogeneities of masonry. Usually, the adopted constitutive relations are based on internal-variable theories, mainly introducing damage or plasticity models, often coupled together, resulting in the following rather general expression:

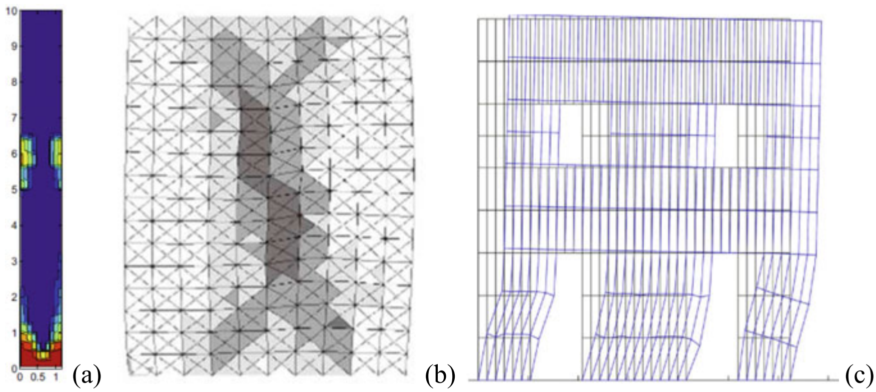

Fig. 3 Examples of phenomenological models: pictures from (**a**) Toti et al. [43], (**b**) Calderini and Lagomarsino [36], and (**c**) Brasile et al. [35]

$$\boldsymbol{\sigma} = (1 - D)\mathbf{C}\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\big) \tag{8}$$

where, according to the common assumption of isotropic damage coupled to plasticity, a single scalar damage variable D is introduced. The damage constitutive law is derived from the fundamental principles of damage mechanics, invoking the principle of the effective stress defined as:

$$\overline{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{(1 - D)} \tag{9}$$

and that of the equivalent strain, which assumes the same strain for the damaged specimen subjected to $\boldsymbol{\sigma}$ and the virgin one subjected to $\overline{\boldsymbol{\sigma}}$. Alternatively, the principle of the equivalent complementary energy can be appealed. Because of the masonry texture, often the hypothesis of isotropic damaging process can be not considered completely realistic. Thus, orthotropic damage models are formulated, where a tensorial representation of the damage variables is introduced, distinguishing between the damaging along the specific material directions related to the constituents texture.

As concerns the plasticity, classical formulations are usually used, as for example Rankine, Mises, Drucker-Prager or even more complex models. Damage-plasticity models are complemented by appropriate incremental laws governing the evolution of the two nonlinear processes, on the basis of properly defined strain and/or stress measures.

Phenomenological approaches adopt non-zero values for masonry tensile strength, although usually much lower than the values of compressive strength. Accordingly, concrete-like strength domains are typically defined. However, it is worth noting that due to the complex mechanical response of masonry, characterized by non-symmetry, anisotropy both in the linear and nonlinear range, and strain-softening, the phenomenological representation of the masonry mechanics is a very challenging task.

An interesting phenomenological model for masonry structures discussed at an AIMETA conference is that proposed by Chiostrini et al. [37]. In particular, they presented a 3D finite element able to deal with mechanical and geometric nonlinearities, developing the expression of the potential strain energy through average functions, and utilizing the Darwin-Pecknold strength domain and a smeared crack approach. Iterative-incremental methods for tracing equilibrium paths were also discussed in Chiostrini [38]. Another 3D finite element for the phenomenological analysis of masonry structures was presented by Esposito et al. [39], using a Drucker-Prager strength domain, and accounting for both cracking and crushing.

Three different numerical approaches based on anisotropic damage were presented at the AIMETA 2005 [34, 36, 40]. Papa et al. [40] utilized equilibrated quadrangular finite elements to have a rigorous check on the stress at the element borders. Bilotta et al. [34] proposed a mixed finite element method based on an orthotropic damage model to evaluate the nonlinear response of masonry panels. Calderini and Lagomarsino [36] developed an anisotropic damage continuum model for historic masonry based on simplified micromechanical hypotheses, also accounting for the cyclic behavior. More recently, an orthotropic continuum damage model was developed by Pelà et al. [41], based on the concept of mapped tensors from the anisotropic field to an auxiliary workspace. This approach allowed the establishment of an implicit orthotropic damage criterion in the real anisotropic space by using the damage criterion formulated in an auxiliary mapped space.

To account for the actual material microstructure in the macromechanical formulation, Addessi et al. [33] implemented a 2D micropolar Cosserat continuum model based on an isotropic damage to describe the in-plane response of masonry structures. This approach allowed to take into account the main inelastic mechanisms characterizing the in-plane response of masonry panels, such as damaging with strain-softening which is naturally regularized. An extension of a nonlocal damage-plasticity formulation to the dynamic regime was proposed by Toti et al. [43]. Such model is able to reproduce the degradation of the mechanical properties (damage), the accumulation of irreversible strains (plasticity), and the cyclic behavior taking into account the loss and recovery of stiffness due to crack closure and reopening. More recently, another nonlocal continuum damage model was proposed by Tesei and Ventura [42] for the nonlinear incremental analysis of masonry structures. In this model, the tensile softening behavior is described through a unique strain-driven nonlocal damage variable, which prevents spurious strain localization.

Aiming at the analysis of full-scale masonry structures, an assumed stress FE formulation in the context of non-associated plasticity was developed by Brasile et al. [35] in the framework of elasto-plasticity. Such a simplified approach appeared able to provide a suitable description of the masonry overall structural response.

5 Multiscale Models and Homogenization

During the early 1990s, an innovative and intriguing approach to the mechanics of masonry took place. This was originally formulated to model the mechanical response of heterogeneous microstructured materials and subsequently widespread in many different fields of mechanics. To date, it is one the most adopted approach for this class of materials.

The main idea of the multiscale modeling approach is to define a fictitious equivalent homogenized medium at the macroscopic scale (macro-scale), whose macromechanical response is derived by studying a representative volume element (RVE) at the microscopic scale (micro-scale), properly selected so as to describe in detail the geometric and mechanical properties of the real heterogeneous material. More than two scales were also considered in the procedures proposed in literature, but two-scale models are those preferred in the framework of masonry modeling (Fig. 4). Indeed, most of the multiscale analyses are performed assuming the geometrical separation of the micro- and macro-scale, so that homogenization can be applied. In other words, it is assumed that the microscopic (local) scale is small enough for the heterogeneities to be well identified and the macroscopic (global) is large enough

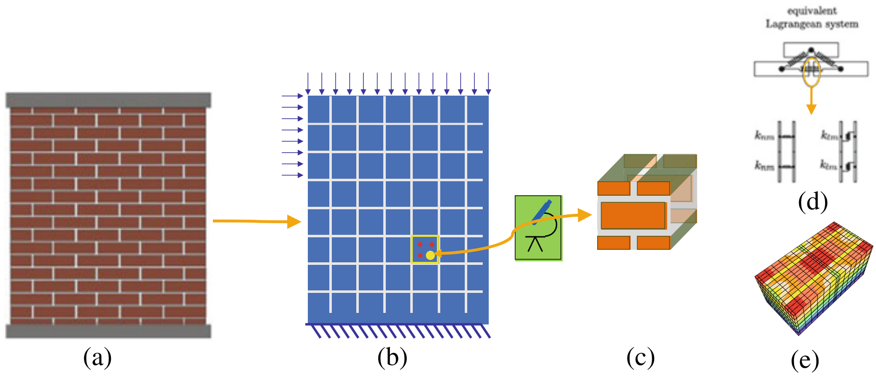

Fig. 4 Two-scale procedure for regular masonry: **a** real heterogeneous wall, **b** effective homogenized medium finite element model, **c** heterogeneous UC, **d** rigid blocks linked by linear/nonlinear interfaces, **e** UC finite element model

for the heterogeneities to be 'smeared-out'. To summarize, multiscale approaches require to develop micromechanical formulations to model the RVE response at the micro-scale and homogenization techniques to link the micro- and macro-scales.

Relying on the heterogeneous nature of masonry, made of the composition of different constituents, bricks/blocks and mortar joints, multiscale modeling represents a suitable and efficient approach to derive its mechanical response. This allows to account for all the micromechanical nonlinear mechanisms causing masonry failure and follow their evolution up to the collapse. Thus, the crack patterns associated with the mortar and/or brick collapse, the possible decohesion between bricks and mortar joints, the unilateral contact and friction phenomena at the interfaces can be suitably modeled, as well as their influence on the global mechanical response correctly considered.

Various Italian research groups deserved particular attention to the multiscale modeling approach and devoted considerable efforts to develop procedures for masonry. Several relevant contributions were produced, many of these presented at AIMETA conferences and published in the AIMETA proceedings.

When dealing with masonry characterized by periodic arrangement of the constituents, generated by the periodic repetition of a Unit Cell (UC), all the data required to apply the homogenization procedure are the geometrical and material properties of the UC. Thus, at the micro-scale, a properly defined boundary value problem is formulated on the UC under kinematic, static, or periodic boundary conditions for evaluating the effective response of the composite material. Indeed, the most effective procedure adopts periodic boundary conditions. However, in case of historic constructions, masonry is often characterized by non-periodic texture (random composites), and the geometrical and/or material properties are only partially known through statistical information. Thus, non-periodic masonry requires proper procedures to define the RVE, which assumes now the meaning of a Statistical

Volume Element (SVE). The homogenization problem for masonry characterized by non-periodic texture was studied and presented by Cluni and Gusella [51].

Focusing on periodic regular masonry, different homogenization procedures were proposed distinguished for the models adopted at the macroscopic structural scale (Fig. 4b) and microscopic UC level (Fig. 4c). Indeed, models using Cauchy continuum at both levels were presented, as well as those adopting enriched/higher order formulations. Masiani et al. [54, 55] presented a micropolar continuum model endowed with a local rigid structure derived by the homogenization of a blocky masonry modeled as a distinct rigid body system with elastic interfaces. In the same framework, Brasile et al. [49] proposed a multi-level strategy to describe the nonlinear response of masonry panels by means of a path-following procedure adopting a discrete model at the micro-scale. Masonry was described as the assemblage of rigid blocks linked by nonlinear damaging interfaces and modeled as a Lagrangean system (Fig. 4d). Ranocchiai and Rovero [57] explored the adoption of different micromechanical models for bi-phase materials, that is multi-layer periodic medium and matrix with elliptic cylindrical inclusions, and various homogenization theories applied to brick masonry walls. Addessi et al. [44, 45] proposed a nonlinear homogenization procedure for in-plane masonry structural problems adopting the Cosserat model at the macroscopic level and the Cauchy formulation for the UC, while Bacigalupo and Gambarotta [48] focused on the use of micropolar and second-order models. Leonetti et al. [53] presented a multiscale procedure using the couple-stress model at the macro-scale where an adaptive strategy able to zoom-in automatically the zones affected by damage initiation was employed.

The described two-scale procedures require to put in communication the macro and micro-level, that is input information have to be passed to the UC to properly define its boundary conditions and state the boundary value problem, and the micromechanical response evaluated in terms of stress state has to be homogenized and passed back to the macro-level. Often the homogenization procedure is strain driven, i.e. the strain evaluated at the macroscopic structural level is considered as the average strain assigned to the UC and the state of stress has to be computed, averaged and transferred to the macroscopic scale. In the framework of Cauchy-Cauchy multiscale procedures, the so-called first order approaches, a kinematic map linking the macro and micro-scale is formulated as follows:

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \mathbf{E} + \boldsymbol{\varepsilon}^*(\mathbf{x}) \tag{10}$$

expressing the microscopic strain field in the UC as the sum of the overall strain $\mathbf{E}$ evaluated at the macroscopic point, which would be the actual strain field in the UC if it were homogeneous, and the perturbation $\boldsymbol{\varepsilon}^*(\mathbf{x})$ which accounts for the presence of heterogeneities. Due to the periodicity of the displacement field in the UC, the following average properties hold for the strain:

$$\langle \boldsymbol{\varepsilon}^* \rangle = 0, \quad \langle \boldsymbol{\varepsilon} \rangle = \mathbf{E} \tag{11}$$

where the brackets denote the average of the field in the UC volume. The overall stress tensor $\mathbf{\Sigma}$ is defined as the average of the microscopic stress tensor as:

$$\mathbf{\Sigma} = \langle \boldsymbol{\sigma} \rangle \tag{12}$$

If enriched/higher-order continuum models are adopted at the macro-scale, similar relations hold, where the overall strain $\mathbf{E}$ contains all the macroscopic strain descriptors to which the overall stress tensor $\mathbf{\Sigma}$ components are work-conjugate. A kinematic operator is then properly defined to express the microscopic strain $\boldsymbol{\varepsilon}$ as function of $\mathbf{E}$. The Hill-Mandel equivalence principle allows to accordingly define the macroscopic stress components in $\mathbf{\Sigma}$.

When nonlinear constitutive models are introduced, the so-called computational homogenization procedures are commonly used, and the described link between macro- and micro- scales is performed step-by-step of the loading history by solving the nonlinear problems at both the global and local levels. This methodology is versatile enough to allow the adoption of any nonlinear constitutive formulation for the masonry constituents at the UC level. Indeed, nonlinear damage and damage-plastic continuum models were adopted for both bricks and mortar, or damage-friction interface models were proposed to model mortar and brick/block-mortar contact considering the first as rigid (Fig. 4d) or linear elastic. Commonly, the finite element method was selected as a suitable tool to solve the nonlinear problems at both scales (Fig. 4b, e), although some alternative techniques were proposed. As an example, La Malfa [52] adopted a meshless formulation for the solution of the UC boundary value problem, where the blocks are considered indefinitely elastic and the mortar joints are simulated by zero-thickness elasto-plastic interfaces.

Although many contributions proposing multiscale procedures were focused on the in-plane failure mechanisms of masonry structure, some relevant studies can also be found concerning the out-of-plane response of columns and walls. Among the others, it is worth mentioning the work by Milani et al. [56] who adopted the limit analysis combined with a homogenization approach to predict masonry out-of-plane collapse. This was based on a simple micro-mechanical model in which finite element discretization was avoided, masonry thickness subdivided in several layers and for each layer fully equilibrated micro-stress fields were assumed. In Cecchi and Milani [50] the kinematic limit analysis approach was proposed for the derivation of the macroscopic failure surfaces of two-wythes masonry arranged in English bond texture. In particular, a 3D system constituted by infinitely resistant bricks connected by joints modeled as interfaces with frictional behavior and limited tensile/compressive strength was identified with a 2D Reissner–Mindlin plate. Addessi et al. [47] proposed a multiscale beam-to-beam finite element model to study the out-of-plane collapse mechanisms of masonry walls under cylindrical bending modes, considering a two-dimensional Timoshenko beam at the macro-scale linked to a repetitive masonry UC at the micro-scale, made by the superposition of a single linear elastic brick and a mortar joint exhibiting a damage-plastic constitutive response. This procedure was subsequently extended in Addessi et al. [46] to account for the nonlinear geometric effects and study the stability of masonry walls under eccentric vertical compressive loads, also in presence of reinforcing layers.

6 Block-Based Models

In block-based models (Fig. 5), masonry is considered as an assemblage of blocks, and their interaction can be modeled through several suitable formulations. Generally, the behavior of the blocks can be considered as rigid, linear or nonlinear, while their interaction is typically nonlinear.

Within the Italian and AIMETA context, the first developments of block-based models were mainly aimed at modeling the dynamics of large block structures, e.g. multi-block columns [68]. In this context, and having as reference the Housner's inverted pendulum, characterized by the following equation of motion:

$$I_\theta \frac{d^2\theta}{dt^2} = -WR\sin(\alpha - \theta) \tag{13}$$

being θ the rotation angle of the block, W and I_θ its weight and moment of inertia, respectively, R the radial distance from the center of rotation to the center of gravity, and α the angle R makes with the vertical, Sinopoli [78] proposed an extension of the Housner's inverted pendulum behavior to account for the dimensional ratio of the stone blocks, allowing to highlight the relevance of dry friction in the mechanical response. Other contributions to this field followed in the subsequent years, see for examples [58, 61, 69, 70, 73], up to the application of rigid plane elements for the dynamic analysis of masonry façades by Casolo [62], then extended to 3D masonry structures in Bertolesi et al. [60].

In the 1990s, Masiani et al. [55] proposed a block-based model composed of rigid bodies assembled with elastic interfaces (preliminarily proposed in Masiani et al.

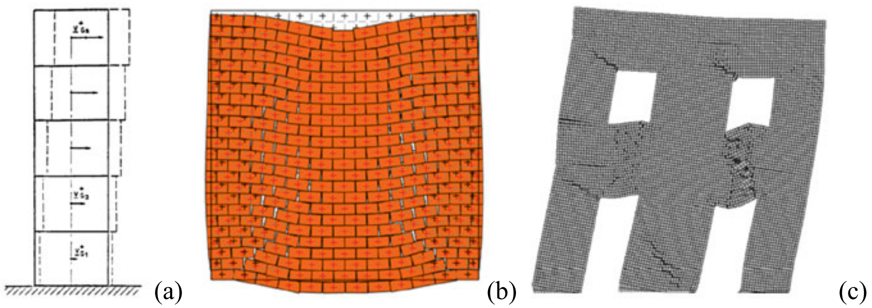

Fig. 5 Examples of block-based models: pictures from (**a**) Augusti and Sinopoli [58], (**b**) Baraldi et al. [59], and (**c**) Minga et al. [72]

[54]), with the aim of obtaining an equivalent Cosserat continuum. Such approach has been extended to the out-of-plane behavior of 3D masonry panels in Cecchi and Rizzi [64]. When assuming the rigid block hypothesis, the displacement of the generic block B is represented by a rigid body motion referred to the translation of its center and the rotation with respect to its center:

$$\mathbf{u}^B(\mathbf{x}) = \mathbf{u}^B + \boldsymbol{\Omega}^B\left(\mathbf{x} - \mathbf{x}^B\right) \tag{14}$$

where $\mathbf{x}^B$ is the position of the block center in the space, $\boldsymbol{u}^B$ is the translation vector of the block B and $\boldsymbol{\Omega}^B$ is its rotation skew tensor collecting block rotations with respect to block local coordinate axes. When assuming deformable blocks, instead, this relationship becomes more complex and is typically solved through the finite element method [75]. The rigid body hypothesis was also adopted by Formica et al. [67], together with damage and friction behaviors in the joints, by Salvatori and Spinelli [77], who assumed no-tension joints with perfectly plastic shear, and by Cecchi and Di Tommaso [65]. In addition, Fileccia Scimemi et al. [66] adopted a double asperity interface model to simulate the response of masonry joints. Finally, rigid and deformable blocks were considered by Cazzani et al. [63], within the discrete element method.

Among the others, a block-based damage-plasticity model with deformable 3D blocks was proposed by Minga et al. [72] to study full-scale masonry structures subjected to cycling loading, and compare the numerical results on in-plane nonlinear behavior of masonry panels with several textures obtained by rigid and deformable blocks carried out by Baraldi et al. [59]. It appears worth to refer also to the numerical study on the effect of joint dilatancy in out-of-plane loaded masonry panels conducted by Godio et al. [71] and to the derivation of a model of imperfect interface with finite strains and damage by asymptotic techniques and its application to the analysis of masonry structures performed by Raffa et al. [74].

7 Conclusions

This chapter illustrated the contribution of the AIMETA community to the development of masonry mechanics over the past five decades. A review of the AIMETA conferences proceedings from the beginning of the 1970’ has been conducted, although not exhaustive for the sake of brevity, highlighting the significant impact of the research in the AIMETA community on masonry mechanics. Indeed, a recognizable contribution can be observed in each type of modeling strategies, with a clear dominating role in the development of no-tension modeling approaches.

It is worth to note that, starting from the 2011 up to the present day, mini-symposia dedicated to the most recent developments in the mechanics of masonry were organized within the AIMETA national conferences, collecting the most relevant contributions of the Italian researchers to the wide field of the modeling of masonry structures.

The journal Meccanica dedicated great attention to the masonry mechanics, collecting scientific advances in this field, as highlighted in the Special Issues "Masonry Construction: Structural Mechanics and Other Aspects", edited by Calladine in 1992, and "New trends in mechanics of masonry", edited by Sacco et al. [76]. Moreover, Meccanica published also many other important contributions on the modeling of masonry material and structures, becoming a clear reference at international level in this field.

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Dynamics and Stability: From an Ancillary to a Leading Role in the History of AIMETA

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Abstract The role of "Dynamics and Stability" in the history of AIMETA Conferences is discussed. It is emphasized that these subjects, initially away from the interests of the scientific community, have assumed increasing importance over time, culminating in the foundation of the AIMETA Group of Dynamics and Stability (GADeS), which today collects scientific contributions from many members belonging to different scientific sectors. The first timid steps taken by "Dynamics and Stability" in AIMETA are recalled in a historical key, and the causes that first slowed down and then determined their growth are conjectured. With reference to Dynamics and to its classic distinction between linear and nonlinear behaviour, the perturbation method is seen as a key to interpreting nonlinear phenomena. With reference to Stability and Bifurcation, the existence of non-communicating worlds, namely static and dynamic bifurcations, is noted in the world scientific panorama. Once again, the perturbation method can be recognized as the tool that acts as a bridge between the two worlds. Finally, the main topics that have been debated in the GADeS meetings and mini-symposia are briefly reviewed. This paper, in addition to representing a historical synthesis and a cross-section of contemporary research in "Dynamics and Stability" within AIMETA, offers critical considerations on the fragmentation of knowledge, and encourages the development of a unifying vision of the two disciplines.

Keywords Dynamics · Stability · Static and dynamic bifurcations · Center manifold · Perturbation method · Structural mechanics

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1 Introduction

Dynamics and Stability are pillars of both theoretical and applied Mechanics. The first describes the evolution, free or forced, of a system, isolated or interacting with the environment; the latter evaluates the robustness of the equilibrium, or more generally, of a stationary state (periodic or quasi-periodic motion). Both disciplines are declined in multiple sub-disciplines, so diversified from each other, to appear as independent. On the one hand, the linear and nonlinear dynamics, the seismic dynamics, the wind response, the impulsive response, the wave propagation, …; on the other, the static stability (buckling and post-buckling), the dynamic stability of autonomous systems (the Hopf, flip, Neimark-Sacker bifurcations), the parametric excitation, the chaos.

As a matter of principle, there is no mechanical problem whose study does not require the use of concepts and methods of Dynamics and/or Stability. One would therefore expect, looking with a careful eye to the history of AIMETA, to notice significant interest from Italian researchers to these two disciplines. Surprisingly, re-reading the indexes of the Conference Proceedings relating to the period 1971–2011, it turns out that their role was marginal and subordinated to other major issues of interest at the time. Stability was intended as a synonym for static bifurcation (buckling, sometimes post-buckling), almost never placed in dedicated sessions, but more frequently discussed in more general contexts of Structural Mechanics. On the other hand, the dynamic stability of nonconservative systems was almost totally absent. Besides, Dynamics was a topic for a few enthusiasts, often relegated to the last session of the last day of the conference, carelessly followed by colleagues who were waiting for the train with suitcase in hand.

The causes of this apparent lack of interest in fundamental issues, which, on the contrary, should have played a unifying role in the varied panorama of the disciplinary competences of AIMETA members, still appear difficult to explain today. Nonetheless, the authors conjecture here the reasons for the phenomenon, in an attempt to shed light on the roots of our scientific history.

1.1 Ancient History

AIMETA, as is well known, collects skills ranging from General Mechanics to Fluid, Machine, Solid and Structure Mechanics. However, the numerically preponderant role of the two scientific sectors that refer to ICAR/08 (i.e., the disciplinary area of Solids and Structures in the Italian university system), is a fact. All the AIMETA Boards of Directors (CD) have worked hard to spread the “mission” of the Association among the affiliates of the three minority sectors, with results that, unfortunately, have not been comforting, so that, to date, the numerical composition remains “stable”, notably unbalanced. The preponderant role of ICAR/08 has therefore markedly addressed the interests of researchers, transferring, in the AIMETA context, the choices, inclinations, beliefs and, sometimes, even prejudices, which pervaded that scientific-disciplinary sector. Dynamics, therefore, which was slow to make its way into ICAR/08, was slow to appear also in AIMETA. Stability, on the other hand, limited to its static aspects, was recognized as an ancient discipline, albeit reserved for a few amateurs and, therefore, was welcomed in the scenario of the great themes of Structural Mechanics, and consequently, in AIMETA. Dynamic stability, on the contrary, which still today (not only in Italy, but in the world) many scholars think that nothing has to do with static stability, was almost completely unknown.

In support of the above assertion, two anecdotes are told. The senior author of this article was lucky enough to attend, as a student, the first edition of the Dynamics of Structures course, which was held in Rome in 1976 by Professor Carlo Gavarini. All the young participants were made aware of the exceptional nature of the event, as there was only one other similar course in Italy, held by Professor Giuseppe Grandori at the Milan Polytechnic. This indicates the fact that the Dynamics of Structures, until then the prerogative of Applied Mechanics and Machines only, entered the context of Structural Mechanics and Engineering late. A second anecdote concerns a discussion, which the senior author himself had at the beginning of the 2000s, with a highly esteemed colleague from Solid Mechanics, who has now passed away, in a selection committee for professorship. The colleague, in examining the titles of an aspiring full professor who had dealt with Dynamics subjects, asked: "But dynamics, is it Structural Mechanics?", thus questioning the relevance of the topic to the core of the scientific-disciplinary sector. This underlines the ancillary role that the same Dynamics played, no more than twenty years ago, in the field of Structural Mechanics.

1.2 Modern History

Recent years (2012–2019) have marked an epochal change regarding the placement of Dynamics in the scientific interests of AIMETA. The authors believe that the establishment of GADeS (AIMETA Group of Dynamics and Stability) represented *the turning point*. The Group was born on a solicitation that Giuseppe Piccardo made to Angelo Luongo, to be a spokesperson at the CD AIMETA, of the need to formalize the growing scientific interest in the themes of Dynamics and Stability with the creation of a new AIMETA Group. The CD, represented by Prof. Paolo Luchini, showed interest and appreciation for the initiative, while expressing some doubts (moreover shared by the promoters themselves) about the success of the operation, especially towards the much desired involvement of all the members of the Association. GADeS was formally founded on October 19, 2012. The first assembly (constituent and scientific) was held in Rome, at the Department of Basic and Applied Sciences for Engineering—Mathematics Section—of the Sapienza University of Rome. 52 sympathizers participated, with significant representation of mechanical engineers and applied mathematicians, in addition to the majority component belonging to structural mechanics, as usual. The first Coordinating Committee was elected, consisting of Profs. Angelo Luongo (coordinator), Sandra Carillo, Walter D'Ambrogio, representing the three souls of the Group. The debut took place at the following AIMETA Conference in Turin (XXI AIMETA Conference, September 17–20, 2013), where, at the mini-symposium organized by GADeS, 46 papers were presented, which occupied all the days of the event. The topics mainly covered the Dynamics and the Control of the Response of Mechanical Systems, the Dynamical Systems and their Stability and Bifurcation. Suddenly, Dynamics had abandoned its minority role in AIMETA to achieve the leading role it deserves. The merit of this was certainly not of the GADeS proponents, who limited themselves to collecting the request of the researchers, but of the many Italian scholars (not only from ICAR/08) who had long shown interest in these disciplinary areas. GADeS, therefore, represented a "home" in which to meet, where everyone could bring their baggage of knowledge, no longer in a personal capacity, but to constitute a recognizable "critical mass". In the following years, the contribution of GADeS to AIMETA remained stable, confirming that the one in Turin was not a singular event. The new coordinating committee, which took the place of the first in 2016, continued the work, urging the development of the scientific subjects that characterize the Group.

Today we can say that Dynamics and Stability are disciplines that contribute significantly to AIMETA's mission of spreading scientific culture in the mechanical field. However, other steps will have to be made to complete the slow transformation processes that have taken place over the past fifty years. In the following, some of these aspects are discussed, focusing on the growth of a desirable unifying vision of the aforementioned sub-disciplines.

2 Dynamics

Dynamics, like many other engineering disciplines, is usually classified into *linear* and *nonlinear* dynamics, as two disjoint worlds. Those who study nonlinear aspects, while knowing basic linear dynamics, turn their attention to the *distinctive features* introduced by nonlinearities, i.e. to all those phenomena that cannot be explained in a linear context. Those who deal with linear problems, on the other hand, often ignore the nonlinear effects, and investigate a multitude of models and application cases, which sometimes reveal unexpected behaviors, and therefore fulfill the need to stimulate scientific curiosity. Thinking today of suggesting the development of a unifying knowledge of the two worlds is an unrealistic undertaking, given the extreme diversification of the themes and, above all, of the applicative interests, which lead to specializations aimed at particular problems. Nonetheless, on these aspects, the "bridging" role played by the *perturbation method*, which we want to comment on, deserves an important consideration.

2.1 The Perturbation Method as a Key to Understanding Nonlinear Phenomena

As is known, the perturbation method, in its various versions (strained parameters, multiple scales, method of averaging, … [1]), reduces a (weakly[1]) nonlinear problem to a succession of linear equations. These are governed by the same operator, and must be solved in sequence, using the information acquired in the previous steps. When applied to a dynamics problem, the (generating) equation that triggers the procedure is constituted by the linear model. The method, therefore, far from representing a mere technical tool, offers an educational and profound key to reading many nonlinear phenomena. These are seen as caused by the occurrence of *resonances* (external, internal, parametric), which arise between the different components of motion, generated by nonlinearities through the creation of multiple frequencies from the basic ones. This vision leads the understanding of nonlinear phenomena (such as the coupling and the transfer of energy from one mode to another, the limitation of the nonlinear response through the creation of limit cycles, …) to the use of a single concept, *the resonance*, which is well known in the linear field. The procedure amply clarifies the role of linear normal modes that contribute to the nonlinear response, and opens the way to the formulation of *reduced models*, with a few degrees of freedom, capable of grasping the essence of phenomena.

On the contrary, it should be noted that, due to the lack of knowledge of perturbation methods, many researchers make recurrent and systematic use of direct integrations of the equations of motion, the results of which often obscure the synthetic understanding of the phenomenon. The lack of a true scientific culture of nonlinear dynamics often leads the researcher to use discrete models with thousands of degrees of freedom, when a few modes (often only two or three) would be sufficient to give qualitatively accurate information and, above all, would be able to reveal the nature of the response. It is worth remembering, in support of this thesis, the animated scientific discussions that developed during the XXIII AIMETA conference in Salerno (2017), around the causes that led to the famous collapse of the Tacoma Narrow Bridge in 1940. The authors of the study, refuting the most accredited interpretation in the literature, namely the occurrence of aeroelastic instability, explained the phenomenon as due to the flexural–torsional modal coupling generated by nonlinearities. However, they did not know how to answer the question of clarifying what was the ratio between the fundamental flexural and torsional frequencies which, if integer (i.e., equal to 2,3, …), is solely responsible for any coupling (from internal resonance), as clearly explained by the perturbation method.

The diffusion in the world scientific community, particularly mechanical, of the perturbation method, as the main tool for the analysis of nonlinear dynamic systems, is due to Ali H. Nayfeh (1933–2017). Author of numerous books and articles on the subject, his work has accompanied the scientific growth of many students around

[1] The smallness of nonlinearity, which limits to consider motions of small but finite amplitude, generally does not represent a (too) strong limitation, but is sufficient to answer most of the engineering problems.

the world. In Italy, the first to early collect his teaching were Angelo Luongo, Giuseppe Rega and Fabrizio Vestroni, who developed some applications, many of which were presented at the AIMETA Conferences. These were followed by other scholars (Stefano Lenci and Walter Lacarbonara among the oldest) and many other young people. Today the Italian school of nonlinear dynamics has an international dimension, worldwide recognized with the appointment of Walter Lacarbonara as Editor-in-Chief of the *Nonlinear Dynamics* journal founded by Ali Hasan Nayfeh.

3 Stability and Bifurcation

Stability and Bifurcation (different concepts, but often confused in the common language) are ancient knowledge, developed in Mechanics but then spread to all physical–mathematical disciplines. In the engineering world, with the singular exception of Systems Theory, the equilibrium bifurcation is almost always understood as a synonym for "Buckling", that is Static Bifurcation. The historical reasons for this lie in the fact that the prevailing attention of researchers has turned to conservative systems (elastic and subject to gravitational forces), whose stability of equilibrium, according to the Lagrange-Dirichlet Theorem, is not influenced by the kinetic energy. The damping, when introduced in these systems, plays a secondary role since, if on the one hand it improves stability transforming it from marginal to asymptotic, on the other it is unable to stabilize unstable equilibria. Therefore, if one ignores the non-conservative external actions (for example the follower forces, or those dependent on the speed), the static approach is correct.

On the other hand, the general theory of stability—Poincarè (1854–1912), Lyapunov (1857–1918)—was developed many years after the pioneering studies on the static stability by Euler (1707–1783). In particular, the fundamental theorem of the *Center Manifold*, which is at the basis of the studies on dynamic bifurcations, has been proved in recent times [2], and it is not yet sufficiently known in the engineering world, remaining for the most part a tool for applied mathematicians.

3.1 Two Non-communicating Worlds

As a result of the evolution of Science in terms of Stability, today there are two distinct worlds on the international scene, apparently not communicating with each other:

- the Elastic Stability community, which draws on the approach of Koiter [3], Budiansky [4], Hutchinson [5], of the English school (Thompson and Hunt [6], Croll and Walker [7], Supple [8]), and which has seen over the years a myriad of contributions, some of which also developed within the AIMETA context;

- the Dynamic Stability community, which operates in the context of the general theory of dynamical systems (Guckenheimer and Holmes [9], Wiggins [10], Troger and Steindl [11]), which has joined AIMETA only in relatively recent years.

The general theory of stability, however, includes the case of static bifurcation, pursuing an economy of thought that can only benefit the rationalization of knowledge. The two theories really differ slightly:

- a bifurcation is static when, as a bifurcation parameter varies, an eigenvalue of the Jacobian matrix, tangent to the equilibrium path, crosses the imaginary axis in zero; a bifurcation is dynamic when a pair of conjugated complex eigenvalues crosses the imaginary axis at points other than zero;
- from a static bifurcation (at least) a new equilibrium branch is born; from a dynamic bifurcation (at least) a family of periodic motions is born.

The same general theory does not attribute the elementary geometric meaning of *branching point* to the bifurcation, suggested for example by the bifurcation diagram of the Euler beam, but the meaning of *instantaneous qualitative modification of the phase portrait* of the system, i.e. of its dynamics. This modification also occurs at the branching point (where the number of equilibrium points changes), but not only at that.

To be able to unify the two worlds, and to look at Stability and Bifurcation as a *unicum*, it is however necessary to abandon the energy criterion of stability and, even more, the static criterion of adjacent equilibrium (very much appreciated by professors and engineering students), and to regard the structure as a dynamical system. An operation that, on the one hand, may appear overabundant compared to the objective, on the other is absolutely necessary when one wants to include non-conservative forces in the model, as in the case of aeroelasticity.

3.2 The Perturbation Method as a "Bridge"

Actually, static and dynamic bifurcations are already shared by a common analytical tool of investigation, which is the perturbation method, although this affinity is often unknown to the authors themselves [12]. The whole theory of post-buckling, aimed at the asymptotic construction of the branched paths, is based on the so-called Method of Static Perturbation [6, 13]. This consists in developing displacement and load in series as a function of a perturbation parameter, equivalent to a curvilinear abscissa along the unknown path. The systematic imposition of compatibility conditions, which remove forces outside the operator's range, allows for the connection between displacement and load. On the other hand, all the perturbation methods developed in nonlinear dynamics (Lindstedt-Poincaré, Multiple Scales, Method of Averaging [1, 14]) can be used in the analysis of dynamic bifurcations. In particular, the multiple scale method appears as the natural extension of the method of static perturbation, where the removal of secular terms plays the same role as static compatibility, since it removes the forces that induce a non-periodic response.

The use of the perturbation method in dynamic bifurcation analysis, however, is not commonly accepted. As mentioned, many researchers use the Center Manifold Theorem to deduce the bifurcation equations of the problem. However, Luongo, Paolone and co-authors [15–18] have shown how the perturbation method provides the same results of the Center Manifold method, in a simpler and more automatic way, even in cases of high degeneracy of the operator (multiple bifurcations, which imply a non-diagonal Jordan canonical form of the Jacobian matrix).

Despite its application complexity, the message transmitted by the Center Manifold Theorem is of incomparable scientific value: close to a bifurcation (i.e. around the critical points in the parameter space), the system evolves on a manifold of the state space of low dimension (usually equal to 1, 2, 3, ...), which is tangent to the critical subspace (i.e. spanned by the critical eigenvectors of the Jacobian matrix), of which it retains the dimension. This means that reduced models, with a very low number of degrees of freedom, are able to capture the essence of the bifurcation phenomenon. These models, however, need to be able to capture the curvature of the manifold, and not just to span the critical subspace. For example, the Nonlinear Normal Modes [19, 20] perform this function. The insufficient knowledge of this very important result, however, still leads many researchers today to tackle the problem through brutal numerical computations, using models with many degrees of freedom.

The Center Manifold Method also has a counterpart in static bifurcation, known as the Lyapunov-Schmidt reduction [11], essentially based on the same considerations. This was Koiter's approach in developing his famous doctoral thesis [3], which marked the beginning of the modern Elastic Stability Theory.

4 A Glance at the GADeS Years

As mentioned in Sect. 1, the GADeS group has given a new home to the topics of Dynamics and Stability within AIMETA. In some cases, GADeS has collected subjects already present within the AIMETA conferences, generally placed under the broad heading of Mechanics of Structures and Mechanics of Machines. In many other cases, GADeS contributed to welcoming new topics, widening the audience of the AIMETA conferences. Since its birth GADeS has been the promoter of three Special Issues (SI) in journals of international importance [21–23]. The material contained in these SI covers a broad spectrum of topics in solid mechanics, structural mechanics and control, in both linear and nonlinear fields, in the presence of environmental or artificial excitations; analytical, numerical and experimental methods are presented, using deterministic as well as stochastic approaches. Therefore, references cited mainly refer to developments of some of the research presented during the GADeS mini-symposia and published in these SI. It is worth noting that the most recent SI [23], published in 2021 on Nonlinear Dynamics journal, was initially conceived for presenting a survey of new studies carried out by researchers participating in the

GADeS group, similarly to the previous two SI. However, the breadth of the topics covered in GADeS and their importance at an international level led the editors to enlarge the Italian viewpoint by inviting worldwide renowned scientists in order to give a broader view on recent advances in stability, bifurcations and nonlinear vibrations that may involve different kinds of mechanical systems. In the following, an attempt to show the main subjects presented during the GADeS meetings and mini-symposia will be made, without any claim to be exhaustive.

- Bifurcation and dynamic stability. This subject is common in the research of GADeS participants, with applications to various fields, using reduced order models. By way of example, the dynamic stability of classic case studies, such as the Ziegler's column and the Nicolai paradox, are discussed [24, 25]; the critical aeroelastic behaviour of bridges [26, 27], cables [28, 29], building [30, 31] is analyzed; the stability of parametrically-excited systems is dealt with [32, 33]. Specific studies relating to wind-structure interaction problems seem to be of considerable interest [34–36].
- Buckling. It is one of the historical topics dealt with in AIMETA, especially regarding thin-walled beams, still active at GADeS meetings [37, 38]. The compressive and the tensile buckling in slender beams [39] is also present, along with instability phenomena in double-layered pipes [40], non-classic interactive buckling in paradigmatic discrete systems [41] and flexural–torsional buckling of beam-like structures [42]. Developments in the framework of the Generalized Beam Theory (GBT) are also addressed [43–45].
- Cable dynamics. The subject, known to very few researchers in the seventies of the last century, was presented for the first time by Luongo, Rega, Vestroni at the V AIMETA conference in Palermo (1980), as regards the perturbation analysis of the free nonlinear oscillations, and is admirably summed up in Rega's review papers [46, 47]. This topic continues to be very active within the GADeS group. In addition to the papers on galloping already mentioned [28, 29], new formulations for cable self-damping [48], mechanical models for specific dampers [49], numerical methods using corotational beam elements [50] and modal interactions in beam–cable systems [51] are dealt with.
- Dynamical phenomena in mechanical systems. It is obviously a very vast subject, impossible to summarize in all its aspects. Among the various interests of the researchers participating in the GADeS meetings it is recalled the friction-induced oscillations [52, 53], the dynamical integrity [54, 55], the dynamics of composite/nanocomposite structures [56, 57], the fractional-order system dynamics [58–60], the hysteretic systems [61–63], the linear and nonlinear analysis of structures [64–70].
- Equivalent beam models. This subject has had a notable impact on GADeS meetings under the impulse of the research group of the University of L'Aquila. Starting from equivalent nonlinear models able to reproduce the dynamic behavior of 3D shear-type structures [71, 72], it has moved on to equivalent Timoshenko beam models [73, 74], which are also able to describe micro-structured bodies [75] and

non-symmetrical layouts [76]. This type of modeling can find large application areas in the context of environmental forces [77, 78].

- Experimental dynamics. This subject, characterized by a strong interdisciplinarity, has had a remarkable development in recent years within GADeS. By way of example, it is reported experimental activities in wind-structure interactions [79–81], in monitoring and earthquake engineering [82–84], in vibro-impact dynamics [85], in dynamics of circular cylindrical shells [86, 87].
- Transient dynamics. This is a subject that has had a great increase in recent years, including through GADeS. It covers classic topics such as vibrations induced by moving loads [88, 89] and moving masses [90–92]. Recently the propagation of uncertainties on serviceability assessment of pedestrian bridges has attracted the attention of various researches [93, 94]. Effects caused by non-stationary wind loading seem a promising research field [95, 96].
- Vibration control systems. A large group of GADeS presentations addresses vibration control through various techniques, as also evidenced by the fact that control appeared within several papers cited up to now. Classic systems such as Tuned Mass Dampers and Tuned Liquid Column Dampers are present [97, 98]. At the same time, Nonlinear Energy Sinks (NES) has received great attention [99–101], together with piezoelectric and hysteretic systems [102, 103].

5 Final Remarks

The growing relevance of Dynamics and Stability in the scientific interests of AIMETA researchers, now coordinated by GADeS, was analyzed in a historical key. Some of the topics discussed at the association's conferences were reviewed, recalling the growth process. However, in the opinion of the authors, this requires further development in the direction of the unification of concepts and methods, and consequent overcoming of barriers, aimed at artificially defining sub-disciplines, which stand in the way of a global and not partial approach to the problem. To this end, the following considerations have been developed.

1. Linear and nonlinear dynamics should be considered as two aspects of the same discipline. Nonlinearities induce qualitatively new effects, which cannot be explained with the linear model. However, the perturbation method, by reducing the nonlinear problem to a succession of linear problems, explains these phenomena with the same simple tools of linear analysis. Above all, it transforms nonlinearities into known forcing, acting on the linear system.
2. Static stability is a particular aspect of dynamic stability, in which an eigenvalue of the Jacobian matrix crosses the imaginary axis in zero, rather than in a generic point. The same concept of bifurcation should be considered not as a branch of the equilibrium path, but as a qualitative change of the phase portrait. Even in stability, the perturbation method allows to deal with static and dynamic bifurcations essentially in the same way.

3. The perturbation method is not merely an operational tool, but is a key to interpreting both static and dynamic nonlinear problems. As a fundamental result, it teaches that, at the bifurcation or near a resonance, the response of a static or dynamic system is described by a very small number of degrees of freedom. This complies with the Center Manifold Theory, according to which the evolution develops on a manifold tangent to the critical subspace. This fundamental notion needs to be spread and matured by many researchers, not experts in the field, who use huge numerical models, which threaten to obscure the deep understanding of the phenomena.
4. As evident from the papers derived from the presentations at the GADeS meetings and mini-symposia, the topics of Dynamics and Stability have expanded considerably from the beginning, and are now characterized by a strong interdisciplinarity. GADeS reflects the rich spectrum of topics covered by the modern researches in "Dynamics and Stability", and fits perfectly into the current trends of Nonlinear Dynamics [104, 105]. Reduced-order modeling remains a common denominator present in almost all the works within GADeS.

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Strain-Difference Based Nonlocal Elasticity Theories: Formulations and Obtained Results

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Abstract The contributions to nonlocal elasticity given by the authors in the last two decades are reported in this article. To better illustrate the above contributions and their pertinence to the nowadays research framework, we start with a scrutiny of the inconsistencies encountered within the Eringen's purely nonlocal model and the remedies required to overcome shortcomings and paradoxical situations known from the literature. It is shown that the so-called strain-difference based nonlocal theories encompassing the mentioned contributions provide effective methods to address boundary-value problems. Applications to plates by nonlocal finite elements and size effects analysis of beams in bending have been reported as illustrative examples of previously obtained results.

Keywords Elasticity theory · Nonlocal elasticity · Eringen's integral nonlocal theory · Eringen's differential nonlocal theory

1 Introduction

The roots of nonlocal elasticity can be traced back to the continuum theories by [1–3], as well as to the multipolar elasticity theory by [4]. Eringen and Edelen [5–9] formulated nonlocal elasticity theories featured by the presence of quantities called residuals with which the nonlocal nature of fields as body forces, mass, stress, entropy, etc., is assessed which makes them rather cumbersome for application purposes.

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Eringen and co-workers (see e.g. [10–13]) simplified the above elasticity theories in such a way that the nonlocal theory and the classic one differ from each other only in the stress-strain constitutive equation, but leaving unaltered the equilibrium equations and the kinematic relationships. Notably, with the nonlocal constitutive equation the stress at a point is expressed as a weighted mean value over a neighbor region of that point. For this purpose, a convolution operation upon the strain field is used, which is featured by a nonlocal kernel being a function of the Euclidean distance between the stress and strain points. Rogula, [14], provided a mathematical definition of the nonlocality concept based upon the existence of an internal length scale as a material parameter. Variational principles within this nonlocal elasticity theory were developed by Polizzotto [15].

The above simplified Eringen's nonlocal model was widely used for both theoretical and engineering applications not only within elasticity, but also in many other fields as plasticity, crack and damage mechanics, dislocation theory, etc. There exists a huge literature on these matters, for which reference is made to more specific works as: [16–18] for plasticity, crack mechanics and damage mechanics; [19] for nonlocal theories of elasticity, thermo-elasticity and electro-magneto-elasticity. The above nonlocal elasticity theory developed by Eringen and co-workers will be referred to as the Eringen's (integral) nonlocal theory, or Eringen's (integral) nonlocal model, in the following.

An important aspect of a nonlocal continuum theory with respect to the classical local one is that, on addressing a boundary-value problem, the former leads to governing integro-differential equations carrying in more computational difficulties than the governing differential equations to which one arrives with the latter. Eringen, [20], provided a method to address an integral nonlocal boundary-value problem by means of a differential equation of which the Green function coincides with the kernel of the integro-differential equation. After this step by Eringen, a new branch of nonlocal elasticity grew up with the name of *differential nonlocal elasticity*. This was widely applied to beam, plate and shell models simulating sensor and actuator devices within micro- and nano-technologies in the purpose to solve various engineering problems as buckling, vibrations and wave propagation problems, along with size effect analysis, for which reference is made to the review papers [21–23].

1.1 Inconsistencies of the Eringen's Nonlocal Model. Remedies

Notwithstanding the notable success of the Eringen's nonlocal theory, some inconsistencies were soon discovered and discussed [24], as described hereafter.

One such inconsistency originates from the Eringen's integral nonlocal model leading to ill-posed boundary-value problems. In fact, the inherent integro-differential governing equation, viewed as integral equation, falls into the category of Fredholm integral equations of the first kind which —as known from integral equation theory

[25]—admits multiple solutions, or no solutions at all. After the contributions by [26, 27], it is known nowadays that a necessary and sufficient condition in order that an integral nonlocal model admits a (unique) solution is that some special boundary conditions (called also "constitutive" BCs, or even "nonlocality" BCs) must be not in contrast with the imposed traction boundary conditions. A remedy to the above drawback was proposed by Eringen, [5, 28], and implemented by many others [15, 29–37]. With this formulation the fully nonlocal model is replaced with a two-phase local/nonlocal mixture model, which leads to Fredholm integral equations of the second kind, hence to well-posed boundary-value problems. Another remedy to the Eringen's integral nonlocal model was proposed by [38] with a new formulation in which the basic concepts of the Eringen's integral nonlocal model are saved, but the strain and stress play therein interchanged roles.

A second inconsistency of the Eringen's integral nonlocal model is more directly related to the differential form of it. Eringen [20] likely proposed this model to address problems with infinite domain (like crack tip singularities, wave propagation, and the like). Peddieson and co-workers [39] first used the differential nonlocal model for size effects analysis of micro- and nano-beams. It was found that the model generally predicts softening effects on the stiffness with increasing the length scale parameter of the beam, but may also predict hardening (as for a cantilever beam under uniform load), or even no size effects at all (as in the so-called "paradox" case, namely, a cantilever beam under point load at the free end), all apparently without a precise rule. The right motivation for which the above anomalous behavior gets out is likely due to the conjugate governing differential equation having a degree equal to that of the integro-differential equation. Therefore the differential-based solution *cannot in general coincide* with the integral-based solution (if it exists), due to the impossibility to implement the nonlocality BCs as side conditions.

A third inconsistency of the Eringen's nonlocal model has been identified with its property of not saving uniform local fields. This implies that, in the case of homogeneous nonlocal elastic material, the stress corresponding to a uniform strain is not uniform in general, except in an infinite domain. In other words, the Eringen nonlocal model does not comply with the *locality recovery condition* [40], that is, the condition for which the material behaves as a local material under a uniform strain field. A milder form of this condition is the *local stress recovery condition* [41], in which the stress is uniform under a uniform strain, but the material still saves some nonlocality features. A remedy to this kind of inconsistency, proposed by Polizzotto and co-workers [41], is in the form of a two-phase local/nonlocal model in which the nonlocal phase is driven by the strain difference measured at the generic point with respect to the reference point where the stress is evaluated. This model called *strain-difference based nonlocal model*, automatically satisfies the local stress recovery condition; it also leads to Fredholm integral equations of the second kind and thus to well-posed boundary-value problems. Another form of strain-difference based nonlocal model was also proposed in [40], which complies with the more stringent locality recovery condition, that is, under uniform strain, not only the stress is uniform, but the inherent Helmholtz energy potential looses its dependence on the length scale parameter.

1.2 Objectives and Outline

The purpose of the present paper is to give an insight over the family of strain-difference based nonlocal models developed by the authors in the last two decades within elasticity. To be concise, the mentioned family is reduced to two basic models, of which one (called "first type") complies with the local stress recovery condition, the other (called "second type") complies with the more stringent locality recovery condition and both models lead to well-posed boundary value problems. The paper is organized as follows. After the introductory arguments in Sect. 1, some preliminary considerations are reported in Sect. 2. Sections 3 and 4 constitute the central part of the paper, in which the two strain-difference based models are presented. Applications of these models to engineering problems are reported in Sect. 5. Section 6 concludes the paper.

2 Preliminaries to Eringen's Nonlocal Elasticity

In this section, the Eringen's nonlocal constitutive model of continuum elasticity is recalled in its fully nonlocal integral form. For this purpose, a 3D (finite) solid body of domain V is considered within a $\mathbb{R}^3$ space, which before deformation is referred to a Cartesian orthogonal co-ordinate system, say $\mathbf{x} = (x_1, x_2, x_3)$. The body is constrained at a portion, say S_c, of its boundary surface $S = \partial V$, in such a way as to impede any rigid motion. The body is also subjected to external actions, which are assumed in the form of body forces, say $\mathbf{b}(\mathbf{x})$ within V (N/m^3) and surface forces, or tractions, say $\mathbf{p}(\mathbf{x})$, applied on the free portion $S_f = S \backslash S_c$ (N/m^2). All these forces vary in time in a quasi-static manner.

Eringen and co-workers, [10–13, 19], proposed a nonlocal constitutive model for elastic materials, which in the common case of homogeneous material, is expressed as

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C} : \int_V g(\mathbf{x}, \mathbf{x}')\boldsymbol{\varepsilon}(\mathbf{x}')\,\mathrm{d}V' \quad \forall \mathbf{x} \in V \tag{1}$$

where the colon product denotes double index contraction operations (e.g. $(\mathbf{C} : \boldsymbol{\varepsilon})_{ij} = C_{ijkl}\varepsilon_{kl}$) and $\mathrm{d}V' = \mathrm{d}V(\mathbf{x}')$. The stress $\boldsymbol{\sigma}(\mathbf{x})$ of Eq. (1) is the "nonlocal" stress arising at the field, or reference, point $\mathbf{x} \in V$, $\boldsymbol{\varepsilon}(\mathbf{x}')$ is the "local" strain acting at the generic source point $\mathbf{x}' \in V$. Also, the two-point symmetric function $g(\mathbf{x}, \mathbf{x}')$ is the *kernel* function (called also *attenuation* or *influence* function after Eringen), which is positive definite and generally is assumed to depend on the field and source points $\mathbf{x}, \mathbf{x}'$ through the Euclidean distance $r = |\mathbf{x} - \mathbf{x}'|$. For mathematical convenience, often in the literature the kernel g is taken in the form of an exponential as

$$g(\mathbf{x}, \mathbf{x}') = k_0 \exp\Big(-\frac{r}{c}\Big), \quad r = |\mathbf{x} - \mathbf{x}'| \tag{2}$$

where c denotes the *length scale parameter*. The coefficient k_0 is determined through the *normalization condition*

$$\int_{V_\infty} g(\mathbf{x}, \mathbf{x}') \, \mathrm{d}V' = 1 \tag{3}$$

where V_∞ is the infinite domain ($\mathbb{R}^3$) in which V is embedded, [16, 19]. In the Introduction, it was pointed out that the Eringen's nonlocal constitutive stress-strain relation (1) leads to ill-posed boundary-value problems whereby there may be either a multiple solution, or more likely no solution at all, due to the impossibility to conciliate the boundary traction conditions with the nonlocality BCs, [26, 27].

In our treatment of nonlocal elasticity, we make reference to a two-phase local/nonlocal model cast in the form

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C} : \left[\xi \boldsymbol{\varepsilon}(\mathbf{x}) + (1-\xi) \underbrace{\int_V g(\mathbf{x}, \mathbf{x}') \boldsymbol{\varepsilon}(\mathbf{x}') \, \mathrm{d}V'}_{\mathbf{E}(\mathbf{x})}\right] \tag{4}$$

This relation strongly appeals to a two-phase local/nonlocal mixture model with the coefficient ξ playing the role of local phase parameter, but also that of an (essentially positive) material constant, not necessarily less than 1. The two-phase model (4) leads to well-posed boundary-value problems. The stress $\boldsymbol{\sigma}(\mathbf{x})$ is there expressed linearly in terms of the local strain $\boldsymbol{\varepsilon}(\boldsymbol{x})$ along with the *strain integral* $\mathbf{E(x)}$ given by the formula

$$\mathbf{E}(\mathbf{x}) = \mathscr{R}(\boldsymbol{\varepsilon})(\mathbf{x}) := \int_V g(\mathbf{x}, \mathbf{x}') \boldsymbol{\varepsilon}(\mathbf{x}') \, \mathrm{d}V' \tag{5}$$

where $\mathscr{R}(\cdot)$ denotes the nonlocal operator acting on $(\cdot)$.

For a correct thermodynamic treatment of the considered two-phase model let us assume, in agreement with the stress-strain relation (4), the existence of an internal energy potential, say $u = u(\boldsymbol{\varepsilon}, \mathbf{E}, \eta)$, where η is the entropy density, [15, 19, 42]. Assuming isothermal conditions for simplicity, the energy balance principle (or first thermodynamics principle) can be cast in a point-wise form as

$$\dot{u} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + P \qquad \text{in } V \tag{6}$$

where P is the (nolocality) *energy residual*, that is, the energy density transmitted to the generic particle from all other particles within the body as a consequence of the nonlocal nature of the material, [9, 15, 19, 42]. The following *insulation condition* has to be satisfied, [9], that is

$$\int_V P(\boldsymbol{\varepsilon}, \dot{\boldsymbol{\varepsilon}}) \, \mathrm{d}V = 0 \tag{7}$$

holding for any deformation mechanism and signifying that the particle system is constitutively insulated within V, that is, no long distance energy is transmitted to the body from the exterior environment due to the nonlocal behaviour of the material.

Let the Helmholtz free energy $\psi = \psi(\boldsymbol{\varepsilon}, \mathbf{E})$ be introduced using the Legendre transform $\psi = u - T\eta$, with $T > 0$ being the (constant) absolute temperature. The energy balance (6) may then be rewritten as

$$T\dot{\eta} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \dot{\psi} + P \geq 0 \qquad \text{in } V \tag{8}$$

Here, the non-negativity sign on the r.h.s. is introduced to enforce the second thermodynamics principle. In this way, inequality (8) identifies itself with the Clausius-Duhem inequality, [43, 44], which differs from its classical counterpart only for the presence of the energy residual P. Was P identically vanishing for any deformation mechanism, then the material would be a simple material.

Let inequality (8) be integrated over V to obtain, recalling (7) and using the appropriate Green identity, the following inequality

$$\int_V \left[\boldsymbol{\sigma} - \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} - \mathscr{R}\left(\frac{\partial \psi}{\partial \mathbf{E}}\right)\right] : \dot{\boldsymbol{\varepsilon}}\, \mathrm{d}V \geq 0 \tag{9}$$

As this has to hold for arbitrary deformation mechanism $\dot{\boldsymbol{\varepsilon}}$, a necessary and sufficient condition of (9) is the state equation

$$\boldsymbol{\sigma} = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} + \mathscr{R}\left(\frac{\partial \psi}{\partial \mathbf{E}}\right) \tag{10}$$

where $\boldsymbol{\sigma}$ represents the Cauchy stress work-conjugate to $\dot{\boldsymbol{\varepsilon}}$. Equation (10) implies that (9) is satisfied as an equality and therefore

$$P = \dot{\psi} - \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} = \frac{\partial \psi}{\partial \mathbf{E}} : \mathscr{R}(\dot{\varepsilon}) - \mathscr{R}\left(\frac{\partial \psi}{\partial \mathbf{E}}\right) : \dot{\varepsilon} \qquad \text{in } V \tag{11}$$

which is the constitutive equation for P.

3 Strain-Difference Based Nonlocal Models of First Type

In this section, we start to provide strain-difference based nonlocal models. Two types of such models will be presented, that is, first type models in the present section, second type models in the next section. In this section the material is characterized by a Helmholtz free energy function of the strain $\boldsymbol{\varepsilon}$ and strain integral $\mathbf{E}$ specified by (5), cast as a quadratic form as

$$\psi = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbf{C}_\infty : \boldsymbol{\varepsilon} + \frac{1}{2}\boldsymbol{\varepsilon} : \mathbf{C}_1 : \mathbf{E} + \frac{1}{2}\mathbf{E} : \mathbf{C}_2 : \mathbf{E} \tag{12}$$

which constitutes an extension of the treatment reported in [41]. Since $\mathbf{E} \to \mathbf{0}$ as the length scale parameter $c \to \infty$, the moduli tensor $\mathbf{C}_\infty$ characterizes the asymptotic behavior of the material; the tensors $\mathbf{C}_1$, $\mathbf{C}_2$ are defined in terms of the moduli tensor of classic anisotropic elasticity, $\mathbf{C}$, that is, $\mathbf{C}_1 = \alpha_1\mathbf{C}$, $\mathbf{C}_2 = \alpha_2\mathbf{C}$, with α_1, α_2 being suitable scalar coefficients. Both $\mathbf{C}_\infty$ and $\mathbf{C}$ are considered nonhomogeneous within V.

Let the body be subjected to external quasi-static actions as specified at the beginning of Sect. 2. We go to determine the governing equations of the inherent boundary-value problem within the framework of small displacements and linearized elasticity using the principle of the minimum total potential energy of nonlocal elasticity, [15], through the functional $\Sigma[\mathbf{u}]$ here cast in the form

$$\Sigma[\mathbf{u}] := \int_V \psi(\boldsymbol{\varepsilon}, \mathbf{E})\,\mathrm{d}V - \int_V \mathbf{b}\cdot\mathbf{u}\,\mathrm{d}V - \int_{S_f} \mathbf{p}\cdot\mathbf{u}\,\mathrm{d}S \tag{13}$$

Here, $\psi(\boldsymbol{\varepsilon}, \mathbf{E})$ is the functional (12), whereas $\mathbf{u}$ denotes the inherent displacements. The functional (13) has to be minimized under the kinematic restrictions relating $\boldsymbol{\varepsilon}$ to $\mathbf{u}$, as well as the imposed displacements $\mathbf{u} = \mathbf{u}_c$ on S_c. Following a straightforward procedure, [15], one easily arrives at the *total Cauchy stress* $\boldsymbol{\sigma}(\mathbf{x})$ cast in the form

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}_\infty(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) + \int_V \mathbf{k}(\mathbf{x}, \mathbf{x}')\boldsymbol{\varepsilon}(\mathbf{x}')\,\mathrm{d}V' \tag{14}$$

The (symmetric) kernel $\mathbf{k}(\mathbf{x}, \mathbf{x}')$ here above is the *nonlocal anisotropic moduli tensor*, that is,

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = \alpha_1\mathbf{k}_1(\mathbf{x}, \mathbf{x}') + \alpha_2\mathbf{k}_2(\mathbf{x}, \mathbf{x}') \tag{15}$$

where

$$\left.\begin{aligned} \mathbf{k}_1(\mathbf{x}, \mathbf{x}') &:= \frac{1}{2}\Big[\mathbf{C}(\mathbf{x}) + \mathbf{C}(\mathbf{x}')\Big]g(\mathbf{x}, \mathbf{x}') \\ \mathbf{k}_2(\mathbf{x}, \mathbf{x}') &:= \int_V \mathbf{C}(\mathbf{z})g(\mathbf{x}, \mathbf{z})g(\mathbf{x}', \mathbf{z})\mathrm{d}V(\mathbf{z}) \end{aligned}\right\} \tag{16}$$

The Cauchy stress $\boldsymbol{\sigma}(\mathbf{x})$ of (14) collects a local contribution at $\mathbf{x} \in V$, along with two types of long distance contributions, one of which is the result of the interaction of every two particles $(\mathbf{x}', \mathbf{x})$ $\forall\mathbf{x}' \in V$, the other of every three particles $(\mathbf{x}', \mathbf{z}, \mathbf{x})$ $\forall(\mathbf{x}', \mathbf{z}) \in V$.

Since the $\boldsymbol{\sigma}(\mathbf{x})$ of (14) does not comply with the locality recovery condition, Eq. (14) is replaced with

$$\boldsymbol{\sigma}(\mathbf{x}) = \alpha_0\mathbf{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) + \int_V \mathbf{k}(\mathbf{x}, \mathbf{x}') : [\boldsymbol{\varepsilon}(\mathbf{x}') - \boldsymbol{\varepsilon}(\mathbf{x})]\,\mathrm{d}V' \tag{17}$$

Equation (17) shows that $\boldsymbol{\sigma}(\mathbf{x}) = \alpha_0\mathbf{C}(\mathbf{x}) : \bar{\boldsymbol{\varepsilon}}$ for $\boldsymbol{\varepsilon}(\mathbf{x}) = \bar{\boldsymbol{\varepsilon}}$ in V. Indeed, the *local stress recovery condition* is satisfied, but not the *locality recovery condition*, since in fact for $\boldsymbol{\varepsilon}(\mathbf{x}) = \bar{\boldsymbol{\varepsilon}}$, ψ can be shown to still have a *nonlocal* form as

$$\psi = \frac{1}{2}\,\bar{\boldsymbol{\varepsilon}} : \mathbf{C}_\infty(\mathbf{x}) : \bar{\boldsymbol{\varepsilon}} + \left[\alpha_1\gamma(\mathbf{x}) + \alpha_2\gamma^2(\mathbf{x})\right]\frac{1}{2}\,\bar{\boldsymbol{\varepsilon}} : \mathbf{C}(\mathbf{x}) : \bar{\boldsymbol{\varepsilon}} \tag{18}$$

which contains c through the function

$$\gamma(\mathbf{x}) = \int_V g(\mathbf{x}, \mathbf{x}')\,\mathrm{d}V' \tag{19}$$

The variational principle invoked previously led us to two forms of the stress-strain relation, namely (14) pertaining to the original *strain-integral model* characterized by an asymptotic moduli tensor $\mathbf{C}_\infty(\mathbf{x})$, and (17), pertaining to the *strain-difference based model* characterized by an asymptotic moduli tensor $\alpha_0\mathbf{C}$, $\alpha_0 > 0$. The variational principle also leads us to the equilibrium equations with which the stress $\boldsymbol{\sigma}$ is required to comply, that is,

$$\nabla\cdot\boldsymbol{\sigma} + \mathbf{b} = 0 \quad \text{in} \quad V, \qquad \boldsymbol{\sigma}\cdot\mathbf{n} = \mathbf{p} \quad \text{on} \quad S_f \tag{20}$$

where $\mathbf{n}$ denotes the unit external normal to $S = \partial V$, along with the kinematic equations, that is,

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}\right) \quad \text{in} \quad V, \qquad \mathbf{u} = \mathbf{u}_c \quad \text{on} \quad S_c \tag{21}$$

Equation (20) and (21), typical of linearized elasticity, associated with (17) form a consistent set of equations governing a well-posed boundary-value problem. It in fact leads to an integro-differential equation of the second differential order in the displacement $\mathbf{u}$ constituting a Fredholm integral equation of the second kind. This admits a unique solution free from any paradoxical condition and additionally satisfies the local stress recovery condition.

4 Strain-Difference Based Nonlocal Models of Second Type

In this section, a strain-difference based nonlocal model complying with the locality recovery condition is reported. Here, the material is characterized by a Helmholtz free energy $\psi(\boldsymbol{\varepsilon}, \mathbf{E}_d)$ defined as

$$\psi = \frac{1}{2}\,\boldsymbol{\varepsilon} : \mathbf{C} : \boldsymbol{\varepsilon} + \frac{1}{2}\,\mathbf{E}_d : (\alpha\mathbf{C}) : \mathbf{E}_d \tag{22}$$

where $\mathbf{E}_d = \mathscr{R}(\mathscr{D}\boldsymbol{\varepsilon})$ = *strain difference integral* in which

$$\mathscr{D}\boldsymbol{\varepsilon}(\mathbf{x}, \mathbf{x}') = \boldsymbol{\varepsilon}(\mathbf{x})' - \boldsymbol{\varepsilon}(\mathbf{x}) \quad \forall(\mathbf{x}, \mathbf{x}') \quad \in V \tag{23}$$

and then

$$\mathbf{E}_d = \mathscr{R}(\mathscr{D}\boldsymbol{\varepsilon})(\mathbf{x}) = \int_V g(\mathbf{x}, \mathbf{x}')\big[\boldsymbol{\varepsilon}(\mathbf{x}') - \boldsymbol{\varepsilon}(\mathbf{x})\big]\,\mathrm{d}V' \tag{24}$$

The strain-difference based nonlocal model here considered is a phenomenological model accounting for inhomogeneity of the moduli tensor $\mathbf{C}(\mathbf{x})$ and of the length scale parameter $c = c(x)$ along with the additional attenuation effects produced by the latter inhomogeneities. As reported in [40], where the mentioned model was developed, the kernel function $g(\mathbf{x}, \mathbf{x}')$ is taken in the form

$$g(\mathbf{x}, \mathbf{x}') = k_0\ \exp\big(-\frac{r_{\mathrm{eq}}}{c_0}\big) \tag{25}$$

where c_0 is the largest value of $c(\mathbf{x})$ in V, whereas r_{eq} denotes the *equivalent distance* defined as

$$r_{\mathrm{eq}} = r + r^* \tag{26}$$

Here, r is the so called *geodetical distance*, that is, the length of the shortest path between $\mathbf{x}$ and $\mathbf{x}'$ without intersecting the boundary surface. For a non-convex domain it is $r \geq |\mathbf{x} - \mathbf{x}'|$, but $r = |\mathbf{x} - \mathbf{x}'|$ for a convex one (no holes, nor rientrant angles, nor cracks). The scalar r^* is a fictitious (non-negative) distance accounting for the additional attenuation effects produced by inhomogeneities of both $\mathbf{C}(\mathbf{x})$ and $c(\mathbf{x})$. However, in the present short review, convex domains are considered, therefore $r = |\mathbf{x} - \mathbf{x}'|$ and $c(\mathbf{x}) = c =$ constant.

The constitutive stress-strain relation can be obtained as in Sect. 3, but using the Helmholtz potential (22). So we obtain the total Cauchy stress as

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} + \alpha\,\mathscr{R}\big(\mathbf{C} : \mathscr{R}(\mathscr{D}\boldsymbol{\varepsilon})\big) \tag{27}$$

After some mathematical operations (not reported here for brevity), one obtains two possible forms for the stress-strain relation, that is, either

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) - \alpha \int_V \mathbf{J}(\mathbf{x}, \mathbf{x}')[\boldsymbol{\varepsilon}(\mathbf{x}') - \boldsymbol{\varepsilon}(\mathbf{x})]\,\mathrm{d}V' \tag{28}$$

or, equivalently,

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) + \alpha \int_V \mathbf{S}(\mathbf{x}, \mathbf{x}')\boldsymbol{\varepsilon}(\mathbf{x}')\,\mathrm{d}V' \tag{29}$$

Here above, $\mathbf{J}$ and $\mathbf{S}$ denote the *nonlocal stiffness tensors* expressed as

$$\mathbf{J}(\mathbf{x}, \mathbf{x}') = \big[\gamma(\mathbf{x})\mathbf{C}(\mathbf{x}) + \gamma(\mathbf{x}')\mathbf{C}(\mathbf{x}')\big]g(\mathbf{x}, \mathbf{x}') - \mathbf{k}_2(\mathbf{x}, \mathbf{x}') \tag{30}$$

$$\mathbf{S}(\mathbf{x}, \mathbf{x}') = \frac{1}{2}\big[\gamma^2(\mathbf{x})\mathbf{C}(\mathbf{x}) + \gamma^2(\mathbf{x}')\mathbf{C}(\mathbf{x}')\big]\delta(\mathbf{x} - \mathbf{x}') - \mathbf{J}(\mathbf{x}, \mathbf{x}') \tag{31}$$

where $\gamma(\mathbf{x})$ is given by (19) ($0 < \gamma(\mathbf{x}) \leq 1$), whereas the tensor $\mathbf{k}_2$ is given by $(16)_2$. Also, $\mathbf{J}$ and $\mathbf{S}$ satisfy the equalities

$$\int_V \mathbf{J}(\mathbf{x}, \mathbf{x}')\, \mathrm{d}V' = \gamma^2(\mathbf{x})\mathbf{C}(\mathbf{x}), \qquad \int_V \mathbf{S}(\mathbf{x}, \mathbf{x}')\, \mathrm{d}V' = \mathbf{0} \qquad \forall\, \mathbf{x} \in \mathbf{V} \tag{32}$$

It is readily seen that for a uniform strain field, say $\boldsymbol{\varepsilon}(\mathbf{x}) = \bar{\boldsymbol{\varepsilon}} = \text{const.}$, Eq. (28) gives $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \bar{\boldsymbol{\varepsilon}}$, that is, the local stress condition is recovered correspondingly. Additionally, since $\mathbf{E}_d\mathscr{R}(\mathscr{D}\bar{\boldsymbol{\varepsilon}}) = 0$ identically, the Helmholtz free energy loses its dependence on the length scale parameter c, which means that the locality recovery condition is satisfied with the strain-difference based nonlocal model of second type. Combining (28), or (29), with the equilibrium equation (20) and the kinematic equations (21) leads, again as in Sect. 3, to an integro-differential equation of the second differential order in the displacement $\mathbf{u}$. This constitutes a Fredholm integral equation of second kind, which admits a unique solution free from paradoxical condition and in addition complying with the locality recovery condition.

5 Numerical Results

The strain-difference based nonlocal model of second type has been implemented into a nonlocal version of the finite element method (NL-FEM) in [45]. Such formulation is based on a nonlocal total potential energy principle given in [40], where the strain-difference based NL-FEM was proposed starting from a variational treatment of a BVP which is a straightforward extension to the strain-difference-based nonlocal model of the general variational principles conceived in [15]. Some numerical findings, related to a nonhomogeneous plate under tension, are given hereafter as an effective application of the NL-FEM. The above quoted papers are referred for theoretical and computational details. The discussed model has been also applied in [46] to the analysis of small-scale Euler-Bernoulli beams in bending. The relevant beam problem is reduced to a set of three mutually independent Fredholm integral equations of the second kind (each independent of the beam's ordinary boundary conditions, only one depends on the given load), which can be routinely solved numerically. Some results concerning a benchmark beam case are given next as a second numerical example, while referring to the above quoted paper for details.

5.1 A Nonhomogeneous Square Plate Under Tension

The square plate under tension depicted in Fig. 1 is analyzed via the NL-FEM. Geometry, material data, boundary and loading conditions are specified in Fig. 1 whose sketch shows that the plate is fixed along the edge at $x = 0$ with assigned displacements $\bar{u}_x = \bar{u}_y = 0$, and suffers uniform given displacements $\bar{u}_x = 0.001\text{cm}$

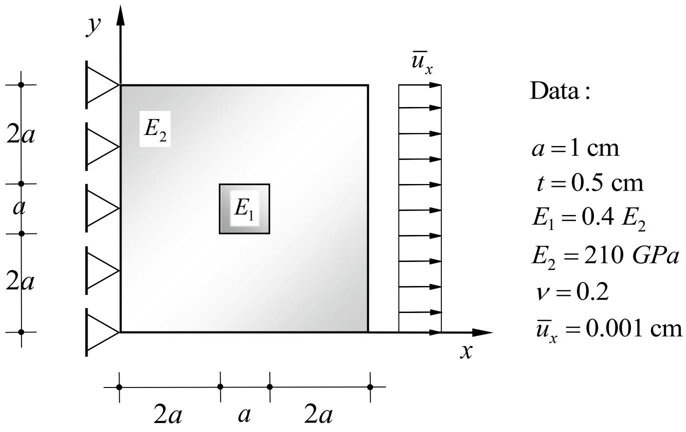

Fig. 1 Nonhomogeneous square plate under tension, after [45], with piecewise constant Young modulus. Geometry (t= thickness), boundary and loading conditions, material data

along the free edge at $x = 5a$. The attenuation function is the bi-exponential of Eq.(2) with $k_0 = 1/2\pi c^2 t$ with an *influence distance* (i.e. the distance beyond which $g(\mathbf{x}, \mathbf{x}') \approx 0$) equal to $11c$. C^0 quadratic, 8−nodes isoparametric Serendipity nonlocal finite elements (NL-FEs) have been used for the numerical analyses. The peculiarity of such NL-FEs is that each element, say the n−th one, aside from the standard (local) element stiffness matrix, say $\mathbf{k}_n^{\text{loc}}$, is equipped with element matrices of *nonlocal nature*, say $\mathbf{k}_n^{\text{nonloc}}$ and $\mathbf{k}_{nm}^{\text{nonloc}}$. The first one accounts for the influence exerted on the n−th element by the nonlocal diffusive processes over the whole domain, it contains the operator $\gamma(\mathbf{x})$ given by (19). The second one is a set of element matrices, all pertaining to the n−th element, precisely: a self-stiffness matrix, obtained for $m = n$, plus all the cross-stiffness matrices with $m \neq n$ being m the generic element *neighbor* of element n. Each $\mathbf{k}_{nm}^{\text{nonloc}}$ (with $n \neq m$) contains the matrices of the Cartesian derivatives of the shape functions of elements m and n so accounts for the influence on the element n of the neigbor elements. Further details are given in [45] and are here omitted for brevity.

In Fig. 2 the plate size a has been proportionally varied, assuming $a = 0.5, 1, 2$ cm, so defining three proportional sized plates with the boundary conditions of Fig. 1 and suffering the displacements $\bar{u}_x$ which have been accordingly proportionally varied with the plate dimension such that $\bar{u}_x = a/1000$. The three solutions obtained with local elasticity coincide, each FE model is a scaled version of the other two. In contrast, the three nonlocal elastic solutions given by the NL-FEM show a decreasing – a flattening – for decreasing specimen dimensions confirming the capacity of the NL-FEM to capture size effects.

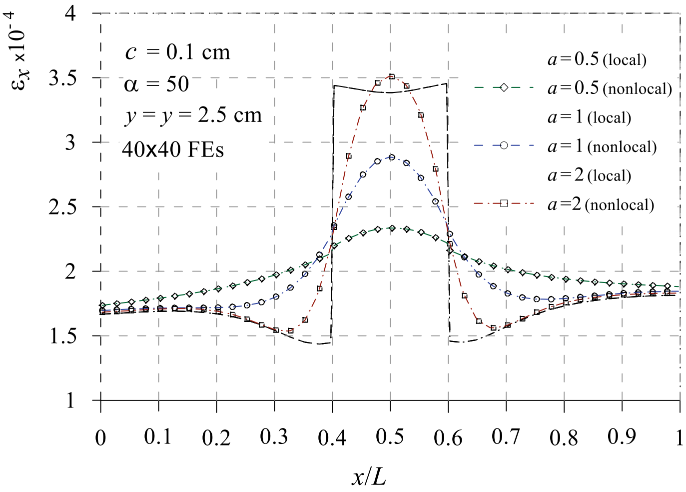

Fig. 2 Nonhomogeneous plate under tension, after [45]. Strain profiles ε_x versus x/L at $y = 2.5$ cm, $c = 0.1$ cm, $\alpha = 50$, mesh of 40×40 FEs and for three different proportional sized plates given by $a = 0.5$, 1 and 2 cm. Local (lines without markers) and nonlocal (lines with markers) solutions

5.2 *Small-Scale Euler-Bernoulli Beams in Bending*

A simple benchmark Euler-Bernoulli beam in bending has been analyzed, precisely a cantilever beam under point load P at the free end. The beam has been assumed homogenous, of length L and referred to orthogonal co-ordinates (x, y, z). The x axis coincides with the beam axis, z is oriented along the beam height, y is in the width direction. The bending plane coincides with the plane (x, z), the (y, z) axes coincide with principal inertia axes of the cross section. The only meaningful strain component is ε_{xx} and the transverse attenuation effects are assumed to be negligible such that the attenuation function g can be considered to be a function of the x co-ordinate only, i.e. $g = g(x, x')$ herein assumed in the bi-exponential form of Eq. (2). The fundamental bending moment/curvature relation featuring the strain-difference based nonlocal model for beams proves to be:

$$M(x) = EI\chi(x) - \alpha \int_0^L J(x, x')\big[\chi(x') - \chi(x)\big]\,\mathrm{d}x' \tag{33}$$

where $M(x)$ and $\chi(x)$ are the bending moment and the curvature, respectively, E is the Young modulus and I the second area moment of the cross-section. Equation (33), counterpart of (28) for the EB beam model, leads to a solving equation for the beam problem which is a Fredholm integral equation of the second kind. The latter, as shown in [46] (see also [47, 48]), can be solved by a splitting strategy that provides a set of three mutually independent Fredholm integral equations of the second kind, all of which holding no matter how the beam is constrained. The solution for the

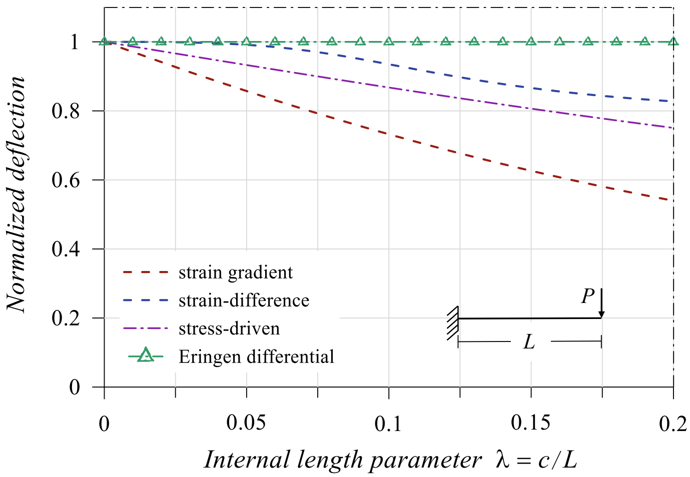

Fig. 3 Cantilever beam subjected to a point load at the free end, after [46]. Normalized deflection at the free end cross section *versus* internal length parameter λ for strain gradient (Polizzotto 2014, [49], dashed line), strain-difference integral (present model, solid line), stress-driven (Barretta et al. 2018, [50], dash dot line) and Eringen differential (Peddieson et al. 2003, [39], solid line with triangles) constitutive behavior

beam problem is indeed achieved to within four constants to be determined by the ordinary boundary conditions characterizing the analyzed specific beam case.

In Fig. 3 the normalized deflection versus the normalized internal length parameter is reported. The obtained result is plotted against the ones given by the strain gradient model [49], the stress-driven model [50] and the Eringen differential [39]. The obtained results show that, with the exception of the Eringen's nonlocal model which is affected by the discussed inconsistencies, the predicted size effects are of stiffening type, a circumstance which seems to confirm the well-known *smaller-is-stiffer* phenomenon. It is worth noting that for "small" values of the internal length c the three methods are in substantial agreement with one another, while for $c \to \infty$, at difference with the other models, the asymptotic behavior predicted by the strain-difference (here not shown for brevity) is of local type, a result in agreement with the expected (local) behavior of a size dependent nonlocal beam model, which for $c \to \infty$ behaves like an atomic lattice model, [20, 51].

6 Conclusion

An overview of a family of strain-difference based nonlocal elasticity theories has been presented. These essentially include two model types, of which one complies with the "local stress recovery condition", the other with the more stringent "locality recovery condition", but both models lead to well-posed boundary-value-problems without computational drawbacks or other paradoxical conditions.

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On the Theory of Inhomogeneities in Continuum Mechanics 1950s–1970s: Elastic Invariants and an Overlooked Paper by C. Eckart

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Abstract The construction of a theory of inhomogeneities in solids is motivated by the attempt to provide a microscopic basis to anelasticity theories. With reference to the divisive continuum theory of defects grown in the years '50–'70s, here I revisit papers that did not follow the trend.

Keywords Inhomogeneities · Cristal defects · Anelasticity

1 Introduction

The topic of distortions in solids has taken an important part in the research of physicists and metallurgists in the second half of the last century since when G. I. Taylor, E. Orowan and M. Polanyi independently formulated their proposal on the role that certain defects (*dislocations*) in crystalline materials may have in the deviation from the elastic behaviour. Illustrious names were involved, some of whom contributed to establish the modern solid state physics in the '40s and later. Much of that work, distilled from several attempts and afterthoughts over the years, is now deposited in classical articles and textbooks, see for instance [4, 18, 21, 25]. Giving a satisfactory account of the vast literature on the subject would be difficult and is out of the purposes here. A sketchy account of the research's evolution with the essential references is found in a plenary lecture held by P. Rodriguez at the Indian Science Congress in 1995, [26].

Together with the above fundamental research, connections with more traditional elasticity was also taken into account. Defects were regarded as singularities in the elastic field and studied within the framework of classical linear elasticity, adopting a straightforward generalization of the notion of dislocation introduced by Volterra and Somigliana at the beginning of the past century. Accordingly, defects were treated as sources of internal stress, "like the electrical charges are the source of the electric field", and the attention was focused on the determination of the related "internal

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stress function". A number of intriguing problems and novel concepts entered the scene of continuum mechanics. Among them, the notion of interaction between a singularity of the elastic field and the field itself, that of mutual force between different defects, that of the movement of a singularity through the body. A general account of the theory and its applications to the most common types of crystal defects is contained in an expository article by Eshelby, [14].

In about the same years a second approach was also introduced, where defects were pictured as continuously distributed and described by suitable densities. This approach was not alternative or in conflict with that mentioned above, but it distinguished for privileging purely geometric aspects of the problem in the attempt to provide a field theory of defects. The way defectiveness was described, in fact, strongly evoked situations covered in the general theory of differentiable manifolds and offered one more occasion for the interpretation of concepts and quantities occurring in that theory in an appropriate physical context. This elegant analogy was first noticed by Kondo and Bilby, and became the leitmotiv of an intense activity that has led to rather abstract theories, where geometry is the main source of inspiration. It would be long to quote major contributions in this area, but, if we set aside too elaborate versions of the theory, a good account of the original ideas can be found in two review articles of Bilby [1] and Kröner [19]. Mention should be also made of an influential work of Noll, see [22], who established a connection with the constitutive properties of the material and contributed to rephrasing the problem in the modern language of continuum mechanics.

Thus, the subject has taken a central role in the last century research on material behavior. Yet, it would be problematic to say that its story has been a story of success, as acknowledged by A. Seeger at a general conference on dislocations [28], sponsored by the National Bureau of Standards.

In chairing the X Panel on: "Future directions for dislocation theory", [28, vol. II, p. 1329], Seeger frankly gave his viewpoint on two of the nine questions posed to the panel. Answering the question on what fruitful interactions can be expected between the ideas of dislocation theory and modern continuum mechanics, he observed that "[…] the dislocation theory contributed substantially to the development of continuum mechanics […] Nevertheless […] in its narrower field the development of new ideas and, even more so, of methods useful for doing calculations in dislocation theory has on the average been far ahead of the approach known as 'rational mechanics' despite statements to the contrary that may be found in the literature." As to the other question on what are the prospects of producing a macroscopic plasticity theory based on meaningful microscopic parameter, Seeger noted that: "If one extrapolates from the past, the answer to the first of these questions can only be: 'poor'." He also added: "In the writer's personal opinion 'the almost complete standstill' in this area is due to not a small degree to the confusion that has been created during the last decade by poorly founded theories of work-hardening and preliminary but over emphasized experimental work on dislocation observation, which have brought the whole field into a state of disrepute."

A similar criticism, although in softer manner and addressed to the way of doing experiments and to certain attempts to bow the dislocation theory in order to interpret them, is also present in Rodriguez's lecture.

The above premise is in order in dealing with a subject that has raised so many expectations, and for so many years, and that has produced profound divisions within the community of continuum mechanics. Having spent some time on the subject, in the following I concentrate on some contributions that did not follow the trend.

After recalling briefly the field theory and what seem to me some of its weak points, in Sects. 3 and 4 I focus on an approach that I first proposed in [5] and then developed jointly with Gareth Parry in a few papers [7–9]. The approach is founded on three keynotes. Firstly, that the model should account for different scales of description, so that even classical concepts of continuum mechanics such as that of material point, for instance, need revision; secondly, that there is no obvious notion of defectiveness in a theory based on smooth fields; thirdly, that in a crystal body it is the mutual position of atoms that counts rather than the destination of the 'material points'. The outcome of these ingredients is a theory where the macroscopic deformation does not play a direct role. Although this looks rather unusual for solids, the successful example of liquid crystals supports the confidence that the problem can be addressed in this way. Issues connected with equilibrium were discussed in [7, 15]; other aspects of the model, such as certain group theoretic implications, have been extensively studied by Parry with various co-authors. Although the founding idea seems sound, the level of generality taken for the model is probably too ambitious and certain conclusions in [7] and [15] indicate that there is something that needs to be better understood. Ericksen [13] has taken the proposal into consideration and has adapted the ideas to elaborate an equilibrium theory of X-ray observations of crystals for the case of no distributed defects. A brief account of his work is included at the end of Sect. 4.

Finally, in Sect. 5, I discuss an inspired article by C. Eckart on anelasticity in solids [11]. Eckart's paper is the last of a series, begun in 1940, on the thermodynamics of irreversible processes. In that article Eckart describes the body as it appears in the present state and introduces a material metric tensor (relative to the current state) at each point. In particular, he assumes that the material metrics may evolve both because of the changes of the current configuration, and because of permanent deformations caused by mechanical forces, as in plasticity, or growth phenomena (although growth was not in Eckart's focus at the time). The approach is very general and is phrased in very modern terms. In particular, it's remarkable that Eckart uses an argument customarily credited to Coleman and Noll to get constitutive restrictions on the stress and the so called anelasticity (rate) tensor from the second law of thermodynamics. In spite of its merits, though, the article was little noticed.[1] My intention is to revisit it and highlight some passages.

[1] In writing, I have found mention of Eckart's paper in a short note by Sadyk and Yavari [27], but it is fair to say that it has been largely overlooked by the major literature.

2 A Flavour of the Continuum Theory of Defects

All the work done on dislocations after Taylor's proposal is based on a seminal paper by Volterra [29]. Looking at that paper suggested the identification of crystal dislocations with corresponding Volterra's distortions, providing the ground for fundamental concepts in the field—the way the dislocation's strength is measured through a Burgers vector [3]; the notion of a strain energy associated with a dislocation (*self energy*); that of an interaction energy between two dislocations and how it can be calculated [4]; that of a force exerted on a dislocation by the surrounding stress field [24]. Some calculations might be done somewhat differently by mechanicians, but all this is consolidated and well accepted. A broad account is given in a review article by Cottrell [4], and in the book by Nabarro [21].

In Volterra's work there is however a point that has contributed to the misunderstanding, especially in those approaches that have considered continuous distributions of dislocations. Volterra's theory is in fact strictly confined to linear elasticity. Namely, it is based on the assumption that the displacements are 'infinitesimal', and on the systematic identification of the actual configuration (with distortions) and reference configuration (at ease, or 'dans l'état naturel'). Although the ideas of Volterra are clear, all this causes confusion, because actual and reference configurations may now have radically different topological properties, for instance. Moreover, unlike what is done in linear elasticity, the strain coming from the imposed distortions are imagined to act on the actual configuration. A consequence is that the meaning of displacement $\mathbf{u}$ changes, because the mapping $\mathbf{x} \mapsto \mathbf{x} + \mathbf{u}(\mathbf{x})$ does not restore the configuration the displacements were computed from, but fractures or overlapping may occur. As a matter of fact, in Volterra's paper the term 'deformation' is used rather informally, without specifying the configuration it refers to, as is noticed also by Nabarro, see [28, vol. I, pag. 675]. As said above, all this is a source of confusion that reflects on much of the later work in the field, see [10].

Volterra did not take into account the possibility that there may be diffuse incompatibilities, see cf. [29, p. 405]. Furthermore, having restricted himself to regular strain fields, he concluded that simply connected bodies do have an "état naturel", and considered distortions that necessarily are specific of the multiply connected ones.

The continuum theory of defects in crystals began with Nye [23], who first introduced the successful notion of *dislocation density tensor* (ddt). His argument is rather direct, but clear. Later interpretations of ddt in terms of the gradient of a 'displacement' $\mathbf{u}$ are much less so for at least two reasons—it is unsaid which is the configuration $\mathbf{u}$ should be measured from, and, just by that reason, $\mathbf{u}$ is not an observable quantity. In spite of this, Nye's paper was the beginning of an abundant literature, most of which is dedicated to geometry and to the description of general properties of non Euclidean manifolds. It is off the purposes of the present note to enter a review of the geometrical theory; an account of the work done by some of the main contributors can be found in the articles of Bilby [1] and Kröner [19].

3 An Alternative Model and the Notion of Defectiveness

After many endeavors to work out doubts and difficulties I had with the geometric theory, in [5, 6] I gave my own way to the topic. The approach described below refers to a crystal model where lattice vectors and mass density are the primitive quantities. The attention is devoted to monoatomic crystals with no visible singularities at microscopic level, as is the case of grain boundaries or twinning surfaces for instance.

Since a crystalline body is made of indistinguishable constituents, the most natural way to represent it is to avoid labelling its constituent parts, and picture it as it is perceived at the present time by suitable tools. Thus, the emphasis is not given to the notion of configuration, but rather to what is called the *state* of the crystal body, defined by the list

$$\Sigma = \{\mathbf{d}_a(\cdot), \rho(\cdot), \mathcal{B}\}, \tag{1}$$

with $\mathcal{B}$ an open simply connected region in the three-dimensional Euclidean space; $\mathbf{d}_a, a = 1, 2, 3$, three smooth fields of linearly independent vectors; and $\rho(\cdot)$ a smooth scalar field with positive values over $\mathcal{B}$. $\mathcal{B}$ is the macroscopic placement of the body in the physical space; ρ (*mass density*) describes the gross distribution of mass throughout $\mathcal{B}$; vectors $\mathbf{d}_a$ (*lattice vectors*) are objects that carry information on the crystalline structure at points of $\mathcal{B}$.

The above description envisages a theory based on observations made at two distinct levels of accuracy, respectively called macroscopic and microscopic in the following. The microscopic scale shall be coarse enough for single defects or atoms do not show up, so that a continuous description is tenable. Therefore, the term lattice vector has not the same meaning as in the molecular theories like Cauchy's. Rather, they are meant as averages of some kind, keeping record of a certain order that is assumed to remain distinguishable at microscopic level, when a mild content of defects is present. These ideas underly, or should do so, any continuum approach to crystals. A clear cut discussion of this aspect is given by Havner [16, 17] in the context of crystal plasticity.

The macroscopic level corresponds to the typical length scale of continuum mechanics. At this level the region occupied by the body appears continuous and its points $\mathbf{x}$ are recognizable during the changes of state. This allows us to use for them the conventional term of "material points" and to define the familiar notions of deformation, velocity, material domains, etc. By regarding $\mathbf{d}_a$ and ρ as functions of $\mathbf{x}$, we are choosing, for the macroscopic scale, the length scale of the continuum description. Accordingly, infinitesimal line or volume elements, and differential calculus in general, reflect notions that refer to the macroscopic length scale. In particular, ρ represents the mass per unit (macroscopic) volume.

The microscopic level, on the other hand, reflects observations made by a microscope that can distinguish features of the lattice, but not single atoms or defects. To be pedantic, one can say that lattice vectors are averaged over distances corresponding to the microscope resolution. In particular, in representing the $\mathbf{d}_a$ as functions of $\mathbf{x}$

one tacitly assumes that they do not vary too wildly within the size of a material neighborhood. Which seems a reasonable restriction on situations that a continuum theory can account for.

The reason for focussing on the notion of state (1) is the prejudice that a body is crystalline if: (i) a lattice is distinguishable; (ii) the constitutive functions are the same at all material points $\mathbf{x}$; (iii) for each $\mathbf{x}$, they depend upon the microscopic arrangement of the atoms at $\mathbf{x}$, rather than upon the macroscopic arrangement of the matter.

The last requirement expresses the view of the classical molecular theories of crystals. Ericksen [13] spends some time on this point. We can interpret it as the assumption that the constitutive functions at $\mathbf{x}$ depend upon the fields $\{\mathbf{d}_a\}$ and ρ for they describe, in some way, the microscopic arrangement of atoms and defects around $\mathbf{x}$. Stretching a little this view, one can imagine to expand the fields $\{\mathbf{d}_a(\cdot)\}$ and $\rho(\cdot)$ at $\mathbf{x}$, and assume that the material response depends on their value and that of their gradients of all order evaluated at $\mathbf{x}$:

$$\sigma(\mathbf{x}) = \left\{ \mathbf{d}_a(\mathbf{x}), \frac{\partial \mathbf{d}_a}{\partial \mathbf{x}}(\mathbf{x}), \ldots, \rho(\mathbf{x}), \frac{\partial \rho}{\partial \mathbf{x}}(\mathbf{x}), \ldots \right\}.$$

The symbol $\sigma(\mathbf{x})$ describes the *local state* of the crystal. A true local theory emerges, if one assumes that the response functions depend on the derivatives of $\mathbf{d}_a$ and ρ up to some finite order.

Let $\{\mathbf{d}^a\}$ be the *dual lattice vectors*, defined by the conditions:

$$\mathbf{d}^a \cdot \mathbf{d}_b = \delta^a_{\ b} \qquad a, b = 1, 2, 3, \tag{2}$$

with $\delta^a_{\ b}$ the Kronecker symbol. Accordingly, the local state is equivalently written as

$$\sigma(\mathbf{x}) = \left\{ \mathbf{d}^a(\mathbf{x}), \frac{\partial \mathbf{d}^a}{\partial \mathbf{x}}(\mathbf{x}), \ldots, \rho(\mathbf{x}), \frac{\partial \rho}{\partial \mathbf{x}}(\mathbf{x}), \ldots \right\}. \tag{3}$$

A deformation of the body is a change of state

$$\Sigma \to \Sigma^* = \{\mathbf{d}^*_a(\cdot), \rho^*(\cdot), \mathcal{B}^*\},$$

where $\mathbf{x}^* = \chi(\mathbf{x})$ is a smooth one-to-one correspondence between material points of $\mathcal{B}$ and $\mathcal{B}^*$, subjected to the restriction that the total mass, $\mathcal{M}_\mathcal{B}$, is the same

$$\int_\mathcal{B} \rho \, d\mathbf{x} = \int_{\mathcal{B}^*} \rho^* \, d\mathbf{x}^* \tag{4}$$

Here, $\mathbf{x}^* = \chi(\mathbf{x})$ represents the *macroscopic deformation* and $\mathbf{F}(\mathbf{x})$ is the macroscopic deformation gradient evaluated at $\mathbf{x}$.

The previous discussion on the difference of scales supports the view that material points and lattice vectors may behave independently of each other when the body undergoes a deformation. Accordingly, one does not expect that the new lattice fields $\mathbf{d}_a^*(\cdot)$ are dictated by $\chi(\cdot)$, in general. On the other hand, in order to assure that we are dealing with the same body, it is reasonable to require that the total mass in Σ and Σ^* is the same. The usual rule of mass conservation

$$\rho^*(\mathbf{x}^*) = (\det \mathbf{F}(\mathbf{x}))^{-1}\rho(\mathbf{x}) \tag{5}$$

is sufficient for this to hold, although condition (5) is more stringent than the nature of the model would require. Ericksen argues on this, cf. [13, Eq. (70) and comments therein].

In the above setting the defect notion is naturally lost, because attention is restricted to smooth fields and smooth deformations. Indeed, that notion can be introduced if one uses old ideas on the role of defects in anelastic deformations.

Continuum approaches to mechanics of crystals are dominated by Cauchy's hypothesis that the macroscopic deformation carries along the lattice changes according to the rule

$$\mathbf{d}_a^*(\mathbf{x}^*) = \mathbf{F}(\mathbf{x})\mathbf{d}_a(\mathbf{x}), \quad \text{with } \mathbf{F} = \nabla\mathbf{x}^*. \tag{6}$$

Thus, the macroscopic deformation gradient becomes the control parameter of the microscopic structure. When (6) is plugged into molecular derivations of the energy associated with a given lattice, one obtains the formal constitutive equations of continuum elasticity.

In a macroscopic theory of crystalline bodies assumption (6) proves adequate for many, if not all, circumstances. Using it, however, is not without consequences because, for instance, the new constitutive equations inherit a periodicity in $\mathbf{F}$ due to the request that the same answer is provided by macroscopic deformations that map the lattice back onto itself. These include, in particular, integer slips along lattice planes.

Taylor indicated how the anelastic behavior may arise from the movement of dislocations more favourably than from solid slips along lattice planes. As is known, the mechanism consists in the passage of a single edge dislocation through the crystal along a specific lattice plane. When the dislocation traverses to the boundary, there is a change in the shape of the body and atoms are displaced, but locally the lattice structure is unchanged in the interior. In other words, there is a macroscopic deformation without a real change of the lattice structure. One may interpret Taylor's conjecture rather broadly and stipulate that the evolution of defects generally implies anelastic deformations. That is, a violation of (6). Taylor's view can be further strengthened by assuming that defects stay frozen under deformations satisfying (6).

In [5] deformations satisfying (5), (6) were called *elastic* and it was assumed that they keep defects unchanged. A consistent notion of defectiveness is then obtained by identifying defectiveness with the totality of quantities, depending on the mass density and lattice vector fields, that stay invariant under this class of deformations. Each of these quantities shall describe some kind of defect to be possibly interpreted. The point of view is close to that of the topological theory of defects in condensed matter physics, cf. Mermin [20], but the choice of the class of topological transformations under which invariance is required is special, and the analysis does provide both a definition and a classification of defects.

4 Elastic Invariants. Ericksen's Theory of X-Ray Observations of Crystals

If one mimics the way how defects are counted over the parts of a crystal, in searching for invariants it is natural to consider quantities, depending on the local state σ, that are defined over the submanifolds of $\mathcal{B}$ and are additive with respect to the union of submanifolds. Accordingly, the attention is addressed to integral quantities of the form

$$\oint_c \mathbf{g}(\sigma) \cdot d\mathbf{x}, \quad \int_{\mathcal{S}} \mathbf{g}(\sigma) \cdot \mathbf{n}\, dS, \quad \int_{\mathcal{V}} g(\sigma)\, dV, \tag{7}$$

defined over oriented circuits c, closed surfaces $\mathcal{S}$ and volumes $\mathcal{V}$ in $\mathcal{B}$ that stay invariant under the elastic deformations. That is, to the most general vector and scalar valued functions $\mathbf{g}$ and g such that

$$\oint_c \mathbf{g}(\sigma) \cdot d\mathbf{x} = \oint_{c^*(c)} \mathbf{g}(\sigma^*(\sigma)) \cdot d\mathbf{x}^*, \quad \int_{\mathcal{S}} \mathbf{g}(\sigma) \cdot \mathbf{n}\, dS = \int_{\mathcal{S}^*(\mathcal{S})} \mathbf{g}(\sigma^*) \cdot \mathbf{n}^*\, dS^*, \quad \text{etc.,}$$

for all material c, $\mathcal{S}$ and $\mathcal{V}$ and all elastic deformations driven by some macroscopic deformation $\mathbf{x}^* = \chi(\mathbf{x})$.

Integrals with the above property were called *elastic invariants*. They are expected to describe, and measure, the strength of defects of some kind contained in, or embraced by, the respective material domains. Either directly, as for the volume invariants in (7), or by applying Stokes and divergence theorems, as for the other two, each of them leads to a density that provides a local measure of defectiveness at the points of $\mathcal{B}$.

A first characterization of the elastic invariants was given in [5, 7]; the analysis was further extended in [8], where it is shown both how to construct invariants of all orders and that there is a finite list of them which provides a full characterization of defectiveness, see [8, Theorem 6]. The reader should refer to those papers for the details.

Let us focus on the *first order invariants*, that is the invariants whose densities contain at most the first order derivatives of the basic fields $\mathbf{d}^a(\cdot)$ and $\rho(\cdot)$. By using the transformation rules of the mass density and the dual lattice vectors:

$$\rho^* = \rho\, (\det \mathbf{F})^{-1} \quad \text{and} \quad \mathbf{d}^{a*} = \mathbf{F}^{-1T} \mathbf{d}^a,$$

under the elastic deformations, the analysis yields that the simplest invariants of the first order are given by

$$\mathfrak{B}^a[c] = \oint_c \mathbf{d}^a \cdot d\mathbf{x}, \quad \mathfrak{J}^{ab}[\mathcal{S}] = \int_{\mathcal{S}} \mathbf{d}^a \times \mathbf{d}^b \cdot \mathbf{n}\, dS,$$
$$\mathfrak{N}[\mathcal{V}] = \int_{\mathcal{V}} \mathfrak{n}\, dV, \quad \mathfrak{S}^{ab}[\mathcal{V}] = \int_{\mathcal{V}} \mathfrak{s}^{ab}\, dV, \tag{8}$$

and

$$\mathfrak{D}^a[c] = \oint_c \mathfrak{m}\, \mathbf{d}^a \cdot d\mathbf{x}, \quad \mathfrak{K}^{ab}[\mathcal{S}] = \int_{\mathcal{S}} \mathfrak{m}\, \mathbf{d}^a \times \mathbf{d}^b \cdot \mathbf{n}\, dS,$$
$$\mathfrak{M}[\mathcal{V}] = \int_{\mathcal{V}} \mathfrak{m}\, \mathfrak{n}\, dV, \quad \mathfrak{G}_a[\mathcal{V}] = \int_{\mathcal{V}} \mathfrak{g}_a\, \mathfrak{n}\, dV. \tag{9}$$

Here, $\mathfrak{n}$, $\mathfrak{s}^{ab}$, $\mathfrak{m}$ and $\mathfrak{g}_a$ are defined by

$$\mathfrak{n} := \mathbf{d}^1 \cdot \mathbf{d}^2 \times \mathbf{d}^3, \quad \mathfrak{s}^{ab} := \nabla \times \mathbf{d}^a \cdot \mathbf{d}^b, \quad \mathfrak{m} := \rho\, (\mathbf{d}_1 \cdot \mathbf{d}_2 \times \mathbf{d}_3), \quad \mathfrak{g}_a := \frac{\partial \mathfrak{m}}{\partial \mathbf{x}} \cdot \mathbf{d}_a. \tag{10}$$

Noticing that $(\mathbf{d}_1 \cdot \mathbf{d}_2 \times \mathbf{d}_3)$ is the volume of the lattice cell, $\mathfrak{m}$ can be regarded as the *cell mass*. Furthermore, $\mathfrak{n}$, which is equal to $(\mathbf{d}_1 \cdot \mathbf{d}_2 \times \mathbf{d}_3)^{-1}$, is the *number of cells per unit volume*. Finally, the quantities $\mathfrak{s}^{ab}$ and $\mathfrak{g}_a$ respectively are the lattice components of $\nabla \times \mathbf{d}^a$ and $\frac{\partial \mathfrak{m}}{\partial \mathbf{x}}$. For convenience, in the above formulae the invariants depending on the cell mass and its gradient have been grouped separately.

List (8) contains known quantities of the classical theory of defects, although obtained from a different perspective, and some new ones.

The $\mathfrak{B}^a[c]$ are the Burgers integrals of the continuum theory of dislocations. According to an interpretation introduced by Bilby et al. [2], the term $\mathbf{d}^a \cdot d\mathbf{x}$ represents the number (with sign) of lattice steps in the a-lattice direction corresponding to the line elements $d\mathbf{x}$. The integration of the differential forms $dy^a = \mathbf{d}^a \cdot d\mathbf{x}$, $a = 1, 2, 3$, along c gives the parametric representation of a curve pictorially interpreted as the image of c in a perfect lattice. Thence, the Burgers numbers $\mathfrak{B}^a[c]$ measure its failure from closure. From Stokes' theorem we get that

$$\mathfrak{B}^a[c] = \int_{\mathcal{S}[c]} \nabla \times \mathbf{d}^a \cdot \mathbf{n}\, dS,$$

with $\mathcal{S}[c]$ any oriented surface with boundary c. Therefore, distributed dislocations reflect the anholonomy of the dual lattice vectors and their strength is locally described by the *Burgers vector*

$$\mathfrak{b}^a := \nabla \times \mathbf{d}^a, \tag{11}$$

that is a measure of the closure failure (per unit area) of infinitesimal circuits on the surface $\mathcal{S}[c]$. In the theory of Bilby, Bullough and Smith quantity $\mathfrak{b}^a$ is often replaced by the *dislocation density tensor*, that with our notations reads as

$$\mathbf{S} := \mathbf{d}_a \otimes \mathfrak{b}^a. \tag{12}$$

The $\mathfrak{s}^{ab}$ defined in (10)$_2$ are then the lattice components of **S**.

The quantity $\mathfrak{N}[\mathcal{V}]$ is new. Since $\mathfrak{n}$ is the number of cells per unit volume, $\mathfrak{N}[\mathcal{V}]$ gives the total number of cells contained in $\mathcal{V}$. In particular, $\mathfrak{n}$ can be taken as a natural measure of the density of vacancies and interstitials, counted together, at the points of the crystal. The other invariants in (9) have less transparent interpretations, except for $\mathfrak{M}(\mathcal{V})$ which gives the total mass $\mathcal{M}_{\mathcal{B}}$ contained in $\mathcal{V}$.

The nature of the model described above is that local states are given by the list (3) and that the free energy density depends upon that list. The systematic charcterization of the invariants and their associated densities serves the purpose to pick out of that list those terms which are specific of some kind of defectiveness, the others being associated with higher-grade elasticity effects. This is discussed in [6].

Notice that the densities $\mathfrak{b}^a$, $\mathfrak{n}$, etc., are determined by $\mathfrak{s}^{ab}$ and $\mathbf{d}_a$, etc. Then, by taking the Galilean invariance into account, one may assume that the free energy density depends upon the variables

$$\sigma = \{\mathbf{d}^a \cdot \mathbf{d}^b, \mathfrak{s}^{ab}/\mathfrak{n}, \mathfrak{m}, \mathfrak{g}_a, \ \ldots\},$$

where dots stand for suitable local descriptors of defectiveness, taken from the complete list explicitly given in [8, Theorem 6]. Or, in a slightly more restrictive theory, that the free energy density depends upon the list

$$\sigma = \{\mathbf{d}^a \cdot \mathbf{d}^b, \mathfrak{s}^{ab}/\mathfrak{n}, \mathfrak{m}, \mathfrak{g}_a\}. \tag{13}$$

With reference to the list (13), for simplicity, observe that the quantities

$$\mathfrak{s}^{ab}/\mathfrak{n} = \mathfrak{s}^{ab^*}/\mathfrak{n}^*, \quad \mathfrak{m} = \mathfrak{m}^*, \quad \mathfrak{g}_a = \mathfrak{g}^*,$$

do not change under elastic deformations, as is easily seen. So, states that share the same values of these quantities correspond to states with the same defectiveness.

Calling for the notion of free energy and the variational criteria for equilibrium raises subtle issues which are partly discussed by Ericksen [12]. In brief, one point is that when speaking of free energy one should compare states that can be reached from one another by reversible, or asymptotically reversible processes, as for viscous materials. But this is not the case in a model where the evolution of defects is associated to the intrisically dissipative mechanisms of plasticity. Thus, one should compare states corresponding to the same defectiveness. That is, states with the same values of $\mathfrak{s}^{ab}/\mathfrak{n}$, $\mathfrak{m}$, $\mathfrak{g}_a$.

It turns out that, for special lattice vector fields, the class of deformations that leave defectiveness unchanged is larger than the class of the elastic deformations. We called the additional ones *neutral* and discussed them in [7]. Neutral deformations are rearrangements of the material that strongly evoke elementary mechanisms of crystal plasticity, see [9]. If this is an appraisal of the analysis done, on one side, to allow for them without suitable penalization in equilibrium criteria yields restrictions on the possible states of stress at equilibrium that are unexpected for solid crystals, see [7, 15]. So, this is a major point left behind.

It's worth to conclude this section by quoting an interesting article by Ericksen [13] that moves from the same viewpoints as the above analysis. There, Ericksen takes into consideration the hint that lattice vectors, instead of the macroscopic deformation, should be the primary variables in a theory of crystals that accounted for the different scales of observations. Accordingly, he works out an equilibrium theory of what he calls X-ray observations of crystals.

Ericksen considered the case of no distributed defects. Therefore, the mass density is given by Eq. $(10)_3$, where now μ is a constant quantity distinctive of the given crystal, since there are no interstitials or vacancies; and the dual lattice vectors are gradients

$$\mathbf{d}^a = \frac{\partial \chi^a}{\partial \mathbf{x}}. \tag{14}$$

of three functionally independent scalar fields χ^a, since the Burgers vectors $\mathfrak{b}^a$ are identically zero in $\mathcal{B}$. Ericksen calls the χ^a *crystallographic coordinates*. One can interpret them as Cartesian coordinates of points in some *crystallographic space*, if he wishes, but it's more appropriate to avoid this and use the current position of points instead, because the notion of deformation involves observations at some grosser scale.

With the above notation, let

$$\varphi = \hat{\varphi}(\mathbf{d}_a, \theta) = \bar{\varphi}(\mathbf{d}^a, \theta)$$

be the free energy per unit mass. Then, standard arguments from molecular elasticity provide for the Cauchy stress the expression

$$\mathbf{t} := \rho \frac{\partial \hat{\varphi}}{\partial \mathbf{d}_a} \otimes \mathbf{d}_a = -\,\rho \mathbf{d}^a \otimes \frac{\partial \bar{\varphi}}{\partial \mathbf{d}^a} = -\,\rho \frac{\partial \bar{\varphi}}{\partial \mathbf{d}^a} \otimes \mathbf{d}^a.$$

Altermately, by introducing the energy per unit volume $w := \rho\,\bar{\varphi}$, one also gets

$$\mathbf{t} = w\,\mathbf{1} - \frac{\partial w}{\partial \mathbf{d}^a} \otimes \mathbf{d}^a.$$

The equilibrium equations are derived from the variational principle:

$$\delta \mathcal{E} = \delta W \quad \text{under the constraint} \quad \delta \mathcal{M}_{\mathcal{B}} = \delta \int_{\mathcal{B}(t)} \bar{\rho}\, dv \bigg|_{t=0} = 0, \tag{15}$$

where $\mathcal{E}$ is the free energy of the body and W the potential of the external forces. Here, $\bar{\rho} = \mathfrak{m}\, \bar{\mathbf{d}}^1 \cdot \bar{\mathbf{d}}^2 \times \bar{\mathbf{d}}^3$ for a given constant cell mass $\mathfrak{m}$.

The variations are calculated for virtual *changes of configuration* from the current state. These are described by functions $\bar{\mathbf{x}} = \bar{\mathbf{x}}(\mathbf{x}, t)$ and $\bar{\chi}^a = \bar{\chi}^a(\bar{\mathbf{x}}, t)$ such that

$$\bar{\mathbf{x}}(\mathbf{x}, 0) = \mathbf{x} \quad \text{and} \quad \bar{\chi}^a(\bar{\mathbf{x}}(\mathbf{x}, 0), 0) = \chi^a(\mathbf{x}).$$

Thus, the virtual variations of the current placement and lattice coordinates to be considered in the variational argument are given by

$$\delta\mathbf{x} := \frac{\partial\bar{\mathbf{x}})}{\partial t}\Big|_{t=0} \quad \text{and} \quad \delta\chi^a := \frac{\partial\bar{\chi}^a}{\partial t}\Big|_{t=0}. \tag{16}$$

Ericksen's calculations are subtle and illuminating. The fact that μ is constant and the $\mathbf{d}^a$ are gradients plays a non minor part in the calculations. The outcome is a non conventional theory where both forces and configurational forces, together with the tight connection between the two, are involved. In [13] Ericksen adapts the ideas to build a theory of twinning in crystals, which makes no use of Cauchy hypothesis, or mention of any macroscopic shear. The approach is then interesting, because it leaves the possibility that atoms shuffle during twinning. For comments on the general theory and its application to crystal twinning, the reader should refer to the original paper.

5 Eckart's Approach to Anelasticity

Finally, let me report on a far-sighted paper by Eckart [11], on a theory of anelasticity, which was largely ignored without reason at the time. The paper aims at indicating how the classical theory of elasticity can be extended to include anelastic phenomena if one removes two cornerstones of the traditional theory: (1) that there is a constant relaxed state; and (2) that there is a relaxability-in-the-large of the elastic body. The latter asserts that the strains in a solid object can be completely relaxed by removing all external forces.[2] Eckart shows that, by postulating only relaxability-in-the-small, the ground is cleared for the construction of a theory of anelasticity.

The starting point is the assumption that at every point of the current configuration two metrics are given: one measuring the actual length of the material element

$$(dl)^2 = \delta_{ij} dx_i\, dx_j;$$

the other its natural length

$$(d\lambda)^2 = g_{ij} dx_i\, dx_j.$$

[2] Quoting from Eckart's paper: "This principle has an interesting history. It apparently entered the theory as a tacit assumption and remained unrecognized, until in 1864 de Saint-Venant criticized Maxwell for ignoring it in the development of a general method for the solution of elastic problems.".

Thus, g_{ij} is a local measure of the strain with respect to the natural state of the material. To get it, Eckart imagines that "…if a small bit of the matter surrounding P is cut out of the larger object, then all strains in this bit will be relaxed, since it will have no forces acting on it.", according to a pictorial representation that is familiar nowadays.

Here and in what follows x_i, $i = 1, 2, 3$, stand for the Cartesian components of $\mathbf{x}$ in some reference frame; also, the indices denote components of tensors in the same frame.

Imagine that $\mathbf{y}(\mathbf{x}, t) = \mathbf{x} + \mathbf{u}(\mathbf{x}, t)$ is a motion of the body. Then, one calculates that

$$\frac{D(dl)^2}{Dt} = \left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right) dx_i dx_j =: 2\dot{u}_{(ij)} dx_i dx_j .$$

Furthermore,

$$\frac{D(d\lambda)^2}{Dt} = \left(\frac{\partial g_{ij}}{\partial t} + \dot{u}_k \frac{\partial g_{ij}}{\partial x_k} + g_{ik}\frac{\partial \dot{u}_k}{\partial x_j} + g_{kj}\frac{\partial \dot{u}_k}{\partial x_i}\right) dx_i dx_j =: 2a_{ij} dx_i dx_j$$

Eckart calls $\{a_{ij}\}$ the *anelasticity tensor*. The principle of a constant relaxed state would require that the relaxed length of dx_i be independent of t, and hence that $a_{ij} = 0$ at all times and places. Of course, the rejection of this principle has the negative effect of leaving a_{ij} physically indeterminate.

It is calculated that

$$a_{ij} = \frac{1}{2}\frac{Dg_{ij}}{Dt} + U_{(ij)} \quad \text{with} \quad U_{(ij)} := \frac{1}{2}\left(g_{ik}\frac{\partial \dot{u}_k}{\partial x_j} + g_{kj}\frac{\partial \dot{u}_k}{\partial x_i}\right). \tag{17}$$

In particular, from

$$U_{(ij)} = \frac{1}{2}\left(g_{ik}\dot{u}_{(k,j)} + g_{kj}\dot{u}_{(k,i)}\right) + \frac{1}{2}\left(g_{ik}\dot{u}_{[k,j]} + g_{kj}\dot{u}_{[k,i]}\right),$$

it follows that $U_{(ij)}$ depends upon the vorticity $\dot{u}_{[i,j]}$ also.

Other relevant equations are

$$\frac{D}{Dt}(\log g) = g^{ij}\frac{Dg_{ij}}{Dt}, \tag{18}$$

with $g := \det g_{ij}$, and

$$\frac{\partial \dot{u}_i}{\partial x_i} = g^{ij}a_{ij} - \frac{1}{2}\frac{D}{Dt}(\log g), \tag{19}$$

where $\{g^{ij}\}$ is the inverse of $\{g_{ij}\}$. Then, by using the equation of mass conservation

$$\frac{D\rho}{Dt} = -\rho\frac{\partial \dot{u}_i}{\partial x_i} \quad \left(\Rightarrow \frac{D}{Dt}(\log v) = \frac{\partial \dot{u}_i}{\partial x_i}\right).$$

with $v := 1/\rho$ the *specific volume*, we get from (19) that

$$\frac{D\log(v\, g^{\frac{1}{2}})}{Dt} = g^{ij} a_{ij}, \tag{20}$$

Note that one consequence of the principle of a constant relaxed state would be that the product $vg^{1/2}$ is constant for each bit of the substance. In the present theory, the quantities v and g become independent, and must both be specified before the state of the substance is known.

Eckart calculates the balance laws: the conservation of momentum is given by

$$\rho \frac{D\dot{u}_i}{Dt} = \frac{\partial S_{ij}}{\partial x_i},$$

with $S_{ij} = S_{ji}$ *Cauchy stress tensor*; and the conservation of energy is

$$\rho \frac{D}{Dt}(\frac{1}{2}\dot{u}_i\dot{u}_i + \varepsilon) + \frac{\partial q_i}{\partial x_i} = \frac{\partial}{\partial x_j}(\dot{u}_i S_{ij}).$$

with ε the *internal energy* per unit mass and q_i the *heat flux*.

Then, if we assume that ε depends upon the specific volume, the (natural) metric tensor g_{ij} and the entropy η, it follows that

$$\begin{aligned} \frac{D\varepsilon}{Dt} &= \frac{\partial \varepsilon}{\partial v}\frac{Dv}{Dt} + \frac{\partial \varepsilon}{\partial g_{ij}}\frac{Dg_{ij}}{Dt} + \frac{\partial \varepsilon}{\partial \eta}\frac{D\eta}{Dt} \\ &= p\frac{Dv}{Dt} + \frac{\partial \varepsilon}{\partial g_{ij}}\frac{Dg_{ij}}{Dt} + \theta\frac{D\eta}{Dt}, \end{aligned} \tag{21}$$

where the thermodynamic definitions of the hydrostatic pressure: $p := \frac{\partial \varepsilon}{\partial v}$, and the absolute temperature: $\theta := \frac{\partial \varepsilon}{\partial \eta}$, have been used.

The calculation of the second term on the right of (21) is more complicated, but it turns out that it can be written as

$$\rho \frac{\partial \varepsilon}{\partial g_{ij}}\frac{Dg_{ij}}{Dt} = -\frac{1}{2}E_{ij}\left(a_{ij} - U_{(ij)}\right),$$

with $U_{(ij)}$ given by (17) and $E_{ij} := -\rho\left(\frac{\partial \varepsilon}{\partial g_{ij}} + \frac{\partial \varepsilon}{\partial g_{ji}}\right)$. By using these and the conservation of momentum in the balance of energy, it can be shown that the first law of thermodynamics takes the form

$$\rho\frac{D\eta}{Dt}+\frac{\partial}{\partial x_i}\left(\frac{q_i}{\theta}\right)=-\left(\frac{1}{\theta}\right)^2 q_i\frac{\partial\theta}{\partial x_i}$$
$$+\left(\frac{1}{\theta}\right)\dot{u}_{(ij)}\left[S_{ij}+p\delta_{ij}-\frac{1}{2}g_{ik}E_{kj}-\frac{1}{2}g_{jk}E_{ki}\right]$$
$$+\left(\frac{1}{\theta}\right)E_{ij}\left[a_{ij}-\frac{1}{2}g_{ik}\dot{u}_{[kj]}-\frac{1}{2}g_{jk}\dot{u}_{[ki]}\right], \tag{22}$$

where the dissipative terms are grouped on the right hand side.

From Eckart's words: "The second law of thermodynamics asserts that, for all motions compatible with the laws of physics, the right side of this equation is greater than zero. If the physical laws are unknown, this imposes restrictions on their possible forms.

One particular set of laws that satisfies these restrictions is …"

$$q_i=-k\frac{\partial\theta}{\partial x_i}$$
$$S_{ij}=-p\delta_{ij}+\frac{1}{2}g_{ik}E_{kj}+\frac{1}{2}g_{jk}E_{ki}+N_{((ij)(kl))}\dot{u}_{(kl)} \tag{23}$$
$$a_{ij}=\frac{1}{2}g_{ik}\dot{u}_{[kj]}+\frac{1}{2}g_{jk}\dot{u}_{[ki]}+M_{((ij)(kl))}E_{kl}.$$

with $k>0$, and the quadratic forms

$$N_{((ij)(kl))}\dot{u}_{(kl)}\dot{u}_{(ij)} \quad\text{and}\quad M_{((ij)(kl))}E_{kl}E_{ij}$$

definite positive.

Here Eckart states quite neatly how to use thermodynamics to get constitutive restrictions. Two more comments are noteworthy. The first is that $N_{((ij)(kl))}\dot{u}_{(kl)}$ is a viscous stress. So, the formula is a generalization of Stokes–Navier formula appropriate for aelotropic substances. The second, that two of the above formulae can be given the form

$$\frac{1}{2}\frac{Dg_{ij}}{Dt}=\frac{1}{2}g_{ik}\dot{u}_{(kj)}+\frac{1}{2}g_{jk}\dot{u}_{(k)]}+M_{((ij)(kl))}E_{kl}+U_{(ij)} \tag{24}$$

Eckart's comment of this is:

"[Eq. (24)] asserts that changes in the strain metric at a given bit of matter are caused, not only by the deformation of the matter, but also by the elastic stresses. It is very interesting to note that the hydrostatic pressure, p, does not contribute to these irreversible changes in the strain metric."

The latter observation is consistent with the well known fact that plastic yielding is not influenced by the hydrostatic pressure.

6 Final Remarks

In conclusion, some words on the reason why choosing inhomogeneities for the fifty years anniversary volume of AIMETA. The subject has obvious connections with ideas born in the Italian school of Mechanics, although I do not know of many Italians involved with the theory of dislocations in the sense of material scientists. The topic is central to explain anelasticity and the attention to it, begun with the dislocation theory, coincides with the early appearance in continuum mechanics of the need to clear the fascinating relation between micro and macro descriptions of the physical world. This theme is particularly felt by mechanicians nowadays.

The subject of inhomogeneities remains an open field of research. This is attested by the abundant literature of the last thirty years, even if I decided not to cover it in this note. I do not underestimate the contributions given to provide answers to a difficult problem, but it is worth saying that while the phenomenological theory of plasticity, and also the rich mathematics involved, have reached a high level of elaboration, a satisfactory theory of the microscopic bases of plasticity is still missing. In the period indicated in the title the subject has grown faulty, in spite of the undoubted quality of many of the people involved. This entailed a certain standstill (Seeger) and the neglect of an absolutely key contribution that anticipated instances that we find, for instance, in the nowadays research on growth. On the latter theme many young Italian researchers are involved. One purpose, in writing this note, was to draw attention to Eckart's paper.

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Configurational Forces on Elastic Structures

Davide Bigoni, Federico Bosi, Francesco Dal Corso, and Diego Misseroni

Abstract The discovery of configurational forces acting on elastic structures and its initial applications are reviewed. Configurational forces are related to the possibility that an elastic structure can change its configuration, thus inducing a variation in the potential energy. This concept has already led to several applications (the elastica arm scale, the dripping of an elastic rod, and the torsional actuator), has been shown to strongly affect stability, and to be related to limbless locomotion. It is believed that these results will open a new research territory in mechanics.

Keywords Eshelbian mechanics · Configurational force · Elastica · Structural stability · Locomotion · Deployable structures

1 Introduction

The concept of configurational force was introduced by Eshelby [1–4] to describe the tendency of defects to move inside solids. In particular, it is postulated that the movement occurs in a way that the total potential energy of the mechanical system decreases, until eventually it reaches a minimum, corresponding to a configuration where these forces vanish. The defects can be massless, such as voids, cracks, vacancies, and dislocations, or can possess a mass, such as inclusions. Within this framework, it may be easier to figure out the movement of a massless defect, e.g. a dislocation in a crystal lattice, than a stiff inclusion in a material. However, the latter can be identified as a portion of material belonging to a phase different from that of the surrounding material, so that the boundary of the inclusion will grow or shrink to minimize the energy.

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The configurational force, also called 'Eshelbian', or 'material', or 'driving', or 'non-Newtonian', is defined as the negative gradient of the total potential energy V of a body with respect to a parameter κ determining the configuration of the defect, namely, $-\partial V(\kappa)/\partial\kappa$.

Well-known examples of configurational forces are the Peach-Koehler interaction between dislocations, the crack-extension force in fracture mechanics, or the material force developing on a phase boundary in a solid under loading. These forces are the central concept in Eshelbian mechanics, a well-consolidated and famous theory (see, for instance, the monographs by Gurtin [5], Kienzler and Herrmann [6], and Maugin [7, 8], and the journal special issues by Dascalu et al. [9], and Bigoni and Deseri [10]).

Despite the broad diffusion of Eshelbian mechanics, examples of configurational forces developing and acting on elastic structures were unknown before the work by Davide Bigoni, Federico Bosi, Francesco Dal Corso and Diego Misseroni, the authors of this article, who opened this field in [11]. The key concept relies on introducing a particular constraint, which allows a change in configuration of the elastic system. When this change leads to a variation of the total potential energy of the structure, an Eshelby-like force develops. A simple constraint permitting the development of a configurational force is the sliding sleeve, which constrains a portion of an elastic rod inserted into it. This constraint prevents rotation and transverse displacement, but allows for axial frictionless movement. The portion of the rod inside the sliding sleeve can be considered as a 'defect', which might change its position, so inducing a variation in the total potential energy and generating a related configurational force.

The simplest configurational force is developed at the end of a sliding sleeve constraining an initially straight elastic rod, subject to a transverse load P, Fig. 1a.

This structure was analyzed in a fully non-linear regime and the configurational force was demonstrated using two different approaches, namely, a variational technique and an asymptotic method, the latter based on the definition of an imperfection in the constraint. Finally, the structure has been realized and instrumented, so that the configurational force has been validated through experimental measures.

The presence of the configurational force was also theoretically derived and experimentally verified within a dynamic framework. More specifically, Armanini et al. [12] have shown that the configurational force deeply influences the dynamics of an elastic rod with a lumped mass at one end and constrained with a frictionless sliding sleeve at the other, as shown in Fig. 2. Moreover, it has also been proven that configurational forces acting at the ends of an elastic rod can strongly influence band gaps in the wave dispersion diagram and introduce a nonlinear coupling between longitudinal and transverse displacements [13].

In this article, the effects of configurational forces on one-dimensional elastic structures are reviewed, together with applications to soft devices (the elastica arm scale, the dripping of an elastic rod, and the torsional actuator), implications on stability (the penetration on an elastic blade), and connections with limbless locomotion (the snaking rod).

It is suggested that the concept of configurational force may lead to a new and exciting research field, *configurational structural mechanics*, connected to the devel-

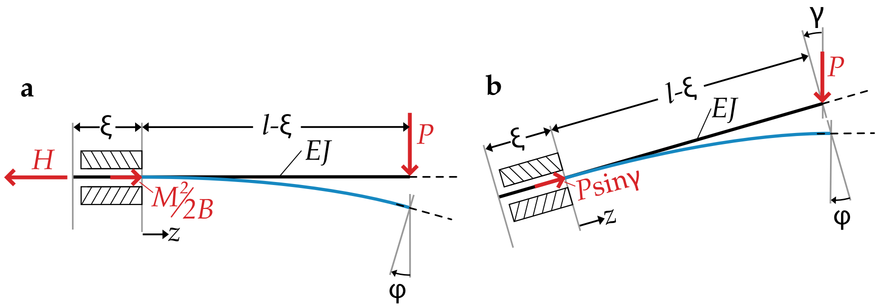

Fig. 1 **a** A simple elastic structure showing the existence of a configurational force. The key to understanding why this force develops is the presence of the sliding sleeve on the left end of an elastic rod of total length l, constraining a length ξ. The elastic rod is subjected to a dead vertical load P on its right end and to an axial dead force H applied at its left end. The presence of the Eshelby-like force $P^2(l-\xi)^2/(2B) = P\phi$ (where $B = EJ$ is the rod's bending stiffness and ϕ is the rotation of the loaded end of the rod) changes the force H at equilibrium, not null whenever the rod is bent. **b** In the absence of the axial force, $H = 0$, the equilibrium of the inclined sliding sleeve (the load has a transverse component equal to $P\cos\gamma$) is obtained only when $\gamma = \phi$. When $\phi < \gamma$ the rod slips inside the sliding sleeve, while, when $\phi > \gamma$, the rod is ejected outside the sliding sleeve

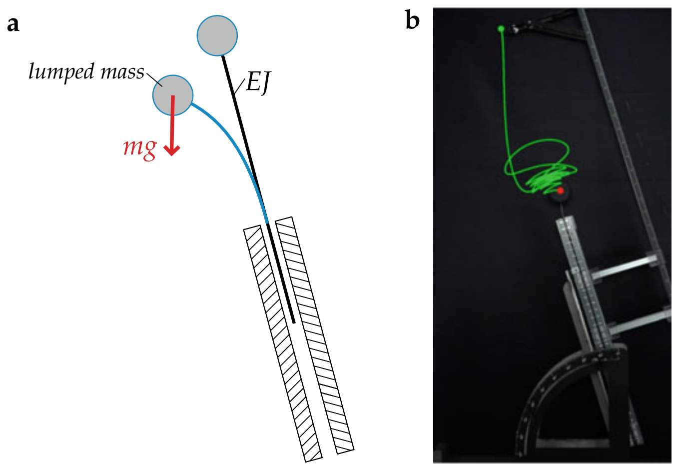

Fig. 2 **a** Schematic of the experiment showing the effects of a configurational force on the dynamics of an elastic rod equipped with a lumped mass. **b** Snapshot of an experiment showing the trajectory of the lumped mass (green line) and evidencing an oscillatory motion during which the elastic rod slips (*into* and *out from* the sliding sleeve) because of the action of a configurational force

opment of devices to be employed in soft robotics, actuation, deployable structures, and deformable sensors.

2 Configurational Force in a Simple Elastic Structure

With reference to Fig. 1a, an elastic rod (straight in its unloaded configuration) of uniform bending stiffness EJ and total length l is considered, constrained by a (frictionless) siding sleeve and subjected to a transverse and an axial force, respectively P and H, applied at the two ends. Under the small rotation hypothesis, the generic configuration of the system is defined by the transverse displacement field $v(z)$, where z is the curvilinear coordinate measured from the sliding sleeve exit. Moreover, the configuration is also described by the configurational parameter ξ, defining the position of the sliding sleeve exit along the rod, and therefore the length $l - \xi$ of the rod undergoing flexural deformations. The total potential energy V can be written as

$$V(v, \xi) = \frac{EJ}{2} \int_0^{l-\xi} \left(v''\right)^2 dz - P \int_0^{l-\xi} v' \, dz - H\xi, \tag{1}$$

where a dash means derivation with respect to the coordinate z. The transverse displacement field is subjected to the following kinematic conditions at the sliding sleeve exit

$$v(0) = v'(0) = 0. \tag{2}$$

Following a variational approach, the transverse displacement v and the length ξ are subjected to variations $\tilde{v}$ and $\tilde{\xi}$ through a small parameter ϵ as

$$v \longrightarrow v + \epsilon \, \tilde{v}, \quad \xi \longrightarrow \xi + \epsilon \, \tilde{\xi}. \tag{3}$$

By considering the kinematic boundary conditions (2), the constraints on the variation $\tilde{v}$ in the transverse displacement field at the sliding sleeve exit are

$$\tilde{v}(0) = \tilde{v}'(0) = 0. \tag{4}$$

Annihilation of the first variation (in ϵ) of the total potential energy V is

$$EJ \int_0^{l-\xi} v'' \tilde{v}'' \, dz - \frac{EJ \left[v''(l-\xi)\right]^2}{2} \tilde{\xi} - P \int_0^{l-\xi} \tilde{v}' \, dz + \left[P v'(l-\xi) - H\right] \tilde{\xi} = 0. \tag{5}$$

Considering Eq. (4), the first integral in Eq. (5) can be evaluated through integration by parts as

$$\int_0^{l-\xi} v''\tilde{v}''\,dz = v''(l-\xi)\,\tilde{v}'(l-\xi) - \int_0^{l-\xi} v'''\tilde{v}'\,dz. \tag{6}$$

and therefore the first variation (5) reduces to

$$\begin{aligned}&-\int_0^{l-\xi} \left(EJ\,v''' + P\right)\,\tilde{v}'\,dz + EJ\,v''(l-\xi)\tilde{v}'(l-\xi)\\&-\left[H + \frac{EJ\left[v''(l-\xi)\right]^2}{2} - Pv'(l-\xi)\right]\tilde{\xi} = 0.\end{aligned} \tag{7}$$

Taking into account Eq. (4), the integral in Eq. (7) can be further integrated by parts as

$$-\int_0^{l-\xi} \left(EJ\,v''' + P\right)\,\tilde{v}'\,dz = \left[EJ\,v'''(l-\xi) + P\right]\tilde{v}(l-\xi) - EJ\int_0^{l-\xi} v''''\,\tilde{v}'\,dz, \tag{8}$$

so that annihilation of the first variation under arbitrary variations $\tilde{v}$ yields to the governing equation of the Euler elastica (in its linearized version)

$$v''''(z) = 0, \qquad z \in [0, l-\xi] \tag{9}$$

complemented, in addition to the kinematic boundary conditions (2), by the static boundary conditions

$$v'''(l-\xi) = -\frac{P}{EJ}, \quad \text{and} \quad v''(l-\xi) = 0, \tag{10}$$

representing the shear and moment conditions at the end of the rod, $z = l - \xi$.

The arbitrariness of $\tilde{\xi}$ allows concluding from Eq. (7) that axial equilibrium does not always correspond to $H = 0$ as it would follow by ignoring the configurational force. More specifically, H corresponds to a non-null value when the rod is bent and is expressed by

$$H = P\phi, \tag{11}$$

where ϕ is the rotation at the loaded end, $\phi = v'(l-\xi)$. Through integration of the linearized elastica, Eq. (9), under the boundary conditions (2) and (10), the loaded end rotation ϕ results

$$\phi = \frac{P(l-\xi)^2}{2EJ}, \tag{12}$$

and therefore the equilibrium of the system evidences the presence of a *configurational force*

$$H = \frac{P^2(l-\xi)^2}{2EJ}. \tag{13}$$

If the elastic rod is inclined at an angle γ with respect to the loading P, so that its transverse and axial components are respectively $P\cos\gamma$ and $P\sin\gamma$, the equilibrium equation (11) would change to

$$H + P\sin\gamma = P\phi\cos\gamma, \tag{14}$$

where the rotation ϕ of the loaded end of the rod is now

$$\phi = \frac{P\sin\gamma(l-\xi)^2}{2EJ}. \tag{15}$$

From Eq. (14), the equilibrium in the case of null force $H = 0$ (Fig. 1b) implies the following geometric condition

$$\phi = \tan\gamma, \tag{16}$$

which, assuming the angle γ to be sufficiently small to allow the retainment of the linear term, leads to

$$\phi = \gamma, \tag{17}$$

implying an orthogonality condition between the applied load and the rod's tangent.

The geometric condition (17) to attain equilibrium has been proven to hold also in the case of large rotations and large angles γ and shows that [11]

- when $\phi = \gamma$ the rod is at equilibrium;
- when $\phi < \gamma$ the rod slips inside the sliding sleeve;
- when $\phi > \gamma$ the rod is ejected outside the sliding sleeve.

3 Elastica Arm Scale

With reference to Fig. 3a, the concept underlying the elastica arm scale is the result of nonlinear equilibrium kinematics of a rod constrained centrally by a sliding sleeve, which induces two configurational forces, with outward direction from each sliding sleeve exit and opposite to each other. Therefore, the deflection of the arms becomes necessary for equilibrium, which would be impossible for a rigid system. The rigid arms of ordinary scales are replaced by a flexible elastic lamina, so that the rod can reach a unique equilibrium configuration when two vertical dead loads are applied at its ends. This configuration has been analytically solved by Bosi et al. [14], showing that the knowledge of a given load value P (at the right end) and the measure of the equilibrium length l_{eq} allows for the identification of an unknown load Q (at the left end).

In a sense, the elastica arm scale is realized through the combination of two different mechanical principles, each one underlying the operation of a specific family of classical scales: (i) equilibrium (on which the steelyard scale is based) and (ii)

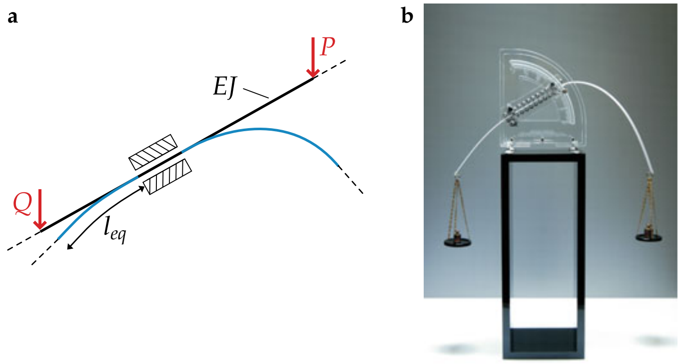

Fig. 3 **a** Schematic of the elastica arm scale based on an inclined sliding sleeve. The equilibrium is attained in the sliding sleeve direction through the contributions of the dead loads P and Q, and the two configurational forces arising from the elastic bending of the rod. **b** A prototype of the elastica arm scale realized by D. Misseroni and donated by the authors to the 'Museo della Bilancia' in Campogalliano (Italy)

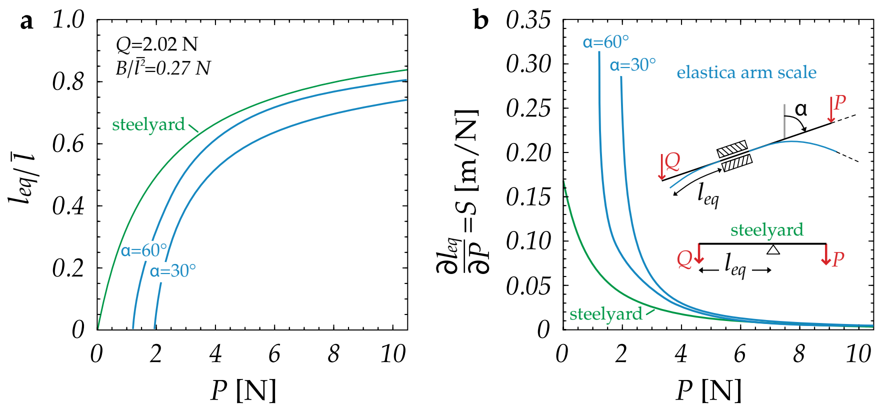

Fig. 4 Comparison between a steelyard scale and the elastica arm scale, the latter inclined at two different angles α. **a** Equilibrium length l_{eq} for different applied loads P. **b** Sensitivity S as a function of the applied load P

deformation (on which the spring scale is based). A comparison between the elastica arm scale and the ordinary steelyard scale is shown in Fig. 4. The equilibrium length l_{eq} and the sensitivity (denoted by S) for different applied loads P are shown in Fig. 4a and b, respectively. It follows that at small given load P, the sensitivity S of the elastica arm scale can become superior to the traditional device.

Prototypes of the elastica arm scale have been designed, realized, and tested at the Instabilities Lab of the University of Trento. The prototype shown in Fig. 3b has been realized by D. Misseroni and donated by the authors to the 'Museo della Bilancia' in Campogalliano (Italy).

The mechanics of the elastica arm scale has been further investigated by O'Reilly [15, 16], who considered the material momentum balance law for rods.

4 Dripping of an Elastic Rod

The application of configurational forces presented in this section addresses the so-called 'self-encapsulation' problem, in which an elastic rod is loaded with a transverse force applied at midspan (between two constraints kept at a fixed distance), with the purpose of reaching a closed deformation loop, thus encapsulating a finite region. Self-encapsulation has connections to micro- or nano-fabrication technologies for deployable structures used in sensors. In this field of application, self-assembly can be achieved through magnetic forces [17], while a self-folding spherical structure has been invented [18] and a dynamic self-encapsulation technique for a thin plate and a rod has already been pointed out [19, 20]. In the former case, only a reduction in the volume of a sphere is achieved, while in the latter self-encapsulation is obtained as a result of both dynamic effects and capillary forces, which are related to the presence of a liquid droplet attached to the rod.

The new idea pursued here is obtained through the use of two sliding sleeves, as illustrated in Fig. 5. These sleeves provide two compressive configurational forces, which tend to 'close' the rod. Since the differential equation of the elastica not only governs the oscillation of a simple pendulum and the deflection of an elastic rod, but also the shape of a droplet, it can be appreciated that the process of encapsulation will be similar to the formation of a droplet, from which the idea of 'dripping of an elastic

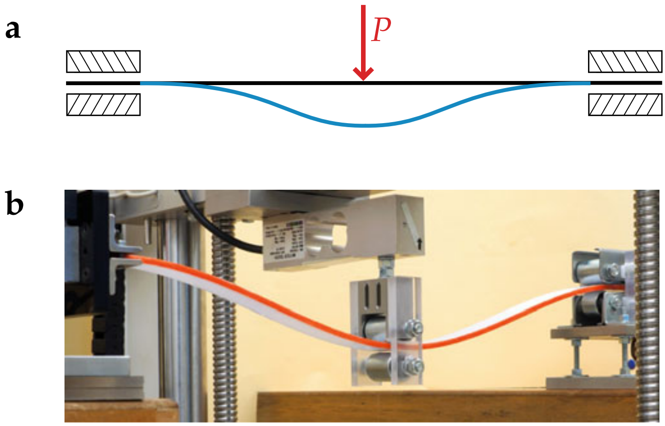

Fig. 5 **a** The encapsulation of an elastic rod, loaded with a transverse force P applied at midspan, is obtained through the use of two sliding sleeves. The sliding sleeves provide two equal and opposite configurational forces, driving the encapsulation mechanism. **b** Experimental setup designed to perform experiments on self-encapsulation

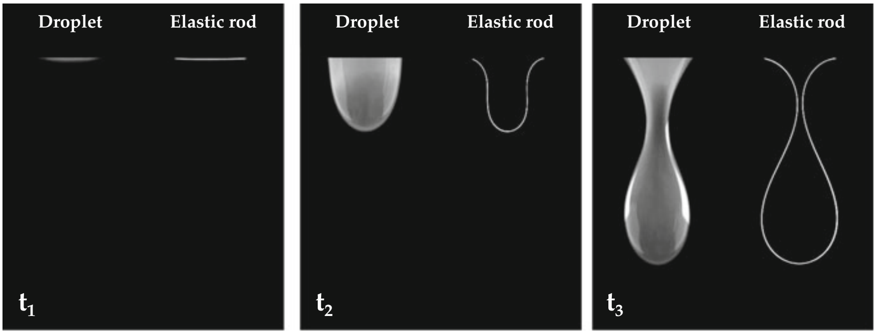

Fig. 6 The dripping of an elastic rod is compared with the progressive formation of an oil drop at three different configurations

rod' arose. The encapsulation driven by Eshelby-like forces was demonstrated to be always possible (for every rod geometrical configuration) by Bosi et al. [21]. The dripping of an elastic rod is shown in Fig. 6, where it is compared with the process of formation of an oil droplet.

5 Penetrating Blades

Configurational forces deeply influence the stability of structures. A paradigmatic example has been presented by Bigoni et al. [22] and is sketched in Fig. 7, where an elastic rod is loaded with a dead vertical force P applied at one end and it can slide inside a sliding sleeve present at the other end, while being axially restrained by a linear spring of stiffness k.

This structure is a generalization of that reported in [23], used to show two counter-intuitive effects: (i) an increase in the stiffness k of the restraining spring *lowers* the buckling load and (ii) the straight configuration of the elastic rod may return to be stable at an axial load higher than that triggering buckling.

During the post-critical deformation of the rod sketched in Fig. 7, a configurational force progressively develops. This force is vertical and directed upwards, so that it lifts the rod and sometimes it can become so large that the rod is expelled from the sliding sleeve. The configurational force also plays an important role in the restabilization problem, when the straight configuration spontaneously returns stable after bifurcation, Bosi et al. [24], or when a rod is injected inside a constraint [25]. Further problems of instability involving configurational forces were analyzed by Liakou et al. [26–28].

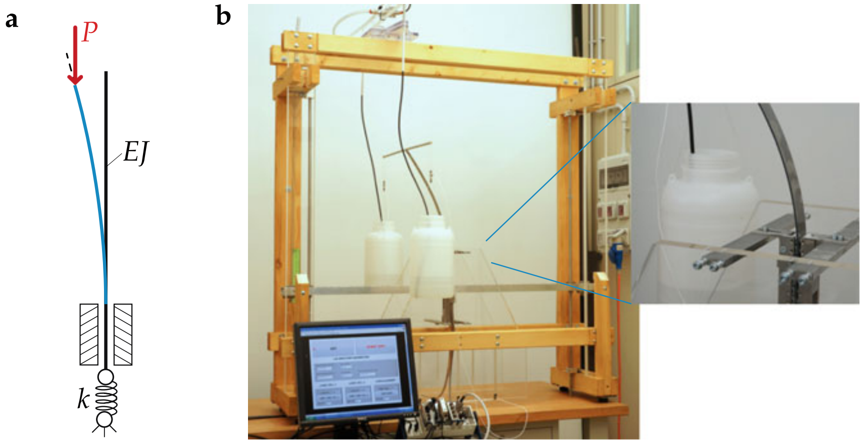

Fig. 7 Instability of a penetrating blade: the elastic rod is loaded axially with a vertical dead force P and it can slide inside a sliding sleeve placed at the other end, while being restrained by a linear spring of stiffness k. Three important features emerge from this problem: (i) an increase in the stiffness of the spring leads to a decrease of the buckling load; (ii) the straight configuration is stable for loads smaller than the buckling load, but can return stable at higher loads; (iii) during the post-critical behaviour a vertical upward configurational force develops, which could even eject the rod from the sleeve. **a** Schematic of the penetrating blade problem. **b** Experimental setup realized for the experimental validation of the features associated with the mechanics of the system

6 Torsional Actuator

The same concept ruling the development of a configurational force in a rod under bending can be applied under a state of torsion. In this case, Bigoni et al. [29] have proven that the configurational force is given by

$$\frac{M^2}{2D}, \tag{18}$$

where M is the twisting moment and D is the torsional rigidity of the rod.

This configurational force may be used to create an actuator, turning a twist into a longitudinal motion, as sketched in Fig. 8.

The actuator represents a purely 'elastic machine' or 'soft device', in which the longitudinal trust is obtained without the use of gears or other transmission mechanisms.

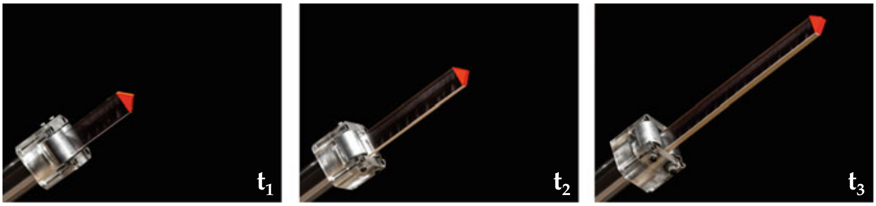

Fig. 8 The torsional actuator, based on the development of a configuration force, transforms a twist into a longitudinal trust through a release of the elastic energy, without any use of gears or other transmission mechanisms

7 Snaking of an Elastic Rod

In the problem of 'snaking', an elastic rod, straight when unloaded, is inside a curved channel. The channel is perfectly frictionless, rigid and tight to the rod. In this situation, a propulsion force is developed from a release of elastic flexural energy and the rod's movement is realized. Such a propulsion force is a configurational force, which was theoretically estimated and experimentally demonstrated by Dal Corso et al. [30]. The obtained propulsion force has been suggested to represent the key in the limbless locomotion, typical of serpentine motion [31–34].

8 Conclusions

The presence of configurational forces on structures was discovered for the first time, analyzed and developed by the authors of this article in the last decade. These forces have been theoretically demonstrated, experimentally validated and implemented in the realisation of novel reconfigurable elastic devices. They have been shown to strongly affect the behaviour of elastic structures and led to the discovery of several new effects. These results inspire new interpretations of configurational forces in solids [35] and pave the way to the new territory of configurational structural mechanics, towards innovative applications of reconfigurable mechanisms in several fields ranging from nanomedicine to aerospace.

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Materials with Memory: Viscoelasticity and Hysteresis

Claudio Giorgi and Angelo Morro

Abstract The paper is devoted to the modelling of materials with memory. Preliminarily some historical developments of viscoelasticity are considered, thus providing a relevant example of material with memory. Next a new approach to the use of the second law, as a restriction on the constitutive equations, is shown to allow a decisively more general setting of materials models. In this setting a far reaching scheme of hysteretic properties is established.

Keywords Materials with memory · Viscoelastic materials · Materials of stress-rate type · Hysteresis

1 Introduction

In continuum physics, materials with memory denote materials where the response at any time t depends on some physical variables at all preceding times. Within these materials, the linear viscoelastic solid is perhaps the first clearly stated model and traces back to Boltzmann [1]. The model elaborated by Boltzmann is based on the following assumptions.

At any point of the body, the stress at any time t depends upon the strain at all preceding times. If the strain up to time t is in the same direction then the effect of memory is to reduce the corresponding stress. The influence of a previous strain on the stress depends on the time elapsed since that strain occurred and is weaker for those strains that occurred long ago. In addition, the stress-strain relation was taken to be linear so that a superposition of the influence of previous strains holds.

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Similar models of solids were developed by Maxwell, Kelvin, and Voigt by arguing on a spring and a dashpot in series or in parallel (Maxwell's or Kelvin-Voigt's models). Really Maxwell [17] had already observed that, while in a elastic solid the relation[1] between the stress F and the strain S may be written $F = ES$, where E is the coefficient of elasticity, in dissipative (viscous) solids the relation may be written

$$\frac{dF}{dt} = E\frac{dS}{dt} - \frac{1}{T}F; \tag{1}$$

an obvious integration shows the dependence of the stress on the past values of the strain. Boltzmann was the first to develop a three-dimensional theory of isotropic viscoelasticity. Later Volterra modelled and investigated anisotropic viscoelasticity.

About the nomenclature, Maxwell was the first to use the term "historical" to describe the feature that the stress at a given time depends not only on the strain at that time but also on the history[2] up to that time. In Volterra's nomenclature [29, 31] such behaviours are regarded as "hereditary phenomena" and "heredity" is the response of the material due to the (strain) history.

Volterra [29, 30] and Graffi [12, 13] found decisive results about energy properties relative to materials with memory. In the sixties Coleman and Noll [2, 4] gave the general scheme of the thermodynamic theory of materials with fading memory. Some developments about viscoelasticity are given in [6, 7].

The theory of materials with memory was thought to provide a wide range of materials models, among them models of hysteretic materials. Though some models of hysteresis have been developed [5], this subject is open to many important improvements. Accordingly, in this paper we follow some historical developments of viscoelasticity within the theory of materials with memory. Next we show that a new approach to the use of the second law, as a restriction on the constitutive equations, allows a decisively more general setting of materials models. In this setting we establish a far reaching scheme of hysteretic properties.

Notation. We consider a body occupying the time dependent region $\Omega \subset \mathscr{E}^3$. The motion is described by means of the function $\boldsymbol{\chi}(\mathbf{X}, t)$ providing the position vector $\mathbf{x} \in \Omega$ in terms of the position $\mathbf{X}$ in a reference configuration R, and the time t, so that $\Omega = \boldsymbol{\chi}(\mathrm{R}, t)$. The symbols ∇ and ∇_R denote the gradient operator with respect to $\mathbf{x} \in \Omega$ and $\mathbf{X} \in \mathrm{R}$. The function $\boldsymbol{\chi}$ is assumed to be differentiable. Hence we can define the deformation gradient $\mathbf{F} = \nabla_R \boldsymbol{\chi}$ or, in suffix notation, $F_{iK} = \partial_{X_K} \chi_i$. The invertibility of $\mathbf{X} \mapsto \mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$ is guaranteed by letting $J := \det \mathbf{F} > 0$. Let $\mathbf{v}(\mathbf{x}, t)$ be the velocity field, on $\Omega \times \mathbb{R}$. A superposed dot denotes time differentiation following the motion of the pertinent point and hence, for any function $f(\mathbf{x}, t)$, we have $\dot{f} = \partial_t f + \mathbf{v} \cdot \nabla f$. Further, $\mathbf{L}$ is the velocity gradient, $L_{ij} = \partial_{x_j} v_i$, and $\dot{\mathbf{F}} = \mathbf{L}\mathbf{F}$. In terms of $\mathbf{F}$ the right Cauchy-Green and the Green-St. Venant deformation tensors are defined by $\mathbf{C} = \mathbf{F}^T\mathbf{F}$, $\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{1})$. The symbol $\mathbf{D}$ denotes the stretching tensor, $\mathbf{D} = \mathrm{sym}\mathbf{L}$, and ρ is the mass density while ε is the infinitesimal strain tensor.

[1] In Maxwell's notation.

[2] The whole set of values.

2 Preliminaries on Viscoelasticity

Following Boltzmann assumptions, in the linear approximation, the Cauchy stress $\mathbf{T}$ is given by the infinitesimal strain ε in the form

$$\mathbf{T}(t) = \mathsf{K}_0\varepsilon(t) + \int_0^\infty \mathsf{K}(s)\varepsilon(t-s)ds,$$

where $\mathsf{K}_0, \mathsf{K}(s)$ have values in the space of fourth-order tensors for any $s \geq 0$ while $\mathbf{T}, \mathsf{K}_0, \mathsf{K}$, and ε are considered at a fixed point of the body. The function K, on $[0, \infty)$, is sometimes called *Boltzmann function*.

To adhere to the standard notation let G on $[0, \infty)$ be defined by

$$\mathsf{G}'(s) = \mathsf{K}(s), \qquad \mathsf{G}_0 = \mathsf{K}_0,$$

the prime denoting the derivative with respect to s. The solution

$$\mathsf{G}(s) = \mathsf{G}_0 + \int_0^s \mathsf{K}(\xi)d\xi$$

is called the relaxation function and G_0 is called the instantaneous elastic modulus. We can then write

$$\mathbf{T}(t) = \mathsf{G}_0\,\varepsilon(t) + \int_0^\infty \mathsf{G}'(s)\varepsilon(t-s)ds. \tag{2}$$

It is assumed that, for solids, both G_0 and the equilibrium elastic modulus

$$\mathsf{G}_\infty = \lim_{s\to\infty} \mathsf{G}(s)$$

are positive definite. Subject to the initial condition $\lim_{s\to\infty}\varepsilon(t-s) = \mathbf{0}$, an integration by parts of (2) results in

$$\mathbf{T}(t) = \int_0^\infty \mathsf{G}(s)\dot{\varepsilon}(t-s)ds. \tag{3}$$

The formulations (2) and (3) are named after Volterra and Boltzmann. Interestingly, in the scalar case the Boltzmann formulation (3) follows directly by integration of (1).

An energy functional E was looked at such that E is minimal at constant histories, $\varepsilon(t-s) = \varepsilon(t)$, $s \in [0, \infty)$ while the stress $\mathbf{T}$, at time t, is obtained by partial differentiation with respect to the present value $\varepsilon(t)$. Volterra [29, 30] in the one-dimensional setting and Graffi in the three-dimensional setting arrived at the Graffi-Volterra free energy

$$\psi_G(t) = \tfrac{1}{2}\varepsilon(t)\cdot \mathbf{G}_\infty \varepsilon(t) - \tfrac{1}{2}\int_0^\infty \{[\varepsilon(t)-\varepsilon(t-s)]\cdot \mathbf{G}'(s)[\varepsilon(t)-\varepsilon(t-s)]\}ds,$$

subject to the main symmetry conditions $\mathbf{G}_\infty = \mathbf{G}_\infty^T, \mathbf{G}'(s) = \mathbf{G}'^T(s)$. The minimum property is obvious in view of the condition $\mathbf{G}'(s) < \mathbf{0}$. The derivative with respect to $\varepsilon(t)$ yields

$$\mathbf{G}_\infty\varepsilon(t) - \int_0^\infty \mathbf{G}'(s)[\varepsilon(t)-\varepsilon(t-s)] = \big[\mathbf{G}_\infty - \int_0^\infty \mathbf{G}'(s)ds\big]\varepsilon(t) + \int_0^\infty \mathbf{G}'(s)\varepsilon(t-s)ds$$
$$= \mathbf{G}_0\,\varepsilon(t) + \int_0^\infty \mathbf{G}'(s)\varepsilon(t-s)ds = \mathbf{T}(t),$$

and hence $\mathbf{T} = \partial_\varepsilon \psi_G$.

3 Second Law Inequality and Viscoelastic Modelling

The balance of energy results in the equation

$$\rho\dot{e} = \mathbf{T}\cdot\mathbf{D} - \nabla\cdot\mathbf{q} + \rho r,$$

where e is the internal energy density (per unit mass), $\mathbf{q}$ is the heat flux (vector), and r is the heat supply. Since the paper of Coleman and Noll [3], the role of the entropy inequality is a general restriction for admissible constitutive relations. Let η be the entropy density. The balance of entropy is assumed to be expressed by the Clausius-Duhem inequality

$$\rho\dot{\eta} + \nabla\cdot\frac{\mathbf{q}}{\theta} - \frac{\rho r}{\theta} \geq 0. \tag{4}$$

In terms of the free energy $\psi = e - \theta\eta$, upon substitution of $\rho r - \nabla\cdot\mathbf{q}$ from the balance of energy we find

$$-\rho(\dot{\psi} + \eta\dot{\theta}) + \mathbf{T}\cdot\mathbf{D} - \frac{1}{\theta}\mathbf{q}\cdot\nabla\theta \geq 0 \tag{5}$$

The conceptual role of the Clausius-Duhem inequality is made formal by saying that *for every admissible thermodynamic process the inequality* (5), *as well as* (7), *must hold for all times t* and points $\mathbf{x}$. The thermodynamic process is characterized by the fields entering the balance equations and related by the constitutive equations.

Later on Müller [25] generalized the form of the second law inequality by observing that the entropy flux need not be the ratio $\mathbf{q}/\theta$ as it is in (4). Letting $\mathbf{j}$ be the entropy flux we express the statement of the second law by saying that the inequality

$$\rho\dot{\eta} + \nabla\cdot\mathbf{j} - \frac{\rho r}{\theta} = \sigma \geq 0 \tag{6}$$

for any admissible thermodynamic process. It is worth remarking that σ as defined by (6) is said to be the *entropy production*. The flux $\mathbf{j}$ has to be determined by investigating the requirements of (6) along with the assumed constitutive equations. The models developed in this paper are consistent with the assumption $\mathbf{j} = \mathbf{q}/\theta$.

In connection with constitutive theories it is convenient to consider the Lagrangian version of the Clausius-Duhem inequality which makes it immediate to establish objective (nonlinear) constitutive functions. Consider the Eulerian Almansi finite strain tensor

$$\boldsymbol{\mathcal{E}} = \mathbf{F}^{-T}\mathbf{E}\mathbf{F}^{-1}$$

and observe that the Cotter-Rivlin tensor rate $\overset{\triangle}{\boldsymbol{\mathcal{E}}}$ coincides with $\mathbf{D}$,

$$\mathbf{D} = \mathbf{F}^{-T}\dot{\mathbf{E}}\mathbf{F}^{-1} = \overset{\triangle}{\boldsymbol{\mathcal{E}}}, \qquad \overset{\triangle}{\boldsymbol{\mathcal{E}}} = \dot{\boldsymbol{\mathcal{E}}} + \mathbf{L}^T\boldsymbol{\mathcal{E}} + \boldsymbol{\mathcal{E}}\mathbf{L}.$$

Moreover

$$\mathbf{T}_{RR} = J\mathbf{F}^{-1}\mathbf{T}\mathbf{F}^{-T}, \qquad \mathbf{q}_R = J\mathbf{F}^{-1}\mathbf{q},$$

while $\nabla_R = \mathbf{F}^T\nabla$. Hence the Clausius-Duhem inequality (5) takes the Lagrangian form

$$-\rho_R(\dot{\psi} + \eta\dot{\theta}) - \mathbf{T}_{RR}\cdot\dot{\mathbf{E}} - \frac{1}{\theta}\mathbf{q}_R\cdot\nabla_R\,\theta \geq 0. \tag{7}$$

Interesting, detailed properties of the viscoelastic model follow by considering time-harmonic processes. In addition we observe that Eq. (2) is to be considered as an approximation of a relation of the form

$$\mathbf{T}_{RR}(t) = \mathsf{G}_0\mathbf{E}(t) + \int_0^\infty \mathsf{G}'(s)\mathbf{E}(t-s)ds.$$

Consider the strain tensor

$$\mathbf{E}(t) = \mathbf{E}_1\cos\omega t + \mathbf{E}_2\sin\omega t, \qquad \mathbf{E}_1, \mathbf{E}_2 \in \mathrm{Sym}$$

and assume $\dot{\theta} = 0$, $\nabla_R\,\theta = \mathbf{0}$. Integration of (7) over $d = n2\pi/\omega$, $n \in \mathbb{N}$, yields [6]

$$\mathbf{E}_1\cdot[\mathsf{G}_0^T - \mathsf{G}_0]\mathbf{E}_2 - \int_0^\infty[\mathbf{E}_1\cdot\mathsf{G}'(s)\mathbf{E}_1 + \mathbf{E}_2\cdot\mathsf{G}'(s)\mathbf{E}_2]\sin\omega s\,ds - \int_0^\infty\Phi(s)\cos\omega s\,ds \geq 0,$$

where $\Phi(s) = \mathbf{E}_1\cdot[\mathsf{G}'(s) - \mathsf{G}'^T(s)]\mathbf{E}_2$. Assume $|\mathsf{G}'(s)| < Ms^{-(1+\delta)}$, $M, \delta > 0$, as $s\to\infty$. Letting $\omega\to\infty$ we find $\mathsf{G}_0^T - \mathsf{G}_0 = \mathbf{0}$; letting $\omega\to 0$ we find $\mathsf{G}_\infty^T - \mathsf{G}_\infty = \mathbf{0}$. Now, $\mathbf{E}_2 = \mathbf{0}$ or $\mathbf{E}_2 = \pm\mathbf{E}_1$ implies

$$\mathsf{G}'_s(\omega) := \int_0^\infty \mathsf{G}'(u)\sin\omega u\,du \leq \mathbf{0}. \tag{8}$$

Accordingly, $\mathbf{G}_0$ and $\mathbf{G}_\infty$ are (required to be) symmetric while the sine transform $\mathbf{G}'_s$ is negative for any $\omega > 0$. Though this is not required by thermodynamics, it is usually assumed that $\mathbf{G}'(s) = \mathbf{G}'^T(s)$, $s \geq 0$.

The result (8) was determined by Graffi [12] for isotropic materials on the assumption that energy is dissipated in hysteretic phenomena. Further developments about viscoelasticity are given in [6–8]. As a comment, a direct connection with the literature is obtained by using (2) along with the approximation $\mathbf{T} \cdot \mathbf{D} \simeq \mathbf{T} \cdot \dot{\varepsilon}$ and $\rho \simeq \rho_R$.

4 A New Approach to Dissipative Phenomena

A wide scheme of dissipative[3] effects is set up by letting the constitutive functions depend on the set of variables

$$\Xi = (\theta, \mathbf{E}, \mathbf{T}_{RR}, \mathbf{q}_R, \dot{\mathbf{E}}, \dot{\mathbf{T}}_{RR}, \dot{\mathbf{q}}_R, \nabla_R\, \theta).$$

The free energy density ψ and the entropy density η are continuous functions of Ξ; in addition ψ is continuously differentiable. Upon evaluation of $\dot{\psi}$ and substitution in (7) we obtain

$$\rho_R(\partial_\theta \psi + \eta)\dot{\theta} + (\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} + \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R$$
$$+ \frac{1}{\theta} \mathbf{q}_R \cdot \nabla_R\, \theta + \rho_R \partial_{\dot{\mathbf{E}}} \psi \cdot \ddot{\mathbf{E}} + \rho_R \partial_{\dot{\mathbf{T}}_{RR}} \psi \cdot \ddot{\mathbf{T}}_{RR} + \rho_R \partial_{\nabla_R \theta} \psi \cdot \nabla_R\, \dot{\theta} \leq 0$$

The linearity and arbitrariness of $\dot{\theta}, \nabla_R\, \dot{\theta}, \ddot{\mathbf{E}}, \ddot{\mathbf{T}}_{RR}$ imply that ψ is independent of $\nabla_R\, \theta, \dot{\mathbf{E}}, \dot{\mathbf{T}}_{RR}$ and hence

$$\psi = \psi(\theta, \mathbf{E}, \mathbf{T}_{RR}, \mathbf{q}_R), \qquad \eta = -\partial_\theta \psi,$$

so that the Clausius-Duhem inequality reduces to

$$(\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} + \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{1}{\theta} \mathbf{q}_R \cdot \nabla_R\, \theta \leq 0. \tag{9}$$

A simple case occurs when $\dot{\mathbf{E}}, \dot{\mathbf{T}}_{RR}$ and the pair $\mathbf{q}_R, \dot{\mathbf{q}}_R$ are independent of one another. It follows

$$\partial_{\mathbf{T}_{RR}} \psi = \mathbf{0}, \qquad \mathbf{T}_{RR} = \rho_R \partial_{\mathbf{E}} \psi, \qquad \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{1}{\theta} \mathbf{q}_R \cdot \nabla_R\, \theta \leq 0.$$

[3] Thermo-viscous and/or elastic-plastic.

Accordingly ψ depends only on θ, $\mathbf{E}$, $\mathbf{q}_R$. In addition, $\mathbf{T}_{RR}$ equals $\rho_R \partial_{\mathbf{E}} \psi$, which can be viewed as the constitutive relation for $\mathbf{T}_{RR}$, as it happens with elastic materials.

If instead $\dot{\mathbf{E}}$ and $\dot{\mathbf{T}}_{RR}$ are related to each other, and independent of $\mathbf{q}_R$, $\dot{\mathbf{q}}_R$, $\nabla_R \theta$, then

$$(\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} \leq 0, \qquad \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{\mathbf{q}_R}{\theta} \cdot \nabla_R \theta \leq 0. \quad (10)$$

It is of interest to look for Eulerian descriptions. To this purpose we consider the Eulerian Almansi finite strain tensor $\boldsymbol{\mathcal{E}} = \mathbf{F}^{-T}\mathbf{E}\mathbf{F}^{-1}$, as well as $\mathbf{T}$ and $\mathbf{q}$. Moreover, by $\mathbf{T} = J^{-1}\mathbf{F}\mathbf{T}_{RR}\mathbf{F}^T$ an $\mathbf{q} = J^{-1}\mathbf{F}\mathbf{q}_R$ we obtain

$$\overset{\square}{\mathbf{T}} := \dot{\mathbf{T}} + (\nabla \cdot \mathbf{v})\mathbf{T} - \mathbf{L}\mathbf{T} - \mathbf{T}\mathbf{L}^T = J^{-1}\mathbf{F}\dot{\mathbf{T}}_{RR}\mathbf{F}^T, \quad \overset{\square}{\mathbf{q}} := \dot{\mathbf{q}} + (\nabla \cdot \mathbf{v})\mathbf{q} - \mathbf{L}\mathbf{q} = J^{-1}\mathbf{F}\dot{\mathbf{q}}_R.$$

$\overset{\square}{\mathbf{T}}$ being the Truesdell tensor rate of $\mathbf{T}$ and $\overset{\square}{\mathbf{q}}$ the Oldroyd vector rate of $\mathbf{q}$. Hence an equivalent scheme is obtained by letting ψ be a function of θ and $\boldsymbol{\mathcal{E}}$, $\mathbf{T}$, $\mathbf{q}$ in the form

$$\psi = \tilde{\psi}(\theta, \boldsymbol{\mathcal{E}}, \mathbf{T}, \mathbf{q}) = \tilde{\psi}(\theta, \mathbf{F}^{-T}\mathbf{E}\mathbf{F}^{-1}, J^{-1}\mathbf{F}\mathbf{T}_{RR}\mathbf{F}^T, J^{-1}\mathbf{F}\mathbf{q}_R).$$

Now,

$$\partial_{\mathbf{E}} \psi = \partial_{\boldsymbol{\mathcal{E}}} \tilde{\psi} \partial_{\mathbf{E}} \boldsymbol{\mathcal{E}} = \mathbf{F}^{-1} \partial_{\boldsymbol{\mathcal{E}}} \tilde{\psi} \mathbf{F}^{-T}, \qquad \partial_{\mathbf{E}} \psi \cdot \dot{\mathbf{E}} = \partial_{\boldsymbol{\mathcal{E}}} \tilde{\psi} \cdot \overset{\triangle}{\boldsymbol{\mathcal{E}}}, \qquad \mathbf{T}_{RR} \cdot \dot{\mathbf{E}} = J\mathbf{T} \cdot \overset{\triangle}{\boldsymbol{\mathcal{E}}},$$

$$\partial_{\mathbf{T}_{RR}} \psi = \partial_{\mathbf{T}} \tilde{\psi} \partial_{\mathbf{T}_{RR}} \mathbf{T} = J^{-1} \mathbf{F}^T \partial_{\mathbf{T}} \tilde{\psi} \mathbf{F}, \qquad \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} = \partial_{\mathbf{T}} \tilde{\psi} \cdot \overset{\square}{\mathbf{T}}.$$

Likewise we evaluate $\dot{\mathbf{q}}_R$, $\partial_{\mathbf{q}_R} \psi$ and $\mathbf{q}_R \cdot \nabla_R \theta$. Consequently inequalities (10) can be written in the Eulerian form

$$\rho \partial_{\mathbf{T}} \tilde{\psi} \cdot \overset{\square}{\mathbf{T}} + (\rho \partial_{\mathbf{E}} \tilde{\psi} - \mathbf{T}) \cdot \overset{\triangle}{\boldsymbol{\mathcal{E}}} \leq 0, \qquad \rho \partial_{\mathbf{q}} \psi \cdot \overset{\square}{\mathbf{q}} + \frac{\mathbf{q}}{\theta} \cdot \nabla \theta \leq 0. \quad (11)$$

Example 1 For definiteness we now examine inequality $(10)_2$ and show that a class of models, of the Maxwell-Cattaneo type [27], follows for heat conduction. Let ψ depend on $\mathbf{q}_R$ via $\xi = |\mathbf{q}_R|^n$, $n \geq 2$. Hence

$$\partial_{\mathbf{q}_R} \psi = n\, \partial_\xi \psi\, |\mathbf{q}_R|^{n-2}\, \mathbf{q}_R$$

and inequality $(10)_2$ becomes

$$(n \rho_R \partial_\xi \psi |\mathbf{q}_R|^{n-2} \dot{\mathbf{q}}_R + \frac{1}{\theta} \nabla_R \theta) \cdot \mathbf{q}_R \leq 0.$$

This inequality holds if

$$(n\rho_R \partial_\xi \psi |\mathbf{q}_R|^{n-2} \dot{\mathbf{q}}_R + \frac{1}{\theta} \nabla_R\, \theta) = -\frac{1}{\kappa(\theta, \mathbf{E}, \xi, |\nabla_R\, \theta|)} \mathbf{q}_R, \tag{12}$$

where κ is a non-negative valued function. Consequently,

$$\kappa n \rho_R \theta \partial_\xi \psi |\mathbf{q}_R|^{n-2} \dot{\mathbf{q}}_R + \mathbf{q}_R = -\kappa \nabla_R\, \theta$$

can be viewed as a Maxwell-Cattaneo equation with the relaxation time

$$\tau = \kappa n \rho_R \theta \partial_\xi \psi |\mathbf{q}_R|^{n-2}$$

where κ is the heat conductivity. If $n = 2$ then

$$\tau = 2\kappa \rho_R \theta \partial_\xi \psi.$$

With a view to the approach developed in the next sections, the choice of the right-hand side of (12) as $-\mathbf{q}_R/\kappa$ corresponds to taking $\mathbf{q}_R^2/\kappa$ as $J\theta$ times the entropy production.

Remark Inequality $(10)_2$ is common to many approaches where both $\mathbf{q}_R$ and $\nabla_R\, \theta$ are independent variables [19–21]. Since, as with $(10)_1$, it is framed in the referential description then it allows the advantage that the time derivative is an objective derivative [22–24]. Moreover it makes consistency with thermodynamics easier than it happens with histories [18] or summed histories [15].

5 The Role of the Second Law and a Representation Relation

The next developments hinge on two new features within the context of constitutive theories. Though the second law, via the entropy inequality, rests on the key role of selecting physically admissible constitutive equations, two different views can be applied. First, an entropy inequality, like (9), is required to hold depending on the degree of arbitrariness of the left-hand side. Formally we write

$$(\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} + \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{1}{\theta} \mathbf{q}_R \cdot \nabla_R\, \theta = -\Gamma, \tag{13}$$

where Γ is $J\theta$ times the entropy production σ. As a consequence of (6), σ and hence Γ are assumed to be non-negative for every admissible set of constitutive equations. Secondly, we let Γ be a function of the set of variables Ξ; hence $\Gamma(\Xi)$ is required to be non-negative and the whole equation is required to hold depending on the degree of arbitrariness.

The view of Γ and σ as constitutive functions traces back to Green and Naghdi [14]. Yet, ignoring, as in [14], that $\Gamma \geq 0$ may lead to unphysical models. Instead, in (6) σ is required to be non-negative but is not considered as a further constitutive function.

The second feature consists on a representation formula, for tensors and vectors, which becomes of interest in connection with thermodynamic requirements.

Let $\mathbf{N}$ be a unit tensor, $\mathbf{N} \cdot \mathbf{N} = 1$, and let I be the fourth-order identity tensor. Hence, for any tensor $\mathbf{Z}$,

$$\mathbf{Z}_{\perp} := \mathbf{Z} - (\mathbf{Z} \cdot \mathbf{N})\mathbf{N} = (\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathbf{N}],$$

is the orthogonal part of $\mathbf{Z}$ relative to $\mathbf{N}$ in that

$$[(\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathbf{Z}] \cdot \mathbf{N} = 0.$$

Consequently

$$\mathbf{Z} = \mathbf{Z}_{\parallel} + \mathbf{Z}_{\perp}, \qquad \mathbf{Z}_{\parallel} := (\mathbf{Z} \cdot \mathbf{N})\mathbf{N}.$$

For any tensor $\mathbf{G}$,

$$[(\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathbf{G}] \cdot \mathbf{N} = 0$$

and hence $(\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathbf{G}$ is orthogonal to $\mathbf{N}$. Consequently, if $\mathbf{Z}_{\perp}$ is undetermined then $\mathbf{Z}$ can be represented by

$$\mathbf{Z} = (\mathbf{Z} \cdot \mathbf{N})\mathbf{N} + (\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathbf{G}. \tag{14}$$

A representation formula like (14) holds when $\mathbf{N}$, $\mathbf{Z}$, $\mathbf{G}$ are vectors provided only that I is replaced by the second-order identity tensor $\mathbf{1}$.

Often thermodynamic requirements result in the knowledge of the inner product $\mathbf{Z} \cdot \mathbf{N}$ while $\mathbf{Z}_{\perp}$ is undetermined. This shows how we can take advantage of the arbitrariness of the tensor $\mathbf{G}$.

6 Models of Viscoelastic Solids

Based on the extended role of the second-law inequality we write (10) in the form

$$(\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} = -\Gamma_T, \quad \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{\mathbf{q}_R}{\theta} \cdot \nabla_R \theta = -\Gamma_q, \tag{15}$$

where Γ_T and Γ_q are non-negative functions of Ξ with $\Gamma_T + \Gamma_q = \Gamma$. Some particular models arising from $(15)_1$ are now examined.

Let $\Gamma_T = 0$ and $\partial_{\mathbf{T}_{RR}}\psi = \mathbf{0}$. Hence

$$(\rho_R \partial_{\mathbf{E}}\psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} = 0.$$

The arbitrariness of $\dot{\mathbf{E}}$ implies

$$\mathbf{T}_{RR} = \rho_R \partial_{\mathbf{E}}\psi,$$

or, in the spatial description, $\mathbf{T}\rho\partial_{\boldsymbol{\varepsilon}}\psi = \rho\partial_{\mathbf{F}}\psi\mathbf{F}^T$. These constitutive equations characterize *hyperelastic* solids.

Let again $\Gamma_T = 0$ whereas $\partial_{\mathbf{T}_{RR}}\psi \neq \mathbf{0}$. Hence inequality $(15)_1$ becomes

$$(\rho_R \partial_{\mathbf{E}}\psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}}\psi \cdot \dot{\mathbf{T}}_{RR} = 0.$$

Accordingly we can say that, by thermodynamics, $\dot{\mathbf{T}}_{RR}$ is subject to

$$\dot{\mathbf{T}}_{RR} \cdot \rho_R \partial_{\mathbf{T}_{RR}}\psi = -(\rho_R \partial_{\mathbf{E}}\psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}}.$$

This in turn suggests that we apply the representation (14) with

$$\mathbf{Z} = \dot{\mathbf{T}}_{RR}, \qquad \mathbf{N} = \frac{\partial_{\mathbf{T}_{RR}}\psi}{|\partial_{\mathbf{T}_{RR}}\psi|}, \qquad \mathbf{G} = \mathsf{G}_{RR}\dot{\mathbf{E}}.$$

We find

$$\dot{\mathbf{T}}_{RR} = [\frac{1}{\rho_R |\partial_{\mathbf{T}_{RR}}\psi|}(\mathbf{T}_{RR} - \rho_R \partial_{\mathbf{E}}\psi) \cdot \dot{\mathbf{E}}]\mathbf{N} + (\mathsf{I} - \mathbf{N} \otimes \mathbf{N})\mathsf{G}_{RR}\dot{\mathbf{E}}.$$

Hence, letting

$$\mathsf{C}_{RR}(\theta, \mathbf{E}, \mathbf{T}_{RR}) = \mathsf{G}_{RR} + \frac{1}{\rho_R |\partial_{\mathbf{T}_{RR}}\psi|^2}\partial_{\mathbf{T}_{RR}}\psi \otimes (\mathbf{T}_{RR} - \rho_R \partial_{\mathbf{E}}\psi - \rho_R \mathsf{G}_{RR}^T \partial_{\mathbf{T}_{RR}}\psi)$$

we can write

$$\dot{\mathbf{T}}_{RR} = \mathsf{C}_{RR}\dot{\mathbf{E}}. \tag{16}$$

A material described by (16) is said to be a *hypoelastic* solid [26, 28]. By the arbitrariness of G_{RR} it follows that there are infinitely many hypoelastic tensors C_{RR} compatible with a given free energy ψ.

6.1 Visco-Thermoelastic Material

By (10) a visco-thermoelastic material can be modelled

$$(\rho_R \partial_{\mathbf{E}} \psi - \mathbf{T}_{RR}) \cdot \dot{\mathbf{E}} + \rho_R \partial_{\mathbf{T}_{RR}} \psi \cdot \dot{\mathbf{T}}_{RR} = -\Gamma_T, \qquad \rho_R \partial_{\mathbf{q}_R} \psi \cdot \dot{\mathbf{q}}_R + \frac{\mathbf{q}_R}{\theta} \cdot \nabla_R \theta = -\Gamma_q.$$

Assuming $|\partial_{\mathbf{T}_{RR}} \psi| \neq 0$, $|\partial_{\mathbf{q}_R} \psi| \neq 0$ and applying the representation formula we can rewrite $\dot{\mathbf{T}}_{RR}$ and $\dot{\mathbf{q}}_R$ in the form

$$\dot{\mathbf{T}}_{RR} = \mathsf{C}_{RR} \dot{\mathbf{E}} - \Gamma_T \mathbf{P}_{RR}, \qquad \dot{\mathbf{q}}_R = \mathsf{K}_R \nabla_R \theta - \Gamma_q \mathbf{p}_R. \tag{17}$$

where

$$\mathbf{P}_{RR} = \frac{\partial_{\mathbf{T}_{RR}} \psi}{\rho_R |\partial_{\mathbf{T}_{RR}} \psi|^2}, \qquad \mathbf{p}_R = \frac{\partial_{\mathbf{q}_R} \psi}{\rho_R |\partial_{\mathbf{q}_R} \psi|^2}.$$

For definiteness, let A and $\mathbf{A}$ be a pair of positive-definite fourth-order and second-order tensors (possibly parameterized by θ), respectively, and let

$$\Gamma_T = [\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})] \cdot \mathsf{A}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})], \qquad \Gamma_q = \mathbf{q}_R \cdot \mathbf{A}\mathbf{q}_R.$$

We can then apply (17) where

$$\mathbf{P}_{RR} = \frac{\mathsf{M}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})]}{|\mathsf{M}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})]|^2}, \qquad \mathbf{p}_R = \frac{\mathbf{M}\mathbf{q}_R}{|\mathbf{M}\mathbf{q}_R|^2},$$

and

$$\begin{aligned}
\mathsf{C}_{RR} &= \mathsf{J}_{RR} + \mathbf{P}_{RR} \otimes \big[\big(\mathsf{M}^{-1} + [\mathcal{G}'(\mathbf{E})]^T - \mathsf{J}_{RR}^T \big) \mathsf{M} \big(\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E}) \big) \big], \\
\mathbf{K}_R &= \mathbf{G}_R - \mathbf{p}_R \otimes \Big[\Big(\frac{1}{\theta} \mathbf{M}^{-1} + \mathbf{G}_R^T \Big) \mathbf{M}\mathbf{q}_R \Big].
\end{aligned}$$

Finally, we choose $\mathsf{J}_{RR} = \mathsf{M}^{-1} + \mathcal{G}'(\mathbf{E})$ and $\mathbf{G}_R = -(\theta \mathbf{M})^{-1}$ and then we have

$$\dot{\mathbf{T}}_{RR} = [\mathcal{G}'(\mathbf{E}) + \mathsf{M}^{-1}] \dot{\mathbf{E}} - \Lambda_T \mathsf{M}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})], \qquad \dot{\mathbf{q}}_R = -\frac{1}{\theta} \mathbf{M}^{-1} \nabla_R \theta - \Lambda_q \mathbf{M}\mathbf{q}_R,$$

where

$$\Lambda_T = \frac{[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})] \cdot \mathsf{A}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})]}{|\mathsf{M}[\mathbf{T}_{RR} - \mathcal{G}(\mathbf{E})]|^2}, \qquad \Lambda_q = \frac{\mathbf{q}_R \cdot \mathbf{A}\mathbf{q}_R}{|\mathbf{M}\mathbf{q}_R|^2}.$$

Note that if $\mathsf{A} = \mathsf{M}^2$ and $\mathbf{A} = \mathbf{M}^2$ then $\Lambda_T = \Lambda_q = 1$.

6.2 Linear Visco-Thermoelastic Material

When K is linear, namely $\mathsf{K}(\mathbf{E}) = \mathsf{G}_\infty \mathbf{E}$, we recover the linear model

$$\dot{\mathbf{T}}_{RR} - \mathsf{G}_0 \dot{\mathbf{E}} + \Lambda_T (\mathsf{G}_0 - \mathsf{G}_\infty)^{-1} (\mathbf{T}_{RR} - \mathsf{G}_\infty \mathbf{E}) = \mathbf{0}, \qquad \dot{\mathbf{q}}_R = -\Lambda_q \mathbf{M}\big(\mathbf{K}\nabla_R \theta + \mathbf{q}_R\big)$$

where

$$\mathsf{G}_0 = \mathsf{G}_\infty + \mathsf{M}^{-1} > \mathsf{G}_\infty, \qquad \mathbf{K} = \frac{1}{\Lambda_q \theta} \mathbf{M}^{-2}.$$

Here G_0 and G_∞ stand for the usual elastic and relaxation moduli, respectively, whereas $\mathbf{K}$ stands for the conductivity tensor. The corresponding free energy is

$$\rho_R \psi = \rho_R \psi_0(\theta) + \tfrac{1}{2}\mathbf{E} \cdot \mathsf{G}_\infty \mathbf{E} + \tfrac{1}{2}[\mathbf{T}_{RR} - \mathsf{G}_\infty \mathbf{E}] \cdot \mathsf{M}[\mathbf{T}_{RR} - \mathsf{G}_\infty \mathbf{E}] + \tfrac{1}{2}\mathbf{q}_R \cdot \mathbf{M}\mathbf{q}_R.$$

6.3 Maxwell-Type Viscoelastic Model

In the special case when $\mathcal{G}(\mathbf{E}) \equiv \mathbf{0}$, we obtain

$$\dot{\mathbf{T}}_{RR} = \mathsf{M}^{-1}\dot{\mathbf{E}} - \Lambda_T \mathsf{M}\mathbf{T}_{RR}, \qquad \dot{\mathbf{q}}_R = -\frac{1}{\theta}\mathbf{M}^{-1}\nabla_R \theta - \Lambda_q \mathbf{M}\mathbf{q}_R,$$

the so-called Maxwell visco-thermoelastic model. The corresponding free energy is

$$\rho_R \psi = \rho_R \psi_0(\theta) + \tfrac{1}{2}\mathbf{T}_{RR} \cdot \mathsf{M}\,\mathbf{T}_{RR} + \tfrac{1}{2}\mathbf{q}_R \cdot \mathbf{M}\mathbf{q}_R.$$

7 Models of Plastic Solids

We go back to Eq. (15) and say that plastic behaviours occur when Γ_T is a function of $|\dot{\mathbf{E}}|$ or $|\dot{\mathbf{T}}_{RR}|$. For formal simplicity we restrict attention to one-dimensional models so that $\mathbf{E} = E\mathbf{e} \otimes \mathbf{e}, \mathbf{T}_{RR} = S\mathbf{e} \otimes \mathbf{e}$. The symbol S for the component of $\mathbf{T}_{RR}$ is consistent with the *engineering stress* considered in the literature as the ratio of the axial force over the reference area ([16], Sect. 74). Accordingly we regard ψ as a function of θ, E, S and write $(15)_1$ in the form

$$\partial_S \psi \dot{S} + (\partial_E \psi - S)\dot{E} = -\hat{\Gamma}(\theta, E, S, \dot{E}, \dot{S}), \tag{18}$$

where $\hat{\Gamma} = \Gamma_T/\rho_R$. At constant temperature $\dot{\psi} = \partial_S \psi \dot{S} + \partial_E \psi \dot{E}$ and hence integration of (18), with $\hat{\Gamma} \geq 0$, as $t \in [t_1, t_2]$, along a closed curve in the $E - S$ plane results in

$$0 \geq \int_{t_1}^{t_2} [\partial_S \psi \dot{S} + (\partial_E \psi - S)\dot{E}]dt = -\int_{t_1}^{t_2} S\dot{E}dt = -\oint SdE,$$

$\oint$ denoting the integral along the closed curve. The positive value of $\oint SdE$ implies that the closed curve is run in the clockwise sense.

Consistent with (18) we let $\hat{\Gamma} = \gamma_E(\theta, E, S, \mathrm{sgn}\dot{E})|\dot{E}|$, so that

$$\partial_S \psi \dot{S} + (\partial_E \psi - S)\dot{E} = -\gamma_E |\dot{E}|. \tag{19}$$

Equation (19) is invariant under the time transformation $t \mapsto ct$, $c > 0$, and hence the associated model is rate-independent because so is γ_E. As we will see, it describes plastic properties when γ_E is smooth and elastic-plastic properties when γ_E is piecewise smooth and vanishes in a suitable open region (called *elastic region*).

Look at time intervals where $\dot{E} \neq 0$. Since $\partial_S \psi \neq 0$, divide (19) by $\partial_S \psi \dot{E}$ to obtain

$$\frac{\dot{S}}{\dot{E}} = \frac{S - \partial_E \psi}{\partial_S \psi} - \frac{\gamma_E}{\partial_S \psi} \mathrm{sgn}\dot{E}. \tag{20}$$

Since $\dot{E} \neq 0$ then changes in time and S changes accordingly. We can then view S as a function of E with

$$\frac{dS}{dE} = \frac{\dot{S}}{\dot{E}}.$$

Let

$$\chi_1 = \frac{S - \partial_E \psi}{\partial_S \psi}, \qquad \chi_2 = -\frac{\gamma_E}{\partial_S \psi}. \tag{21}$$

Accordingly, χ_1 is a function of E and S parameterized by θ, whereas χ_2 can also depend on $\mathrm{sgn}\dot{E}$. The uniaxial stress-strain slope is then given by

$$\frac{dS}{dE} = \chi_1 + \chi_2 \mathrm{sgn}\dot{E}. \tag{22}$$

If $\gamma_E = 0$, and hence $\chi_2 = 0$, we have

$$\frac{dS}{dE} = \chi_1(\theta, E, S,).$$

This means that the slope $dS/dE = \chi_1$ of the hypo-elastic curve[4] depends on both E and S; we assume that

$$\chi_1 > 0. \tag{23}$$

The slope of the stress-strain curve is assumed to be non-negative. Hence we assume also that

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[4] The elastic regime is recovered by letting $\gamma_E = 0$, $\partial_S \psi = 0$, $S = \partial_E \psi(\theta, E)$.

$$\chi_1 + \chi_2 \,\mathrm{sgn}\dot{E} \geq 0. \tag{24}$$

The conditions (23) and (24) follow from physical arguments about the stress-strain curve but are not required by thermodynamics.

Models of elastic-plastic, rate-independent materials are now established by specifying the functions χ_1, χ_2. In this regard we observe that χ_1 is fully determined by the free energy ψ while χ_2 depends also on γ_E. Consequently different models can be determined by means of the same free energy ψ but different functions γ_E. The hysteretic properties are shown to depend on γ_E; that is why γ_E is referred to as *hysteretic function*. To establish the functions χ_1, χ_2 we argue as follows. In the elastic regime, i.e. in the absence of hysteretic effects ($\chi_2, \gamma_E = 0$), we have

$$\partial_S \psi \dot{S} + (\partial_E \psi - S)\dot{E} = 0, \qquad \frac{dS}{dE} = \frac{S - \partial_E \psi}{\partial_S \psi} = \chi_1; \tag{25}$$

hence χ_1 can be viewed as the *elastic differential stiffness*. Equation $(25)_1$ is in fact a constitutive equation of the form

$$\dot{S} = \hat{S}(S, E, \dot{E}),$$

characterizing hypoelastic materials [28]. We denote by *elastic region* the set of points (E, S) such that $\gamma_E = 0$.

We now characterize the hysteretic regime by letting

$$\gamma_E \neq 0, \qquad \partial_S \psi \neq 0, \qquad \frac{dS}{dE} = \frac{S - \partial_E \psi}{\partial_S \psi} - \frac{\gamma_E}{\partial_S \psi} \mathrm{sgn}\dot{E}.$$

7.1 The Helmholtz Free Energy

To determine the function ψ appearing into the model equations (21)–(22) we start with the generic assumption

$$\psi(S, E) = \mathcal{L}(S - \mathcal{G}(E)) + \mathcal{F}(S) + \mathcal{H}(E),$$

$\mathcal{L}, \mathcal{G}, \mathcal{F}$, and $\mathcal{H}$ being undetermined differentiable functions, possibly dependent on the temperature θ. Substitution of $\partial_S \psi$ and $\partial_E \psi$ in (21) yields

$$\chi_1 = \frac{S + \mathcal{L}'(S - \mathcal{G}(E))\mathcal{G}'(E) - \mathcal{H}'(E)}{\mathcal{L}'(S - \mathcal{G}(E)) + \mathcal{F}'(S)}, \quad \chi_2 = -\frac{\gamma_E}{\mathcal{L}'(S - \mathcal{G}(E)) + \mathcal{F}'(S)}.$$

Borrowing from the elastic properties we assume

$$\chi_1 = g(E) \geq 0.$$

Consequently we obtain the requirement

$$g(E)\{\mathcal{L}'(S-\mathcal{G}(E))+\mathcal{F}'(S)\}=S+\mathcal{L}'(S-\mathcal{G}(E))\mathcal{G}'(E)-\mathcal{H}'(E).$$

For definiteness, let

$$\alpha = g(E)-\mathcal{G}'(E)$$

be a nonzero constant. Hence we have

$$\mathcal{L}'(S-\mathcal{G}(E))\alpha+g(E)\mathcal{F}'(S)-S+\mathcal{H}'(E)=0.$$

Accordingly we assume

$$\mathcal{L}'(S-\mathcal{G}(E))=\frac{1}{\alpha}(S-\mathcal{G}(E)), \qquad \mathcal{F}'(S)=0.$$

The remaining condition is

$$\mathcal{H}'(E)=\mathcal{G}(E).$$

Moreover we have

$$\chi_2=\frac{\gamma_E\alpha}{S-\mathcal{G}(E)}.$$

In view of these conditions, the free energy ψ is given by

$$\psi(S,E)=\frac{1}{2\alpha}[(S-\mathcal{G}(E)]^2+\mathcal{H}(E) \tag{26}$$

where $\mathcal{H}'(E)=\mathcal{G}(E)$; really ψ may depend also on the temperature θ possibly via an additive term. The characteristic function χ_1,χ_2 are given by

$$\chi_1=\mathcal{G}'(E)+\alpha\geq 0, \qquad \chi_2=-\frac{\alpha\gamma_E}{S-\mathcal{G}(E)}. \tag{27}$$

Accordingly, the model is fully characterized by the elastic function $\mathcal{G}(E)$, the positive parameter α, and the hysteretic function γ_E. Since $\alpha>0$ then the differential stiffness χ_1 is greater than the slope of the elastic function $\mathcal{G}(E)$. The condition $g(E)=\mathcal{G}'(E)+\alpha$ allows us to establish significant models also with $\mathcal{G}(E)\equiv 0$; in that case the elastic differential stiffness is the constant α.

7.2 An Elastic-Plastic Model

Let $\mathcal{G}(E)\equiv 0$ and hence

$$\psi=\frac{S^2}{2\alpha}, \qquad \chi_1=\alpha.$$

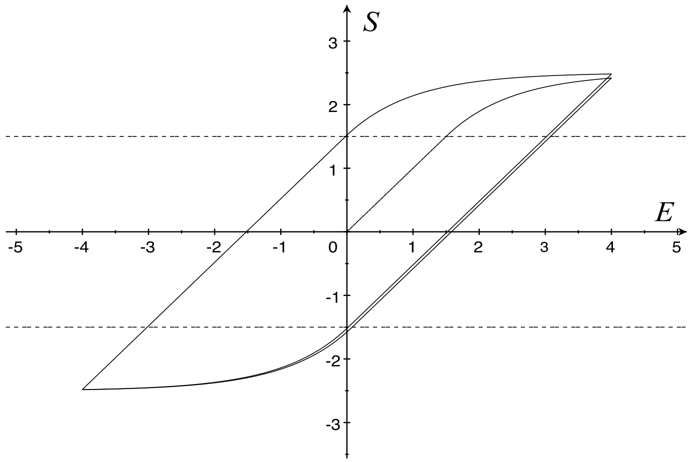

Fig. 1 Elastic-plastic model: hysteresis loops (solid) and yield-strength bounds $|S| = S_y$ (dashed)

Let $S_u > S_y > 0$ and choose the hysteretic function in the form

$$\gamma_E(S, \mathrm{sgn}\dot{E}) = \begin{cases} \frac{|S|-S_y}{S_u-S_y}|S| & \text{if } |S| \geq S_y \text{ and } \mathrm{sgn}(S\dot{E}) > 0 \\ 0 & \text{if } |S| < S_y \text{ or } |S| \geq S_y,\ \mathrm{sgn}(S\dot{E}) < 0. \end{cases}$$

In view of (22) we find that

$$\frac{dS}{dE} = \begin{cases} \frac{\alpha(S_u-|S|)}{S_u-S_y} & \text{if } |S| \geq S_y \text{ and } \mathrm{sgn}(S\dot{E}) > 0 \\ \alpha & \text{if } |S| < S_y \text{ or } |S| \geq S_y,\ \mathrm{sgn}(S\dot{E}) < 0. \end{cases}$$

This model is quite simple, it is characterized by the positive parameters α, S_y, S_u. Despite the simplicity, the model is quite realistic; it combines a plastic flow with asymptotic strength along with an elastic-plastic behaviour. The loops are placed within the strip $|S| < S_u$. Inside the open strip $|S| < S_y$ the body behaves elastically and hence the strip $|S| < S_y$ is the elastic region. Within the strips $|S| \in (S_y, S_u)$ the elastic behaviour occurs only during unloading; instead during loading the material behaves plastically.

Figure 1 shows the hysteresis loops obtained by solving the system of equations

$$\begin{cases} \dot{E} = \omega\mathfrak{E}\cos\omega t, \\ \dot{S} = (dS/dE)\dot{E}, \end{cases}$$

where $\alpha = 1$, $S_y = 1.5$, $S_u = 2.5$, $\mathfrak{E} = 4$.

Further models are developed in [11].

8 Conclusions

The paper shows developments of concepts and models of materials with memory mainly exemplified by viscoelastic solids. The characterization of the relaxation function, and particularly the requirement (8), is a result that follows directly as a consequence of the second law inequality on the constitutive properties within the now classical approach of rational thermodynamics.

The role of the second law is then reviewed in this paper by emphasizing that the entropy production can be regarded as a constitutive function, subject to be non-negative valued. This view introduces a further constitutive function which then allows more general constitutive models based on rate-type equations, which then avoid the recourse to the use of memory functionals. Hysteresis is a remarkable example of process framed within the new approach [9, 10]. In essence, for elastic-plastic materials it emerges from the thermodynamic analysis that the elastic behaviour is modelled by a free-energy function while the hysteretic loops are characterized by the entropy-production function.

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Fractional Calculus in Visco-Elasticity

Mario Di Paola and Antonina Pirrotta

Abstract The fractional calculus is now popular in the engineering community because of its capability, especially to predict the visco-elastic response of the various materials in both time and frequency domain. In the present paper results of the research group of Palermo, in this setting, are briefly summarized.

Keywords Fractional calculus · Fractional visco-elasticity · Springpot

1 Introduction

Since the increasing number of applications, in any engineering field, in which fractional calculus recently proved to be attractive, several research efforts have focused on the response determination of systems in which fractional operators appear. For instance, employment of fractional operators has been proposed in the context of biophysics, bioengineering, finance, control theory, image and signal processing, electrical circuits and material sciences. A rather detailed account of diverse recent theoretical advances and applications of fractional calculus in the various fields can be found in the books by Sabatier et al. [1], Hilfer [2]. Pioneering studies, on the use of fractional calculus and fractals in solid mechanics, have been performed by the research group of Torino [3].

In this context, since the findings of Bagley and Torvik [4, 5] who firstly casted fractional calculus in a theoretical framework to describe the behavior of visco-elastic materials, visco-elasticity has proved to be one of the most promising field for fractional calculus applications of engineering interest.

In the past the "classical" models as Maxwell and Kelvin-Voigt ones or more complex combinations of such units composed by springs and dashpots have been

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used to capture visco-elastic phenomena like relaxation and/or creep (Flügge [6], Pipkin [7], Christensen [8]). Such elementary models show some inconsistencies. (i) Experimental relaxation and creep functions are more or less well fitted by different compositions of springs and dashpots. This is a very serious problem since the inverse of the constitutive law $\sigma = L[\varepsilon]$ may not be written as $\varepsilon = L^{-1}[\sigma]$, where L is a linear differential operator; (ii) whatever the number and combinations of elementary units are, the kernel of hereditary integrals is of exponential type and then for a constant load (creep-test) for $t \to \infty$ the strain takes an asymptotic value. Such a behavior is not observed in real experiments that show an increasing trend as $t \to \infty$.

From these observations it may be stated that visco-elastic models based upon combinations of spring and dashpots may capture the real behavior only for short observation time. A more realistic description of creep and /or relaxation is given by a power law function with real order exponent (Nutting [9], Gemant [10], Di Paola et al. [11]). As soon as we assume a power law function for creep the constitutive law relating deformation and stress is ruled by a Riemann Liouville fractional integral with order equal to that of the power law, and vice versa, starting from the relaxation function, the σ-ε constitutive law is ruled by its inverse operator that is the Caputo's fractional derivative. Moreover, also the behavior for $t \to \infty$ is captured with a power law fractional constitutive law. Such a model is called fractional hereditary model since fractional operators are involved and readers are referred to Samko et al. [12], Podlubny [13], Hilfer [2]. For these reasons in the second part of the last century a lot of researches have been carried out enforcing the knowledge of fractional hereditary materials (Gonsovski et al. [14], Stiassnie [15], Bagley and Torvik [16, 17], Schmidt and Gaul [18], Mainardi and Gorenflo [19], Evangelatos and Spanos [20]).

As in fact shown in Di Paola et al. [11], derivatives of non-integer order naturally appear in the stress–strain equations of any linear visco-elastic material, if creep and relaxation functions of the power-law type are considered in the Boltzmann superposition integral, as suggested since 1921 by Nutting [9] and confirmed through experimental studies [11].

Further, fractional derivative based models to describe damping behavior of materials and systems have been considered by Koeller [21], Mainardi [22], Shen and Soong [23], Pritz [24], Papoulia and Kelly [25], Pirrotta et al. [26] and Schiessel et al. [27]. Extension to tridimensional fractional visco-elasticity may be found in [28].

In this regard, a natural application of engineering interest relates to the study of continuous visco-elastic beams comprising a fractional derivative element, and is used to describe their constitutive behavior. This problem has in fact been addressed in many papers from different viewpoints. In Yao et al. [29], for instance, the quasi-static analysis of Euler–Bernoulli beams described by fractional Kelvin visco-elastic model has been proposed and solution obtained using Laplace transformations. Even though the derivations are correct, since solution is performed through Laplace transform, no physical implication of the hereditary model based upon fractional hereditary materials comes out. Avoiding resorting to Laplace domain, Di Paola et al. [30] studied the fractional Euler–Bernoulli beam operating in time domain, highlighting many observations which would have remained hidden in the Laplace one. This issue will be presented in Sect. 3.

2 Visco-Elastic Fractional Constitutive Law

Visco-elastic phenomena are mostly described by relaxation $E(t)$ and creep function $D(t)$. In particular, $E(t)$ can be interpreted as the stress history for a unit strain $\varepsilon(t) = U(t)$, and $D(t)$ represents the strain history for a unit stress $\sigma(t) = U(t)$, ($U(t)$ being the unit step function).

At the beginning of the last century, Nutting [9] observed that $E(t)$ is well suited by a power law decay

$$E(t) = \frac{C_\beta}{\Gamma(1-\beta)} t^{-\beta}; 0 < \beta < 1 \tag{1}$$

where $\Gamma(\cdot)$ is the Euler-Gamma function, C_β and β are characteristic coefficients depending on the material at hand. Once $E(t)$ is determined in the form according to Eq. (1) the function $D(t)$ is given as

$$D(t) = \frac{1}{C_\beta \, \Gamma(1+\beta)} t^{\beta}; 0 < \beta < 1 \tag{2}$$

The result of Eq. (2) is obtained simply taking into account that $E(s)D(s) = s^{-2}$ where $E(s)$ and $D(s)$ are the Laplace transform of $E(t)$ and $D(t)$, respectively, and s denotes the Laplace parameter.

Due to Boltzmann superposition principle, compare e.g. Flügge [6], Pipkin [7], the stress history, for an assigned strain history $\varepsilon(t)$ may be easily derived in the form

$$\sigma(t) = \int_0^t E\left(t - \bar{t}\right)\dot{\varepsilon}\left(\bar{t}\right) d\,\bar{t} \tag{3}$$

Conversely the strain history, for an assigned stress history $\sigma(t)$ is given as

$$\varepsilon(t) = \int_0^t D\left(t - \bar{t}\right)\dot{\sigma}\left(\bar{t}\right) d\,\bar{t} \tag{4}$$

Equations (3) and (4) are valid if the system starts at rest at $t = 0$, otherwise $E(t)\varepsilon(0)$ and $D(t)\sigma(0)$ have to be added in Eq. (3) and in Eq. (4), respectively.

As soon as we assume that the kernel in the convolution integrals Eqs. (3) and (4) are given as in Eq. (1), respectively, the fractional constitutive law of the visco-elastic material results in the form

$$\sigma(t) = C_\beta \left({}_C D_{0^+}^{\beta} \varepsilon\right)(t) \tag{5}$$

and

$$\varepsilon(t) = \frac{1}{C_\beta}\left(D_{0^+}^{-\beta} \sigma\right)(t) \tag{6}$$

where the symbol $\left({}_C D_{0^+}^{\beta} \varepsilon\right)(t)$ is the Caputo's fractional derivative represented as

$$\left({}_C D_{0^+}^{\beta} \varepsilon\right)(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{\dot{\varepsilon}(\bar{t})}{\left(t-\bar{t}\right)^{\beta}} d\,\bar{t} \tag{7}$$

while $\left(D_{0^+}^{-\beta} \sigma\right)(t)$ is the Riemann–Liouville fractional integral defined as

$$\left(D_{0^+}^{-\beta} \sigma\right)(t) = \frac{1}{\Gamma(\beta)} \int_0^t \frac{\sigma(\bar{t})}{\left(t-\bar{t}\right)^{1-\beta}} d\,\bar{t} \tag{8}$$

From Eqs. (7) and (8) we may recognize that for $\beta = 0$ and $\beta = 1$ the purely elastic and viscous fluid behavior is recovered, respectively.

It is worth stressing that the Caputo's fractional derivative coincides with the Riemann–Liouville fractional derivative only for quiescent systems or for systems that operate from $t = -\infty$. In all other cases, results in terms of the Riemann–Liouville or Caputo's fractional derivative are quite different to each another, and fractional differential equations involving Riemann–Liouville fractional derivative show some inconsistencies in terms of initial conditions (Samko et al. [12], Podlubny [13], Hilfer [2], Evangelatos and Spanos [20]). Contrary, such a problem disappears when working in terms of Caputo's fractional derivative. In any case the fractional visco-elastic model involves Caputo's fractional derivative and no problems appear for the ensuing derivations.

Exact mechanical models of the fractional operators may be found in [31–33].

In the remainder of the paper fractional operators are performed only with respect to time, thus no distinction between partial fractional operators in time and space has to be considered.

3 Fractional Visco-Elastic Euler–Bernoulli Beam

Let us consider an isotropic homogeneous visco-elastic Euler–Bernoulli beam of length L, Fig. 1, referred to the axes (x, y, z) with origin located at the centroid of the cross section, and (x, y) principal axes of inertia of the cross section. All external spatially distributed loads, denoted as $q_y(z, t)$, are assumed to act in y-direction, thus orthogonally to the z-axis, and the analyzed transverse displacement, $v(z, t)$, is also oriented in y-direction.

Usually phenomenological visco-elastic models are based on springs and dashpots, the first obeys Hooke's law, the second Newton's law.

Visco-elastic behavior, indeed, is an intermediate behavior between the two ones afore mentioned. Then, as mentioned in [11], the constitutive law in Eqs. (5) and (6) interpolates the purely elastic behavior ($\beta = 0$) and the purely viscous behavior ($\beta = 1$), and is termed in literature as springpot element depicted in Fig. 2 and thus it is the most suitable model for visco-elastic behavior.

In the context of the underlying continuous beam problem, the constitutive laws in Eqs. (5) and (6) are rewritten as

$$\sigma(y, z, t) = C_\beta\left({}_C D^{\beta}_{0^+}\varepsilon\right)(y, z, t); \, 0 < \beta < 1 \tag{9}$$

$$\varepsilon(y, z, t) = \frac{1}{C_\beta}\left(D^{-\beta}_{0^+}\sigma\right)(y, z, t); \, 0 < \beta < 1 \tag{10}$$

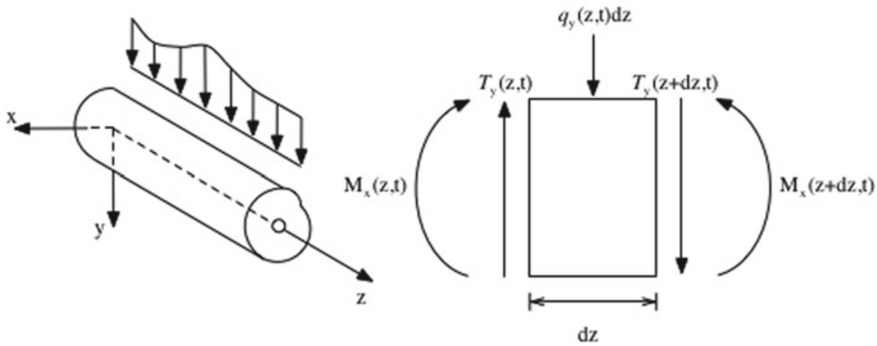
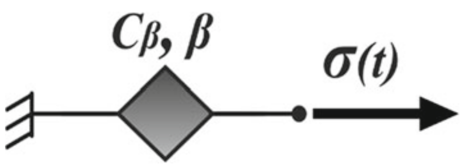


Fig. 1 Euler- Bernoulli beam; **a** layout of the beam; **b** free body diagram of a beam element

Fig. 2 Springpot element: fractional model



while the dynamic equilibrium of a visco-elastic Euler- Bernoulli beam is expressed in the form:

$$\rho(z)\frac{\partial^2 v(z,t)}{\partial t^2} + C_\beta \frac{\partial^2}{\partial z^2}\left[I_x(z)\frac{\partial^2}{\partial z^2}\left[\left({}_C D^\beta_{0^+} v\right)(z,t)\right]\right] = q_y(z,t) \quad (11)$$

In Eq. (11), $\rho(z)$ is the mass per unit length and $I_x(z)$ is the moment of inertia of the cross section with respect to the x-axis.

Once the governing equation of motion is known, the flexural vibrations, solution of this differential Eq. (11), are expressed through the linear combination of the eigenfunctions $\Phi_k(z)$ that are dependent on the constraints only (boundary conditions):

$$v(z,t) = \sum_{k=1}^{\infty} q_k(t)\Phi_k(z) \quad (12)$$

Further, the coefficients of this linear combination, which are functions of time, are the modal coordinates $q_k(t)$ and depend on the initial conditions. In the following, numerical solution will be commented through a given example.

For a simply supported beam with $\rho(z) = \rho$, $I_x(z) = I_x$ and $\phi_j(z) = \sqrt{\frac{2}{l}}\sin\left(\frac{j\pi z}{l}\right)$, by means of a Galerkin approach, a set of decoupled fractional differential equations is given in the form

$$\ddot{q}_k(t) + \overline{\omega}^2_{\beta,k} D^\beta q_k(t) = p_k(t) \quad (15)$$

where

$$p_k(t) = \rho^{-1}\sqrt{\frac{2}{l}}\int_0^l p_y(z,t)\sin\left(\frac{k\pi z}{l}\right)dz \quad (16)$$

and $\overline{\omega}_{\beta,k}$ is an anomalous coefficient (which can be viewed as a frequency if $\beta = 1$) $\left[\overline{\omega}_{\beta,k}\right] = T^{\beta-2}$, given as

$$\overline{\omega}_{\beta,k} = \sqrt{C_\beta \frac{I_x k^4 \pi^4}{\rho l^4}} \quad (17)$$

For $\beta = 0$ $\overline{\omega}_{0,k}$ coalesces with ω_k (k-th natural radian frequency of the beam). In frequency domain the transfer function of Eq. (15), labeled as $H_{\beta,k}(\omega)$, is readily found as

$$H^{-1}_{\beta k}(\omega) = -\omega^2 + \overline{\omega}^2_{\beta,k}(i\omega)^\beta = \overline{\omega}^2_{\beta,k}|\omega|^\beta\left(\cos\left(\frac{\beta\pi}{2}\right) + i\sin\left(\frac{\beta\pi}{2}\right)\right) - \omega^2 \quad (18)$$

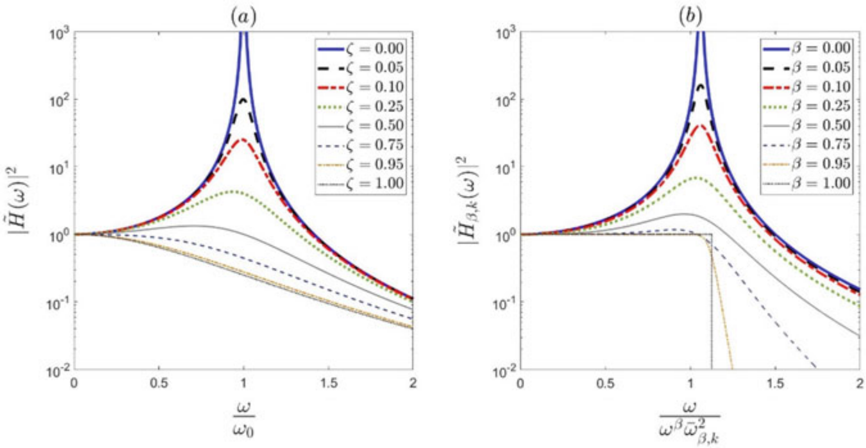


Fig. 3 Dimensionless transfer function: **a** classical differential equation $\ddot{q} + 2\zeta\omega_0\dot{q} + \omega_0^2 q = f(t)$; **b** fractional differential equation $\ddot{q} + \overline{\omega}^2_{\beta,k} D^\beta q = f(t)$

From this equation it may be observed that working in frequency domain the presence of fractional differential equation does not present any additional difficulty with respect to that present in classical differential equations in which the two phases (pure solid and pure fluid) are separated. The dimensionless transfer functions of the classical case $\left|\tilde{H}(\omega)\right|^2 = \left|H(\omega)\omega_0^2\right|^2$ is depicted in Fig. 3a for different values of ζ while that of the fractional case $\left|\tilde{H}_{\beta,k}(\omega)\right|^2 = \left|H_{\beta,k}(\omega)\omega^\beta \overline{\omega}^2_{\beta,k}\right|^2$ is depicted in Fig. 3b.

From these figures it may be asserted that the dimensionless transfer functions for the two cases are quite similar, moreover the presence of the damping ratio $\zeta > 0$ in the classical case and the presence of the fractional derivative with $\beta > 0$ in the fractional case move the system away from infinite resonance. However, in all the classical books the uncertainty of the evaluation of the viscosity parameter ζ is overcome by considering the real constitutive law of the phenomenon at hands. Moreover, since for all the last century the decay of the vibration has been always represented by the term $2\zeta\omega_k\dot{q}_k(t)$ and the elastic part by $\omega_k^2 q_k(t)$, it is difficult to abandon this way of thinking. In many papers the two classical terms $2\zeta\omega_k\dot{q}_k(t)$ and $\omega_k^2 q_k(t)$ are retained and a new term is enforced involving the fractional derivatives without any specification of the presence of the fractional term.

In some cases, the structure is composed by different materials like laminated glasses [34]; in these cases, the stratified glasses are connected with a polymeric film. The presence of two different materials with very different time scales, the glass is almost elastic ($\beta = 0.01$) while the value of β for polymer is of order $\beta = 0.25$, produces different terms in the fractional equation. Other examples of the coexistence of more fractional terms in the equations of motion are beams lying on rubbers, railways resting on viscoelastic soils and so on.

An important property of fractional term stressed in [20] is pointed out in Fig. 3, that is the fractional derivative term can alter simultaneously both the resonant frequency and the degree of damping of the system. Precisely for this latter important physical aspect in [35, 36] the fractional derivative term inside the equilibrium equation of motion of the free liquid surface inside a tube is proper for capturing the sloshing motion that alters the resonant frequency.

It is worth noting that extension to fractional visco-elastic Timoshenko beam is detailed in [37–39], where taking advantages of Mellin transform method [40], the problem of fractional Timoshenko beam model is assessed in time domain by expressing the equation of the elastic curve through a single relation. In [38] important information to engineering designers are reported, introducing exact linking relationships between elastic Euler–Bernoulli beam response and fractional visco-elastic Timoshenko beam response. Extension to the fractional visco-elastic Reddy beam are reported in [41].

4 Variable-Order Fractional Operators for Viscoelastic Materials

A critical discussion on the actual meaning of variable-order fractional operators, in the context of viscoelasticity, is reported in [42]. Two variable-order fractional operators have been proposed in literature [43–45] with the aim of representing the mechanical behavior of viscoelastic materials under varying environmental conditions (as changes in temperature or viscoelastic aging effects [46, 47]) and more generally to capture changes in mechanical properties during a general loading program [48, 49]. In [42] it is shown that, when the change in values of the fractional order is independent of state variables and depends only on varying environmental conditions, only one of the two existing models is meaningful in the context of viscoelasticity (specifically, the model relying on Caputo's variable order fractional derivative); while the other is based on an inconsistent application of the Boltzmann superposition principle. It is also shown that, when the change in values of the fractional order depends on state variables (stress or strain), both models rely on a non-standard application of the Boltzmann superposition principle. In order to fill this gap and change perspective, a novel formulation is proposed in [50] to build, step by step, the strain response of a viscoelastic material to a given stress input under varying temperature conditions. The associated operator does not violate the past strain history the material had been subjected to and relies on a consistent application of the Boltzmann superposition principle. Essentially, the novel formulation is a step by step integration scheme for fractional viscoelastic materials with variable fractional order, under a given stress input, starting from known initial conditions in terms of the strain history. In [42, 51] the novel formulation is extended to more general conditions of time-varying mechanical properties, including viscoelastic aging effects and nonlinear effects arising during a general loading program. As a further result, in

[42] it is proved that the operators involved in the proposed formulation and in the meaningful existing model, when the change in values of the fractional order is independent of state variables, are inverses of each other.

5 Other Applications in Biomechanics

Fractional-order differential equations have also been proposed in biomechanics to provide a non-linear formulation of the material hereditariness of tendons and ligaments of the human knee [52, 53] whereas a multiaxial linear hereditary model has been introduced in the context of biomimetic ceramics [54]. Some other contributions have been provided in a stochastic biomechanics setting to capture the parameter fluctuation of the constitutive equations of tendons and ligaments hereditariness [55]. Fractional differential equations have also been used to model the apparent non-linear viscosity coefficients arising in non-newtonian description of blood flow circulation in small vessels [56] as well as in a "small-on-large" model of cell membranes instabilities under rafts formations [57]. Additionally fractional-order differential equations have proved to rule evolutive phenomena in hereditariness of long bones [58] modelled as fractal media as well as non-fickian diffusive models along fractal pathways [59].

6 Conclusions

Every material during creep test is very well fitted by a simple power law of the kind t^{β} and this in the Boltzmann superposition principle leads to fractional operators. Unfortunately, the parameter β strongly depends on various factors like the temperature as well as the level of stress during the experiments.

Experimental campaign for a correct behavior of triaxial stress is still an open problem as in fact the parameter β of the power law derived from the tensile test is different from that obtained by the shear test and this produces that also ν depends on time.

As a conclusion from a mathematical point of view, integration of fractional operators is totally overcome but a lot of experiments are necessary for a proper description of the parameters in such equations.

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Contributions and Challenges on the Computational Modeling of Damage and Fracture

Claudia Comi and Umberto Perego

Abstract Computational modeling of damage and fracture propagation is a challenging problem, still attracting the attention of many research groups throughout the world. Over the years, the Italian community in structural mechanics, gathered in the scientific fori provided by Aimeta congresses and by the meetings of its interest groups (in particular GIMC and, later, GMA), has provided a huge amount of innovative contributions to the subject. The variety and richness of these contributions make it impossible to provide an exhaustive account in this short note. Rather, we want to try retracing the main stages of the evolution of the Aimeta community research in this particular field from the limited viewpoint of the activity carried out in our research group at the Politecnico di Milano, along the last 30 years.

Keywords Damage · Strain localization · Regularization · Fracture

1 Introduction

One century has elapsed since the pioneering work of Alan Arnold Griffith [22] on the energy approach to fracture. In 1927, in the fourth edition of his fundamental treatise on the mathematical theory of elasticity [24], at page 121 Love was still writing "…. The conditions of rupture are all but vaguely understood …". Many things have changed since then and, thanks to the seminal contributions of Westergaard, Irwin, Rice and many others, we have now understood a lot about the mechanics of fracture. The same can be said for the mechanics of damage, starting from the fifties with the early work of Kachanov [23] and the subsequent contributions by Rabotonov, Lemaitre and others, which have made useful and accurate mathematical models available to the engineering community. Despite these progresses, the prediction of the evolution of damage in a structure, up to its final condition of failure, can still

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be considered a challenging task, attracting the attention of thousands of researchers around the world.

The purpose of this short note is to provide a brief account of the research activities carried out over the last 30 years by our research group at the Politecnico di Milano on the computational modeling of damage and fracture. Since the beginning, these activities have been strongly influenced by the developments taking place within the Aimeta community and have found a natural forum for presentation and discussion in the congresses and conferences of Aimeta and its Groups of Interests, in particular the Italian Group of Computational Mechanics (GIMC) and the Group of Mechanics of Materials (GMA). The Italian mechanics community has been particularly active and prolific on this subject and it is impossible for us to summarize here the huge variety of important results that have been contributed over the last three decades. What we propose is therefore a survey of the developments in this field, though limited to the particular viewpoint of our contributions.

Many other colleagues in our research lab at the Politecnico have been active and have contributed along these years to the field of damage and fracture. For the same reasons as above, we cannot however go analytically through the details of these contributions. Some of them (Stefano Mariani, Aldo Ghisi, Roberto Fedele) have collaborated with us and will appear in the references. Among the others, we just mention Alberto Corigliano, for his work on the formulation of cohesive models for mixed-mode delamination [1, 2] and on the multiscale failure of microsensors [26]; Anna Pandolfi, for her work on the simulation of crack propagation and fragmentation [27, 29, 30]; Attilio Frangi and Giorgio Novati for their work on boundary elements simulation of brittle fracture [20]; Gabriella Bolzon (see, e.g., [4, 5]); Giuseppe Cocchetti (see e.g. [18]); Raffaele Ardito, for damage identification in concrete dams [3]. Finally, we mention the work of Giulio Maier, scientific father and mentor, whose vast and fundamental contributions to the mechanics of materials and structures cannot be summarized at all.

2 Damage Models

2.1 Bi-Dissipative Model for Concrete

Towards the end of the nineties, research was actively in progress on the modeling of concrete failure. Due to the quasi-brittle nature of concrete, linear elastic fracture mechanics tools could not be satisfactorily applied. Other theories, based on cohesive and/or smeared crack approaches and on damage models, were diffusely investigated, a lot of attention being devoted to the issue of pathological mesh dependence in the softening stage of material response. At that time, these different approaches were the object of numerous contributions and intense discussion in the Aimeta community.

Within this context, in 1999, at the XII GIMC in Naples and later, in the same year, at the XIV Aimeta Congress in Como, we presented a simple model for concrete [11],

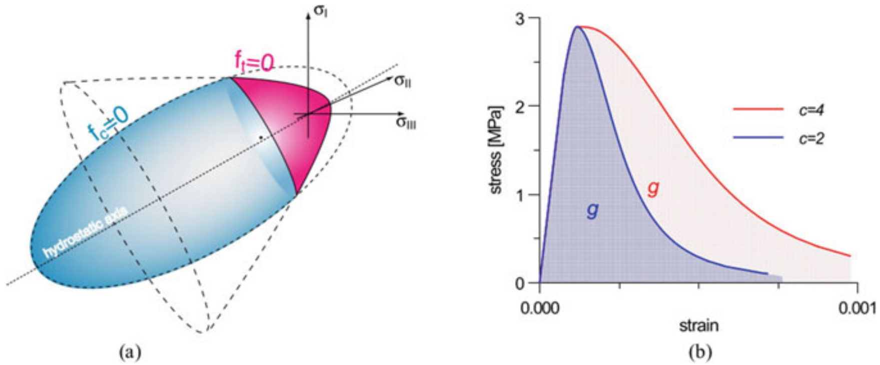

Fig. 1 **a** Damage activation functions and elastic domain in principal stress space; **b** fracture energy scaling

intended to be robust and suitable for large scale computations of engineering structural problems, and to reproduce accurately the following main macroscopic features of concrete behavior: stiffness degradation; strength reduction (softening); different behavior, with different fracture energies, in tension and compression; apparent strength and ductility increase in compression under increasing lateral confinement; unilateral effect, i.e. stiffness recovery when the loading condition is reversed from tension to compression; degradation of material properties characterizing the behavior in tension due to previous development of damage in compression.

The proposed isotropic damage model considered two separate dissipation mechanisms, depending on two monotonically increasing scalar damage variables, one in tension and the second in compression (hence the name bi-dissipative, see Fig. 1a). Reasonably accurate and convergent results with a simple computer implementation, avoiding mesh dependence of global results in the presence of strain localization in the softening regime, were achieved by the so called fracture energy regularization technique, as an alternative to the more rigorous, but more complex nonlocal approaches, which were intensively studied at the time. To facilitate the scaling of the material fracture-energy density in the model, a special form of the hardening-softening functions was also proposed (Fig. 1b).

2.2 Model for Concrete Affected by Alkali-Silica Reaction

Besides mechanical loading, long-term chemical reactions between the different constituents of concrete, possibly in the presence of aggressive environments, can cause the degradation of concrete structures. Starting from 2008, we studied in particular the phenomena of the alkali-silica reaction (ASR) and of the external sulfate attack (ESA). The interest in those phenomena was driven by several research projects

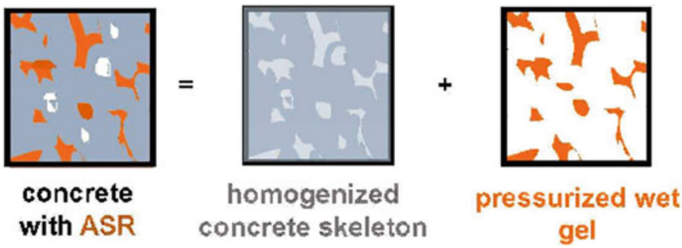

Fig. 2 Scheme of the bi-phase material representing concrete affected by ASR: solid skeleton and pressurized wet gel

on the safety assessment of concrete dams conducted with different Italian groups, active in the Aimeta community.

The ASR is a chemical reaction occurring in concrete between the alkali of the cement paste and non-crystalline silica, which is found in some kind of aggregates. The main product of the reaction is a gel, which expands in the presence of water, initially filling up the pre-existing concrete pores and then causing micro-cracking and overall expansion of the concrete structure. To predict the mechanical effects of this reaction, we formulated a bi-phase chemo-thermo-damage model [8]. The basic idea of the model is shown in Fig. 2: concrete affected by ASR is modeled, according to Biot's theory of porous media, as a heterogeneous material at the meso-scale, constituted by two elastic-damageable phases: the gel produced by the chemical reaction, which expands in time, and the homogenized concrete skeleton, which is subjected to tensile effective stresses causing damage. The response of the homogenized concrete skeleton to effective stresses is described by the model recalled in the previous section.

The model was validated through comparison with experimental tests reported in the literature and applied to the prediction of degradation in dams.

Extensions to consider the effects of anisotropy [14] and of partial saturation [9] were proposed and discussed at the XIX, XX, and XXI Aimeta Congresses.

2.3 *Model for Concrete Affected by Sulfate Attack*

The external sulfate attack (ESA) is another important cause of concrete degradation. It consists of a complex set of reactions between sulfate ions (coming from the external environment) and the hydrate calcium aluminates present in the cement paste. The final reaction product is the secondary ettringite that, forming within the hardened matrix, can generate swelling and microcracks formation inside the material.

Figure 3a schematically shows the effect of ESA: when sulfates penetrate, leaching and swelling occur in the zone in contact with the aggressive solution. Beyond this

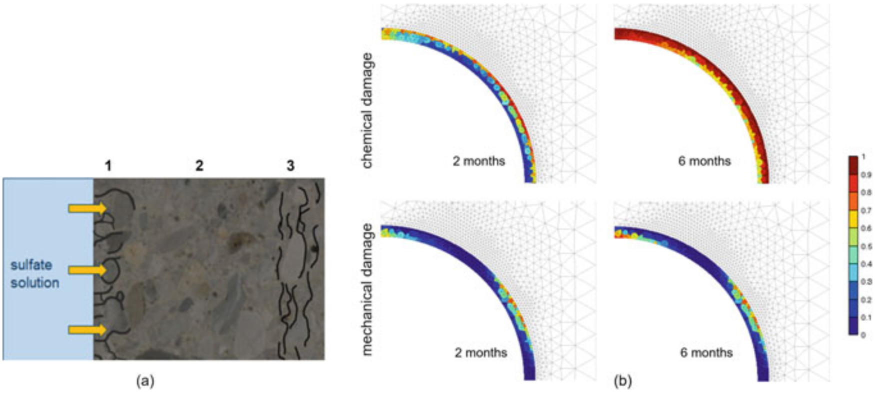

Fig. 3 **a** Schematic view of the degradation and formation of three zones in a specimen affected by ESA; zone 1: leaching and ettringite formation, zone 2: expansion due to ettringite formation, zone 3: microcraking due to tensile stresses. **b** Chemical and mechanical damage evolution in a tunnel lining due to ESA

zone, the concrete is intact and swells due to ettringite formation, causing tensile stresses and microcracks formation in the most internal parts.

To simulate the mechanical effects of ESA, similarly to what proposed in the case of ASR, we developed a multi-phase material model, also accounting for partially saturated conditions [6]. A reactive-diffusion model was used to compute the sulfate molar concentration and the amount of formed ettringite. The ettringite formation implies a volume increase and, once the initial porosity is filled, it induces a volumetric deformation. Two phenomenological isotropic damage variables describe the chemical degradation and the stress-induced degradation. The model has been validated by simulating experimental tests under isothermal conditions presented in the literature, and then used to simulate the behavior of a reduced-scale model of a tunnel lining. Figure 3b shows the chemical and mechanical damage distributions after 2 and 6 months. The chemical damage progressively develops, starting from the surface in contact with the aggressive soil and then spreads through the whole thickness of the tunnel lining. The mechanical damage in tension concentrates at the upper part of the vault and reaches high values already after 2 months of exposure. Another highly damaged part develops at the interface between concrete and soil at one third of the arch.

3 Strain Localization and Regularization Methods

In parallel with the formulation of complex damage models intended to describe with high accuracy various degradation phenomena, in the 90s an important international scientific debate started about the localization phenomena occurring in damageable materials and softening models, with the consequent ill-posedness of the boundary value problem. To restore well-posedness, different regularization strategies were proposed and discussed at Aimeta and GIMC Congresses, starting form the XII Aimeta Congress of Naples in 95. Here we briefly account of gradient-dependent and non-local formulations.

3.1 Gradient-Dependent Models

In local continuum damage models or in plasticity models with softening, when localization occurs, damage or plastic strains grow in a band whose thickness tends to zero, leading to the wrong prediction of material failure without dissipation. To avoid this un-physical behavior, a characteristic internal length, which characterizes the material micro-structure, must be introduced in the model. In [15], an isotropic gradient-enhanced damage model is proposed in which the loading function not only depends on the damage value, but also on its Laplacian. The coefficient of this latter term depends on the material internal length and fixes the width of the localization zone. Due to the presence of the gradient term in the loading function, in finite element analysis, not only the displacements, but also the damage field must be modeled and proper boundary conditions on damage must be added. The finite element mixed formulation of the finite-step was obtained through the Hu-Washizu variational principle. This strategy proved to be effective to regularize the problem.

From the computational point of view, the gradient regularization is quite demanding due to the coupling between different points in the corrector phase. However, for particular forms of the loading function, the corrector phase can be recast in the form of a linear complementarity problem, which can be solved very efficiently e.g. by Mangasarian's algorithm. The more recent phase-field approach to quasi-brittle and ductile fracture can be seen as a more rigorous formulation of the same type of approach.

3.2 Nonlocal Models

Another popular approach to introduce an internal length is the nonlocal integral formulation. The key idea is to consider in the definition of the constitutive behavior of a point its interaction with other points inside a proper neighborhood defined by the material length. Different formulations are obtained by different choices of the non-local variable, which is replaced by its averaged mean over the neighborhood. The resulting models should ensure the well-posedness of the initial boundary value problem and possibly be not too computationally demanding.

The choice of replacing the strain invariants with their non-local counterparts in the loading functions proposed in [7] proved to comply with the above requirements

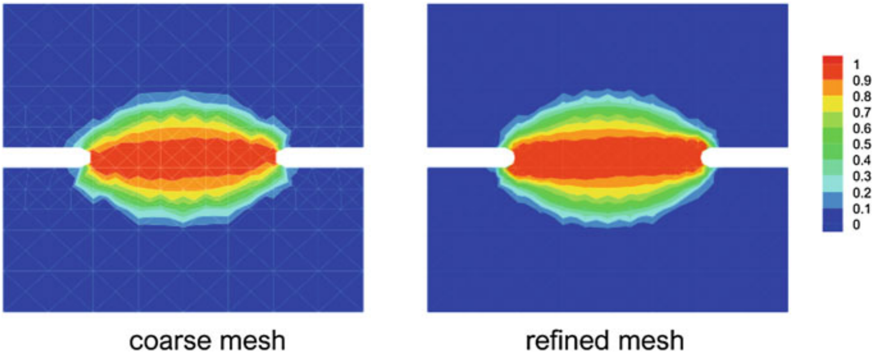

Fig. 4 Notched tension test: damage distribution for coarse and refined meshes

and effectively regularized the model described in 2.1. Figure 4 shows the mesh independence of the damage pattern in a tension test.

The numerical aspects related with the non-local formulation were further discussed in [12, 13] and presented in 2000 at the GIMC meeting in Brescia.

4 Modeling of Crack Propagation and Delamination

4.1 Finite Element Models for Crack Propagation

The finite element modeling of crack propagation is a complex problem in view of the necessity to follow the evolution of the displacement discontinuity. Smeared crack, or continuum damage approaches, as those discussed in the previous section, remove the evolving discontinuity, at the cost of the necessity to resolve the evolving band, with the ensuing computational cost.

In the Extended Finite Element Method (XFEM), the difficulty connected with the finite element modeling of an evolving discontinuity is overcome by augmenting the displacement interpolation basis through ad hoc assumed local functions, incorporating a displacement discontinuity. In addition, local known features of the exact solution can be explicitly added to the standard FE approximation fields, as in the case of brittle fracture. In the case of quasi-brittle fracture, the tractions transmitted between the two sides of the discontinuity in the process zone avoid the stress singularity in the tip region, eliminating the need of this latter enhancement. Rather, they require considering a cohesive behavior along the propagating discontinuity in the process zone. This has been achieved in [25] by enhancing the approximation fields via quadratic polynomials, discontinuous across the fracture surface, using in 2D three-node constant strain triangles. Figure 5 shows the results for a three-point-bending test and for varying position of the initial notch.

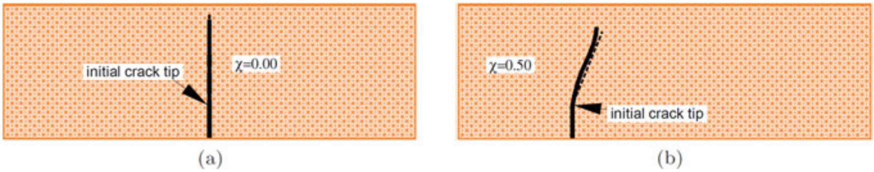

Fig. 5 Mixed-mode three-point-bending test: comparison between available experimental (dashed lines) and numerical (continuous lines) crack paths for different position χ of initial notch: **a** $\chi = 0.00$; **b** $\chi = 0.50$

A similar approach has been successfully implemented in [16] for the simulation of fracture processes in quasi-brittle functionally graded materials (FGMs), endowed with elastic and toughness properties gradually varying in space. These XFEM approaches have also been presented and discussed at the XV and XVII Aimeta Congresses, in Taormina 2001 and Firenze 2005, respectively.

A particular type of problems in fracture mechanics is represented by the blade cutting of thin shells. The process of cutting is a highly nonlinear and complex problem, dominated by the co-existence of several geometric scales. The first one is the scale of the thickness of the thin-walled structure, which is usually orders of magnitude smaller than the in-plane global dimensions. The second is given by the material critical size of the process zone, where the inelastic phenomena preceding crack propagation take place. Another small scale is determined by the curvature radius of the cutting blade, which can be of the order of microns, or even less, in the case of a sharp blade.

In cutting problems, the main crack propagation direction is dictated by the blade movement, so that the mesh can be a priori adjusted to follow the correct propagation path, without the need to use extremely refined meshes. A new type of cohesive interface element, to be interposed between adjacent separating solid-shell elements, the so-called "directional" cohesive element, has been proposed in [17, 21, 28] and discussed at several Aimeta Congresses (XX, Bologna 2011, XXI Turin 2013, XXII Genoa 2015) and GIMC/GMA conferences (XVIII GIMC Siracusa 2010, XXI GIMC/VIII GMA, Lucca 2016).

When a suitable fracture criterion is met at a given node, this is duplicated and a massless string is introduced in the model in correspondence of each pair of separating nodes (Fig. 6). The string is a straight segment, naturally endowed with a length coinciding with the distance between the nodes and transmitting a cohesive force defined by the specific cohesive traction-separation law adopted. The string element is a well defined geometric entity and its contact against the cutting blade can be checked throughout the analysis duration. When a point of a string element is detected to be in contact with the blade, the string is subdivided into two string elements (Fig. 6). In this way, the cohesive forces are transmitted between the crack flanks along directions taking into account the presence of the cutter. When the current total length of the string exceeds the limit value, the string is removed. The proposed method with directional cohesive elements has been applied to blade cutting simulation of a

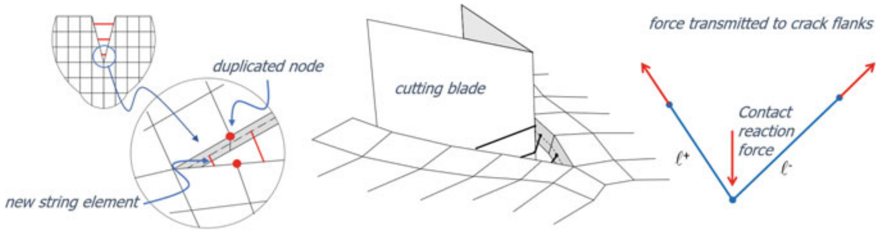

Fig. 6 Insertion procedure for directional cohesive elements: node duplication, string insertion, contact detection, cohesive force definition

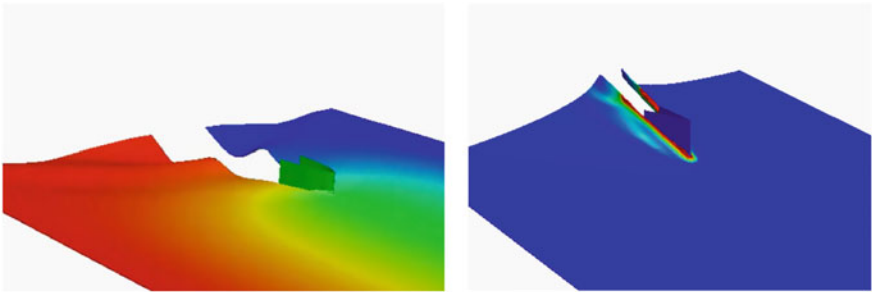

Fig. 7 Blade cutting of a rubber sheet (left) and of a steel plate (right)

number of different configurations and materials (Fig. 7) with excellent accuracy of results.

4.2 *From Regularized Damage to Cohesive-Crack*

Continuum approaches prove efficient in the first stage of spreading of damage in localized regions. However, the width of the band of growing damage tends to reduce, making the computational costs excessive for real-life structures. If the mesh is adaptively refined during the analysis, a significant computational burden is still due to the continuous convection of current results from the old mesh to the new one. Another important drawback of the non-local continuum approach is that non-local interaction remains active also when damage tends to one, leading to an unrealistic spread of the fully damaged band. These difficulties suggested to consider the possibility of a transition from a continuum damage model, in the early stage of loading, to a discrete description of crack evolution, as in the XFEM approach. In this way, the advantages of both formulations could be retained, avoiding their main drawbacks.

The debate on these issues has been intense, in particular at the beginning of the 2000s, at the III Joint Conference of the Italian Group of Computational Mechanics and the Ibero-Latin Association of Computational Methods in Engineering, in

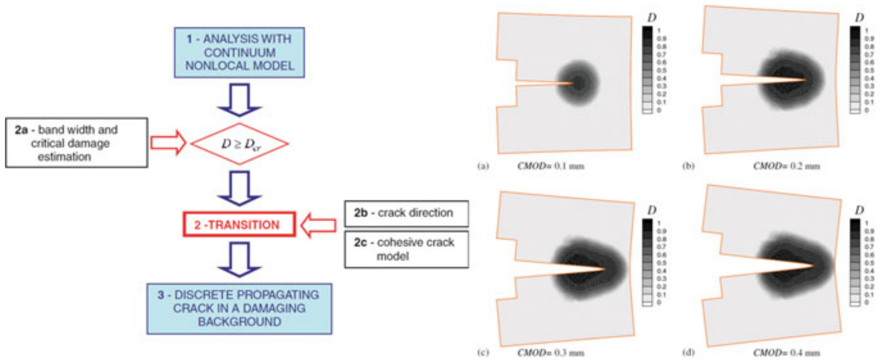

Fig. 8 Left. Schematic flow of the procedure. Right. Wedge-splitting test: damage evolution and transition to fracture

Giulianova, 2002, and at the XVI Aimeta Congress, in Ferrara, 2003. On these occasions, we presented an integrated strategy to model the transition from a continuum description of damage evolution to a discrete model for cohesive crack propagation [10].

The strategy proposed in this work consists of three steps, as shown in the left part of Fig. 8. In a preliminary step, a critical value of damage is computed for each element, based on its size and on the minimum number of elements considered necessary to resolve the localization band. In step (1) a finite element analysis with standard finite elements and a non-local continuum damage model is carried out to avoid mesh dependence. Step (2) consists of three substeps: in step (2a) the current localization bandwidth corresponding to the accumulated value of damage is estimated based on a perturbation analysis; in step (2b), when the critical damage is exceeded in an element, a discrete cohesive crack is introduced in the element according to an 'extended' finite element approach, by enriching the displacement interpolation with discontinuous functions satisfying the partition of unity concept; in step (2c) the cohesive law of the crack is defined through an energy balance, in such a way that the energy not yet dissipated in the damage band is transferred to the cohesive interface. In step (3) the cohesive crack is let to propagate. The result of the application of the methodology to the simulation of a wedge-splitting test is shown in the right part of Fig. 8.

4.3 Interface Models for Mixed-Mode Delamination of Composites

For layered composites or adhesive junctions, delamination or debonding are typical failure modes, where the delamination front is forced to propagate along the low toughness interface, rather than realigning along a mode I direction, so that mixed

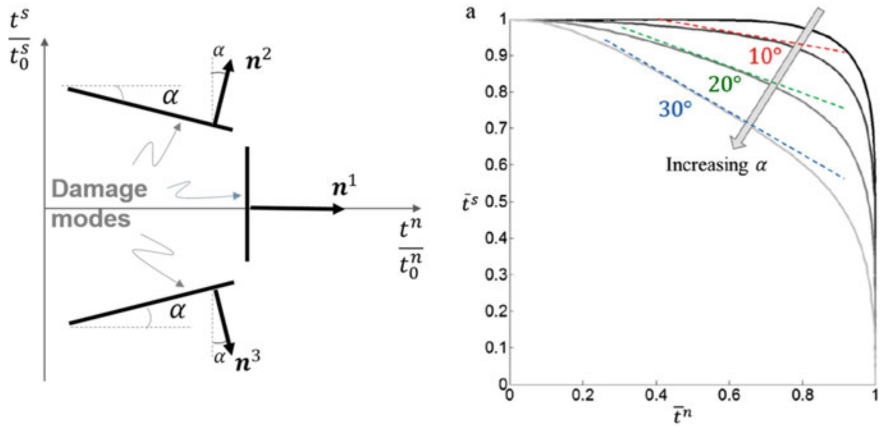

Fig. 9 Damage modes identification in normalized tractions space (left). Activation domain in normalized tractions space (right)

mode is the most frequent loading condition. An important feature of mixed-mode delamination is that the fracture energy depends on the active fracture mode and may be significantly different in mode I and mode II. Experimental evidences show that the higher complexity of the fracture surface, with a resulting greater effective area, accounts for the increasing fracture energy in passing from mode I to mode II delamination. The debate on how to treat the fracture energy variation with the mode ratio has been particularly intense over the years, with the proposal of many different approaches.

A new, thermodynamically consistent framework for the formulation of cohesive interface models has been proposed in [19] and presented and discussed at Aimeta Congresses (XXIII, Salerno 2017, XXIV, Rome 2019). While a free energy decomposition in terms of normal and shear components is usually considered, in the proposed model the damage activation locus is obtained by a decomposition driven by the identification of three damage modes (Fig. 9).

In contrast to the majority of available models, the definition of equivalent opening displacements and tractions is not required and the framework is particularly suited for the simulation of complex, non-proportional loading paths. The model requires only few parameters, which can be identified based on tests in pure mode I and II and in mixed mode, for varying mode-ratio.

A new, rigorous and exhaustive validation protocol for delamination models has also been proposed, based on three different types of tests: consistency tests, whereby the model is tested at material point level on intentionally complex loading paths; accuracy tests, whereby the model is applied to estimate the fracture energy evolution with the mode ratio; evolutionary tests, whereby the model is used for the simulation of experimental tests. The proposed model has been shown to provide excellent results in all these test types.

5 Conclusions and Perspectives

The computational modeling of damage and fracture can be considered to a certain extent to be a mature field. However, a number of different factors call for more advanced analysis tools, driving the research efforts in new, still not completely explored research directions. A growing worldwide demand for safe and durable products calls for refined design and verification concepts, capable to account for possible defect growth at different scales and for a variety of extreme loading conditions, in a multiphysics, coupled context. From the environmental point of view, a growing attention and consciousness towards the depletion of natural resources calls for extreme design in terms of weight and material consumption, with new bio-inspired and engineered materials, requiring mechanical models capable to span across a multitude of scales. On the other hand, growing competition in the global market calls for new paradigms in design, for innovative and less expensive engineering realizations, requiring design and analysis tools that are robust, fast, accurate and easily usable in an engineering context. Finally, computer architectures are steadily evolving towards distributed computing models (not to mention the still to come quantum computing), calling for highly parallelizable algorithms, to be combined with artificial intelligence tools to manage data-driven material models at different scales.

We are confident that the Aimeta community, with the value of the expertise matured along its history and the growing richness of new talents joining the association will be able to match these challenges and to continue to provide valuable contributions to science and society.

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Structural Dynamic Identification and Damage Detection

Fabrizio Vestroni and Antonino Morassi

Abstract Dynamic methods are a powerful tool for studying the behaviour of existing structures and their health conditions. The practical application, however, often raises subtle questions related to the accuracy and completeness of experimental data, the complexity of the mechanical modelling and, ultimately, the inverse nature of the problems that leads to ill-conditioning and non-uniqueness. This chapter addresses some of these aspects, and presents a short overview of the topic, with particular emphasis on dynamic structural identification and damage detection.

Keywords Structural identification · Damage detection · Inverse problems

1 Introduction

Great improvements have been made in modelling the dynamic behaviour of structural systems in last decades. Nevertheless, when the investigation involves complex structures or when the aim of the analysis also includes the in-service response of existing structures, reliable results cannot be obtained by means of a mathematical model solely. Usually simplifying assumptions are introduced in the model and the values of the main parameters can only be approximate. For existing structures, the model is developed on the basis of a survey of the construction which is by no means complete. On the other hand, an accurate evaluation of the in-service response could be important in monitoring conditions where the distance between experimental and predicted response is observed.

These reasons led to the confidence that experimental results should be necessary to improve the reliability of the mathematical model. Moreover, the growing interest in the experimental investigation has been also supported by improvements

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in methodology and, mainly, in experimental setup with a considerable reduction in their costs. However, the availability of experimental data is always limited, well below what is necessary to furnish exhaustive information, because few points can be instrumented, and the number of conditions that can be investigated are few. Thus, it was immediately clear that the best use of experimental data can be achieved through a close comparison with the analytical results, also because only thus is it possible to obtain information on the local characteristics of the structure.

In engineering, such as in computer science or modelling engineering, where the models have a mathematical origin rather than physical bases, the demand to rigorously respond to the above issue was the flowering of a new line of research: the system identification. In mechanical engineering, in a broad sense, where the models are better structured and based on the theory of structures, the same demand materialised later, leading to many activities in the field of structural identification [14, 15, 33, 36, 55, 57].

In this chapter attention is limited to two main topics of this large research area: dynamic structural identification, in the framework of inverse problems [37], and damage identification [54]. The latter can be considered as a sub-problem of the main one, but very important for the health monitoring of structures, after having developed the issue of dynamic structural characterization [74]. Some results obtained by the authors are reported, framed in the international literature, as well as a selection of Italian contributions, due to problem of space.

2 Structural Identification and Inverse Problems

The usefulness of the comparison of analytical and experimental results stimulated the appearence of studies on the dynamic characterization of structures, with the aim to obtain models more appropriate to predicting the dynamic response. A short list of papers, mainly by Italian researchers, is included [5, 16, 20, 30, 34, 35, 53, 59, 60, 62, 71]. The comparison requires a scientific approach and system identification guarantees the best use of a priori analytical information and experimental results.

By referring to mechanical systems, structural identification is a process which defines a model $\mathcal{M}$, within a class $S(\mathcal{M})$, that fits at best the experimental response, assuming a judgment criterion. Of course, due to modelling and measurement errors, a difference between the model predicted and experimentally measured data cannot vanish. Thus, it is not possible to determine the true model within $S(\mathcal{M})$, but only the closest possible model to the real one.

Regarding the experimental data, dynamic measurements which are easy to obtain are considered; thus, structural dynamic identication is dealt with here.

The variety of approaches in structural identification is very large, even limiting attention to linear structures: it depends on the class of interpretative models, the criteria for optimal estimate and the experimental results, which can be represented by different response quantities. Moreover, structural identification belongs to the category of inverse problems, and as such could call for specific treatments to deal with ill-conditining and possible indeterminacy.

The objectives of structural identification can be multiple:

- to determine an analytical model based on experimental information only;
- to improve the mathematical model based on a priori information making use of experimental data;
- to evaluate the characteristics of the system under study, mainly to monitor the integrity status over time.

It has been always considered little realistic to obtain a confident and complete model based on experimental data only. The prevailing interest has been directed towards the second objective because the experimental data are in any case limited and it would be unrealistic to disregard the a priori information which gives an initial estimate of the model of the structure. Moreover, this step can be used in the third objective of structural health monitoring.

2.1 Models

The most suitable models to meet the different aims of identification are the *parametric models*, where the identification concerns a certain number of their parameters whose optimal estimate will depend on experimental available data. Among parametric models, modal models and physical models must be mentioned. In *modal models* [31, 48, 74] the only assumption is that the dynamics of the structure is governed by uncoupled second order linear differential equations and the modal parameters are the unknown to be identified. In *physical models* [32] more important use is made of a priori information coming from the theory of structures, and the aim of identification is an updating the model by adjusting the values of selected parameters. The appealing aspect of these models is that the parameters have a physical meaning and this can be very useful in health monitoring where the evolution of local properties is of interest.

The equations of motion of a linear discrete system, or discretized continuous system, with N degrees-of-freedom can be written in matrix form

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{p}(t), \quad t > 0, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively, and $\mathbf{p}(t)$ is the vector of external forces. If the damping matrix $\mathbf{C}$ is proportional to a linear combination of $\mathbf{M}$ and $\mathbf{K}$ (classical damping), then the system will have the same real eigenvectors of the undamped system. The equations of motion can be decoupled in the eigenvector space by describing the displacement motion in terms of eigenvectors, namely $\mathbf{u}(t) = \mathbf{\Phi}\mathbf{q}(t)$, where $\mathbf{q}(t) = (q_1(t), \ldots, q_N(t))^T$ are the modal coordinates and the columns of $\mathbf{\Phi}$ are the eigenvectors $\mathbf{\Phi}^r$, leading to the N uncoupled modal equations:

$$\ddot{q}_r(t) + 2\xi_r\omega_r\dot{q}_r(t) + k_r q_r(t) = \mathbf{\Phi}^r \cdot \mathbf{p}(t), \quad t > 0, \quad r = 1, \ldots, N. \tag{2}$$

The dynamics of the system is described as a linear superposition of the response of N single-degree-of-freedom systems, and the prediction of the response is given as $\mathbf{U}(\omega) = \mathbf{H}(\omega)\mathbf{P}(\omega)$, where $\mathbf{U}(\omega)$ and $\mathbf{P}(\omega)$ are the Fourier transform of the displacement and force vectors, respectively, and the $N \times N$ matrix $\mathbf{H}(\omega)$ is the frequency response function. The generic term $H_{ij}(\omega)$, the displacement in the ith node caused by a unit force applied in the jth node, is given by

$$H_{ij}(\omega) = \sum_{r=1}^{N} \frac{1}{k_r} \cdot \frac{\Phi_i^r \Phi_j^r}{1 - \left(\frac{\omega}{\omega_r}\right)^2 + 2i\xi_r \frac{\omega}{\omega_r}}. \tag{3}$$

A knowledge of the modal characteristics of the structure, natural frequencies ω_r, mode shapes components Φ_i^r and damping coefficients ξ_r, makes it possible to evaluate $\mathbf{H}(\omega)$ and predict the response to any external force. The identification of the modal parameters from experiments has been denoted as *Experimental Modal Analysis* [54].

Various procedures to determine the modal properties of a structure have been introduced in frequency and time domains. In the frequency domain the modal unknown parameters $\mathbf{x}^T = (\omega_r, \Phi_i^r, \xi_r)$ are determined by minimizing the error between the experimental and the analytical frequency response function:

$$e_{ij}(\mathbf{x}) = \int_{\omega_a}^{\omega_b} \left(H_{ij}^{exp}(\omega) - H_{ij}(\omega, \mathbf{x})\right)^2 d\omega. \tag{4}$$

Frequently do not exist the conditions to obtain a complete modal model which can satisfactorily predict the structural response due to small number of sensors. In any case, the identification of modal parameters is important because frequencies and even few components of eigenvectors are the experimental quantities normally used in the identification of physical models [23, 53, 74]. Moreover, they are the basic quantities of the dynamic characterization of a structure.

Reference is made before to the procedure for obtaining modal quantities based on a forced response, where input (the force) and output (the response) are measured. In several cases, mainly with civil constructions, the recourse to external forces to excite the structure can be complicated and demanding. This has led to a novel approach where ambient vibrations are used and modal quantities are determined only by the measured response after introducing some assumptions on the broad-banded excitation. The technique is just denoted *Output-Only Modal Analysis* and does not offer the accuracy of the input-output techniques. It is not further discussed due to the length limit [7, 61].

Finite element models are typical examples of physical models. The identification of a finite element model of a structure is very appealing because it is just the model actually used in direct problems where the response to a known excitation is sought. The response furnished by a finite element model depends on a series of coefficients, closely linked to physical quantities, such as masses and stiffness of single elements or group of elements. The identification of the structural model consists in the optimal estimate of these parameters, collected in a vector $\mathbf{x}$, minimizing the difference between experimental and analytical values of the observed response quantities. This approach, which combines the best use of modelling and experimental information on the structure, is frequently designated as *model updating* [23, 32, 55] because it is closer to a successive adjustment of the model rather than a general structural identification.

The quantities involved in the comparison are usually response quantities, mainly modal quantities, frequencies and eigenvectors components; the difference to minimize is the output error $\mathbf{e}(\mathbf{x}) = \mathbf{U}^{exp}(\omega) - \mathbf{U}(\omega, \mathbf{x})$, which also for an elastic structure depends nonlinearly on the unknown parameters $\mathbf{x}$. It is worthy noting that the vector of the observed quantities $\mathbf{U}^{exp}$ can contain only some components of mode shapes, not necessarily the complete mode, thus the preliminary identification of modal models does not require determining the complete modes of the structure but only the components of the instrumented points. With this approach the observed modal quantities are not directly measured, but they result from the preliminary experimental modal analysis.

2.2 Estimators

The evaluation of the optimal estimate of parameters can be pursued in different ways, following a deterministic or probabilistic criterion, and selecting accordingly the scalar objective function. The relationship between observed quantities $\bar{\mathbf{z}}$, which are the measured data, and the analytical quantities provided by the model $\mathbf{z}(\mathbf{x})$, which depends on the parameters $\mathbf{x}$, contains an error $\mathbf{n}$:

$$\bar{\mathbf{z}} = \mathbf{z}(\mathbf{x}) + \mathbf{n}, \tag{5}$$

due to the presence of modelling and experimental errors. Thus, parameters $\mathbf{x}$ cannot be directly determined, but an estimator which furnishes an optimal value of $\mathbf{x}$, given measured quantities $\bar{\mathbf{z}}$, must be assumed to minimize the errors.

Among the estimators proposed, the *Bayesian approach* takes into account the random characteristics of measurements and parameters, without the complexity of a fully probabilistic approach; moreover, it furnishes other cost-effective contributions, as shown below [4, 21, 43, 68, 69]. With this approach, the best estimate $\widehat{\mathbf{x}}(\mathbf{z})$ is which that maximizes the a posteriori probability density function $p(\mathbf{x}|\bar{\mathbf{z}})$. When $\mathbf{x}$ and $\mathbf{n}$ have a normal distribution and $\boldsymbol{\Sigma}_{\mathbf{x}}$ and $\boldsymbol{\Sigma}_{\mathbf{n}}$ are positive definite covariance matrices of $\mathbf{x}$ and $\mathbf{n}$, which take account of the reliability of the measurements and the a priori information on parameters, respectively, $p(\mathbf{x}|\bar{\mathbf{z}})$ attains its maximum for a value of $\widehat{\mathbf{x}}$ which minimizes the function [69]:

$$l(\mathbf{x}) = (\bar{\mathbf{z}} - \mathbf{z}(\mathbf{x}))^T \boldsymbol{\Sigma}_{\mathbf{n}}^{-1} (\bar{\mathbf{z}} - \mathbf{z}(\mathbf{x})) + (\mathbf{x} - \mathbf{x}_0)^T \boldsymbol{\Sigma}_{\mathbf{x}}^{-1} (\mathbf{x} - \mathbf{x}_0). \tag{6}$$

The first term represents the difference between measured and model-predicted quantities, while the second takes account of the distance between $\mathbf{x}$ and the initial estimate $\mathbf{x}_0$, both weighted by the inverse of the covariance matrices.

With respect to a deterministic estimator, which is based on the least square method, the objective function (6) differs because, in the first term, the weights of different terms take into account the reliability of the measurements. However, the main difference is the second term which introduces a constraint on the variation interval of parameters that is stronger the higher the confidence on $\mathbf{x}_0$ is. The main benefit of the method is the possibility of obtaining analytical expressions of certain quantities which make it possible to evaluate the importance and independency of different parameters, the optimal selection of quantities to observe and, more generally, the ill-conditioning characteristics by means of the assumed interpretative model and pseudo-experimental data [12, 21, 23, 69]. It is useful to consider the dispersion of the estimate $\mathbf{x}$ around the optimal solution $\widehat{\mathbf{x}}$, because its geometrical shape gives meaningful information on the solution reliability. In a Bayesian framework the dispersion is described by the a posteriori covariance matrix:

$$\boldsymbol{\Sigma} = \left(\boldsymbol{\Sigma}_{\mathbf{x}}^{-1} + \mathbf{S}^T \boldsymbol{\Sigma}_{\mathbf{n}}^{-1}\mathbf{S}\right)^{-1}, \tag{7}$$

where $S_{ij} = \frac{\partial z_i}{\partial x_j}$ is the sensitivity matrix evaluated in $\widehat{\mathbf{x}}$. This equation shows that the a posteriori covariance matrix $\boldsymbol{\Sigma}$ is made by two contributions, one is simply its a priori counterpart covariance matrix, while the other is the covariance matrix of noise modified by the sensitivity of the observed quantities with respect to the parameters. Its value strongly depends on the choice of observed quantities and parameters, and as such it can be used in their optimal selection.

Function $\mathbf{z}(\mathbf{x})$, furnished by a finite element model of the structure, describes a relationship between p observable quantities and a wide set of q parameters; in practice, due to limitations in experiments and in dimension of the inverse problem, only $m < p$ components z_i are observed and only $n < q$ parameters x_i to identify can be considered.

When an optimal selection of observed quantities and parameters is pursued, it is convenient to only consider the second term in brackets of (7), called information matrix or Fisher matrix $\mathbf{A}$, which is directly dependent on $\mathbf{z}$ and $\mathbf{x}$ through the sensitivity matrix $\mathbf{S}$. First $n < q$ parameters are chosen as those associated to the greatest diagonal elements of $\mathbf{A}$; since they cannot be independent, the eigenvalues of $\mathbf{A}$ are determined and the parameters associated to the highest eigenvalues are assumed. After the choice of parameters, the selection of the observed quantities must be pursued with the aim of reducing the covariance matrix, considering again the second term of (7) only, because the first does not depend on the measurements. This can be achieved by selecting the set of quantities which minimizes $\mathbf{A}^{-1}$, for example the trace of $\mathbf{A}^{-1}$, that is the sum of its diagonal terns. In conclusion, it must be noted that the overall procedure is only approximated because the matrix $\boldsymbol{\Sigma}$ should be determined in $\widehat{\mathbf{x}}$ and not in $\mathbf{x}_0$; however, the information obtained for the selection of $\mathbf{z}$ and $\mathbf{x}$ is very useful.

3 Damage Identification

Increasing safety standards have motivated research and technology to evaluate the health conditions of a structure: a central point is the identification of damage. Different levels of knowledge can be defined, with increasing insight, that is detection, localization and quantification, useful for the structural assessment. Here, identification of damage means *localization* and *quantification* of damage, which occurs in different ways, but always corresponds to a reduction of the stiffness characteristics of the structure [1, 15, 42, 54].

As already mentioned in the previous section, in structural identification, once parameters of structural model and observed quantities have been defined, the problem consists in determining the optimal estimate of parameters such that the measured response quantities $\bar{\mathbf{z}}$ to known input and those predicted by the model are as close as possible. If the conditions of the structure are assumed as those of the undamaged structure, the identified parameters will be denoted as $\mathbf{x}_U$, which are associated to the modal quantities $\mathbf{z}_U$. In a damaged state, the response quantities $\mathbf{z}_D$ exibit a variation $\Delta\mathbf{z}_D$ with respect to $\mathbf{z}_U$ and the identification of damage requires an evaluation of the damage parameters $\mathbf{x}_D$, different from $\mathbf{x}_U$ by $\Delta\mathbf{x}_D$, which is the quantity directly related to damage.

Two main approaches can be pursued to damage identification. The first consists in two successive model parameters identification in the undamaged and damaged conditions, $\mathbf{x}_U$ and $\mathbf{x}_D$, leading to a degree of damage as the relative variation in the parameters $(x_{Ui} - x_{Di})/x_{Ui} = \Delta x_{Di}/x_{Ui}$. Starting from a model of the undamaged structure with parameters $\mathbf{x}_U$, the second approach consists in identification of the modification $\Delta\mathbf{x}_D$ such that the model is able to give a best match of the experimental variations $\Delta\mathbf{z}_D$ of the observed quantities [74].

In both cases, once the parameters $\mathbf{x}_U$ are determined, the solution can be sought as a minimization problem of a suitable objective function: $l(\mathbf{x}_D, \bar{\mathbf{z}}_D)$ in the first case, where the optimal values of $\mathbf{x}_D$ are determined on the basis of the measured quantities $\bar{\mathbf{z}}_D$, and $l(\Delta\mathbf{x}_D, \Delta\bar{\mathbf{z}}_D)$ in the second case, where the optimal values of $\Delta\mathbf{x}_D$ are determined on the basis of the measured quantities $\Delta\bar{\mathbf{z}}_D$.

Two cases of damage identification are treated separately below: damage diffused over the structure and damage located in a small region of the structure.

3.1 Identification of Diffused Damage

As a paradigmatic damage problem, let us consider the small axial vibration of a straight elastic beam under free-free boundary conditions,

$$\begin{cases} (au')' + \lambda\rho u = 0, \quad x \in (0, L), & (8) \\ a(0)u'(0) = 0 = a(L)u'(L), & (9) \end{cases}$$

and assume that the damage reflects into a change (reduction) of the axial stiffness $a = a(x)$, whereas the mass density per unit length $\rho = \rho(x)$ remains unchanged. The diagnostic problem consists in determining the function $a = a(x)$ from the knowledge of a finite number of natural frequencies $\omega_n = \sqrt{\lambda_n}$ of the beam, $n = 1, \ldots, N$.

From the mathematical point of view, this problem falls into the class of the *inverse eigenvalue problems with finite data*. A celebrated result proved in [41] shows that if the axial stiffness a is prescribed over the half beam $(L/2, L)$, then the unique determination of a in the other half beam $(0, L/2)$ requires *all* the infinite eigenvalues of the beam. It is therefore evident that a first serious problem is the non-uniqueness of the solution, that is the possible presence of a very large set of functions $a(x)$ which corresponds to beams with different axial stiffness but with exactly the same lower eigenvalues.

General studies focussed on inverse eigenvalue problems with finite data are relatively few. Among them, the contribution given in [3] shows that to deal with the problem of reconstruction it is vital to determine the weakest topology in which the available set of eigenvalues are continuous with respect to the unknown coefficient $a(x)$, since, otherwise, one would attempt to extract more information from the spectral data than it contains. In particular, since such inverse problems cannot be solved uniquely, a solution can be a coefficient $a(x)$ which has the correct spectral behavior and greater closeness to the measured data. A noticeable consequence of the theory in [3] is that from a finite number of eigenvalues one can only extract some averaged information on the unknown coefficient, not a pointwise information, that instead would be desireable. With reference to our model problem, the only information about $a(x)$ which can be extracted numerically from a finite number of eigenvalues is an approximation of the expression $\int_0^x a(s)ds$ for every $x \in [0, L/2]$. To obtain pointwise information on the coefficient $a(x)$, it is necessary to numerically differentiate the integral estimate, and it is precisely this step that is often ill-conditioned.

With a view to the above questions, in [52] the reconstruction problem for damage identification in the axially vibrating rod (8)–(9) was investigated from a different point of view. Under the assumption that the damaged configuration is a perturbation of the undamaged one, that is the stiffness variation Δa is small in L^2-norm, the inverse eigenvalue problem with finite data is linearized around the undamaged configuration to have, up to higher order terms,

$$\Delta\lambda_n = \int_0^{L/2} \Delta a (u_n')^2 dx, \tag{10}$$

where $u_n(x)$ is the nth eigenfunction of the undamaged rod, $\int_0^L \rho u_n^2 dx = 1$, $n = 1, \ldots, N$, and $\Delta\lambda_n$ is the difference between perturbed and unperturbed nth eigenvalue. The eigenfrequency shifts caused by the damage are then correlated with the generalized Fourier coefficients $< \Delta a, (u_n')^2 >$ of the unknown stiffness variation evaluated on the family of functions $\{(u_n')^2\}$. When it is a priori known—as in the present example—that the damage belongs to the interval $(0, L/2)$, the measurement of the first N eigenfrequency shifts, roughly speaking, allows for the determination of the first N generalized Fourier coefficients of the stiffness change. The numerical procedure is based on an iterative algorithm, and a proof of local convergence was obtained in [28] for a similar class of inverse problems.

The a priori assumption of stiffness change over one half of the beam can be removed by adding the first M antiresonant frequencies of the point frequency response function measured at one end of the beam [26]. Therefore, always retaining the assumption of small damage, it is possible to construct an approximation of the stiffness coefficient by means of a generalized Fourier sum of order $N + M$ whose coefficients are evaluated in terms of the eigenvalues belonging to two (suitable) partial spectra. A comprehensive series of numerical simulations and experimental tests were carried out to validate the predictions of the theory and the reliability of the method. Referring to the above mentioned papers for more details, the identification of more or less diffused damage in longitudinally vibrating rods was in good agreement with the theory, provided that eigenfrequency shifts are bigger—in the averaged sense—than modeling/measurement errors. An extension of the method to the identification of damage in bending vibrating beams can be found in [27].

Finally, it should be recalled that the idea of connecting the Fourier coefficients of the $a(x)$ with the frequency shifts is more deep and traces back to the cornerstone contribution in inverse spectral theory given in [6], see also [39] for numerical applications. In the context of damage identification in elastic beams, a reconstruction method based on generalized Fourier coefficients in a pinned-pinned beam with a crack at mid-span was presented in [75].

3.2 Identification of Concentrated Damage

3.2.1 A Beam Model

In this case damage is due to localized phenomenon related to aging or to stress exceeding in a few sections. In a usual monitoring of the health conditions a small number of damages should be expected, frequently just one, between one check and another. In any case the unknown quantities are very limited in number, as they are the location and stiffness reduction of damaged sections. A concentrated damage is represented by an open crack or a reduction of the mechanical properties of a small beam element, and is described by a decrease of stiffness, more or less localized, and as such linear behaviour is assumed before and after the damage. For each damage, the vector $\mathbf{x}$ collects the parameters of position s, intensity β and extension b [17, 74]. When the extension is very small, there can be only two parameters, position and intensity [9, 45, 58, 73]. In the latter case a frequently adopted model for the damage is a rotational spring, with a stiffness k related to damage intensity [17, 56]. In some cases it can be useful to consider a continuous description of the damage to facilitate analytical developments [19].

3.2.2 Identification Methods

When damage is due to one crack, described by a rotational spring of stiffness k at the cross-section of abscissa s, the characteristic equation provides a relationship between s and k, which, for a uniform simply supported beam of unit length, is explicit with respect to k and reads [17, 73]:

$$kg_1(\mu) + g_2(\mu, s) = 0, \tag{11}$$

where μ is the eigenvalue, $g_1(\mu) = 4\sin(\mu)\sinh(\mu)$, $g_2(\mu, s) = \mu\{\sinh(\mu)(\cos(\mu) - \cos(s\mu)) + \sin(\mu)(\cosh(\mu) - \cosh(s\mu))\}$. For a given damage, described by k and s, Eq. (11) is satisfied for each $\mu = \mu_r$. On the other hand, given a μ_r, a function $k_r(s) = -g_2(\mu_r, s)/g_1(\mu_r)$ is obtained; thus, the condition $k_i(s) = k_j(s)$ furnishes the solution, k and s, when it is unique. For a simply supported uniform beam this is true when using the first and second eigenvalues, since the related $k_1(s)$ and $k_2(s)$ cross in one location s with the same k (up to symmetry with respect to the mid-point of the beam). This is not true for higher eigenvalues; in any case two or more eigenvalues uniquely define the two damage parameters [13, 72].

Two procedures for damage identification may be followed. The first is based on Eq. (11) and is denoted as the *modal equation procedure*. The second is based on the direct output comparison of (5) and is denoted as the *response quantity procedure* [17]. In the real world a noise is present due to experimental and modeling errors. Therefore, the problem of finding the best damage parameters can be conveniently formulated as a minimum problem. In the response quantity procedure based on output error equation, it is convenient to directly introduce an objective function $l(\mathbf{x})$ depending on the difference between experimental and analytical variations of modal quantities from the undamaged to the damaged states. For the case of a diffused damage described by three parameters (position s, extension b and intensity β), and assuming the frequencies as measured quantities, the simplest expression of $l(\mathbf{x})$ reads:

$$l(s, b, \beta) = \sum_r \left(\frac{\Delta\omega^D_{exp,r}}{\omega^U_{exp,r}} - \frac{\Delta\omega^D_r(s, b, \beta)}{\omega^U_r} \right)^2. \tag{12}$$

The minimum of $l(\mathbf{x})$ can be pursued directly in the parameter space or can be obtained in two phases, which makes a useful representation possible. For any possible damage position s, the value of the objective function

$$\widetilde{l}(s) = \min_{b,\beta} l(s, b, \beta) \tag{13}$$

as a function of parameter s only, is first determined by minimization of $l(s, b, \beta)$ with respect to parameters b and β. Then, the minimum of $\widetilde{l}(s)$ gives the solution of damage identification, the parameter s and the related b and β determined in the previous minimization at the s position. In both cases a number of frequencies, suitably selected, equal to the number of unknown parameters, are sufficient to obtain a unique solution. Of course a higher amount of measured data improves the reliability of solution.

In realistic applications a finite element model is widely used to describe the structure. Notwithstanding that it was clear that the unknown quantities in damage identification are very limited in number, i.e. the locations and stiffness reductions of damaged sections, sometimes in a discretized problem the set of stiffness property of all elements are assumed as unknowns, leading to an undetermined problem. According to the analysis of the continuous model, if one damage or two damages have to be identified in a beam, the class of solutions to consider is those where one or two elements are characterised by a reduced stiffness coefficient, and the coefficients of all the other elements are equal to the undamaged values [72]. Also in this case the best approach to solve the inverse problem is to tackle it as a minimization problem where, if a beam is affected by one damage, the unknown $x_i = \Delta k_i / k_{Ui}$, with $\Delta k_i = k_{Ui} - k_{Di}$, and k_{Ui} and k_{Di} undamaged and damaged stiffnesses of the ith element, respectively, gives a measure of damage in the ith element. The procedure in two steps, previously recalled, is very effective: for each ith element the optimal estimate of x_i is determined by the minimum of $l(x_i)$ and then the minimum of the obtained $\widetilde{l}(x_i)$ gives the searched position and intensity of damage [13, 73].

3.2.3 Identification of Small Cracks

As stated above, beams with uniform cross-section allow the frequency equation to be expressed in a closed form, and this property makes damage identification simpler. In particular, a fairly complete theory is now available for the identification of a single open crack in a beam, either under axial or bending vibration, when the damage is small, that is when the damaged system can be considered as a perturbation of the undamaged one, which is the case of greatest interest [8, 22, 29, 70]. Let us focus our attention on uniform beams under longitudinal vibration and assume that the crack is modeled by a translational elastic spring, of stiffness k, located at the cross-section of abscissa s, with $1/k$ small enough.

One of the first rigorous results, based on a perturbation analysis, is in [56], where it was proved that the crack in a free-free rod can be uniquely localized (up to a symmetric position) by the first two frequencies of the longitudinal vibration. A different approach to the identification of a single small crack in a rod with a variable profile was proposed in [51]. Working on the weak formulation of the eigenvalue problem, it was shown in [50] that the first order change $\delta\lambda_n$ in a generic eigenvalue λ_n (e.g., the resonant frequency squared) is given by

$$\delta\lambda_n = -\frac{N_n^2(s)}{k}, \tag{14}$$

where $N_n(s)$ is the axial force in the nth vibration mode of the undamaged rod, evaluated at the cracked cross-section of abscissa s. It follows that the ratio of the first order changes to two eigenvalues (when $\delta\lambda_m < 0$)

$$\frac{\delta\lambda_n}{\delta\lambda_m} = \frac{N_n^2(s)}{N_m^2(s)} \tag{15}$$

is a function of s only, and therefore it may be possible to find the position of the crack s corresponding to a given (measured) value of $\delta\lambda_n/\delta\lambda_m$. In particular, for a free-free rod with regular profile $a = a(x)$ and symmetric with respect to the mid-point, it was shown in [51] that the knowledge of the first and second eigenfrequencies ($m = 1$, $n = 2$) uniquely determines the position of the crack, up to a symmetric position. Moreover, simple closed-form expressions are available both for the position and the severity of the damage in case of uniform beams.

It should be noted that the indeterminacy induced by the symmetry of the rod can be removed by using the first resonant frequency of the free-free rod and the first antiresonant frequency of the driving point frequency response function measured at one end of the rod [25].

The theory has been extended to the identification of a crack in bending vibrating bars, and applications of the above results to experimental data on cracked steel beams are reported in the above mentioned papers. Generalizations to beams with variable profile and a single crack, either under axial or bending vibration, are available in [64, 65]. Regarding the identification of finite number of small cracks in rods and beams using eigenfrequencies only, an analysis of the literature and an original reconstruction algorithm are shown in [67].

3.2.4 Identification from Mode Data

In the previous sections, the emphasis was restricted to the use of natural frequencies for damage identification. Now, attention is briefly devoted to the use of eigenfunctions and, moreover, modal curvatures and nodal positions of the vibration modes.

In the output error equation only frequencies are referred to above, but, as has been said, any other quantities of the response can be suitably used; remaining in the family of modal quantities, attention has been given over time to modes first and then to modal curvatures. Both quantities are local quantities and for this reason their measure is less reliable than eigenfrequency data. They are generally introduced as added measured quantities, also because at least the components at the positions of the accelerometers are already available by the setup used to measure frequencies. The mode components are certainly useful to improve the estimate of the unknown parameters, for example they can eliminate the indeterminacy of symmetric solutions, not discernible when using frequencies only. However, they cannot give satisfactory contributions to damage localization which is an important step of damage identification in eliminating multiple possible solutions [44, 46, 47, 63].

In this context curvatures were seen as the most sensitive quantities to damage, and to a certain extent it is true, but they should be derived by modes and derivation introduces noise [19]. Recently the use of strain sensors was satisfactorily exploited to give a direct measure of curvatures [2]. Another weak point is the circumstance that their variation due to damage is concentrated in an area restricted to damage position and practically vanishes elsewhere, so it is difficult to detect these variations. Moreover, different from what one may imagine, variations of modal curvatures are not directly related to the distribution of damage, unless for concentrated damage, and a filtering technique must be used [19]. In any case modal curvatures, along with modes, are the subject of recent papers and represent, together with frequencies, a variety of measured quantities all candidates to be used together in damage identification [10, 11, 18, 66].

Another observed quantity is the change in nodal positions, the use of which in reconstruction problems started with the fundamental papers [40, 49]. The detection of a crack, simulated by an elastic spring of stiffness k, in a longitudinally vibrating rod was considered in [38]. As a consequence of Sturm's theorems, it was found that, when the mode is sensitive to damage, the nodes move toward the damage location s. Namely, nodes of the rod which, in the undamaged state, are to the left of s, move to the right, while those to the right of s move to the left. The direction and amount by which the nodes move may be used to estimate the position and severity of the damage. An experimental study based on these results may be found in [38]. The analogous problem for a flexurally vibrating beam was considered in [24]. By means of counterexamples, it was shown that the monotonicity property linking changes in node position and crack location found in [38] does not hold in the bending case. However, the direction and amount of nodal points change may be useful in predicting damage location, as confirmed by experiments [24].

4 Concluding Remarks

This chapter addresses some topics of the wide area of research devoted to structural identification and damage detection. In particular, a general framework of the subject is given dealing with some topical points: the interpretative models to describe the structural behaviour, the estimators to obtain the best estimate of model parameters, the type of experimental data, uniqueness issues of the related inverse problems and tools to govern their peculiar difficulties in damage identification. An analysis of the recent literature suggests that practical use of structural identification methods still requires extensive study, prioritizing a multi-disciplinary approach and opportunities from the great advances in the technological, processing and theoretical fields.

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Experimental Dynamic Substructuring: Significance and Perspectives

Walter D'Ambrogio and Annalisa Fregolent

Abstract Dynamic substructuring allows to describe an assembled structural system in terms of component subsystems. In experimental dynamic substructuring, the model of at least one (sub)system derives from experimental tests: this allows to consider systems that may be too difficult to model. The degrees of freedom (DoFs) of the assembled system can be partitioned into internal DoFs (not belonging to the couplings) and coupling DoFs. A possible application of experimental dynamic substructuring is substructure decoupling, i.e. the identification of the dynamic model of a structural subsystem embedded in a structural system known from experiments (assembled system) and connected to the rest of the system (residual subsystem) through a set of coupling DoFs. Coupling DoFs are often difficult to observe, either because they cannot be easily accessed or because they include rotational DoFs. However, whilst coupling DoFs and in particular rotational DoFs are needed when coupling together different subsystems, they are not essential in substructure decoupling, because the actions exchanged through the coupling DoFs are already included in the dynamic response of the assembled system. The most promising fields in substructure coupling are: coupling with configuration dependent interface and nonlinear coupling with localized nonlinearities. With reference to substructure decoupling, the most remarkable topics are: interface optimization, configuration dependent coupling conditions, and joint identification.

Keywords Experimental dynamic substructuring · Substructure decoupling · Interface DoFs

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1 Introduction

In the framework of dynamic substructuring, substructure coupling consists in the identification of the dynamic behavior of an assembled structural system, starting from the dynamic behavior of the component subsystems. The degrees of freedom (DoFs) of the assembled system can be partitioned into internal DoFs (not belonging to the couplings) and coupling DoFs. Many well-established techniques exist to perform substructure coupling when all substructures are modeled theoretically, see for instance [8, 12, 18] and subsequent literature. However, in many cases, the model of at least one subsystem derives from experimental tests, mainly because complex subsystems may be too difficult to model. In this case, one speaks of experimental dynamic substructuring.

A general framework for dynamic substructuring is provided in [15], where primal and dual assembly are introduced. Furthermore, coupling can be performed in the physical domain, in the frequency domain (Frequency Based Substructuring), or in reduced domains (e.g. the modal domain). When using the modal domain or other reduced domains, truncation problems may arise. In Frequency Based Substructuring, Frequency Response Functions (FRFs) are used to avoid modal truncation problems.

A well known issue in experimental substructure coupling is related to rotational DoFs. Whenever coupling DoFs include rotational DoFs, the related rotational FRFs must be obtained experimentally. This becomes a quite complicated task when measuring only translational FRFs, as shown in [20]. Several techniques for measuring rotational responses have been devised, see e.g. [1, 21].

Substructure decoupling represents another possible application of experimental dynamic substructuring. It can be defined as the identification of the dynamic model of a structural subsystem embedded in a structural system known from experiments (assembled system) and connected to the remaining part of the system (residual subsystem) through a set of coupling DoFs. Decoupling is a need for subsystems that cannot be measured separately, but only when coupled to their neighboring substructure(s) (e.g. fixtures needed for testing or subsystems in operational conditions).

Coupling DoFs are often difficult to observe, either because they are not easy to access or because they include rotational DoFs. However, whilst coupling DoFs and in particular rotational DoFs are needed when coupling together different subsystems, they are not essential in substructure decoupling [9]. In fact, the actions exchanged through the connecting DoFs are already embedded in the dynamic response of the assembled system.

Contact problems can also be tackled using a numerical computation technique based on dynamic substructuring, if time varying coupling conditions are assumed. In [2, 6] a sliding contact interface between a rigid suspended bar and a lumped mass is considered, without and with friction. In [3, 7] a sliding contact interface between a horizontal and an oblique cantilever beam is considered, without and with friction, using a basic contact assumption, in order to deal with friction induced vibrations not involving instabilities. In [4, 5] the sliding contact between a horizontal and an oblique cantilever beam is considered again, using more realistic contact assumptions that take into account the deformation of the contacting bodies in order to analyze friction induced instabilities.

In the present contribution, Sect. 2 describes the recent advances of research on experimental substructure coupling, focusing on configuration dependent Frequency Response Function; Sect. 3 deals with substructure decoupling, focusing on interface optimization; finally, Sect. 4 discusses the future perspectives of experimental dynamic substructuring.

2 Experimental Substructure Coupling

Let us consider a dynamic system made up of n coupled subsystems. Each subsystem can be described either in the physical domain using mass, stiffness and damping matrices or in the frequency domain using the dynamic stiffness matrix.

2.1 Frequency Based Substructuring

In the frequency domain, the equation of motion of a linear time-invariant subsystem r may be written as:

$$\mathbf{Z}^{(r)}(\omega)\boldsymbol{u}^{(r)}(\omega) = \boldsymbol{f}^{(r)}(\omega) + \boldsymbol{g}^{(r)}(\omega) \tag{1}$$

where:

$\mathbf{Z}^{(r)}$: dynamic stiffness matrix of subsystem r;
$\boldsymbol{u}^{(r)}$: vector of degrees of displacements of subsystem r;
$\boldsymbol{f}^{(r)}$: vector of external forces on subsystem r;
$\boldsymbol{g}^{(r)}$: vector of connecting forces with other subsystems (internal constraints).

The equation of motion of the n subsystems to be coupled can be written in a block diagonal format, by omitting the frequency dependence:

$$\mathbf{Z}\boldsymbol{u} = \boldsymbol{f} + \boldsymbol{g} \tag{2}$$

with

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{(1)} & & \\ & \ddots & \\ & & \mathbf{Z}^{(n)} \end{bmatrix}, \quad \boldsymbol{u} = \begin{Bmatrix} \boldsymbol{u}^{(1)} \\ \vdots \\ \boldsymbol{u}^{(n)} \end{Bmatrix}, \quad \boldsymbol{f} = \begin{Bmatrix} \boldsymbol{f}^{(1)} \\ \vdots \\ \boldsymbol{f}^{(n)} \end{Bmatrix}, \quad \boldsymbol{g} = \begin{Bmatrix} \boldsymbol{g}^{(1)} \\ \vdots \\ \boldsymbol{g}^{(n)} \end{Bmatrix}$$

The compatibility condition at the interface DoFs implies that any pair of matching DoFs $u_l^{(r)}$ and $u_m^{(s)}$, i.e. DoF l on subsystem r and DoF m on subsystem s must have the same displacement, that is $u_l^{(r)} - u_m^{(s)} = 0$.

This condition can be generally expressed as:

$$\mathbf{B}\boldsymbol{u} = \mathbf{0} \tag{3}$$

where each row of **B** corresponds to a pair of matching DoFs.

The equilibrium condition for internal constraint forces implies that, when the connecting forces are considered at a pair of matching DoFs, their sum must be zero, i.e. $g_l^{(r)} + g_m^{(s)} = 0$: this holds for any pair of matching DoFs. Furthermore, if DoF k on subsystem q is not a connecting DoF, it must be $g_k^{(q)} = 0$: this holds for any non-interface DoF.

Overall, the above conditions can be expressed as:

$$\mathbf{L}^T\boldsymbol{g} = \mathbf{0} \tag{4}$$

where the matrix **L** is a Boolean localisation matrix.

Equations (2)–(4) can be put together to obtain the so-called three-field formulation, describing the coupling between any number of subsystems:

$$\begin{cases} \mathbf{Z}\boldsymbol{u} = \boldsymbol{f} + \boldsymbol{g} \\ \mathbf{B}\boldsymbol{u} = \mathbf{0} \\ \mathbf{L}^T\boldsymbol{g} = \mathbf{0} \end{cases} \tag{5}$$

In the dual formulation [15, 22], the total set of DoFs is retained, i.e. each interface DoF is present as many times as there are substructures connected through that DoF. The equilibrium condition $g_l^{(r)} + g_m^{(s)} = 0$ at a pair of interface DoFs is ensured by choosing, for instance, $g_l^{(r)} = -\lambda$ and $g_m^{(s)} = \lambda$. Therefore, the equilibrium of internal constraint forces can be ensured by writing the connecting forces in the form:

$$\boldsymbol{g} = -\mathbf{B}^T\boldsymbol{\lambda} \tag{6}$$

where $\boldsymbol{\lambda}$ are Lagrange multipliers corresponding to connecting force intensities.

The equilibrium of internal constraint forces (4) is thus written:

$$\mathbf{L}^T\boldsymbol{g} = -\mathbf{L}^T\mathbf{B}^T\boldsymbol{\lambda} = \mathbf{0} \qquad \forall\boldsymbol{\lambda} \tag{7}$$

Since Eq. (7) is always satisfied by any set of connecting force intensities $\boldsymbol{\lambda}$, the system of equations (5) becomes:

$$\begin{cases} \mathbf{Z}\boldsymbol{u} + \mathbf{B}^T\boldsymbol{\lambda} = \boldsymbol{f} \\ \mathbf{B}\boldsymbol{u} = \mathbf{0} \end{cases} \tag{8}$$

To eliminate $\boldsymbol{\lambda}$, from the first of Eq. (8), it can be written:

$$\boldsymbol{u} = -\mathbf{Z}^{-1}\mathbf{B}^T\boldsymbol{\lambda} + \mathbf{Z}^{-1}\boldsymbol{f} \tag{9}$$

which substituted in the second of Eq. (8) gives:

$$\mathbf{B}\mathbf{Z}^{-1}\mathbf{B}^T\boldsymbol{\lambda} = \mathbf{B}\mathbf{Z}^{-1}\boldsymbol{f} \quad \Rightarrow \quad \boldsymbol{\lambda} = \left(\mathbf{B}\mathbf{Z}^{-1}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{Z}^{-1}\boldsymbol{f} \tag{10}$$

Substituting $\boldsymbol{\lambda}$ in the first of Eq. (8), it is obtained:

$$\begin{aligned} &\mathbf{Z}\boldsymbol{u} + \mathbf{B}^T\left(\mathbf{B}\mathbf{Z}^{-1}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{Z}^{-1}\boldsymbol{f} = \boldsymbol{f} \\ \Rightarrow \quad &\boldsymbol{u} = \left(\mathbf{Z}^{-1} - \mathbf{Z}^{-1}\mathbf{B}^T\left(\mathbf{B}\mathbf{Z}^{-1}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{Z}^{-1}\right)\boldsymbol{f} \end{aligned} \tag{11}$$

By considering that:

$$\mathbf{Z}^{-1} = \mathbf{H} = \begin{bmatrix} \mathbf{H}^{(1)} & & \\ & \ddots & \\ & & \mathbf{H}^{(n)} \end{bmatrix} \tag{12}$$

being $\mathbf{H}^{(r)} = [\mathbf{Z}^{(r)}]^{-1}$ the Frequency Response Function (FRF) matrix of the r-th subsystem, Eq. (11) can be rewritten:

$$\boldsymbol{u} = \left(\mathbf{H} - \mathbf{H}\mathbf{B}^T\left(\mathbf{B}\mathbf{H}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{H}\right)\boldsymbol{f} \tag{13}$$

Therefore, by considering that the FRF matrix $\mathbf{H}_c$ of the coupled system satisfies the relation $\boldsymbol{u} = \mathbf{H}_c\boldsymbol{f}$, it is:

$$\mathbf{H}_c = \mathbf{H} - \mathbf{H}\mathbf{B}^T\left(\mathbf{B}\mathbf{H}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{H} \tag{14}$$

Note that the rows and columns corresponding to the coupling DoFs appear twice in $\mathbf{H}_c$. Obviously, only independent entries are retained.

2.1.1 Configuration Dependent Interface

An interesting extension of the substructuring approach is to consider systems built from time-invariant component subsystems subjected to configuration dependent coupling conditions, such as those encountered when a relative motion exists between two coupled bodies.

In this case, compatibility and equilibrium conditions become configuration dependent. If χ denotes a given configuration, the compatibility condition can be generally expressed as:

$$\mathbf{B_C}(\chi)\boldsymbol{u}(\chi) = \mathbf{0} \tag{15}$$

where each row of $\mathbf{B_C}(\chi)$ concerns a pair of matching DoFs at configuration χ.

The equilibrium condition $g_l^{(r)}(\chi) + g_m^{(s)}(\chi) = 0$ at a pair of interface DoFs is ensured by choosing, for instance, $g_l^{(r)}(\chi) = -\lambda(\chi)$ and $g_m^{(s)}(\chi) = \lambda(\chi)$. Therefore, the overall interface equilibrium can be ensured by writing the connecting forces in the form:

$$\boldsymbol{g}(\chi) = -\mathbf{B_E}^T(\chi)\boldsymbol{\lambda}(\chi) \tag{16}$$

where $\boldsymbol{\lambda}(\chi)$ are Lagrange multipliers corresponding to connecting force intensities and $\mathbf{B_E}(\chi)$ is different from the signed Boolean matrix $\mathbf{B_C}(\chi)$ used to enforce the compatibility condition, because $\mathbf{B_E}(\chi)$ must also account for friction forces at the interface.

2.2 Configuration Dependent Frequency Response Function

A configuration dependent frequency response function of the coupled system can be computed as follows. When a configuration dependent interface is considered, Eq. (8) can be rewritten as:

$$\begin{cases} \mathbf{Z}\boldsymbol{u}(\chi) + \mathbf{B_E}^T(\chi)\boldsymbol{\lambda}(\chi) = \boldsymbol{f} \\ \mathbf{B_C}(\chi)\boldsymbol{u}(\chi) = \mathbf{0} \end{cases} \tag{17}$$

A procedure similar to that outlined in Eqs. (9)–(13) can be followed to eliminate $\lambda(\chi)$ from the first of Eq. (17) and to obtain, finally, an expression of the FRF matrix of the coupled system with configuration dependent interface:

$$\mathbf{H}_c(\chi) = \mathbf{H} - \mathbf{H}\mathbf{B_E}^T(\chi)\left(\mathbf{B_C}(\chi)\mathbf{H}\mathbf{B_E}^T(\chi)\right)^{-1}\mathbf{B_C}(\chi)\mathbf{H} \tag{18}$$

In Fig. 1, a beam on beam system is considered, where the upper oblique beam 1 is subjected to a vertical load F_y and to a constant velocity v_x in the horizontal direction. Therefore, the contact point C moves from the fixed end to the free end of the horizontal beam. More details about the system characteristics are available in [3]. Figure 1 shows the configuration dependent drive point FRF at the contact point C, computed according to Eq. (18). It can be noticed that the low level pattern highlights configuration dependent anti-resonance locations, while the high level pattern accounts for configuration dependent resonance location.

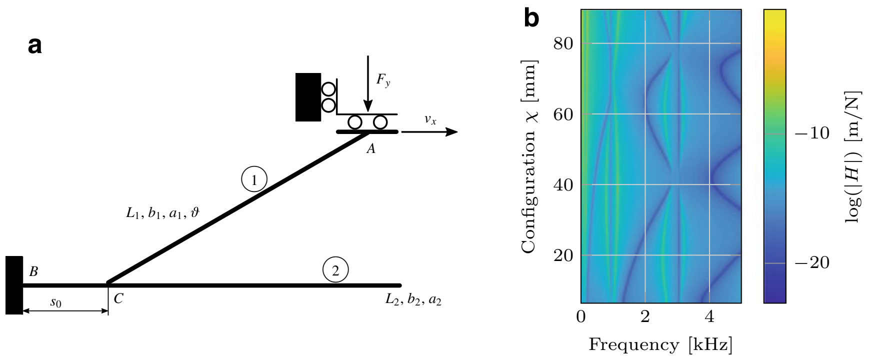


Fig. 1 Configuration dependent beam on beam model. **a** Beam on beam model [3]. **b** $|H_{CyCy}(\omega, \chi)|$

3 Decoupling

As stated in the Introduction, substructure decoupling consists in the identification of a dynamic model of a structural subsystem, starting from the FRFs of the assembled system RU and from a dynamic model of the so-called residual subsystem R. The unknown subsystem U (N_U DoFs) is joined to the residual subsystem R (N_R DoFs) by n_c coupling DoFs through which constraint forces (and moments) are exchanged (see Fig. 2). The degrees of freedom of the assembled structure (N_{RU} DoFs) can be partitioned into coupling DoFs (c), internal DoFs of substructure U (u) and internal DoFs of substructure R (r).

The FRFs of the unknown substructure U can be predicted from those of the assembled structure RU by taking out the dynamic effect of the residual subsystem R. In principle, this can be accomplished by considering a negative structure, i.e. by adding to the assembled structure RU a fictitious substructure with a dynamic stiffness opposite to that of the residual substructure R (see Fig. 3). The effect of

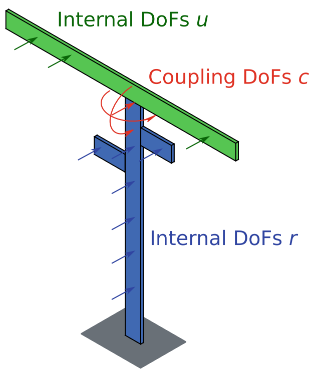


Fig. 2 Assembled system RU, with the unknown subsystem U (green) and the residual subsystem R (blue) [11] (For interpretation of the references to colour in the text, the reader is referred to the web version of this chapter)

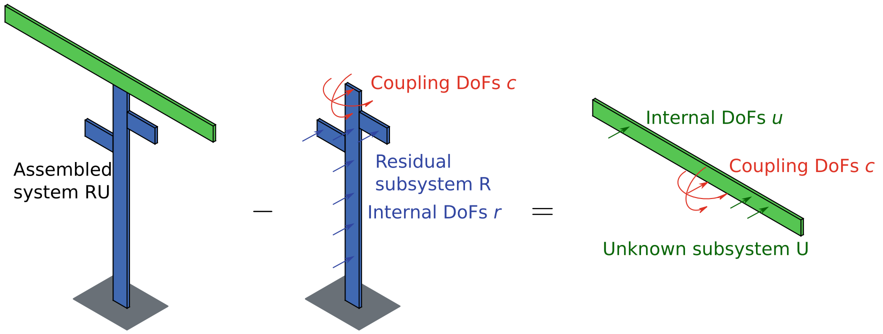


Fig. 3 Scheme of the direct decoupling problem [11]

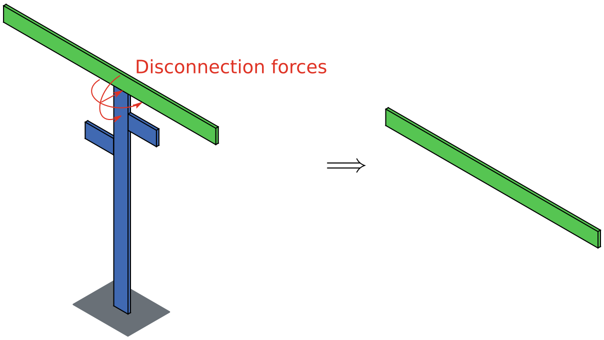


Fig. 4 Trivial set of disconnection forces (and moments) acting on the assembled structure (corresponding to the standard interface) [11]

the negative structure is to add a set of disconnection forces (and moments) to the external forces acting on the assembled system in order to uncouple the unknown substructure from the assembled structure. However, the set of disconnection forces is not unique. In fact, several sets of disconnection forces can be devised:

- a trivial set, consisting of disconnection forces acting at the coupling DoFs and opposite to the constraint forces (see Fig. 4); in this case, disconnection forces may include moments opposite to the constraint moments.
- non trivial sets of disconnection forces acting at different DoFs but able to cancel the constraint forces at the coupling DoFs (see Fig. 5); in this case, disconnection forces applied to internal DoFs must be able to provide a moment about the rotation axes.

In both cases, the dynamic equilibrium of the assembled structure RU is:

$$\mathbf{Z^{RU}} \boldsymbol{u}^{\boldsymbol{RU}} = \boldsymbol{f}^{\boldsymbol{RU}} + \boldsymbol{g}^{\boldsymbol{RU}} \tag{19}$$

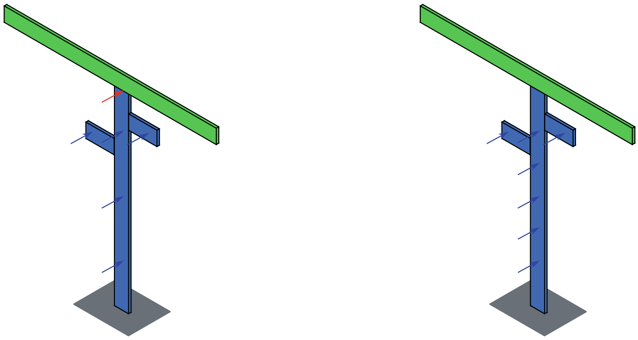


Fig. 5 Non trivial sets of disconnection forces corresponding to a mixed interface (left) and to a pseudo interface (right) [11]

where $\boldsymbol{g}^{\boldsymbol{RU}}$ is the vector of disconnection forces applied to the assembled structure by the negative structure, $\mathbf{Z}^{\mathbf{RU}}$ is the dynamic stiffness matrix of the assembled structure RU, $\boldsymbol{u}^{\boldsymbol{RU}}$ is the vector of degrees of freedom of the assembled structure RU, $\boldsymbol{f}^{\boldsymbol{RU}}$ is the external force vector on the assembled structure RU.

Similarly, the dynamic equilibrium of the negative structure is expressed as:

$$-\mathbf{Z}^{\mathbf{R}}\boldsymbol{u}^{\boldsymbol{R}} = \boldsymbol{f}^{\boldsymbol{R}} + \boldsymbol{g}^{\boldsymbol{R}} \tag{20}$$

where $-\mathbf{Z}^{\mathbf{R}}$ is the dynamic stiffness matrix of the negative structure, and $\boldsymbol{u}^{\boldsymbol{R}}$, $\boldsymbol{f}^{\boldsymbol{R}}$ and $\boldsymbol{g}^{\boldsymbol{R}}$ are defined as for the assembled structure.

In order that Eqs. (19)–(20) can be put together to represent the unknown substructure U, disconnection forces $\boldsymbol{g}^{\boldsymbol{RU}}$ and $\boldsymbol{g}^{\boldsymbol{R}}$ must be in equilibrium, and compatibility between degrees of freedom $\boldsymbol{u}^{\boldsymbol{RU}}$ and $\boldsymbol{u}^{\boldsymbol{R}}$ must hold at the interface between the assembled structure RU and the negative structure. Note that such interface includes both the coupling DoFs between substructures U and R, and the internal DoFs of substructure R (the blue part of the structure in Fig. 3). However, it is not necessary to retain the full set of these interface DoFs, because it is sufficient that the number of interface DoFs be not less than the number of coupling DoFs n_c. Therefore, several options for interface DoFs can be considered:

- standard interface, including only the coupling DoFs (c) between substructures U and R, e.g. those corresponding to the disconnection forces (and moments) in Fig. 4;
- extended interface, including also a subset of internal DoFs ($i \subseteq r$) of substructure R;
- mixed interface, including subsets of coupling ($d \subset c$) and internal DoFs ($i \subseteq r$), e.g. those corresponding to the disconnection forces in Fig. 5 (left);
- pseudo interface, including only internal DoFs ($i \subseteq r$) of substructure R, e.g. those corresponding to the disconnection forces in Fig. 5 (right).

The use of a mixed or pseudo interface allows to replace rotational coupling DoFs with translational internal DoFs.

Compatibility at the (standard, extended, mixed, pseudo) interface implies that any pair of matching DoFs, i.e. DoF l on the coupled system RU and DoF m on subsystem R, must have the same displacement, that is $u_l^{RU} - u_m^R = 0$. Let S_C be the set of N_C interface DoFs on which compatibility is enforced. The compatibility condition can be expressed as:

$$\mathbf{B_C}\,\boldsymbol{u} = 0 \qquad \text{where:} \quad \boldsymbol{u} = \begin{Bmatrix} \boldsymbol{u^{RU}} \\ \boldsymbol{u^R} \end{Bmatrix} \tag{21}$$

where $\mathbf{B_C}$ has size $N_C \times (N_{RU} + N_R)$ and each row corresponds to a pair of matching DoFs.

Let S_E be the set of N_E interface DoFs on which equilibrium is enforced. Note that sets S_E and S_C can be different (non-collocated approach [22]); the traditional approach, when $S_E \equiv S_C$, is called collocated. The equilibrium of disconnection forces implies that for any pair of matching DoFs, i.e. DoF r on the coupled system RU and DoF s on subsystem R, their sum must be zero, that is $g_r^{RU} + g_s^R = 0$. In the dual assembly, the total set of DoFs is retained, and the equilibrium at a pair of matching DoFs is ensured by choosing $g_r^{RU} = -\lambda_r$ and $g_s^R = \lambda_r$. Therefore, the overall interface equilibrium can be ensured by writing the disconnection forces in the form:

$$\boldsymbol{g} = -\mathbf{B_E}^T\,\boldsymbol{\lambda} \qquad \text{where:} \quad \boldsymbol{g} = \begin{Bmatrix} \boldsymbol{g^{RU}} \\ \boldsymbol{g^R} \end{Bmatrix} \tag{22}$$

where $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers corresponding to disconnection force intensities, and $\mathbf{B_E}$ is a $N_E \times (N_{RU} + N_R)$ matrix.

Having defined $\mathbf{B_C}$ and $\mathbf{B_E}$, an expression of the FRF $\mathbf{H^U}$ of the unknown subsystem can be derived using the same procedure as in Sect. 2.2, leading to:

$$\mathbf{H^U} = \mathbf{H} - \mathbf{H}\mathbf{B_E}^T \left(\mathbf{B_C}\mathbf{H}\mathbf{B_E}^T\right)^+ \mathbf{B_C}\mathbf{H} \tag{23}$$

where $+$ denotes the pseudo-inverse. This operation is necessary because it can be $N_C \neq N_E$. To obtain a determined or overdetermined matrix for the generalized inversion operation, the number of rows of $\mathbf{B_C}$ must be greater or equal than the number of rows of $\mathbf{B_E}$, i.e.

$$N_C \geq N_E \geq n_c \tag{24}$$

3.1 Interface Optimization

Disconnection forces $\boldsymbol{g}^{RU}$ must be able to uncouple the unknown substructure from the assembled system, i.e. they are such as to cancel constraint forces at the coupling

DoFs. If coupling DoFs include rotational DoFs, constraint forces include moments about some given axes.

As stated at the beginning of Sect. 3, the set of disconnection forces is not unique. Therefore, non trivial sets of disconnection forces, typically not including moments, can be selected taking into account the further objective of avoiding ill conditioning problems.

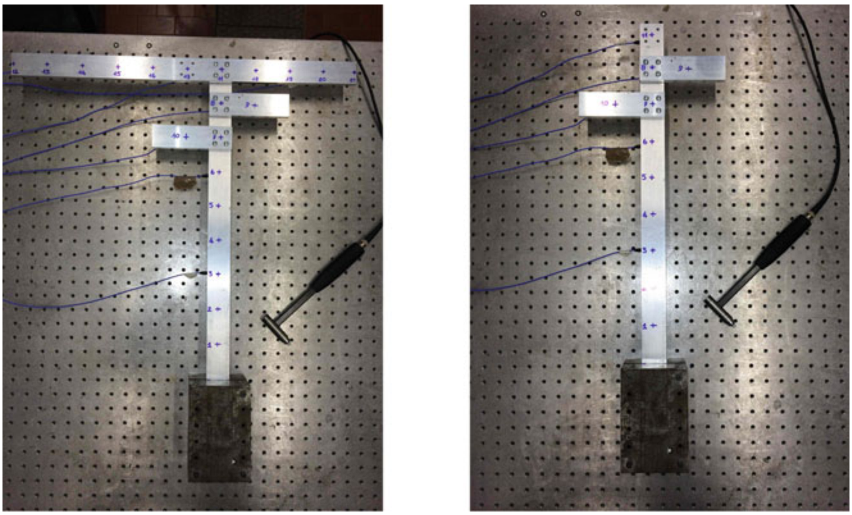
In Eq. (23), the product $\mathbf{B_C H B_E}^T$ that has to be inverted is defined as interface flexibility matrix. In fact, as shown in [9], it can be rewritten as:

$$\mathbf{B_C H B_E}^T = \hat{\mathbf{H}}^{\mathbf{RU}} - \hat{\mathbf{H}}^{\mathbf{R}} \tag{25}$$

where $\hat{\mathbf{H}}^{\mathbf{RU}}$ and $\hat{\mathbf{H}}^{\mathbf{R}}$ are the FRFs of the assembled structure and of the residual substructure at interface DoFs. The interface flexibility matrix depends on the choice of interface DoFs and it can be ill conditioned for some set of interface DoFs. Therefore, the conditioning of the interface flexibility matrix must be taken into account in the choice of interface DoFs.

An application is performed on the assembled system shown in Fig. 6a. The residual subsystem R consists of a cantilever beam with two offset short arms (Fig. 6b). The unknown substructure U is the beam bolted to the free end of the cantilever beam. The joint involves both translational and rotational DoFs. More details are available in [10].

The experimental FRF matrix of the assembled system is obtained at seven locations (3, 6, 9, 10, 11, 13, 20) along the z-direction orthogonal to the plane of the



(a) Assembled system. (b) Residual subsystem.

Fig. 6 Experimental test bed for decoupling [11]

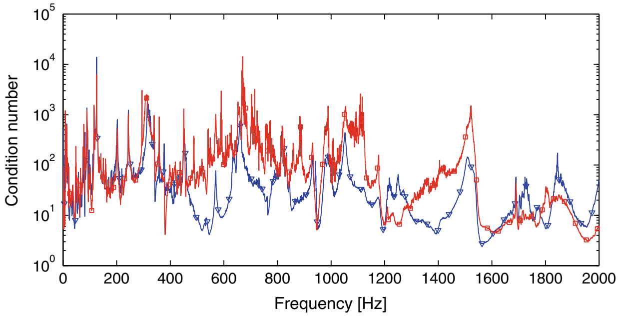


Fig. 7 Condition number of the interface flexibility matrix using experimental data and interface DoFs: $11z$, $3z$, $9z$ (–▽–) (blue); $11z$, $3z$, $10z$ (–□–) (red) [10]

structure. For the residual subsystem, the experimental FRF matrix is similarly measured at five locations (3, 6, 9, 10, 11). Coupling DoFs are: $11z$, $11\theta_x$ and $11\theta_y$, that is one translational DoF and two rotational DoFs around the horizontal and vertical directions in the plane of the structure.

There are 3 coupling DoFs, so that $n_c = 3$. As stated in Eq. (24), it must be $N_E \geq n_c = 3$, where $n_c = 3$ is the minimal number of DoFs. Therefore, in principle, any set of at least 3 DoFs among the measured DoFs on the residual subsystem is a possible set of interface DoFs. It can be quite significant to compare minimal sets of DoFs.

As shown in [10], either DoF $9z$ or DoF $10z$ must be included among the interface DoFs so that the corresponding disconnection force is able to provide a moment around DoF $11\vartheta_y$. The objective is to select, between DoFs $9z$ and $10z$, the most appropriate in terms of the condition number of the interface flexibility matrix. It may be convenient, although not strictly necessary, to keep one of the coupling DoFs, in this case DoF $11z$, among the interface DoFs.

Therefore, two minimal sets of DoFs are compared, each one of then representing a mixed interface: $11z$, $3z$, $9z$ and $11z$, $3z$, $10z$.

The condition number of the interface flexibility matrix $\hat{\mathbf{H}}^{\mathbf{RU}} - \hat{\mathbf{H}}^{\mathbf{R}}$ is considered for each of the two sets of interface DoFs (Fig. 7). It can be noticed that the condition number is lower when DoF $9z$ is considered instead of DoF $10z$, except in a limited frequency range, i.e. 1200–1300 Hz. Therefore, according to this criterion, DoF $9z$ should be preferred to DoF $10z$.

FRFs of the unknown substructure are predicted using the two sets of minimal interface DoFs defined previously. To check decoupling results, FRFs are measured also at three locations (11, 13, 20) of the unknown subsystem U. The FRF of the unknown substructure U, predicted using interface DoFs $11z$, $3z$, $9z$, is shown in Fig. 8: the result is quite clean considering that experimental data are being used. The FRF of the unknown substructure U, predicted using interface DoFs $11z$, $3z$, $10z$, is shown in Fig. 9: the result is much worse than in the previous case, especially

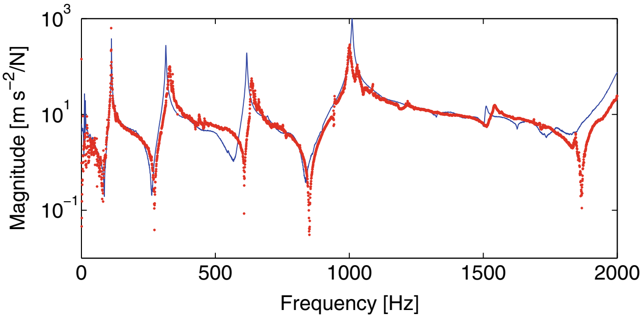


Fig. 8 $H^U_{11z,11z}$: measured (—) (blue), predicted from experimental FRFs using coupling DoF $11z$ and internal DoFs $3z$, $9z$ ($***$) (red) [10]

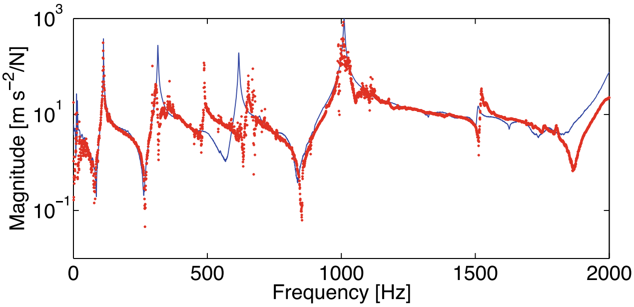


Fig. 9 $H^U_{11z,11z}$: measured (—) (blue), predicted from experimental FRFs using coupling DoF $11z$ and internal DoFs $3z$, $10z$ ($***$) (red) [10]

in the frequency band 400–1000 Hz, where the condition number of the interface flexibility matrix using interface DoFs $11z$, $3z$, $9z$ is significantly lower than using interface DoFs $11z$, $3z$, $10z$.

4 Perspectives

Perspectives of experimental dynamic substructuring differ according to whether substructure coupling problems or substructure decoupling problems are considered. With reference to substructure coupling problems, the most promising fields are those of coupling with configuration dependent interface and of nonlinear coupling with localized nonlinearities.

Coupling with configuration dependent interface is discussed in Sect. 2.1 and a simple example of configuration dependent Frequency Response Function is shown in Sect. 2.2: the example considers numerically simulated FRFs, but the procedure can be readily extended to experimental FRFs.

Nonlinear coupling with localized nonlinearities, i.e. linear structures connected by nonlinear joints is an ongoing field of research. The focus is about the effects of nonlinear connections on the dynamics of an assembly in which the coupled subsystems can be assumed as linear. This is suitable in many engineering systems where the nonlinearity introduced by the connecting element is much more relevant than those of the substructures to be coupled, as for bolted joints, wire rope isolators, turbine blade-rotor connections, etc. The advantages of nonlinear dynamic substructuring, where the nonlinear connecting elements are modeled using nonlinear normal modes, are shown in several papers [16, 17].

The most remarkable topics in substructure decoupling are: interface optimization and disconnection force identification; decoupling with configuration dependent coupling conditions; joint identification, including nonlinear joints.

Interface optimization, discussed in Sect. 3.1, concerns how to select the best set of DoFs in order to have a good conditioning of the interface flexibility matrix. In fact, by considering pseudo-interfaces, measurements on the connecting DoFs can be avoided. This is useful when for instance connecting DoFs are difficult to access. Identification of disconnection forces is discussed in [11].

Substructure decoupling with configuration dependent coupling conditions is a very challenging task, that can lead to interesting results for instance when only the coupling conditions are configuration dependent, while the subsystems to be coupled are not, as for a lifting crane or a Cartesian robot.

Among the many techniques that can be devised to deal with joint identification, substructure decoupling is—in the authors' opinion—one of the most promising. Typically, the envisaged process involves subsequent steps of substructure coupling and decoupling: model mixing techniques [14, 19] can be used to merge experimental measurements and theoretical models, and nonlinear joints can be identified by experimentally varying the excitation level [13].

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Bluff-Body Aerodynamics: Research Challenges from Wind Engineering

Guido Buresti and Giuseppe Piccardo

Abstract The evaluation of wind-induced loads poses challenging problems of bluff-body aerodynamics because, from an aerodynamic point of view, almost all civil structures are bluff bodies. In order to devise physically sound design procedures, considerable research is still necessary as regards many important topics, a few of which are briefly reviewed in the present paper by mainly focusing on recent developments. Vortex shedding response and galloping are first considered, as well as their possible interaction, which is a complex and open issue. The interference between bluff bodies and its ensuing effects on the aerodynamic loads and on the possible aeroelastic phenomena are then considered, pointing out the still unsatisfactory level of the relevant present information. Finally, the problem of the prediction of the loads acting on bodies subjected to accelerating flows, like those that may often occur in thunderstorms, is tackled. In particular, the present available procedures to evaluate acceleration-induced forces are critically described, together with a discussion on their potential importance in wind engineering.

Keywords Bluff-body aerodynamics · Wind engineering · Vortex shedding · Galloping · Interference · Accelerating flows

1 Introduction

When considering the relative motion between a fluid and a body, and the consequent loads caused by this motion, one has first to analyse the differences between aerodynamic and bluff bodies. We may then say that aerodynamic bodies are those that are characterized by the presence of completely attached thin boundary layers over their whole surface, and by thin and generally steady vorticity-containing wakes

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trailing downstream. Conversely, bluff bodies display a more or less premature separation of the boundary layer from their surface and wakes having significant lateral dimensions and generally unsteady velocity fields (see, e.g., [6]).

These different flow configurations have fundamental consequences as regards the available methods to predict the aerodynamic loads. Indeed, for an aerodynamic body the flow field and the related loads may be analysed and predicted using a simplified iterative procedure, which implies consecutive solutions of the outer irrotational (and thus potential) flow and of the boundary layer equations. This procedure, which has largely been adopted by aeronautical engineers to develop useful predictions without having to solve the full non-linear Navier–Stokes equations, may now be carried out rapidly and efficiently by using the present available computational resources. In fact, one might say that it is a fortunate circumstance that aeronautical engineers have to design bodies that must produce high lift with a small drag penalty, which are exactly those for which the simplified procedure may be applied.

The same good luck is not shared by technicians involved in the prediction of the forces acting on bluff bodies, like civil engineers facing the hard task of predicting both the loads induced by wind on the structures they design and their often disastrous dynamical effects. Indeed, the occurrence of boundary layer separation implies that, in principle, we can no longer apply the above-described simplified procedure. Consequently, resort must be made either to experiments or to the numerical solution of the Navier–Stokes equations. Therefore, the estimate of the loads acting on bluff bodies immersed in a cross-flow is definitely more complex than for aerodynamic bodies. Particularly challenging is also the prediction of the aeroelastic phenomena typical of bluff bodies, such as the response to vortex shedding and the galloping instability.

Due to the complexity of the possible shapes and configurations of civil structures, wind engineers have to cope with difficult problems of bluff-body aerodynamics, for which they must devise adequate design practices even when the present understanding of the physical aspects of those problems is not yet satisfactory. This is usually done through ad-hoc and often heuristic procedures, which allow suitable (and hopefully conservative) predictions to be carried out by using the available information. On the other hand, from the point of view of an aerodynamicist, wind engineering may be considered as an inexhaustible source of fascinating research challenges. This paper aims at providing a concise outline of a few of those challenging topics, limiting the reference list to basic investigations, available reviews and recent works. In particular, after a brief recall, in Sect. 2, of some of the aerodynamical features that are peculiar to wind engineering, in Sect. 3 attention is given to two aeroelastic phenomena typical of bluff bodies, namely response to vortex shedding and galloping, together with their possible interaction. Section 4 deals with the interference between bluff bodies, describing in some detail a particular wake-induced vibration, while Sect. 5 is devoted to the possible consequences of flow acceleration on the aerodynamic forces acting on a bluff body, which is a subject of direct relevance to thunderstorm-induced loads. Some final comments are provided in Sect. 6.

2 Wind Engineering and Bluff-Body Aerodynamics

The birth of wind engineering may be traced back to the late 70s of the last century, when scientists involved in the study of wind and its effects on structures felt the need to broaden their scientific and cultural horizons [31]. According to the definition given by Cermak [8], "Wind engineering is best defined as the rational treatment of the interactions between wind in the atmospheric boundary layer and man and his works on the surface of earth". Therefore, it is intrinsically multi-disciplinary, comprising subjects ranging from aerodynamics to atmospheric physics, from structural mechanics and engineering to environmental and urban sustainability issues. Furthermore, as regards aerodynamics, it is clear that wind engineers face very different problems compared to those concerning aircraft design. Indeed, besides having to deal with bluff bodies, they must consider that the wind of the atmospheric boundary layer is characterized by a non-constant vertical profile and by a turbulence intensity which is generally much higher than the one encountered in aircraft aerodynamics (with values up to 35% near the ground for particularly rough sites). Moreover, increasing importance has been assumed in the last decade by non-stationary events, such as thunderstorms, which typically display unsteady mean value and variance, together with wind profiles showing substantial differences compared to those of synoptic (stationary) winds (see Fig. 1). However, the nose shape typical of thunderstorm profiles is limited to quite short time intervals during the thunderstorm ramp-up and peak stages [7].

The Davenport stationary wind loading chain (e.g., [14]) provides a firm basis for the assessment of wind loads on structures. In order to carry out predictive evaluations of the wind-induced response, some essential assumptions are usually made: (a) small ratio between turbulent and mean wind velocity components, and (b) quasi-steady approach in the evaluation of the forces induced by the wind. The first hypothesis permits to neglect the quadratic (and higher) terms in the instantaneous components

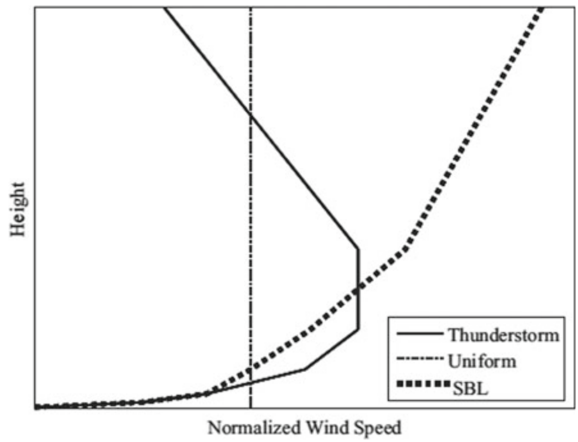

Fig. 1 Generalized vertical profiles for thunderstorm (ramp-up) and stationary boundary layer (SBL) winds (from [17])

of the wind speed and, therefore, to treat the wind actions as linear functions of the turbulence components. The quasi-steady theory assumes that the forces acting on a moving body may be obtained from wind tunnel tests in stationary conditions, which means that they are completely determined by the instantaneous position of the body and the instantaneous relative velocity field. Consequently, any memory effect is neglected and the wind forces acting on an oscillating structure at any instant are equal to those on an identical stationary structure placed in the same relative wind [25].

Accepting the validity of the aforementioned hypotheses, the aerodynamic forces acting on a moving bluff body in turbulent flow (i.e., buffeting and aeroelastic forces) can be expressed by a model derived for the case of a fixed body in uniform flow, substituting the steady mean wind velocity with the instantaneous (function of time) flow-cylinder relative velocity. In this approach, the aerodynamics is assessed statically and the aeroelasticity may be accounted for in a purely analytical way. As we will outline in the following Sections, there are various situations in which this type of approach is definitely questionable.

3 Vortex Shedding Response, Galloping and Their Interaction

The alternate shedding of vortices from bluff bodies is certainly one of the most studied subjects of bluff-body aerodynamics and its main features have been described in several reviews (e.g., [5]). The vortex shedding frequency is given by the relation $f_v = StU/h$, where U is the free-stream velocity, h is the cross-flow dimension of the body and St is the Strouhal number, which is inversely proportional to the ratio between wake width and h. Many slender structures exposed to wind are subjected to the unsteady forces caused by vortex shedding and, especially, to a possible dynamical response when their natural frequencies happen to approach $f_{v.}$ The first important feature of the response to vortex shedding is that it is intrinsically non-linear, because the oscillation of a cylinder at a frequency near f_v may significantly alter the fluid dynamic field around the body, and thus the forces acting on it. Furthermore, when the velocity of the flow is such that f_v is close to the natural frequency f_n of the cylinder, a synchronization between the two frequencies may take place (*lock-in* phenomenon) and the shedding of the vortices occurs exactly at f_n, being captured by the structural oscillation. This happens starting at a value of the reduced velocity $U_r = U/(f_n h)$ near $1/St$ and for a range of velocities whose amplitude is inversely proportional to the Scruton number, $Sc = 4\pi\xi m/\rho h^2$, where ρ is the density of air, ξ is the damping ratio, and m the cylinder mass per unit length (or the equivalent one related to a critical mode). Finally, the amplitude of the response that ensues in this range increases with decreasing Scruton number, but not indefinitely. Indeed, a limit amplitude exists, of the order of the body cross-flow dimension, which is caused by a profound modification of the shedding process, leading to an effective damping of the oscillations even for vanishing values of *Sc*.

Several models were developed to predict the response to vortex shedding; reviews and references may be found in Sarpkaya [27], Williamson and Govardhan [39], Pagnini et al. [23], Tamura [38]. A tentative classification was given by Païdoussis et al. [24], who differentiated between forced, fluid-elastic and coupled system models. In the first, the influence of the response amplitude on the exciting forces is not considered, apart from a possible increase of the correlation of vortex shedding. In the fluid-elastic models the exciting forces are assumed to depend on the response amplitude, and the best values of various parameters are chosen in order to obtain a good agreement with available data. However, there is no attempt to connect the observed trends of the aerodynamic forces to the physics of the flow and, in particular, to the wake and vortex shedding features. Coupled system models try to express this connection by introducing a “wake oscillator”, whose dynamics is described through autonomous differential equations. Nonetheless, parameters are again present that must be assigned through a best fit with experimental results; therefore, also these models might still be considered heuristic or phenomenological and the reasons for their possible suitability are then open to further scrutiny.

Galloping is an instability phenomenon that occurs for certain bluff bodies, like rectangular cylinders, which show a sufficiently negative slope of the C_L—α curve, where C_L is the cross-flow (or lift) aerodynamic coefficient and α is the flow angle of attack. In that case, if the body undergoes a downward motion, so that α is positive, the resultant force may have a component in the direction of motion, as happens in Fig. 2a. This is due to the higher convex curvature of the lower boundary of the wake, with consequent higher velocities and lower pressures in that region compared to the upper wake boundary. The pressures on the lateral faces of the body are not equal to those on the wake boundaries, but they may be expected to be connected to them, and thus a downward force may arise due to the pressure difference between the two lateral faces. Obviously, if the ratio between the sides of the cross-section is such that the lower shear layer may reattach, as shown in Fig. 2b, a higher pressure will be present on the lower face, causing an upward global force.

The analytical prediction of the occurrence of galloping is classically based on the quasi-steady assumption, and the derived necessary condition, in terms of

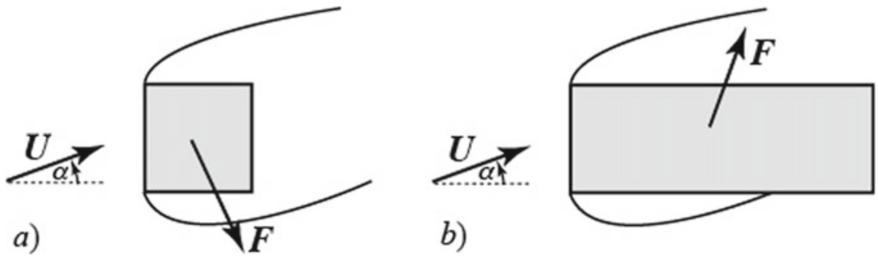

Fig. 2 Flow field around rectangular cylinders

aerodynamic characteristics of the body, is that $\partial C_L/\partial\alpha + C_D < 0$ (Den Hartog criterion), where C_D is the drag coefficient. If this condition is satisfied, the critical velocity U_g at which the global damping of the oscillations becomes negative may be readily obtained and is found to be directly proportional to the Scruton number (e.g., Païdoussis et al. [24]). Galloping may occur with complex features and in multiple degrees of freedom; detailed analyses and generalizations of the quasi-steady approach to describe those conditions may be found in Piccardo et al. [25].

It should be stressed that the quasi-steady assumption is acceptably satisfied only when the frequency of the galloping oscillations is definitely lower than the frequency of vortex shedding, i.e. when $U_g/(f_n h) >> 1/St$. Indeed, in that case there is no significant interaction between galloping and vortex shedding response and the two phenomena may be treated through separate approaches. Conversely, it is possible that the particular geometry of the body and a sufficiently low value of the Scruton number cause this interaction to occur, with a galloping-type response that starts at values of the reduced velocity coinciding with the expected ones for the response to vortex shedding. A significant example is given by the experimental results of Mannini et al. [18] for the case of a rectangular cylinder with a 3:2 ratio between along-wind and cross-flow dimensions. Figure 3 shows, for different values of the Scruton number, the results of their wind tunnel tests in terms of response amplitude as a function of the ratio between the flow velocity and the velocity U_v at the start of the vortex shedding response. As can be see, when Sc is sufficiently low, the typical finite range of velocities for the vortex shedding oscillations disappears, and the response increases with increasing velocity, as is typical of the galloping phenomenon.

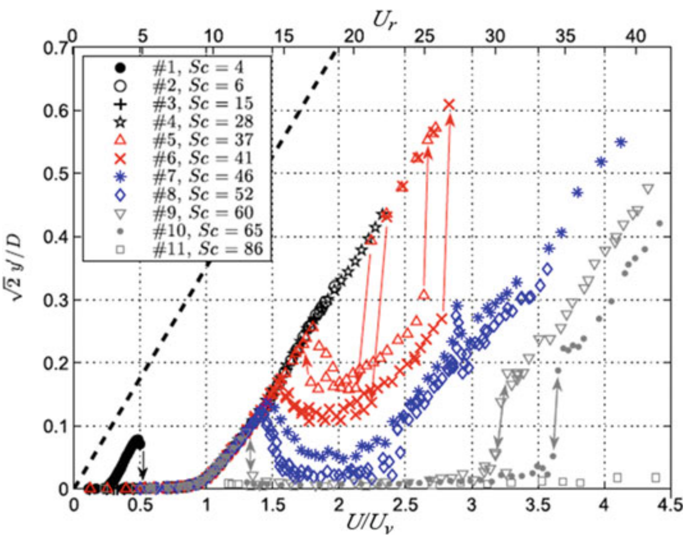

Fig. 3 Response diagrams for a 3:2 rectangular cylinder with various Sc (from [18])

The details of the physics associated with the complex types of behaviour of Fig. 3 are not yet clear, although Mannini et al. [18] provided a detailed description of the phenomenological features of the oscillations in the different cases. A procedure to predict vortex-induced oscillation, galloping and their interaction was developed by Mannini et al. [19] by modifying a previous non-linear wake-oscillator model proposed by Tamura and co-workers (see Tamura [38]). The model introduces an oscillating "equivalent wake lamina" to describe the dynamics of the near wake, and the forces caused by its motion are obtained through relations that derive from the thin airfoil theory. In spite of this questionable assumption, satisfactory predictions of the model are described, provided an adequate choice is done of the values of at least two parameters, which are expected to be a function of the shape of the body cross-section. Recently, Han et al. [10] proposed a method in which a wake-oscillator model is coupled to a quasi-steady approach to describe not only the vortex-induced vibrations and the galloping of a square cylinder but also their interaction. Reasonable results were obtained when the model parameters were derived from appropriate tests. Nonetheless, in the situations in which an interaction occurs, the use of the quasi-steady approach is certainly controversial.

In any case, it is clear that the present physical comprehension of the interaction between vortex-induced oscillations and galloping is still definitely limited. Unfortunately, also the available prediction procedures, having different degrees of empiricism, cannot yet be considered as sufficiently general and widely reliable.

4 Interference Between Bluff Bodies

The problem of the interference between bluff bodies is of great importance for wind engineering applications. Indeed, civil structures are seldom isolated and the presence of other bodies placed upstream or sidewise may cause on a certain body not only different values of the aerodynamic loads, compared to those acting if it were isolated, but also, and often more importantly, the presence of mean and fluctuating load components that would be absent without body interference. These effects are essentially due to the changes produced on the flow field by neighbouring bodies and, in particular, by the geometrical and dynamical features of their wakes.

The numerous and complicated flow configurations that may arise due to body interference are well exemplified in the comprehensive review by Sumner [35], which concerns two infinite circular cylinders in cross-flow. Quite different situations may occur for tandem, side-by-side or staggered arrangements, which are characterized by critical spacings and possible hysteretic behaviour. For the staggered configuration, nine different flow patterns were even identified, with various and complex types of wake interaction and vortex dynamics. The effect of Reynolds number, up to very high values, was also recently analysed for the tandem configuration by Schewe and Jacobs [29] and by Schewe et al. [30].

The interference between square cross-section cylinders, which are perhaps nearer to those typical of high-rise buildings, was also studied, particularly through wind tunnel tests (see, e.g., Du et al. [9], and the references therein). However, finite cylinders, rather than infinite ones, are the rule in wind engineering and the presence of free ends may significantly alter the interference effects and also introduce new vortical structures in the flow [36]. It was indeed found that it may be questionable to extrapolate infinite-cylinder data to finite cylinders, particularly for cylinders of relatively small aspect ratio or when they are closely or moderately spaced. Information on the interference effects of finite square prisms representing prototype buildings is also reported by Sy et al. [37].

An ample discussion on the parameters and features affecting the interaction between adjacent buildings was provided in the review by Khanduri et al. [13], who stressed the importance, as regards the aerodynamic loads acting on bodies in interference, of the type of the upstream terrain, of the wind direction and of the geometry, arrangement and spacing of the involved buildings. The various loads may decrease or increase and, furthermore, new components may arise which are not present when a certain body is isolated, such as cross-flow forces or torsional moments. All these effects should be considered at the design stage for stationary structures, but perhaps even more important are the effects of interference on the possible aeroelastic phenomena. Indeed, not only the response to vortex shedding and galloping may be altered, but new dynamical conditions may also arise.

As a single paradigmatic example, we will concentrate on the wake-induced vibrations (WIV) described by Assi et al. [2] for a pair of circular cylinders with diameter D placed in tandem. Indeed, the downstream cylinder, having natural frequency f_n, was found to be subject to cross-flow oscillations with different characteristics according to the value of the centre-to-centre distance between the cylinders, x_0. In particular, while for large separations the response occurs in a finite range of reduced velocities $U/(f_n D)$, as happens for a usual vortex-shedding response, for lower values of x_0/D the response amplitude either remains constant or keeps increasing with increasing velocity, similarly to what happens for a galloping-type oscillation (see Fig. 4). The latter behaviour, however, cannot be a classical galloping because, if the rear cylinder is displaced transversely away from the centreline, a restoring force appears, which acts to return the cylinder to its original position and suggests stability rather than instability. Assi et al. [2] carried out a careful analysis of the results and concluded suggesting that WIV is essentially caused by an unsteady interaction between the vortices shed from the upstream cylinder and the motion of the oscillating downstream cylinder. A further tentative interpretation of the excitation mechanism was given in Assi et al. [3], while the case of staggered cylinders was considered by Assi [1], finding that WIV decreased for increasing lateral separation. Although all the above tests were carried out at very low Scruton numbers, probably rarely encountered in wind engineering, they demonstrate the possibility of complex aeroelastic phenomena completely dependent on the interference between bluff bodies. Therefore, one might well consider their possible occurrence, perhaps with lower maximum amplitudes, also for multiple light structures subjected to wind.

Further experimental data on the aeroelastic effects caused by the interference between two high-rise buildings with square cross-section were recently reported in Lo et al. [16], where an updated reference list on this subject may also be

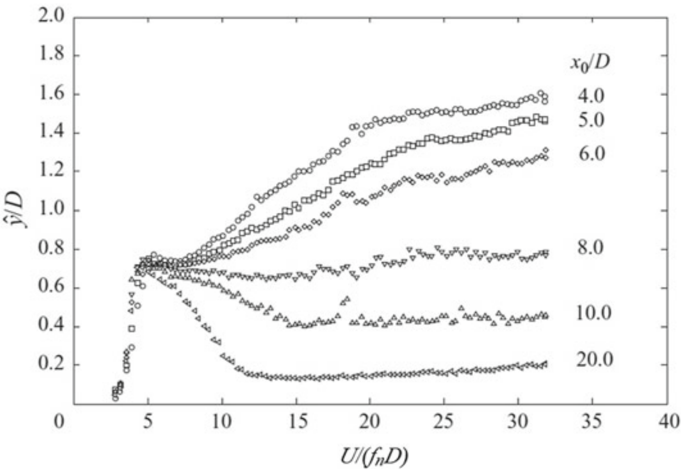

Fig. 4 WIV of downstream cylinder at various x_0/D separations (from [2])

found. Nonetheless, it is quite clear that further investigations are still necessary to reach a level of understanding of the static and dynamic effects of the interference between bluff bodies that may permit to face the many complex geometrical and flow configurations that may be found in wind engineering.

5 Thunderstorms and Accelerating Flows

The prediction of the aerodynamic loads in thunderstorms poses new and complex challenges to the designer because of the significant differences compared to the case of synoptic winds (see Solari [32], Solari et al. [33, 34]). One of the peculiar features of thunderstorm winds is the possible presence of significant accelerations, as can be seen in Fig. 5, in which two examples of the velocity records reported in Zhang et al. [42] are shown (the black lines represent time-moving averages). As reported also by Brusco [4], maximum absolute values of the acceleration of the order of 1 m/s^2 may be found in thunderstorms. The objective of predicting the forces in those conditions is far from easy, because the aerodynamics of bluff bodies in accelerating flows is, to say the least, a still definitely open research subject. In particular, the difficulty lies in the intrinsic fluid dynamical complexity of the topic and in the scarcity of available experimental or numerical data, especially for the flow conditions relevant to wind engineering.

Generally, the along-wind forces acting on a fixed body immersed in a unidirectional unsteady wind $U(t)$ are expressed by means of the Morison equation [21, 22], which is here reported in a form in which the sign of the velocity is assumed not to vary:

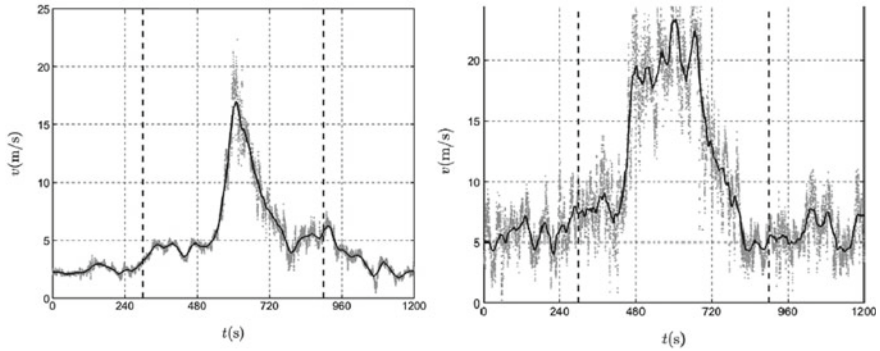

Fig. 5 Two examples of thunderstorm velocity records (from [42])

$$F_x(t) = \frac{1}{2}\rho A U^2 C_D + \rho V \frac{\partial U}{\partial t} + \rho V_r C_a \frac{\partial U}{\partial t}, \quad (1)$$

where A is a reference surface, C_a is the added mass coefficient, V is the body volume and V_r is the reference volume used for the definition of the added mass coefficient.

The two terms proportional to the acceleration are, respectively, the Froude-Krylov and the added mass forces. The former derives from the pressure gradient that is present in an accelerating flow and which acts on each volume of the fluid. Therefore, the same pressure field would act over the surface of any solid body present in the flow and produce a force that is directly proportional to the acceleration and to the displaced mass of fluid. Consequently, the Froude-Krylov force is zero only if the volume of the body is negligible. Conversely, the added mass force acts on a body due to the variation of the global kinetic energy of the flow caused by the change of the relative velocity between body and fluid; therefore, the added mass is connected with the relative acceleration. The reference volume appearing in the added mass term does not necessarily coincide with the body volume. Indeed, the added mass does not vanish when the volume of the body is zero; thus, in that case a finite arbitrary reference volume may be chosen and this choice obviously determines the value of the added mass coefficient.

As an example, we may assume the body to be a cylinder with finite cross-section S, length L and cross-flow dimension h, so that the reference area and volume may be taken to be $A = Lh$ and $V_r = V = LS$. If we further introduce the so-called inertia coefficient $C_m = 1 + C_a$, we may easily recast relation (1) as follows:

$$F_x(t) = \frac{1}{2}\rho L h U^2 C_D \left(1 + 2\frac{C_m}{C_D}\frac{S}{h}\frac{\partial U/\partial t}{U^2}\right). \quad (2)$$

The second term inside the parenthesis gives the relative importance of the force directly dependent on the acceleration compared to a drag force term having the same form as the one acting in steady flow with equal instantaneous velocity. Considering the usual wind velocities and accelerations and the fact that the ratio C_m/C_D may be expected to be of unitary order of magnitude, it is apparent that this term may generally be significant only if the body is elongated in the flow direction, i.e. if the along-wind dimension $l = S/h$ is large.

Several comments are now appropriate on the reliability of relations (1) or (2). Indeed, the validity of the Morison equation, which was originally introduced to tentatively describe the forces on piles immersed in waves, is open to discussion (see, e.g., [26]). Apart from the assumed formal dependence of the force on the two terms, respectively linked to the square velocity and to the acceleration of the fluid, the main questionable point concerns the values that should be given to the drag and added mass coefficients. In particular, there is no reason to assume, in principle, that these coefficients be independent of time and of the flow parameters. In effect, the forces acting on a body in cross-flow are strictly dependent on the structure of the wake and, in particular, on the amount, configuration and dynamics of the vorticity shed from the body, which also carries the memory of the flow. In other words, there is a profound difference, for instance, between a body immersed in a purely oscillatory flow and one in a constant flow which undergoes rapid velocity variations of different sign. This may be easily understood by referring to a cylinder with a regular shedding of vortices in its wake. In a cyclic oscillatory flow the direction of the velocity changes continuously, causing a complex dynamics of the vortices shed in the previous half cycle, which depends on the ratio between the periods of flow oscillation and of vortex shedding. Conversely, when a body is immersed in a unidirectional flow with a velocity that may experience a sudden acceleration or deceleration, as shown in the thunderstorm records of Fig. 5, the vortices shed in the wake are not expected to have a retrograde motion, even if their intensity, layout and shedding frequency will depend on the variations of velocity and acceleration.

Unfortunately, most of the available data on the coefficients to be introduced in the Morison equation refer only to purely sinusoidal oscillatory flows, with application to wave-induced forces (e.g. [26, 28]), and it is then definitely problematic to extrapolate them to wind engineering applications. Furthermore, in a transient flow there is no sound foundation for the common practice of using the values of the drag coefficients obtained in wind tunnel tests in steady flow and of the added mass coefficients deriving from the potential flow theory, even if there may be reasons to expect that, in most cases, this procedure presumably leads to conservative design predictions. It is then advisable that ad hoc investigations be carried out with realistic unsteady flow conditions, not only to devise sensible design procedures but, especially, to increase the present physical understanding of the role of the fluid velocity variations on the wake configuration and, consequently, on the fluid dynamic forces. Thanks also to the appearance of wind tunnel facilities capable of producing significant values of flow acceleration, a certain number of indications on the variation of the forces and of the vortex shedding features in those conditions are now indeed starting to be available (see [4], and the references therein).

6 Final Comments

The task of an engineer is to devise adequate design procedures, irrespective of their level of connection with real fluid dynamical features. Therefore, heuristic or phenomenological models may certainly be appropriate, provided they are used within their ranges of application and accurately verifying the suitability of their assumptions for the considered cases. However, this may not always be easy to accomplish in wind engineering, considering the extremely high number of different configurations which may arise as regards shape and arrangements of the concerned civil structures. Thus, it may be problematic to extrapolate the available knowledge deriving from existing data or models and it may be an extremely demanding effort to devise experimental or numerical campaigns to obtain the necessary design data. On this respect, a significant help may derive from new techniques of data analysis founded on AI-based schemes, such as machine learning. Indeed, these techniques may significantly reduce the number of tests that are necessary to predict, for instance, the value of the loads that may act on a body in interference conditions (e.g., [11], [12]) or the cross-flow oscillations of a cylinder [15]. Another important point is that when applying a certain design procedure, which may often be a complex one, it is a usual occurrence that certain input data, such as the aerodynamic load coefficients, be not known exactly. In that case, techniques of uncertainty quantification (e.g., [20]) may be extremely useful, and may permit to determine the effects on the predicted outputs of the uncertainty in the various parameters.

On the other hand, the task of an aerodynamicist is to pursue an increased understanding of the involved physical phenomena, in order not only to try to develop new and more realistic design procedures, but also to attain a deeper insight on the complex flow configurations that characterize separated flows. Thus, a researcher will be mainly interested in the reasons why certain loads are generated or particular flow features appear. To achieve this objective, investigations should not be seen as just a collection of data on the variation of the loads as a function of the many parameters affecting the flow. Conversely, all possible efforts should be made to connect the observed trends to the characteristics of the aerodynamic field. To this end, an invaluable help may derive from numerical simulation, which may provide a description of the complete flow field, including quantities, like vorticity, which may hardly be obtained from experiments. Thus, the synergy between numerical simulations and wind tunnel tests may probably be the most promising source of physical understanding. Furthermore, clues on the origin of flow features and aeroelastic phenomena may derive from a close scrutiny of the possible reasons why a certain model produces good predictions. Indeed, the fact that a simplified model works, while perhaps in principle it should not, may suggest that it is taking some basic physical mechanism into account, even if not necessarily in the way the researchers who produced it may have thought. Finally, any explanation on the origin of the loads that a particular flow pattern may produce on a body must always be in agreement with consolidated theoretical findings. For instance, the principle of total vorticity conservation and the connection between fluid dynamic loads and time variation of the moments of vorticity (e.g. [6, 40, 41]) should always be satisfied and might also be a guide to develop new physical interpretations.

Dedication

The present work is dedicated to the memory of Giovanni Solari, an eminent scientist, a special person, a dear friend.

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Stability Analyses to Predict Tidal Sedimentary Patterns

Paolo Blondeaux and Giovanna Vittori

Abstract The results on the prediction of (i) the conditions leading to the appearance of periodic sedimentary patterns forced by tidal currents on the sea bottom and (ii) the geometrical characteristics of these bottom forms are summarized. Attention is focused on the results obtained by means of stability analyses, even though a few results obtained by means of numerical simulations are also mentioned. The characteristics of ripples, dunes, sand waves, long bed waves and sand banks are described along with the mechanisms that originate these different sedimentary patterns.

Keywords Tidal currents · Sea bottom · Sedimentary patterns

1 Introduction

Many sedimentary patterns (bedforms), which can be observed on the continental shelves or on a river bed, are repetitive both in space and time and it is possible to associate to them a characteristic wavelength L^* and a characteristic period T^* (hereinafter a star denotes a dimensional quantity). The reader who is interested in a description of the main characteristics of the bedforms observed on the sea bottom or on a river bed can read the monograph of Blondeaux et al. [6].

Nowadays, it is widely accepted that the appearance of periodic bedforms is due to the instability of the plane interface between the sea/river bottom, made up of cohesionless sediments, and the water flowing over it. Different physical mechanisms are commonly proposed to explain the appearance of the different sedimentary patterns even though, in a recent contribution, Vittori and Blondeaux [33] discussed the theories of formation of river dunes and tidal sand waves and showed that the different physical mechanisms proposed to explain their formation show more similarities than it was believed. Certainly, because of the different length scales that character-

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ize the different bedforms, different approaches are used to investigate the processes that lead to the formation of ripples, megaripples, dunes, bars, sand waves, long bed waves and sand banks.

In the present contribution, we focus our attention on the bedforms that are generated on the continental shelves by the currents forced by tide propagation. Even narrowing the focus on the bedforms generated by tidal currents, the topic is extremely vast and it is not possible to describe in detail the results available in the literature. Consequently, we will limit ourselves to a brief description of the approaches used to study tidal bedforms and a summary of the main results, providing some references, which can be consulted by the reader who wants to deepen the subject.

As already pointed out, the appearance of periodic bedforms is assumed to be the result of the instability of the flat configuration of the sea bottom above which the water flows and induces the sediment motion. The stability of the flat bottom is investigated by determining the time development of a random perturbation of the flat bed. The growth/decay of the bottom perturbation is determined by assuming that its initial amplitude is ‘small’ and by linearizing the morphodynamic problem. Therefore, it is possible to consider separately the different components of the bottom waviness

$$z^* = \eta^* \left(x^*, y^*, z^*, t^*\right) = \frac{h_0^*}{2}\epsilon A(t^*) \exp\left[i\left(\alpha_x^* x^* + \alpha_y^* y^*\right)\right] + \text{c.c.}. \qquad (1)$$

In (1), (x^*, y^*, z^*) is a Cartesian coordinate system, with the x^*- and y^*-axes lying on the average bottom and the z^*-axis being orthogonal to it and pointing upwards, and h_0^* is the mean water depth. The parameter ϵ is a measure of the ratio between the amplitude of the bottom perturbation and the local water depth and is assumed to be much smaller than one ($\epsilon \ll 1$). Moreover α_x^* and α_y^* are the wavenumbers of the generic component of the bottom waviness and $A(t^*)$, which is assumed to be of order 1, describes the time development of its amplitude (c.c. indicates the complex conjugate of the previous term). The growth of the most unstable component, if any, is assumed to generate the observed bottom forms.

Of course, the bottom waviness induces perturbations of (i) the flow field, (ii) the bottom shear stress and (iii) the (volumetric) sediment transport rate. To determine the time development of the perturbation of the sea bottom, which is supposed to be made of cohesionless sediments of uniform size d^* and density ρ_s^*, it is necessary to determine the flow generated by a tidal current over a wavy bottom, which is assumed to be fixed. Indeed, the time scale T_M^* of the evolution of the bottom perturbation (the morphodynamic time scale) is much longer than the hydrodynamic time scale T_H^*. The different spatial and temporal scales, which characterize the different bedforms, imply the use of different approaches to solve the hydrodynamic problem. In particular, to study the hydrodynamic phenomena that take place in large areas of the Earth surface but characterized by a length scale significantly smaller than the Earth radius, it is appropriate to introduce the f-plane approximation (see for example [21]) and to consider momentum equations where the Coriolis contributions related to the Earth rotation are taken into account.

If the problem is written in dimensionless form using the water depth h_0^*, the inverse of the angular frequency ω^* and the amplitude U_0^* of the velocity oscillations induced by tide propagation as length, time and velocity scales, respectively, four dimensionless parameters appear, which are $\hat{r} = U_0^* / \left(\omega^* h_0^*\right)$, $\hat{\delta} = \sqrt{\nu_{T0}^*/\omega^*}/h_0^*$, $\hat{Fr} = U_0^*/\sqrt{g^* h_0^*}$, $\hat{\Omega} = \Omega^*/\omega^*$. The parameter $\hat{r}$ is the ratio between the amplitude of fluid displacement oscillations in the horizontal direction and the local water depth and it turns out to be order 10^2. The parameter $\hat{\delta}$ is the ratio between the order of magnitude of the thickness of the bottom boundary layer generated by the tidal wave and the local depth and, in shallow see, it turns out to be of order one. The Froude number $\hat{Fr}$ can be written as $\left(U_0^*/\omega\right)\left(\omega/\sqrt{g^* h_0^*}\right)$ and can be thought as the ratio between the amplitude of fluid displacement oscillations in the horizontal direction and the order of magnitude of length of the tidal wave. Finally, $\hat{\Omega}$ is the ratio between the angular velocity of the Earth rotation Ω^* and the angular frequency of the tidal wave ($\hat{\Omega} \cong 0.5$ for a semidiurnal tide, $\hat{\Omega} \cong 1$ for a diurnal tide). Often, the problem can be simplified by assuming that the Froude number $\hat{Fr}$ is much smaller than one. Indeed, the assumption $\hat{Fr} \ll 1$ allows the rigid lid approximation to be introduced.

The complete equations are often simplified when large scale bedforms are considered (e.g. long bed waves and sand banks). In this case, the water depth turns out to be much smaller than the wavelength of the bedforms and the shallow water approximation can be introduced. Hence, the depth averaged values of the velocity components can be considered and different dimensionless variables are introduced.

Once the flow field is determined, it is necessary to evaluate the sediment flux. The sediment flux due to the sediments that move close to the bed, rolling, sliding and saltating over the resting bed (bed load) is usually evaluated by means of empirical formulae. The sediment flux due to the sediments that move far from the bottom being trapped within the large scale turbulent vortices (suspended load) is usually evaluated by solving an advection-diffusion equation for the sediment concentration.

Then, mass conservation of the sediments (Exner equation) implies that a spatial increase of the sediment transport rate implies a decrease of the bed elevation and vice versa. This mass balance equation gives rise to an amplitude equation for A, which can be written in the form

$$\frac{dA}{dt^*} = \Gamma^* A \tag{2}$$

where $\Gamma^* = \Gamma_r^* + i\Gamma_i^*$ is a complex quantity which depends on both the hydrodynamic and morphodynamic parameters of the problem.

When it is reasonable to consider a steady forcing flow, the value of Γ^* is independent of time and the amplitude equation (2) can be easily integrated. Then, it is straightforward to discriminate between growing ($\Gamma_r^* > 0$) and decaying ($\Gamma_r^* < 0$) components of the bottom perturbation. Moreover, it is possible to determine the orientation and wavelength of the periodic bedforms predicted by the analysis.

However, tide propagation gives rise to a time dependent (periodic) flow and Γ^* turns out to be a periodic function of time. Therefore, different contributions to the time development of $A(t^*)$ can be identified. The most important contributions are related to the real and imaginary parts of the time average $\overline{\Gamma}^*$ of Γ^* defined by $\overline{\Gamma}^* = \int_0^{T_0^*} \Gamma(\hat{t}^*)d\hat{t}^*$, T_0^* being the period of the tide. The decay/amplification of the bottom waviness is controlled by the value of $\overline{\Gamma}^*{}_r$ while $\overline{\Gamma}^*{}_i$ controls the migration of the bottom forms. The further contributions to $A(t^*)$, which are related to $\Gamma^* - \overline{\Gamma}^*$, describe the oscillations (during the tidal period) of the bottom profile around its averaged position. Such oscillations are small and, in general, are not considered.

2 An Overview of the Results of the Stability Analyses

The sedimentary patterns generated by tidal currents are characterized by spatial and temporal scales that range from centimetres to kilometres and from minutes to centuries. Hence, the theoretical models used to investigate the process which leads to the appearance of the different tidal bedforms are based on different approaches. Therefore, in the following, we consider separately the formation of (i) ripples and dunes, (ii) sand waves, (iii) long bed waves and (iv) sand banks, even though the different bedforms may coexist and interact. Since ripples and dunes are characterized by values of the morphodynamics time scale T_M^* that are smaller than the period T_0^* of the tide, they can be assumed to be generated by a steady (slowly variable) current and there is no need to account for the oscillatory character of the tidal current. On the other hand, sand waves, long bed waves and sand banks should be studied taking into account the periodic character of the tidal currents, since the value of T_M^* of these bedforms is larger than T_0^*.

2.1 *Ripples and Megaripples/Dunes*

Both ripples and megaripples/dunes are small scale undulations of the sea bottom. The first bedforms are characterized by wavelength of the order of tens of centimetres whereas the second bedforms have typical wavelength of the order of metres. As already pointed out, it is reasonable to investigate the process that leads to the formation of the ripples and dunes generated by tidal currents by considering a sequence of steady flows as in [11, 13]. Indeed, the characteristics of the small scale bedforms generated by tidal currents do not significantly differ from those observed along a river course (see Fig. 1).

The hydrodynamic and morphodynamic problems can be solved by expanding any unknown function as a power series of ϵ. This procedure leads to a sequence of problems at the different orders of approximation. At the leading order ($O(\epsilon^0)$), the velocity profile is described by the well known logarithmic law for both the hydraulically rough and smooth regimes. At O(ϵ), an eigenvalue problem was obtained by Colombini and Stocchino [13], who solved it by forcing an eigenre-

Fig. 1 Ripples on the bottom of the Hunter river (New South Wales, Australia). Current direction is from the top-right to the bottom-left. Photo courtesy of Michael C. Rygel

lation which provides the value of Γ^*. Colombini and Stocchino [13] obtained Γ^* as a function of the ratio between the Shields parameter and its critical value and the dimensionless wavenumber α, for different values of the sediment Reynolds number $R_p = \sqrt{(\rho_s^*/\rho^* - 1)g^* d^{*3}}/\nu^*$, ρ_s^* and ρ^* being the densities of the sediments and of the water, respectively, g^* the acceleration of gravity, d^* the sediment size and ν^* the kinematic viscosity of the water. Their results, supported by a comparison with the experimental observations described in [16], show that for large values of R_p (e.g. $R_p = 20$), only one unstable region appears for values of α typical of dunes, i.e. α of order 1. For smaller values of R_p ($R_p = 14$), a second unstable region appears for α of order 10. This range of wavenumbers can be related to ripples. When the value of the sediment Reynolds number is further decreased, the two unstable regions merge but two different relative maxima of the growth rate can still be identified down to a value of R_p equal to 10 (see Fig. 2). Then, a further descrease of R_p leads to a single maximum. Of course, the linear approach described in [13] cannot predict if one mode would eventually prevail over the other. However, it is worth pointing out that experimental observations [16] show that ripples and dunes may coexist. Therefore, the results of the analyses described in [11, 13] suggest that ripples and dunes are two different modes that can be excited individually or simultaneously. On the other hand, Fourriere et al. [15] proposed a model that shows that the most unstable wavelength of the bottom perturbations, forced by a uniform steady current, corresponds to ripples. According to Fourriere et al. [15] dunes are the result of a nonlinear process (ripple coarsening). In any case, the physical mechanism which drives the growth of ripples and dunes is the phase shift between the perturbation of the bottom profile and the perturbation of the bed shear stress.

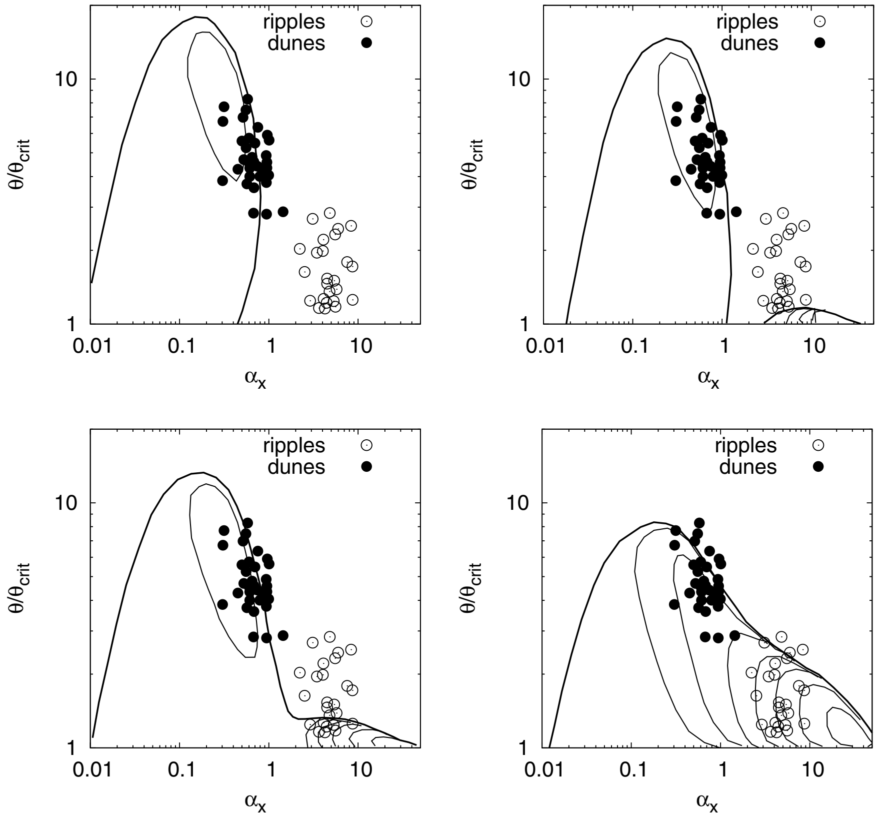

Fig. 2 Positive growth rate in the $(\alpha_x, \theta/\theta_{crit})$-plane. The thick line indicates the isoline characterized by vanishing values of Γ_r. (Top left panel), $R_p = 20$; (top right panel), $R_p = 14$; (bottom left panel), $R_p = 12$; (bottom right panel), $R_p = 5$. The dots indicate the experimental data of Guy, Simons and Richardson (1966), 'Summary of alluvial channel data from flume experiments, 1956–61', US Government Printing Office

2.2 Sand Waves

Sand waves are large scale bottom undulations characterized by wavelengths of the order of hundreds of metres with crests almost orthogonal to the direction of the tidal current. One of the first contributions to the study of the mechanism that leads to the appearance of sand waves was given by Hulscher [17], who showed that the appearance of these bedforms is driven by the presence of steady recirculating cells generated by the interaction of the oscillatory tidal current with the bottom waviness. Hulscher [17] quantified turbulence mixing by introducing an eddy viscosity that is assumed to be constant over the water depth. Since the turbulence mixing tends to vanish close to a rigid wall, the model by Hulscher [17] neglects the bottom layer and introduces a partial slip condition of the fluid at the bottom. A more accurate model is that of Besio et al. [5], who used the approach proposed by Blondeaux and Vittori [8, 9] to evaluate the flow field. In particular, the turbulence generated by tidal currents was modelled by introducing an eddy viscosity that grows linearly with the distance from the bottom in the region close to the sea bottom, it reaches a maximum and it decreases and assumes small values close to the free surface. Moreover, since the analysis of Besio et al. [5] was aimed at studying both sand waves and sand banks and the dynamics of the latter bedforms is affected by Coriolis terms and the local time derivatives of the velocity components, Besio et al. [5] considered the contribution of these terms to the hydrodynamic problem.

Even though it is possible to consider tidal currents generated by the superposition of two or more tidal constituents (e.g. semidiurnal and diurnal constituents), Besio et al. [5] assumed that the solution at the leading order of approximation was dominated by just one tidal constituent, as made by Blondeaux et al. [10]. The solution of the hydrodynamic problem was obtained by expanding any unknown as a power series of ϵ. When terms of order ϵ of continuity and momentum equations are considered, partial differential linear equations for the flow and free surface perturbations are obtained. Since sediment continuity equation states that $dA(t^*)/dt^*$ is proportional to $A(t^*)$ through the ratio between the hydrodynamic and the morphodynamic time scales and this ratio turns out to be much smaller than one, the terms of the momentum equations that are proportional to the time derivative of $A(t^*)$ can be neglected with respect to the terms proportional to $A(t^*)$. Moreover, the periodicity of the basic flow suggests to write the unknown functions as Fourier series of time. By plugging the Fourier series into the hydrodynamic problem, a linear system of coupled ordinary differential equations can be derived, where the unknown functions depend only on the vertical coordinate. The solution can be numerically obtained with the procedure used by Vittori [31]. At last the equation, which provides the time development of the amplitude of the bottom perturbation, follows from the empirical predictors used to evalute the sediment transport rate and sediment continuity equation.

The results show that the bedforms, which tend to appear, are characterized by crests that are orthogonal to the major axis of the tidal ellipse. Indeed, the maximum value of the amplification rate is attained for vanishing values of $\alpha_{\hat{y}}$, $\hat{x}$ and $\hat{y}$ being two horizontal axes such that $\hat{x}$ is aligned with the major axis of the tidal ellipse. This theoretical finding agrees with field observations (see [1, 29]). Also the wavelength of the fastest growing mode predicted by the stability analysis agrees with the wavelength of the observed bottom forms. Moreover, since the velocity oscillations considered by Hulscher [17] and Besio et al. [5] are symmetric, the sand waves predicted by their stability analyses do not migrate. Later, Nemeth et al. [26] made a linear analysis similar to that of Hulscher [17] but they added a residual (steady) current to the main oscillatory tidal current. Hence, Nemeth et al. [26] found that the bedforms predicted by their analysis migrate in the direction of the steady current. However, field observations exist that show that sand waves can migrate also against the residual current. The migration of the bedforms in the direction opposite to that of the steady velocity component is possible because of the nonlinear relationship between the sediment trasport rate and the current velocity and it was investigated by Besio et al. [2, 3], who considered a tidal current generated by the superposition of the M2, M4 and Z0 constituents, which are supposed to be the dominant ones. The first two constituents induce oscillations of the tidal current characterized by periods of about 12 and 6 h, respectively, while the Z0 constituent is characterized by a steady velocity component.

Up to now, only tidal currents which change on a temporal scale of the order of hours have been considered. However, the interaction between the lunar constituents with the solar constituents makes the tidal current to change also on much longer temporal scales, e.g. the temporal scale of the neap-spring cycle. As discussed in [10], the changes of the velocity field induced by a slow modulation of the tidal current can be determined, along with the time development of the bottom profile, if the presence of three distinct temporal scales is recognized. The first temporal scale is the tidal period equal to $T_0^* = 2\pi/\omega^*$, which is of the order of hours. The second temporal scale is the period of the tide modulation equal to $T_1^* = 2\pi/\Omega_{sn}^* \cong 15$ days, Ω_{sn}^* being the angular frequency of the spring-neap cycle. The third and last temporal scale is the morphodynamic time scale $T_2^* = (1 - p_{or})h_0^{*2}/[d^*\sqrt{(\rho_s^*/\rho^* - 1)g^*d^*}]$, which depends on the local water depth and sediment characteristics (p_{or} is the porosity of the sea bottom). Considering the typical values of these temporal scales, it appears reasonable to assume $T_0^* \ll T_1^* \ll T_2^*$ and to introduce the small parameters $\delta_1 = T_0^*/T_1^*$ and $\delta_2 = T_0^*/T_2^*$. Then, using a multiple scale approach [25], the following time variables, $t_0 = t = \omega^* t^*$, $t_1 = t^*/T_1^*$, $t_2 = t^*/T_2^*$ can be introduced and the unknown functions, which appear at the different orders of approximation ($O(\epsilon^0)$, $O(\epsilon)$, ...), are assumed to depend on t_0, t_1 and t_2 beside on the spatial coordinates. Sediment continuity equation suggests that A can be decomposed into a part of order one, which depends only on t_2, and a further contribution, which depends on t_0, t_1 and t_2 but it is much smaller than one: $A = A_0(t_2)\,[1 + \delta_2 A_1(t_0, t_1, t_2)]$. The function $A_0(t_2)$ describes the growth/decay of the bottom perturbation, which takes place on the morphodynamic time scale, whereas $A_1(t_0, t_1, \ldots)$ describes the oscillations of the bottom configuration, which take place during the tidal cycle and the spring-neap cycle. However, because of the small values of δ_2, the bottom oscillations described by A_1 are of small amplitude and can be neglected. Since significant bottom changes take place on the morphodynamic time scale t_2 only, the velocity and pressure fields can be assumed to respond instantaneously to the variations of the bottom configuration. Then, the equation that provides the time development of the amplitude of the bottom perturbation follows from sediment continuity equation. A comparison between the model findings and the characteristics of the sand waves observed in the North Sea was made by considering the data collected by Menninga [24]. A fair agreement among the theoretical predictions and the field observations was found. In particular, the analysis was able to predict the migration of the bedforms which is in the direction of the residual current or against it, depending on the strength of the different tidal constituents.

2.3 Long Bed Waves

Long bed waves are periodic bottom forms characterized by crest-to-crest distances, which are larger than those of sand waves and smaller than those of sand banks and, more importantly, by crests which form an angle with the direction of the tidal current, which ranges from about $-65°$ to about $35°$. Moreover, these bedforms are characterized by a morphodynamic time scale that is much longer than the period of the tide and significant changes of the bottom configuration take place only after a large number of tidal cycles. Therefore, it is convenient to introduce the two different time variables t_0 and t_2 already defined. Of course, the oscillations of the bottom profile during the tidal cycle have negligible effects and they can be safely neglected to determine the long term behaviour of the bottom configuration. The equation that provides the time development of the amplitude $A(t_2)$ of the generic component of the bottom perturbation follows from sediment continuity equation and it allows to discriminate the perturbation components which grow from those which decay and to evaluate the component characterized by the fastest growth (see Blondeaux et al. [7]). The model of Blondeaux et al. [7] was tested against the field data described in [20] and fair results were obtained. Indeed, different relative maxima of $\overline{\Gamma}_r$ were identified and the predicted wavelengths well agree with the values of the different bedforms observed in the field. The presence of multiple relative maxima and the emergence of long bed waves appear to be related to the relative weakness of the tidal current, the ellipticity of the tidal current and the anisotropy of the slope-induced sediment transport. Indeed, for the same values of the parameters, but for stronger tidal velocities and/or unidirectional tidal currents and/or isotropic slope-induced sediment transport, the amplification rate Γ_r^* has only one significant maximum that corresponds to sand banks.

2.4 Sand Banks

As described by Dyer and Huntley [14], sand banks are large scale bottom forms that are characterized by crests that are almost aligned with the major axis of the tidal ellipse and by a crest-to-crest distance of the order of 10 km (see Fig. 3). To predict the appearance of sand banks and their geometrical characteristics two approaches are used. The first approach is based on the shallow water approximation whereas the second approach considers the full three-dimensional equations. First, we discuss the results that are obtained by means of the three-dimensional approach [5]. When the values of $\overline{\Gamma}_r^*$ are plotted versus the wavenumbers $(\alpha_{\hat{x}}, \alpha_{\hat{y}})$, for values of the hydrodynamic and morphodynamic parameters chosen to reproduce the site in the North Sea described in [22, 23], two maxima appear. One maximum describes sand waves and the other maximum describes sand banks. Indeed, field data collected along the Calais-Dover strait show that both sand waves and sand banks are observed in the site. The wavelength of the observed sand waves, which have crests practically

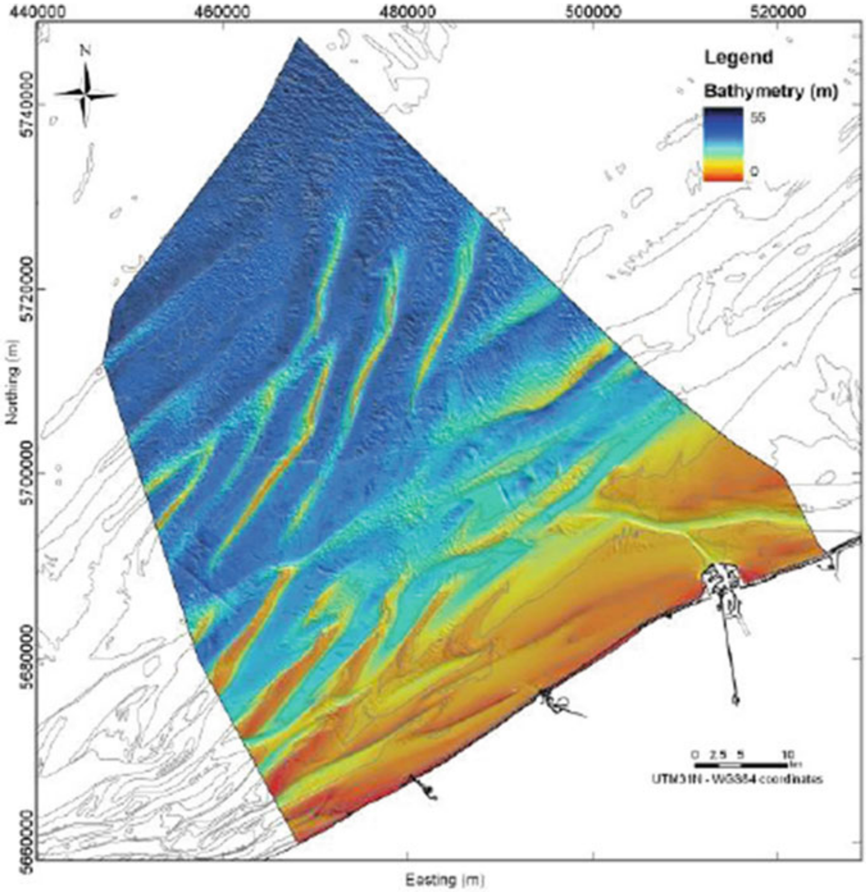

Fig. 3 The digital elevation model of the Belgian continental shelf based on single beam measurements (courtesy of Vera van Lancker)

orthogonal to the main tidal current, falls between 350 and 750 m and the sand banks are almost aligned with the main axis of the tidal ellipse but slightly clockwise rotated and are characterized by an average crest-to-crest distance of about 8 km. One of the maxima predicted by the model of Besio et al. [5] takes place for $\alpha_{\hat{x}} \simeq 0.38$ and for a vanishing value of $\alpha_{\hat{y}}$. The second maximum takes place for $\alpha_{\hat{x}} = 0.013$ and $\alpha_{\hat{y}} = 0.03$. Hence, the stability analysis predicts the appearance of two different bedforms. The first bedform has crests orthogonal to the direction of the tidal current and its wavelength is about 355 m. The second bedform has crests slightly clockwise rotated with respect to the major axis of the tidal ellipse and its wavelength is about 5.5 Km. It is easy to associate the first mode to the sand waves observed by Le Bot et al. [22, 23] and the second mode to the sand banks that are present nearby (Sandettie bank, Ruytingen bank, Dyck bank).

Once it is verified that the three-dimensional approach provides reliable predictions of the tidal bedforms, let us look at the results provided by the shallow water approach. Hulscher et al. [18] were among the first investigators to study the process that leads to the appearance of sand banks by using the shallow water approximation. For a unidirectional tide, the fastest growing mode predicted by Hulscher et al. [18] is characterized by crests almost aligned with the main axis of the tidal ellipse and the wavelength of the predicted bedforms is in agreement with the field observations. However, the analysis of Hulscher et al. [18] shows that the first mode that becomes unstable, when the parameters are varied, corresponds to an ultra-long bottom form. Better predictions were found by Hulscher et al. [18] by considering circular tides. Indeed, the first mode to become unstable, when the parameters are varied, is characterized by a finite wavelength but, since a circular tide has no preferred direction, no preferred orientation of the selected bedforms was provided by the theoretical analysis of Hulscher et al. [18].

Since a depth-averaged approach cannot predict the deviation of the sediment transport rate from the direction of the depth averaged velocity, Besio et al. [4] developed a model based on the shallow water approximation but they added a correction to the sediment transport direction to account for this three-dimensional effect. Moreover, the solution procedure used by Besio et al. [4] takes into account the cascade process that gives rise to a large number of harmonic components in the velocity field, that are generated by the interaction of the oscillatory tidal current with the bottom waviness. Finally, the approach proposed by Besio et al. [4] allows to consider unidirectional as well as elliptical and circular tides. When the real part $\overline{\Gamma}_r^*$ of the averaged amplification rate $\overline{\Gamma}^*$ is plotted versus $\alpha_{\hat{x}}^*$ and $\alpha_{\hat{y}}^*$, for a clockwise rotating tidal current, the most unstable perturbation generated by a circular tide has no preferred orientation. For a unidirectional tide, the crests of the most unstable mode are counter-clockwise rotated with respect to the direction of the tidal current. For values of the ratio e between the minor and major axes of the tidal ellipse falling between 0 and 1, the most unstable mode is always characterized by crests that are counter-clockwise rotated with respect to the main tidal current.

Of course the results obtained for the same values of the parameters but for a counter-clockwise rotating velocity vector do not differ from the previous ones when a unidirectional tide is considered. Moreover, for a circular tide only quantitative differences are present. However, for an elliptical tide, qualitative differences are present. In fact, for values of e falling between 0.02 and 0.6, the maximum of the amplification rate is characterized by positive values of both $\alpha_{\hat{x}}^*$ and $\alpha_{\hat{y}}^*$ and the predicted sand banks are clockwise rotated. Hence, the results obtained by Besio et al. [4] are in agreement with the findings of Besio et al. [5] and show that both a three dimensional analysis and a shallow water model predict the appearance of large scale bedforms, the orientation of which depends on the clockwise/counter-clockwise rotation of the tidal velocity vector.

Even though a linear stability analysis can predict the crest-to-crest distance and the orientation of sand banks, little is known on the mechanism that leads to equilibrium conditions. The extension of the linear analysis made by Huthnance [19] provides an equilibrium profile of the sand banks but the hydrodynamics is described by a simplified approach. Roos et al. [27] developed a numerical approach that fully resolves the morphodynamics of the fastest growing mode. However, the numerical approach was applied only considering a unidirectional tidal current and the solution procedure implies significant computational costs.

A weakly nonlinear stability analysis is an alternative tool to determine the equilibrium profile attained by the unstable bottom perturbations when the parameters are close to their critical values. Indeed weakly nonlinear stability analyses have been successfully used in hydrodynamic stability and have been also applied to morphodynamic problems [12, 28, 32].

However, difficulties in the formulation of a weakly nonlinear stability analysis of the growth of sand banks for a unidirectional tidal current arose, because the first linear analyses predicted vanishing values of the wavenumber of the most unstable mode close to the critical conditions. In other words, close to the critical conditions, the bottom forms predicted by the linear analyses were characterized by an infinite wavelength. This problem was overcome by Tambroni and Blondeaux [30] who employed a sediment transport predictor that provides vanishing values of the sediment transport rate when the bottom shear stress is smaller than its critical value. The introduction of a critical value of the Shields parameter allowed to obtain better predictions of the sediment transport rate and of the morphodynamic phenomena when the bottom shear stress is close to the initiation of sediment motion. In particular, close to the critical conditions and for tidal currents characterized by a low ellipticity, Tambroni and Blondeaux [30] found that the wavelength of the most unstable mode turns out to be finite. This result opened the possibility to carry out a weakly nonlinear stability analysis. The time development of the most unstable mode was determined by Tambroni and Blondeaux [30] for values of the parameters that differ from the critical ones by a small amount ε. For such values of the parameters, the bottom waviness turns out to be of order $\varepsilon^{1/2}$ and the amplitude of the most unstable component of the bottom perturbation is provided by

$$\frac{dA}{dt^*} = a_1^* A + a_2^* |A|^2 A. \tag{3}$$

Equation (3) is of Landau-Stuart type and can be integrated in closed form to obtain the time development of a small bottom perturbation and its equilibrium amplitude ($|A_{1e}| = \sqrt{-\text{Real}(a_1^*)/\text{Real}(a_2^*)}$).

3 Conclusions

Tidal currents flowing over a cohesionless sea bottom give rise to the formation of bedforms of different length scales ranging from tens of centimetres to tens of kilometres. Examples of these bedforms are ripples, megaripples, sand waves, long bed waves and sand banks. The results of stability analyses, which can predict the appearance of these bedforms and their main geometrical characteristics, are reviewed.

Needless to write that the hydrodynamics is described assuming the flow regime to be turbulent and considering Reynolds equations either neglecting Coriolis effects, when the dynamics of small bedforms is analysed, or taking them into account, when the largest bedforms (e.g. sand banks) are analysed. The linear stability analyses provide reliable predictions of the wavelength and orientation of a large number of the periodic bedforms observed in tidal seas. In a few cases, a weakly nonlinear stability analysis can be used to estimate the equilibrium amplitude of the bedforms even though in many cases the equilibrium bottom configuration can be predicted only taking into account strong nonlinear effects and using a numerical approach.

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Wall Turbulence and Turbulent Drag Reduction

Paolo Luchini and Maurizio Quadrio

Abstract Fluid flow turbulence is probably the last unsolved mystery of classical physics. Research groups are active all over the World with the double aim to accumulate ever more precise empirical knowledge (using a synergy of experiments and numerical simulations), and to frame this empirical knowledge into a consistent theory that may prove of predictive value for applications. Italian Mechanics strongly participates in this effort. Here progress is described in the comprehension of wall-bounded turbulence and the reduction of turbulent skin-friction drag, in their four aspects of the shape of the mean velocity profile, the statistics of turbulent fluctuations, devices of passive drag reduction and devices of active drag reduction. An inevitable bias will be seen for the topics to which the present authors have contributed directly, but these are also the results that we can more easily entice the reader with.

Keywords Fluid mechanics · Turbulence · Drag reduction

1 Introduction

To sketch a history of our understanding of turbulence and turbulent flow, at any level of detail, is a daunting task. In this chapter we take up the limited goal of sketching part of the Italian contributions to the subject, with a focus on our own contributions. Discussion is limited to the specific type of turbulence that takes place in the vicinity of a solid wall. Since the most practically relevant manifestation of the onset of turbulence in wall-bounded flow is its increased skin-friction drag, developments in studies, concepts and technologies for turbulent skin-friction reduction will also be given particular emphasis.

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The notion that turbulence near a wall possesses a certain degree of structure emerged half a century ago, and is slowly superseding the concept (still embedded in some current RANS turbulence models) that a turbulent flow is just random and unpredictable overall, and approaches the laminar state once the very near-wall layer is reached. Lexically, this corresponds to the terminology shift where this layer, once called laminar sublayer, is now referred to as the viscous sublayer.

Around the middle of the past century, early flow visualizations provided the first evidence of the streaky structure of the flow in the near-wall region. The qualitative nature of such early visualizations notwithstanding, they exposed the organized character of the flow, and immediately raised the question of the dynamical significance of such organized motions. Attempts to create a link between visualizations and Reynolds stresses quickly led to the notion of near-wall "events" (the dominant ones being called ejections and sweeps). Their contribution was typically categorized via the quadrant analysis, which is still in use, and quantifies the contribution of different events to the Reynolds shear stresses starting from a time history of a pointwise measurement of the longitudinal and wall-normal velocity components, typically performed with a two-component hot-wire probe. This is where the first conceptual models of turbulence started to appear. Conditional sampling and, more importantly, conditional averaging techniques (the latter implying the ability to store a large measurement dataset and process it later on, something that became available at the same time as computers gradually evolved in power) marked one further step in the characterization of wall turbulence, leading to additional means to characterize the importance of near-wall coherent structures.

The last step connecting the pioneering era to modern times is the advent of direct numerical simulation (DNS) of wall turbulence. It's generally marked to begin with the publication of results from the first DNS of a turbulent channel flow, by Kim Moin and Moser in 1987 [28], following nearly a decade of preparation and development of the numerical approach that predated the actual availability of the necessary computing power. DNS revolutionized our way of looking at wall turbulence, since its limitations are largely complementary to those of the experimental approach, and opened up novel ways of studying its physics.

2 The Structure of the Mean Velocity Profile

The mean velocity profile is definitely the most important quantity of interest. Nearly a century ago, Prandtl recognized by his mixing-length argument that the mean turbulent velocity profile in a pipe or channel would have to be approximately logarithmic in shape. The theory was then refined by von Kármán, and given its present-day form based on dimensional analysis by Millikan. Their argument applies, in a suitable range of distance from the wall, to basically all wall-bounded turbulent flows.

The essence of the Prandtl–von Kármán–Millikan theory sits in scale separation between a "viscous" layer where the wall-normal coordinate z is of the order of the viscous length[1] $\ell \equiv \nu/u_\tau$, and a "defect" layer where z is of the order of the macroscopic scale h of the flow (say, the half-distance between parallel walls or the radius of a pipe, or the thickness of a boundary layer). The ratio h/ℓ coincides with the shear-based Reynolds number $\mathrm{Re}_\tau \equiv u_\tau h/\nu$, and thus scale separation arises naturally in the turbulent asymptotic limit of $\mathrm{Re}_\tau \to \infty$. Dimensional analysis dictates the functional form of the velocity profile in either layer, and the ansatz that the velocity function be independent of both ℓ and h where the two layers overlap, for $\ell \ll z \ll h$, leads to the conclusion that in this region the functional form of the velocity profile $u(z)$ must be logarithmic with universal coefficients κ (named von Kármán's constant) and B:

$$\frac{u}{u_\tau} \equiv u^+ = \frac{1}{\kappa}\log(z^+) + B, \quad \text{with} \quad z^+ \equiv \frac{u_\tau z}{\nu}. \tag{1}$$

This century-old formula is one of the mainstays of turbulence theory and is taught in all basic textbooks about it, yet it has been the target of continued debate, with a large fraction of scientists contending the universality of constant B, a smaller crowd contending the universality of κ, and a few loudly proposing outright alternative formulas which are not logarithmic. In fact the fit of (1) to both experimental and numerical-simulation data exhibits measurable deviations with geometry, and a worldwide effort is ongoing to explain such deviations and to measure von Kármán's constant, a measurement made difficult by the very presence of deviations even for those scientists who believe κ (and/or B) is indeed a universal constant of Nature. Short of denying (1) altogether, one must evince that deviations from (1) only disappear at values of the Reynolds number that are beyond those achieved by up-to-present experiments and numerical simulations. Trust in (1) would be much higher, and a precise measurement of von Kármán's constant from present data would become feasible, if the observed deviations could be explicitly accounted for in the form of higher-order corrections to the asymptotic theory underpinning the logarithmic law. Research has thus focused on one hand upon enlarging the Reynolds-number range that experiments and numerical simulations can afford, on the other upon higher-order asymptotic theories able to explain the present data at present values of Re.

An important effort towards increasing the Reynolds-number range of velocity-profile measurements is the CICLoPE project. Based near Predappio (Italy) inside a long tunnel escavated during war times by the aeronautical Caproni industries, and managed by the University of Bologna under the supervision of A. Talamelli and G. Bellani, CICLoPE (Centre for International Cooperation in Long Pipe Experiments) is an international cooperation that built a 115 m-long circular pipe of 0.9 m $\pm$ 0.1 mm diameter where air can flow at up to 60 m/s. The facility was inaugurated in 2015 and has been hosting a large number of projects since. Details can be found at https://www.euhit.org/infras/ciclope/.

[1] With ν denoting kinematic viscosity, and $u_\tau \equiv \sqrt{\tau_{\text{wall}}/\rho}$ the characteristic velocity of turbulent fluctuations, based on the wall shear stress τ_{wall} and fluid density ρ.

A simple higher-order extension of the logarithmic law that allows a single set of κ and B constants to fit velocity profiles in different geometries starting at $\mathrm{Re}_\tau \gtrsim 400$ was proposed by one of the present authors [36]. Already noticed by Afzal [1] and Jimenez and Moser [25] had been that if the pressure gradient p_x is assumed as a perturbation parameter in an asymptotic theory of the velocity profile, a dimensional argument leads the dimensionless pressure gradient to contain the first power of the wall-normal coordinate z; in other words, it leads to a velocity correction that is a linear function of z. Neither author, however, actually determined a value for the coefficient of this linear function or pushed the argument to its practical consequences for the experimental and numerical validation of the logarithmic law. From an overview of experimental and numerical data of a wide range of sources, Luchini [32, 36] determined that the best fitting value of its coefficient (named A_1) is near unity, leading to the conjecture that it may actually be $A_1 = 1$, and thus to the higher-order logarithmic law

$$u^+ = \frac{1}{\kappa}\log(z^+) + B - \frac{p_x}{\tau_{\mathrm{wall}}} z. \tag{2}$$

Equation (2) fits the velocity profile in all tested geometries (Couette and Poiseuille plane flow, Poiseuille circular pipe flow, boundary layer) with common values

$$\kappa = 0.392, \quad B = 4.48 \tag{3}$$

of the classical coefficients (most notably with a single value of the B coefficient, which most previous analyses, even those assuming that κ is universal, concluded to change with geometry [40]). Von Kármán's constant κ thus comes out in agreement with a specific one of the previous proposals, the value of 0.39 extracted by the Australian school [42] from experimental measurements of the friction law in place of the velocity profile, and in contrast to historical estimates of $0.40 \div 0.41$. In addition, interpolating formulas for the wall layer and the defect layer were also provided (see Box 1 and Eqs. 27, 29, 30 of [32]), arriving at a uniform formulation that describes the velocity profile over the entire range of $0 \le z \le h$ and all geometries (Eq. 1 and Fig. 2 of [38]). Particularly satisfying was the comparison of the present theory with the independently performed Hi-Reff pipe-flow experiment [19], reproduced as Fig. 1 here. An application range was also determined, that this formulation works well within available experimental accuracy for all $\mathrm{Re}_\tau \gtrsim 400$, and that the logarithmic portion of it prevails in the range $200\ell \lesssim z \lesssim 0.5h$ (whereas competing theories [44] surmise that an asymptotic regime is only attained for Re_τ greater than approximately 30,000).

Further developments ensued in yet more recent times. Once taken for granted that deviations from the turbulent logarithmic law are to be ascribed to the pressure gradient, one can quantitatively compare them to analogous deviations from the rectilinear, zero-pressure-gradient Couette profile that take place in laminar flow. Luchini [37] highlighted that this deviation is of opposite sign in laminar and in turbulent flow. That is, if in laminar flow the presence of a favourable, negative pressure gradient decreases the flow rate for a given wall shear stress (or equivalently,

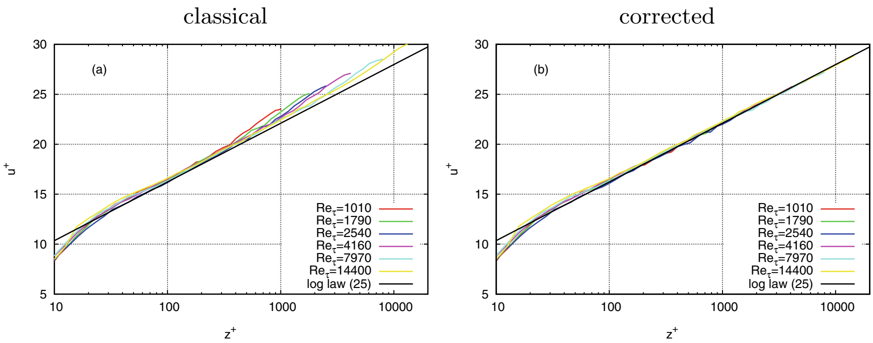

Fig. 1 Fit of the classical logarithmic law (1) and of the corrected logarithmic law (2) to the Hi-Reff pipe-flow experiments. From [38]

increases the wall shear stress for a given flow rate), in turbulent flow it increases the flow rate for a given wall shear stress (decreases the wall shear stress for a given flow rate). Astounding as this behaviour is, it is actually consistent with (one could say, it explains) some previous equally astounding observations: in particular, the conclusion by Johnstone et al. [26] that a mixing-length turbulence model based on the wall shear stress offers a better adherence to reality than a mixing-length model based on the local shear stress (whereas considerations based on the locality of eddy viscosity would favour the latter, and those authors were actually surprised at their own result); and the conclusion by Russo and Luchini [54] that the response of channel flow to a vertically varying volume force with zero mean (meant, in that context, to mimic the action of a wavy bottom) is of opposite sign in laminar and in turbulent flow. These observations have far-reaching implications for the future of turbulence modelling, because most current turbulence models based on eddy viscosity are bound to predict a same-sign behaviour of laminar and turbulent flow in all of these configurations.

3 Statistical Characterization of Wall Turbulence

Besides the mean velocity profile there are other statistical quantities of interest in wall turbulence, most prominently the second-order moments of velocity fluctuations; these relate to turbulent energy, a quantity of practical interest. The experiments we already mentioned endeavoured to measure these quantities at the same time as the mean velocity profile. So did direct numerical simulations; the canonical wall-bounded flows (the plane channel, either pressure or shear-driven, and the cylindrical pipe) were considered early in DNS history, and several Italian groups took part in the international effort at progressively increasing the computational scale of the simulations and the achievable Reynolds number. Among several important contributions, one may recall the many ones by P. Orlandi and coworkers [9], the recent, record-setting pipe flow DNS at $\mathrm{Re}_\tau = 6000$ by Pirozzoli et al. [46], and the open-source codes made available to the research community for massively parallel and GPU-based computations, for both incompressible [58] and compressible [10] flow.

The present authors, while contributing their own DNS code and approach [34, 48] which is however not aimed towards high-Re simulations, have focused their efforts upon less conventional descriptions of the same class of flows. For example they were among the first to consider the full space-time structure of the turbulence statistics [50], thus separating the physical correlation from the spurious effects of the periodic computational box. They also pioneered the (admittedly simple) idea of considering DNS output (say, the mean velocity at a particular wall distance) as affected by measurement error, as any experimentalist would be naturally inclined to do, and to propose to augment DNS results with an error bar representative of the uncertainty related to the statistical-averaging process. In the presence of homogeneous directions, one typically uses statistical averages over finite time and space windows to arrive at estimates of the true expected value, and a rational criterion to properly select such windows is necessary. Albeit at different levels of complexity, this concept [21, 45, 54, 55] is progressively becoming accepted by the community.

An original statistical quantity was proposed by the authors [49] in order to describe a turbulent channel flow in terms of its complete mean impulse response to a perturbation applied at the wall. This tensorial quantity (that can in principle also be measured experimentally) is a symmetric second-order tensor where the independent variables are the time delay and the 3 spatial coordinates expressing separation from the point at the wall where perturbation is applied. It can be easily defined in the laminar case, while in the turbulent case this quantity is important whenever one attempts to control the flow by using linear control theory, for which the mean response constitutes the best model of the dynamical system. The very definition of the impulse response for a turbulent flow involves two non-trivial conceptual steps: first the mean response must be defined, as the instantaneous one is unavoidably bound to diverge; and then the response must be actually measured, by circumventing the practical difficulty that linearity of the response requires small forcing perturbations at the wall, which may easily be overwhelmed by the comparatively large noise of turbulent fluctuations. An ingenious measurement strategy was devised [35], exploiting the fact that passing a white noise through a linear system and measuring the correlation between its input and output yields the impulse response of the system. Figure 2 displays the difference between the correct response, accounting for turbulent diffusion, and the one obtained by simply using linearized equations about the mean turbulent velocity profile. A similar impulse-response concept, extended to measure the response to a volume forcing, was later used in the already mentioned work [54].

A further innovation to be mentioned here is a general description of the second-order moments of velocity fluctuations, which naturally links together the concept of energy cascade in the space of scales, along the lines of the Richardson–Kolmogorov theory, and the concept of Reynolds stresses varying with the distance from the wall,

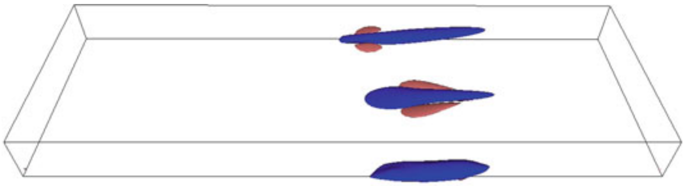

Fig. 2 Streamwise component of the longitudinal-velocity response to a wall-normal velocity impulse. The figure plots, at a time separation of $\tau^+ = 30$ after the impulse, isosurfaces of the positive (red) and negative (blue) response in the three cases of laminar flow (top), laminar flow linearized about the mean turbulent profile (center) and full averaged turbulent response (bottom). Taken from [49]

yielding fluxes in physical space. In fact, the single-point budgets for the Reynolds stresses [39] lack information about the scales involved in their fluxes, and miss the multi-scale nature of turbulence, whereas a spectral decomposition does discern different scales, but fails to provide direct information on their role in production, transfer and dissipation of turbulent kinetic energy. About 20 years ago, Hill [24] generalized the classic Kolmogorov equation for the second-order structure function (or, equivalently, the Kármán–Howart equation for the correlation tensor) from homogeneous and isotropic turbulence to inhomogeneous turbulent flows. The generalized Kolmogorov equation was the tool of choice for studying the scale energy in several flows, ranging from simple wall-bounded flows to shear layers; Italian contributions [13–15] are prominent in this thread. Most recently, Gatti et al. [20] further extended the concept to deal with anisotropy. They derived exact budget equations for the second-order structure function *tensor*: the anisotropic generalised Kolmogorov equations (AGKE) describe the production, transport, redistribution and dissipation of each Reynolds stress component occurring simultaneously among different scales and in space, i.e. along directions of statistical inhomogeneity. AGKE provide a natural definition of scales in inhomogeneous directions, and describe fluxes across such scales too. AGKE are being used to describe the structure of turbulence, in the compound space of scales and positions, starting from the simple turbulent Poiseuille flow, to the same flow at higher Re, where it is found that AGKE quantities start very soon to highlight the structure of the outer turbulent cycle. Couette flow has been investigated too [12], producing an explanation of the complex direct/inverse and ascending/descending cascade taking place in a turbulent Couette flow, where the near-wall cycle coexists with large streamwise vortices which fill the entire gap between the walls. A sample result is shown in Fig. 3, where fluxes of the off-diagonal component $\langle \delta u \delta v \rangle$ in the subspace (r_z^+, r_y^+, Y^+) are drawn together with the source term that denotes net production of shear stress.

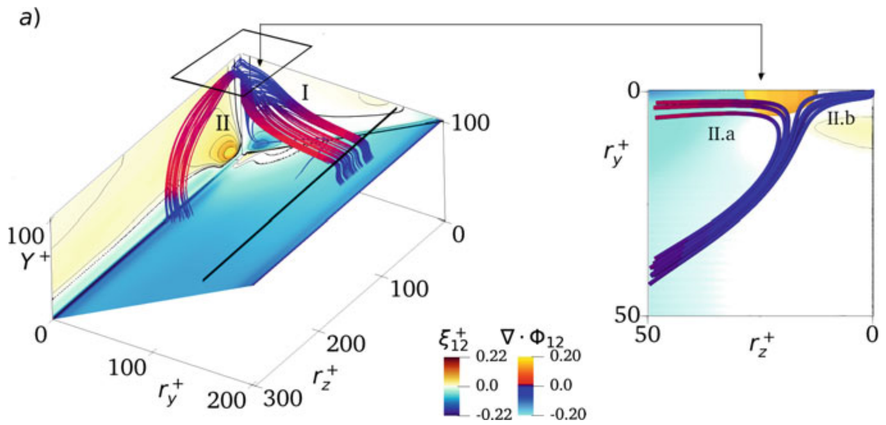

Fig. 3 Spatial and scale fluxes of the structure function $\langle \delta u \delta v \rangle$ in a turbulent Couette flow at $Re_\tau = 100$. Flux lines are drawn together with a colormap (on the bounding planes) of the source term, in a three-dimensional space where the coordinates are the wall-normal distance Y^+ and the spanwise and wall-normal separations r_z^+ and r_y^+. Taken from [12]

4 Passive Drag Reduction and Riblets

The reduction of turbulent drag in wall-bounded flow is an obvious technological goal. The most practical means of turbulent-drag reduction is passive, consisting of static modifications of the solid wall surface or of the fluid's composition, as opposed to active (discussed in next section), which involves moving parts and energy expenditure.

Most effective among the methods of passive drag reduction is the injection of tiny amounts of soluble long-chain polymers, which can provide up to 80% skin-friction reduction [57] by a non-Newtonian mechanism which still today cannot be said to be totally understood. Examples of Italian research contributions on this topic are [8, 16, 17]. The drawbacks of this method are, of course, that it can only be used in liquids and that polymers are consumed continuously and dispersed into the working fluid. It has found important applications in oil pipelines, although these will not be reviewed here.

Wall-surface modifications provide a more modest advantage, of the order of 10% reduction at best, but can be applied in any fluid, and are the most promising drag-reduction devices for aeronautical, marine or terrestrial vehicle applications. The present authors, in particular, have contributed to clarifying the operating mechanism of the so called "riblets", long and fine ridges (or grooves, if seen on the negative side) directed parallel to the flow on the wall surface. It appears at first sight counterintuitive that such a non-smooth surface may have a lower drag than a perfectly smooth one, but similar surfaces exist in nature: the empirical idea that riblets could reduce drag came about early, after the observation that similar structures are present on the scales of shark skin (M. O. Kramer patented it in 1939). Only later, proceeding from the observation that the drag-reducing action of riblets occurs in a size range ($\approx$15 spanwise in units of the length ℓ defined near (1), also known as "wall units") which is relatively small compared to the near-wall structures of turbulence ($\approx$100 wall units), Bechert and Bartenwerfer [6] and Luchini et al. [33] proposed that their action could be explained through a virtual displacement of the wall represented by the viscous "protrusion height".

In the viscous layer that extends for the first few wall units above the surface, the physics of a turbulent flow is dominated by viscous forces (both in its mean value and in its fluctuations). Therefore the Navier–Stokes equations can be simplified in this region to the linear Stokes equations and, owing to linearity, longitudinal and transverse components can be studied separately and assigned, each, a separate protrusion height, this being the intercept with $u = 0$ of the straight line that the Stokes velocity profile tends to as $z \to \infty$. (For a general surface shape the protrusion height, also known as slip length, becomes a 2×2 symmetric tensor and the longitudinal and transverse protrusion heights are its principal values; see e.g. [31].)

Because of their simple definition based on Stokes flow, the longitudinal and transverse protrusion heights are unique for a given geometry, and scale linearly with spatial dimensions. In other words, the protrusion heights are geometrical parameters. Several examples of their values were provided in [33]. The protrusion heights are often measured from the riblet tips, but this is just a convention; the origin of the reference frame is arbitrary, and eventually only the difference of the two protrusion heights affects the flow. As an empirical computational observation this difference turns out to be very sensitive to sharpness of the tips, increasing with it; in fact, sufficient sharpness is not always easy to achieve in reality, and experiments conducted with razor blades [7] have shown that indeed the obtainable drag reduction is very sensitive to sharpness.

The physical explanation of the drag-reducing effect of riblets emerges from the observation that the differential action of riblets upon the longitudinal mean flow and the, prevalently transverse, turbulent eddies effectively pushes the eddies away from the wall and thus diminishes the near-wall turbulence level. This explanation was made quantitative by introducing the following ansatz [29, 30]: *the law of the wall describing the mean-flow profile of the turbulent stream is modified by the presence of riblets only through a displacement of its origin by an amount equal to the difference of the two protrusion heights.* This ansatz derives its rationale from the idea that the turbulent fluctuations, and thus the Reynolds stresses, above riblets are the same that would persist if a plane wall was present at the position of the *transverse* protrusion height, whereas the mean flow (integral of the Reynolds stress) has its integration constant determined so that it vanishes at the *longitudinal* protrusion height.

Since in wall units the viscous tract of the velocity profile is $u^+ = z^+$, with a coefficient that becomes unity in this particular nondimensionalization, a displacement of the entire profile by an amount $\Delta z^+ = \Delta h^+$, where Δh^+ is the protrusion-height difference expressed in wall units, entails a velocity increase $\Delta u^+ = \Delta z^+ = \Delta h^+$. The displacement being rigid, this velocity increase remains constant for all z, and once attained the region where (1) prevails, it amounts to an increase of the B constant by $\Delta B = \Delta h^+$.

The classical theory of turbulent skin friction (and just as well its most recent variations: see e.g. Sect. 6 of [32]) dictates the following formula for the skin-friction coefficient c_f:

$$(c_f/2)^{-1/2} = \kappa^{-1} \log\left[(c_f/2)^{-1/2}\mathrm{Re}/2\right] + B + C - D \tag{4}$$

where κ and B are the constants that appear in the logarithmic law of the wall (1), and C and D are characteristic constants of the outer layer unaffected by the wall's texture. From an expansion of (4) for small ΔB there follows Eq. (4) of [30]:

$$-\frac{\Delta c_f}{c_f} = \frac{\Delta B}{(2c_f)^{-1/2} + (2\kappa)^{-1}} = \frac{\Delta h^+}{(2c_f)^{-1/2} + (2\kappa)^{-1}}, \tag{5}$$

which provides a general quantitative expression of the relative skin-friction reduction as a function of the protrusion-height difference in wall units Δh^+. Equation (5) was verified by comparing it to an extensive number of test results [5] and found to agree reasonably well with the initial slope of the experimental drag curve if account is taken of the actual curvature radius of the riblet tips used in the experiments.

With increasing riblet size (usually measured by their spanwise repetition period s^+), the assumption that the action of riblets stay bounded to the viscous sublayer begins to fail. Drag reduction then ceases to be proportional to Δh^+. Eventually drag reduction saturates, and drag starts to increase again when the period s^+ grows beyond a threshold which empirically is $s^+ \approx 15$ [7]. The curve is not far from being parabolic, with a maximum drag reduction 1/2 of the value given by (5) at $s^+ = 15$. What happens, in fact, is that the riblets' action no longer takes place in the portion of the velocity profile where $u^+ = z^+$, and therefore ΔB no longer equals Δh^+. Nevertheless the first half of (5) remains valid in a much larger range of sizes, as large as a logarithmic region exists, which is the case for the great majority of drag-reducing and drag-increasing devices. Even the classical drag increase produced by random sand roughness is nothing else than a negative ΔB, and the next section will show how ΔB is a suitable parameter to describe the drag-reducing performance of active control as well.

5 Active Techniques of Drag Reduction

In comparison to passive control, active control for skin-friction drag reduction carries the obvious drawbacks of extra complexity and extra energy expenditure. However, especially for relatively simple open-loop strategies, these drawbacks may be compensated by the larger savings. One family of active and open-loop techniques, referred to as spanwise forcing as it forces the boundary layer by injection of spanwise momentum in the near-wall region, has seen a major part of its development propelled by the Italian community [47].

With active control, the non-zero energy costs need to be accurately weighed against energy benefits before declaring a strategy effective. It is not uncommon to see massive "drag reduction" figures reported in the literature, with flow control strategies that cost way more than they save! The increasing importance of active control led to the formalization of the flow-control problem for skin-friction drag reduction [18, 23], where a theoretical framework is developed for assessing drag reduction performance while properly accounting for the energy cost of the control. It is now clearly stated that laminar flow becomes the theoretical best, once control energy is accounted for, and that driving a successful flow-control strategy towards either maximized savings or maximized performance is something left to the specific application. Awareness of the importance of energy considerations, related to spanwise forcing, is steadily increasing [41].

In its early days, spanwise forcing was not particularly attractive in terms of global energy savings. The spanwise-oscillating wall (SOW), i.e. the simplest form of spanwise forcing, consists of an alternating movement of the wall in the spanwise direction, as

$$v(x, y, 0, t) = A \sin(\omega t)$$

where v is the spanwise component of the velocity at the wall, i.e. at $z = 0$, A is the maximum velocity along the cycle, and ω is the oscillating frequency. Known has been since long time [11] that in a turbulent wall-bounded flow the sudden application of a spanwise pressure gradient causes a drop in the streamwise friction, and that a sinusoidally varying spanwise pressure gradient (or, equivalently, a sinusoidal spanwise movement of the wall) makes this effect sustained in time. However, the impressive values of skin-friction reduction reported in the early SOW studies (up to 40–50%, see [27]) were not balanced against the energy cost of the actuation. Baron and Quadrio [4] were first to measure the energy expenditure of SOW, and found that—in an idealized scenario where actuation has unitary efficiency—a tiny amount of net savings is achievable, provided that the forcing amplitude remains moderate. Although the practical appeal of SOW remained scarce, this result was in principle extremely interesting, insofar as it attested the possibility to interact with the complex turbulence dynamics using a simplistic control, and to achieve an overall positive gain, thus motivating the continuation of the research effort. The SOW technique was later comprehensively assessed by Quadrio and Ricco in [51].

More than one decade later, and still leading the way of the international efforts, the present authors established [56] that a purely spatial oscillation is equivalent (in fact, slightly superior) to the temporal one, thus opening the way to exploitation of the spanwise-forcing concept with passive mechanisms. The following forcing type was studied:

$$v(x, y, 0, t) = A \sin(kx)$$

where the temporal oscillation at frequency ω is replaced by a spatial oscillation along the streamwise direction with wavelength $2\pi/k$.

The breakthrough came in 2009, with the discovery by Quadrio, Ricco and Viotti [53] that a combined spatio-temporal oscillation brings superior drag reduction prop-

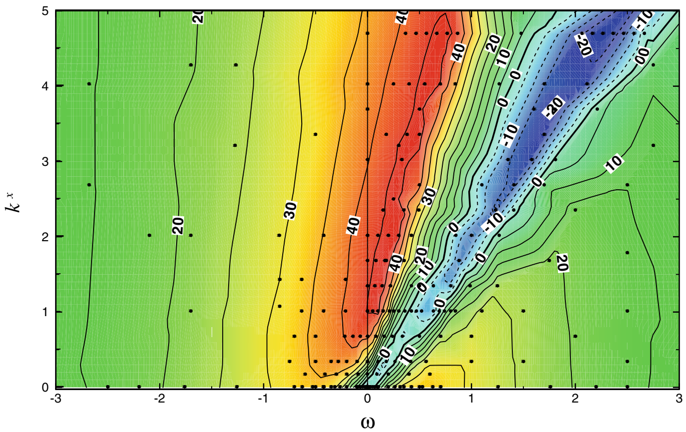

Fig. 4 Map of the drag reduction (red)/increase (blue) as a function of wavenumber and frequency of the forcing, for $A^+ = 12$ in a turbulent channel flow at $\mathrm{Re}_\tau = 200$. Figure taken from [53]

erties and a vastly improved energy balance. They introduced the following forcing:

$$v(x, y, 0, t) = A \sin(kx - \omega t) \quad (6)$$

and found that the combined forcing, which takes the form of a streamwise-traveling wave of spanwise wall velocity, exhibits several interesting properties, most prominently a relatively small energy required for the forcing, and leads to a maximum *net* drag reduction of more than 20%. A map of measured drag reduction as a function of wavenumber and frequency is plotted in Fig. 4. Drag reduction was found to be well predicted by properties of the spanwise boundary layer created by the forcing, named the "Generalized Stokes Layer" [52] as it contains as a special case the classic Stokes layer that develops when an indefinite wall oscillates beneath a still fluid.

Only one year after writing [53], the same group at Politecnico di Milano demonstrated the streamwise-travelling-wave concept experimentally, with a test that so far has yielded one of the largest measured drag reductions ever (neary 40%), and probably the largest net saving. The experiment was designed in a pipe flow configuration, and the wall forcing was realized through an alternate azimuthal movement of thin ring-like slices of the pipe, independently actuated via a shaft-and-belt system (see Fig. 5). Use of water as the working fluid, and the relatively low value of Re, allowed design and implementation of a mechanical actuator that cannot scale up to realistic applications, but was definitely effective in a proof-of-principle experiment.

The combination of large benefits and relative ease of implementation make the spanwise forcing in the form of streamwise-travelling waves one of the best candidates for applications. The most relevant issue that remains still open is the lack of a suitable actuator with the required high efficiency, low cost, low weight, and high control authority (large velocities, actuation frequencies, etc.): good candidates

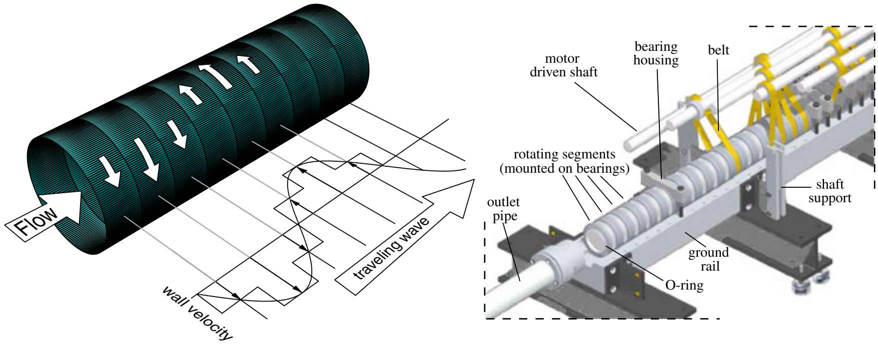

Fig. 5 Conceptualization of the actuation strategy to implement forcing (6), and sketch of the experimental setup for the drag-reduction experiment of the PoliMI pipe. Figures taken from [2]

indeed exist, but none is ready for industrial applications. Several other concerns, though, have been successfully addressed in the meantime. An important one was that, until recently, all the available information on spanwise forcing, collected via DNS or experiments, was related to low-Re flows. Extrapolating available information to application-level Re indicated a rapid decrease of the performance of drag reduction. This was until a large-scale campaign of numerical experiments carried out by Gatti and Quadrio [22] demonstrated that spanwise forcing is equivalent to a drag-decreasing roughness, discussed above in Sect. 4, and shares with it its Reynolds-number dependence embodied in (5), which was extended to account for larger values of ΔB. They carried out more than 4,000 DNS at two well-separated values of Re, and made it possible to extrapolate drag reduction at any Re: according to their data, the reduction of turbulent friction attainable on an airplane in cruise flight via spanwise forcing remains remarkable, in the order of 30%.

Finally, it is emerging that a reduced skin friction (by spanwise forcing, or by other means) has the potential to bring in additional benefits when the application involves a body with complex shape (an airplane, for example) where turbulent friction is only one of the sources for aerodynamic drag. The original suggestion originated once again in Italy, thanks to RANS simulations of riblets on an airplane carried out by Mele and Tognaccini [43], and was later investigated with DNS. Banchetti et al. [3] applied spanwise forcing to the incompressible flow over a bump, and found benefits for pressure drag too; ongoing work is demonstrating the same concept on a wing section in transonic flight, where spanwise forcing is successful in altering the position of the shock wave.

6 Concluding Remarks

In this chapter four aspects of turbulent flow in proximity of a wall have been reviewed, all four being the subject of very active and ongoing research: the shape of the mean velocity profile and the applicability limits and precise values of the coefficients of the classical logarithmic law, the statistics of turbulent fluctuations and their multi-dimensional characterization in space-time, the passive (static) surface modifications that may offer some reduction in skin-friction drag, and the active (moving-wall) modifications that may offer a larger reduction at the expense of greater complication. These are just examples of the open research problems that the mystery of turbulence exposes, and we hope that the progress made up to now may be an incentive for the Italian Mechanics community to keep on devoting energies towards their solution.

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Theory and Simulation of Dynamic Nucleation in Metastable Fluids

Mirko Gallo and Carlo Massimo Casciola

Abstract We discuss a mesoscopic approach we recently devised to deal with the nucleation of a daughter phase in a metastable mother fluid, with application to bubble/droplet nucleation in metastable liquids/vapors. By numerical solution of the relevant stochastic system of partial differential equations the model is shown able to deal both with homogenous and heterogenous nucleation over surfaces of different wettability and geometry and to couple the nucleation process of the new phase with the large-scale flow dynamics. Indeed, the approach we discuss bridges the gap between the atomistic scale where nucleation takes place and the macroscopic dynamics, reaching an unprecedented range of length and time scales in a problem where thermal fluctuations are among the crucial ingredients. The review intends to drive the reader through the rich theory that, starting with the basic aspects of statistical mechanics and density functional theory, touches upon the subject of non-homogenous fluids and their dynamics. Details on the physical relevance of the proposed examples as well as many technical aspects, both concerning the theory and the numerical implementation, are purposely left out of consideration addressing the interested reader to the recently published literature.

Keywords Nucleation · Phase transitions · Rare events · Metastability · Fluctuating hydrodynamics · Diffuse interface · Bubbles

1 Introduction

The present chapter deals with an aspect of fluid motion that has longly been left out of consideration of even the most sophisticated models and remained out of reach of the most powerful numerical simulations: the effect of phase change, and topological

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changes more in general, on the flow dynamics. Among the many beautiful examples, one may think of, we will focus here on multiphase fluid flows, like when interfaces exist between a liquid and a gaseous phase. Space limitations do not allow to review of the amazing simulations—often in turbulent conditions—that can be found in the literature, where the Navier-Stokes equations are solved for each fluid phase accounting for their coupling across a highly convoluted interface endowed with surface tension. It will be sufficient to recall that, when different fluid phases already exist, current techniques can reproduce with high fidelity their evolution. There is however a crucial aspect that did not receive the attention it deserves, namely the appearance of a new phase in an otherwise homogeneous fluid. This happens, e.g., when liquid droplets appear in a flowing, vapor-rich gaseous mixture (condensation) or bubbles form in a streaming liquid due to temperature increase (boiling) or pressure decrease (cavitation).

There are reasons for such delays in our modeling capability. Phase transitions are traditionally the realm of physics. Any modern book on statistical mechanics devotes a full chapter to the subject, mostly focusing on the fundamental aspects of critical phenomena and their scaling theory, see the comprehensive review by Halperin and Hohenberg. The basic aspects of nucleation theory, i.e. the theory describing the formation of the embryos of the new phase, are nowadays reasonably well developed. However, the available approaches, e.g. the so-called Classical Nucleation Theory, see the book by Debenedetti devoted to metastable liquids, do not capture many important features of nucleation, in particular when the process takes place in correspondence of solid surfaces, with their chemical inhomogeneities and geometrical defects, and couples with macroscopic dynamics. The computer power partially come to our rescue, allowing the simulation of the nucleation process using atomistic models (Molecular Dynamics, MD), where the trajectories of elementary atoms/molecules are integrated to extract the desired statistical information, Fig. 1.

In their simplest form, classical atomistic systems comprise a collection of point particles (the atoms) with a given mass m, position $\mathbf{q}_k$ and momentum $\mathbf{p}_k = m\dot{\mathbf{q}}_k$, $k = 1, \ldots, N_p$. The Hamiltonian $H = H(\mathbf{q}, \mathbf{p})$ will include internal interactions among the particles and an external potential which confines the particles to a certain domain $\mathcal{D}$. The evolution is governed by the system

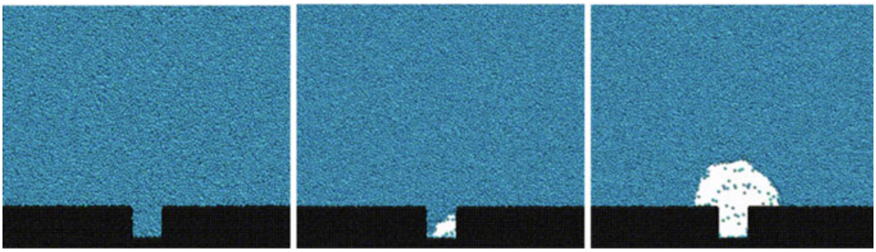

Fig. 1 Vapor nucleation catalyzed by an indentation over an otherwise atomically flat wall (molecular dynamics simulation). Nucleation in the metastable liquid is favored by geometrical and chemical inhomogeneities that make the process fast enough to be captured by brute force simulations

$$\dot{\mathbf{q}}_k = \frac{\partial H}{\partial \mathbf{p}_k}, \quad \dot{\mathbf{p}}_k = -\frac{\partial H}{\partial \mathbf{q}_k} + \text{Thermostat} + \text{Barostat}, \tag{1}$$

where the thermostat enforces constant temperature and the barostat controls the pressure. In many cases, the phase transition takes place on time scales unaffordable by computer simulations. For this reason, specialized, rare event techniques have been developed. This aspect is left out of the present review and only briefly touched upon in the last section.

Despite its fundamental role, the atomistic approach alone is however not sufficient. Its more severe limitation is the small size of the sample and the short time scale of the affordable simulations scales.

To summarize, we are able to deal with multiphase flows on two mutually exclusive planes: on the one hand, we simulate multiphase flows with macroscopic, continuum models, but only when the phases already coexist. On the other hand, we can address the nucleation process on the atomistic scale but cannot couple nucleation to the macroscopic dynamics. When we are required to encompass the entire phenomenology—nucleation, growth of the new phase, and coupling to macroscopic dynamics—we are at a loss. A possibility to overcome the impasse is to try and bridge the gap between the atomistic description and the continuum formulation.

In this context, the purpose of the present chapter is to review recent work we have been doing in the last years to develop a model and the related algorithms to consistently include the nucleation dynamics in a continuum setting. This will allow the simulation of multiphase flows encompassing the whole phenomenology, from the formation of the new phase embryos to the large scale dynamics where, overall, the system obeys the Navier-Stokes equation augmented by capillary effects.

The main players in the nucleation game are metastability and thermal fluctuations. For instance, it is well known that liquids can be significantly over-heated or stretched before a breakdown takes place and a cavity filled of vapor forms. The liquid can be trapped in a metastable condition, meaning that the system is locally stable with respect to perturbations, and can transition to a new phase if a (free-) energy barrier is overcome. Such perturbation can be purely mechanical, like when we shake a soda can before opening it. Here we are interested instead in spontaneous transitions, triggered by the thermal fluctuations that naturally occur in the fluid. In broad terms, a spontaneous transition is a rare event, i.e. something that, more often than not, does not take place.

When it comes to the liquid-vapor transition, our favorite example here, the transition time is normally of the same order of the macroscopic scales of the flow, but still enormously larger than the atomistic scale. The challenge we have undertaken—an effort that we like to review here—is describing this process in the context of continuum fluid mechanics. Four are the basic ingredients we need: (i) an approach based on fields that allows the formation of the new phase and describes the associate flow topology change; (ii) a proper way to introduce thermodynamically consistent thermal fluctuations; (iii) the coupling between fluctuations and the continuum field; (iv) tools to deal with the rare-event issue when required. We will be concerned with the first three aspects, in particular, with the purpose of providing a description that, although as rigorous as possible, could be appreciated also by non-specialists.

Among the different angles under which the subject can be viewed, here we have privileged the theory. Specific cases are mainly used as examples, referring the reader to the published literature for additional details.

2 The Diffuse Interface Description

The first ingredient is a deterministic, mesoscopic model accounting for the topological changes like formation, dissolution, and coalescence of new phase enclosures.

2.1 Equilibrium of Non-homogeneous Fluids

A microscopically-founded continuum model is provided by the (classical) density functional theory (DFT) [11]. The theory describes the density field on the basis of the statistical mechanics of the underlying atomistic system. The (grand canonical) probability distribution function of the microstate at temperature θ and subject to the chemical potential μ under the action of an external field $\phi(\mathbf{x})$ is

$$f(\mathbf{q}, \mathbf{p}, N_p|\theta, \mu, [\phi]) = \frac{1}{N_p!\hbar^{3N_p}\Sigma(\theta, \mu, [\phi]))} \exp\left(-\frac{H_{N_p} - \mu N_p}{k_b\theta}\right), \quad (2)$$

where $\hbar = h/(2\pi)$, with h the Planck's constant, k_b is the Boltzmann's constant, $H_{N_p} = \sum_{k=1}^{N_p} |\mathbf{p}_k|^2/(2m) + \phi(\mathbf{q})$ is the Hamiltonian of the system when formed by N_p particles, and

$$\Sigma(\theta, \mu, [\phi]) = \exp\left(-\frac{\Omega(\theta, \mu, [\phi])}{k_b\theta}\right) = \sum_{N_p=0}^{\infty} \int d\mathbf{q}^{N_p} d\mathbf{p}^{N_p} \frac{\exp\left(-\frac{H_{N_p}-\mu N_p}{k_b\theta}\right)}{N_p!\hbar^{3N_p}} \quad (3)$$

is the grand canonical partition function with $\Omega(\theta, \mu, [\phi])$ the grand potential. After defining the microscopic particle (number) density $\hat{\rho}(\mathbf{x}; N_p, \mathbf{q}, \mathbf{p}) = \sum_{k=1}^{N_p} \delta(\mathbf{x} - \mathbf{q}_k)$, the expected (macroscopic) density is (minus) the functional derivative of the grand potential with respect to the external field, $\rho(\mathbf{x}, \theta, \mu, [\phi]) = \langle\hat{\rho}\rangle = -\delta\Omega(\theta, \mu, [\phi])/\delta\phi(\mathbf{x})$. The free energy follows as the (functional) Legendre transform of the grand potential with respect to the field,

$$F[\theta, \rho] = \Omega(\theta, \mu, [\phi]) + \int \frac{\delta\Omega(\theta, \mu, [\phi])}{\delta\phi(\mathbf{x})} (\phi(\mathbf{x}) - \mu)\, dV. \quad (4)$$

It is a universal functional of the density field (i.e. independent of the field ϕ) and determines the equilibrium statistical properties of the atomistic fluid. In terms of the above universal free energy functional, the equilibrium density profile is determined by the equation $\delta F[\theta, \rho]/\delta\rho(\mathbf{x}) = \mu - \phi(\mathbf{x})$.

Unfortunately, the universal free energy functional, Eq. (4), which is easily determined for an ideal gas, is unknown in general—it is the holy grail of the theory of liquids. DFT boils down to providing approximations of different accuracy to the unknown universal functional. However, in many cases, if the intermolecular interactions are short-range, a (truncated) gradient expansion can be sufficient to approximate the free energy,

$$F_{vdw}[\theta, \rho] = \int f(\theta, \rho, \nabla\rho)d^3\mathbf{x} = \int \left(f_b(\theta, \rho) + \frac{1}{2}\lambda(\theta)\nabla\rho \cdot \nabla\rho \right) dV \qquad (5)$$

leading to the so-called square-gradient approximation, originally proposed by Van der Waals [3]. In this expression f_b is the free energy density at particle density $\rho(\mathbf{x})$ and λ is the so-called capillary coefficient.

Under the above approximation, the density obeys the equation[1]

$$\frac{\partial f_b}{\partial \rho} - \nabla \cdot (\lambda \nabla \rho) = \mu_0 \,, \qquad (6)$$

where a constant bulk density (corresponding to liquid or vapor) is assumed at large distance ($\lim_{|\mathbf{x}|\to\infty} \rho = \rho_b$). For a one-dimensional problem, the solution is easily obtained by quadrature and consists of a smooth transition between the high-density liquid and the low-density vapor that takes place across a thin layer, the diffuse interface [1]. As the ratio of layer thickness to external scale shrinks to zero, the excess grand potential localized in the transition layer becomes the usual surface tension.

2.2 *Free Space Dynamics of Non-homogeneous Fluids*

Once the equilibrium properties of the inhomogeneous fluid are established, dynamics follows by postulating mass, momentum, and energy conservation,

---

[1] Given a functional of the form $\Phi[h] = \int_{\mathcal{D}} \psi(h(\mathbf{x}), \nabla h(\mathbf{x}))d^n V(\mathbf{x})$, its first variation is $\delta\Phi = \int_{\mathcal{D}}(\frac{\partial\psi}{\partial h}\delta h + \frac{\partial\psi}{\partial\nabla h} \cdot \delta\nabla h)d^n V(\mathbf{x}) = \int_{\mathcal{D}}(\frac{\partial\psi}{\partial h} - \nabla \cdot \frac{\partial\psi}{\partial\nabla h})\delta h d^n V(\mathbf{x}) + \int_{\partial\mathcal{D}} \mathbf{n} \cdot \frac{\psi}{\partial\nabla h}\delta h d^{n-1}S(\mathbf{x})$. Under many circumstances, depending on the boundary conditions, the last integral over the boundary of the integration domain vanishes altogether (i.e. when $h_{|\partial\mathcal{D}}$ is assigned, implying $\delta h_{|\partial\mathcal{D}} = 0$, or when $\mathbf{n} \cdot \frac{\partial\psi}{\partial\nabla h}{}_{I_{\partial\mathcal{D}}} = 0$). In these cases, the first variation is $\delta\Phi = \int_{\mathcal{D}} \frac{\delta\Phi}{\delta h}\delta h d^n V(\mathbf{x})$, where the function $\frac{\delta\Phi}{\delta h} = \frac{\partial\psi}{\partial h} - \nabla \cdot \frac{\partial\psi}{\partial\nabla h}$ is called the functional derivative of Φ with respect to h. For general boundary conditions, the boundary terms arising from the integration by parts need to be retained.

$$
\begin{aligned}
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) &= 0, \\
\frac{\partial (\rho_m \mathbf{u})}{\partial t} + \nabla \cdot (\rho_m \mathbf{u} \otimes \mathbf{u}) &= \rho_m \mathbf{g} + \nabla \cdot \mathbf{T} + \nabla \cdot \delta \boldsymbol{\tau}, \\
\frac{\partial (\rho_m e)}{\partial t} + \nabla \cdot (\rho_m e \mathbf{u}) &= \nabla \cdot (\mathbf{u} \cdot \mathbf{T}) + \rho_m \mathbf{g} \cdot \mathbf{u} - \nabla \cdot \mathbf{q} - \nabla \cdot \delta \boldsymbol{\sigma},
\end{aligned} \tag{7}
$$

where $\rho_m = m\rho$ is the mass density, $\mathbf{u}$ the velocity field, e the energy density, $\mathbf{T}$ and $\mathbf{q}$ the stress tensor and the energy flux, respectively, with $\mathbf{g}$ the external force per unit mass. The terms $\delta\boldsymbol{\tau}$ and $\delta\boldsymbol{\sigma}$ will be discussed in a later section.

Familiar thermodynamic considerations based on the Clausius-Duhem inequality on entropy production combined with the local equilibrium assumption [10], lead to the splitting of thermodynamic fluxes into an equilibrium (reversible) and a dissipative contribution, e.g., $\mathbf{T} = \boldsymbol{\Sigma}_{rev} + \boldsymbol{\Sigma}_{dis}$. The equilibrium part is uniquely determined by the equation of state (e.g. the pressure gradient appearing in the Navier-Stokes equation is $\nabla \cdot \boldsymbol{\Sigma}_{rev} = -\nabla p = -\rho_m \nabla \mu + s \nabla \theta$, where s is the entropy per unit mass) whereas the dissipative part should lead to a positive production of entropy like, e.g., for the Newtonian constitutive equations of a viscous fluid.

For a fluid described by the free energy functional (5), the divergence of the conservative stress is

$$
\nabla \cdot \boldsymbol{\Sigma}_{rev} = -\rho_m \nabla \left(\frac{\delta F}{\delta \rho_m} \right) + \frac{\delta F}{\delta \theta} \nabla \theta \tag{8}
$$

see [13, 14] for the explicit expressions of the total stress tensor and energy flux. Within homogeneous phases Eq. (8) reverts back to the standard pressure gradient term, but at interfaces additional stresses associated with capillarity (e.g. with strong density gradients) take care of the interface dynamics.

Figure 2 shows the numerical solution of the Capillary-Navier-Stokes equations (7) just described for the spherical collapse of a nanobubble triggered by a sudden increase of the liquid pressure and show the potential of the approach to deal with the extreme phenomena that take place at bubble collapse [14].

2.3 Wall Wettability and Heterogeneous Fluids at Solid Boundaries

In order to include solid walls in the model, the free energy functional (5) must be augmented with the interfacial energy of the fluid-solid interface,

$$
F_{vdw}[\theta, \mu, \rho] = \int \left(f_b(\theta, \rho) + \frac{1}{2} \lambda(\theta) \nabla \rho \cdot \nabla \rho \right) dV + \oint_w f_w(\rho, \theta) dS, \tag{9}
$$

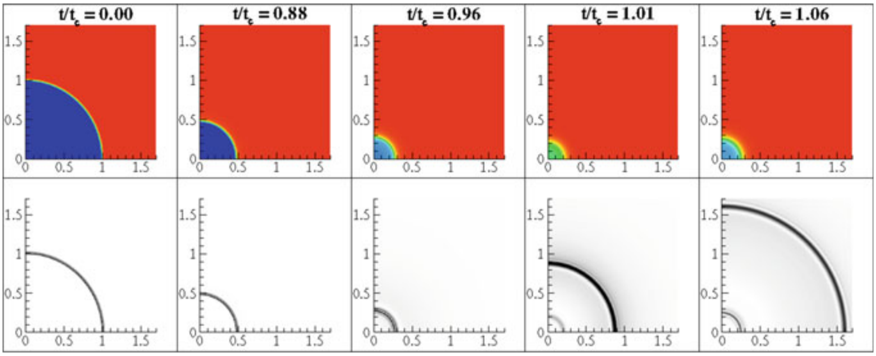

Fig. 2 Collapse of spherical nanobubble (reproduced from [14]). The top pictures display the density field at different times along the collapse process induced by the increase of the external pressure (blue—vapor density, red—liquid density). Time is normalized with bubble collapse time. The diffused interface where the density smoothly changes between the extremal values is apparent. The bottom pictures display the intensity of density gradient which visualize the bubble interface and the shockwave launched in the liquid at bubble collapse

Fig. 3 Left: density field sketch for a bubble deposited over a solid surface. Center: Equilibrium configuration of bubble in contact with a lyophobic wall. Right: Equilibrium configuration of bubble in contact with a lyophilic wall. Reproduced from [7]

where the last (surface) integral extends to the fluid-solid interface. The equilibrium density profile is the solution of Euler-Lagrange equations, Eq. (6) for the bulk fluid, combined with the surface equation

$$\lambda \frac{\partial \rho}{\partial n} + \frac{\partial f_w}{\partial \rho} = 0. \tag{10}$$

The wall free energy density f_w controls the wall wettability and determines the contact angle of the liquid/vapor interface [7]. Figure 3 shows the sketch of the density field of a vapor bubble over a surface, left panel, with the center and right panels showing the computed density fields for a lyophobic and a lyophilic wall, respectively.

Figure 4 shows the collapse of a vapor bubble near a solid wall triggered by an incoming pressure wave [13]. The asymmetry of the system induces the topological transformation of the bubble from spherical to toroidal. During this phase, a strong jet is produced which impinges normally to the wall. The sequence of bubble collapses and rebounds leads to a system of interacting compression (shock) waves.

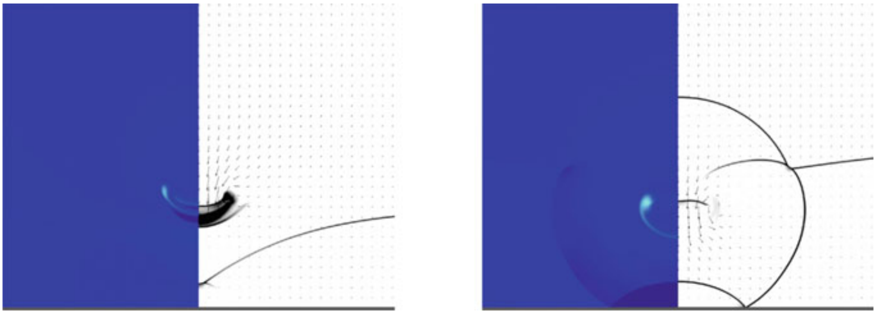

Fig. 4 Bubble collapse near a solid wall induced by an incoming pressure wave. The left part of each plot reports the density field. The right part the velocity field. During the collapse the bubble undergoes a topological change from spherical to toroidal and string pressure waves are emitted

3 Thermal Fluctuations and the Einstein-Boltzmann Principle

As discussed in the Introduction, thermal fluctuations are a crucial ingredient of the phase transition. The purpose of the present section is to describe their effect on a capillary fluid in the context of continuum mechanics.

Let us consider a microcanonical (classical) system of N particles of mass m with microstate pdf $\mathrm{p}(\Gamma) = \delta(H(\Gamma) - E_0)/(N!\hbar^{3N} Z)$, where $Z = \exp(S_0/k_b)$ is the partition function. The Hamiltonian (binary interaction are assumed for the sake of definiteness) is $H = \sum_{k=1}^{N} \left(|\mathbf{p}_k|^2/(2m) + \sum_{r=1, r\neq k}^{N} \Phi_{12}(|\mathbf{q}_k - \mathbf{q}_r|)\right)$ and the particles are confined to a domain $\mathcal{D}$. For a given partition of the domain in n_c cells such that $\cup_{c=1}^{n_c} C_c = \mathcal{D}$, and $C_c \cap C_{c'} = \emptyset$ for $c \neq c'$,

$$N_c(\Gamma) = \sum_{k=1}^{N} \int_{C_c} \delta\left(\mathbf{x} - \mathbf{q}_k\right) dV, \qquad \mathbf{u}_c(\Gamma) = \frac{\sum\limits_{k=1}^{N} m \mathbf{v}_k \int_{C_c} \delta\left(\mathbf{x} - \mathbf{q}_k\right) dV}{\sum\limits_{k=1}^{N} m \int_{C_c} \delta\left(\mathbf{x} - \mathbf{q}_k\right) dV},$$

$$U_c(\Gamma) = \sum_{k=1}^{N} \left(\frac{1}{2} m |\mathbf{v}'_k|^2 + \epsilon_k\right) \int_{C_c} \delta\left(\mathbf{x} - \mathbf{q}_k\right) dV \qquad (11)$$

are the particle number, the coarse-grained (cell mass-center) velocity, and the internal energy in cell c. In Eq. (11) $\epsilon_k = 1/2 \sum_{r=1, r\neq k}^{N} \Phi_{12}(|\mathbf{q}_k - \mathbf{q}_r|)$ is the interaction energy ascribed to the k-th particle and $\mathbf{v}'_k = \mathbf{p}_k/m - \mathbf{u}_c$ the particle velocity relative to the center of mass. The values of $N_c(\Gamma)$, $\mathbf{u}_c(\Gamma)$, and $U_c(\Gamma)$—so called collective variables—identify a set of microstates. Their deviation with respect to the most probable state represents a fluctuation.

The pdf of the above observables can be factored in terms of marginal pdf of (coarse-grained) velocity and conditional pdf of internal energy and particle number,

$$\mathrm{p}\left(\hat{\mathbf{u}}_1, \ldots, \hat{U}_1, \ldots, \hat{N}_{n_s}\right) = \mathrm{p}\left(\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_{n_s}\right) \mathrm{p}\left(\hat{U}_1, \ldots, \hat{N}_{n_s} | \hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_{n_s}\right). \quad (12)$$

Since the most probable values for the velocities, $\hat{\mathbf{u}}_c^*$, is zero by symmetry, for small fluctuations, the velocity pdf can be approximated by a quadratic expansion,

$$\mathrm{p}\left(\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_{n_s}\right) \simeq \frac{1}{Z} \exp\left(-\frac{1}{2k_b} \sum_{c,c'=1}^{n_c} \delta\hat{\mathbf{u}}_c K_{c,c'} \delta\hat{\mathbf{u}}_{c'}\right), \quad (13)$$

where $\delta\hat{\mathbf{u}}_c = \hat{\mathbf{u}}_c - \hat{\mathbf{u}}_c^*$, the inverse Boltzmann constant has been added for convenience, and the negative sign renders $K_{c,c'}$ a positive definite matrix defined by the velocity-velocity correlations, $\langle \delta\hat{\mathbf{u}}_c \delta\hat{\mathbf{u}}_{c'} \rangle = k_b K_{c,c'}^{-1}$.

In order to proceed further with the conditional pdf of internal energy and particle number, we appeal to the local equilibrium assumption in the form of the two following hypothesis: (i) the (conditional) pdf of internal energy and number of particles is independent of the coarse-grained velocity

$$\begin{aligned}\mathrm{p}(\hat{U}_1, \ldots, \ldots, \hat{N}_{n_c} | \hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_{n_c}) = \int d\Gamma \mathrm{p}(\Gamma) \delta(\hat{U}_1 - U_1(\Gamma)) \delta(\hat{N}_{n_c} - N_{n_c}(\Gamma)) \\ \exp\left(-\frac{S_0}{k_b}\right) \int d\Gamma \delta(\hat{U}_1 - U_1(\Gamma)) \delta(\hat{N}_{n_c} - N_{n_c}(\Gamma)) \frac{\delta\left(H(\Gamma) - \mathcal{E}_0\right)}{N! \hbar^{3N}} = \\ \exp\left(\frac{S(\hat{U}_1, \ldots, \hat{N}_{n_c}) - S_0}{k_b}\right),\end{aligned} \quad (14)$$

except for the available amount of energy $\mathcal{E}_0$ which is reduced by the (coarse grained) kinetic energy, $\mathcal{E}_0 = E_0 - \sum_c 1/2m\hat{N}_c|\hat{\mathbf{u}}_c|^2$. The integral in the second line is the phase space volume available to the system for the prescribed fluctuation, hence it is the entropy of the fluctuation; (ii) the entropy of the fluctuation is identified with the (additive) thermodynamic entropy, $S(\hat{U}_1, \ldots, \hat{N}_{n_c}) = \sum_c^{n_c} S_c^{\mathrm{Th}}(\hat{U}_c, \hat{N}_c)$. Equation (14) relating the pdf of a fluctuation to its entropy is often called the Einstein-Boltzmann principle.

As done for the velocity, the quadratic expansion of the entropy around the most probable state $\mathbf{u}_c^*(\mathbf{x})$, $U_c^*(\mathbf{x})$, $N_c^*(\mathbf{x})$, obtained by maximizing the constrained entropy

$$\begin{aligned}S_c(\hat{U}_1, \ldots, \hat{N}_{n_c}) = \\ S(\hat{U}_1, \ldots, \hat{N}_{n_c}) + k_1\left(E_0 - \sum_{c=1}^{n_c}\left(\hat{U}_c + \frac{1}{2}m\hat{N}_c|\hat{\mathbf{u}}_c|^2\right)\right) + k_2\left(N_0 - \sum_{c=1}^{n_c}\hat{N}_c\right),\end{aligned} \quad (15)$$

where k_1 and k_2 are Lagrange multipliers, leads to the Gaussian approximation

$$\mathrm{p}\left(\hat{\mathbf{u}}_1, \ldots, \hat{U}_1, \ldots, \hat{N}_{n_s}\right) = \frac{1}{Z} \times \exp\left(\frac{1}{2k_b}\sum_{c,c'=1}^{n_c} \delta\hat{\mathbf{u}}_c K_{c,c'} \delta\hat{\mathbf{u}}_{c'}\right) \times$$
$$\frac{1}{2k_b}\left(\left.\frac{\partial^2 S}{\partial\hat{U}_c\partial\hat{U}_{c'}}\right|_* \delta\hat{U}_c\delta\hat{U}_{c'} + 2\left.\frac{\partial^2 S}{\partial\hat{U}_c\partial\hat{N}_{c'}}\right|_* \delta\hat{U}_c\delta\hat{N}_{c'} + \left.\frac{\partial^2 S}{\partial\hat{N}_c\partial\hat{N}_{c'}}\right|_* \delta\hat{N}_c\delta\hat{N}_{c'}\right) \quad (16)$$

Increasing the system size at constant cell volume leads to a field theory where the pdf of the coarse grained velocity $\mathbf{u}(\mathbf{x})$, internal energy density $u(\mathbf{x})$, and particle number density $n(\mathbf{x})$ fields is

$$\mathrm{p}\left[\mathbf{u}(\mathbf{x}), u(\mathbf{x}), n(\mathbf{x})\right] = \frac{1}{Z}\exp\left[-\frac{1}{2k_b}\int (\delta\mathbf{u}(\mathbf{y})K(\mathbf{y},\mathbf{x})\delta\mathbf{u}(\mathbf{x}) + \right.$$
$$-\delta u(\mathbf{y})\left.\frac{\delta^2 S}{\delta u(\mathbf{y})\delta u(\mathbf{x})}\right|_* \delta u(\mathbf{x}) - 2\delta u(\mathbf{y})\left.\frac{\delta^2 S}{\delta u(\mathbf{y})\delta n(\mathbf{x})}\right|_* \delta n(\mathbf{x}) + \quad (17)$$
$$\left. -\delta n(\mathbf{y})\left.\frac{\delta^2 S}{\delta n(\mathbf{y})\delta n(\mathbf{x})}\right|_* \delta n(\mathbf{x})\Big)\,\mathrm{d}V_{\mathbf{y}}\mathrm{d}V_{\mathbf{x}}\right],$$

with the functional derivatives evaluated at equilibrium, $\mathbf{u}_*(\mathbf{x})$, $u_*(\mathbf{x})$, $n_*(\mathbf{x})$.

For a classical fluid, the entropy is given by the integral of the entropy density

$$S_{cf}\left[u(\mathbf{x}), n(\mathbf{x})\right] = \int s_b\left(u(\mathbf{x}), n(\mathbf{x})\right)\mathrm{d}V_{\mathbf{x}}, \quad (18)$$

where the subscript b stands for "bulk". In these conditions, using mass density and temperature as independent variables,

$$\left.\frac{\partial^2 s_b}{\partial u^2}\right|_* \delta u^2 + 2\left.\frac{\partial^2 s_b}{\partial u\partial n}\right|_* \delta u\delta n + \left.\frac{\partial^2 s_b}{\partial n^2}\delta n^2\right|_* = -\frac{c_T^{*\,2}}{\rho_m^*\theta^*}\delta\rho_m^2 - \frac{\rho_m^* c_v^*}{\theta^{*2}}\delta\theta^2, \quad (19)$$

the pdf becomes

$$\mathrm{p}\left[\mathbf{u}(\mathbf{x}), u(\mathbf{x}), n(\mathbf{x})\right] = \frac{1}{Z} \times$$
$$\exp\left[-\frac{1}{2k_b}\int\left(K_*|\delta\mathbf{u}(\mathbf{x})|^2 + \frac{c_T^{*\,2}}{\rho_m^*\theta^*}\delta\rho_m(\mathbf{x})^2 + \frac{\rho_m^* c_v^*}{\theta^{*2}}\delta\theta(\mathbf{x})^2\right)\mathrm{d}V_{\mathbf{x}}\right], \quad (20)$$

where, as expected of an equilibrium classical fluid, the velocity-velocity is diagonal, $K(\mathbf{x},\mathbf{y})|_* = K_*\delta(\mathbf{y}-\mathbf{x})$.

For capillary fluids the same procedure applies, starting from the free energy (5) that provides the entropy density

$$s_{wdv} = -\frac{\delta F[\rho_m, \theta]}{\delta\theta} = s_b(\rho_m(\mathbf{x}), \theta(\mathbf{x})) = s_b(u_b(\mathbf{x}), \theta(\mathbf{x})), \quad (21)$$

and the internal energy

$$U_{vdw}[\rho_m, \theta] = F_{vdw}[\rho_m, \theta] - \int \theta \frac{\delta F_{vdw}}{\delta \theta} \mathrm{d}V_{\mathbf{x}} = \int \left(u_b(\rho_m(\mathbf{x}), \theta(\mathbf{x})) + \frac{\lambda}{2} \nabla \rho_m(\mathbf{x}) \cdot \nabla \rho_m(\mathbf{x}) \right) \mathrm{d}V_{\mathbf{x}}, \quad (22)$$

where $u_b = f_b + \theta s_b$. The essential difference with a classical fluid is the capillary contribution to the total internal energy density $u = u_b + \lambda/2|\nabla\rho|^2$. The constrained entropy functional is in this case

$$S_c = \int \left\{ s_b(u_b(\mathbf{x}), \rho_m(\mathbf{x}) + k_1 \left(e_0 - (u_b(\mathbf{x}) + \frac{\lambda}{2}|\nabla \rho_m(\mathbf{x})|^2 + \frac{1}{2}\rho_m(\mathbf{x})|\mathbf{u}(\mathbf{x})|^2) \right) + k_2 \left(\rho_0 - \rho_m(\mathbf{x})\right) \right\} \mathrm{d}V_{\mathbf{x}}, \quad (23)$$

with the extremal field obeying the equations

$$\frac{\partial s_b}{\partial u_b} - k_1 = 0,$$
$$\frac{\partial s_b}{\partial \rho_m} - k_1 \left(\frac{1}{2}|\mathbf{u}|^2 - \nabla \cdot (\lambda \nabla \rho_m) \right) - k_2 = 0. \quad (24)$$

After identifying the Lagrange multipliers as $k_1 = 1/\theta_*$ and $k_2 = -\mu_*/\theta_*$ and neglecting the velocity in a first order expansion around the equilibrium state, the second equation is reduced to Eq. (6), $\mu_b - \nabla \cdot (\lambda \nabla \rho_m) = \mu_*$.

Using Eq. (19), the quadratic expansion around the equilibrium state reads [6]

$$\mathrm{p}\left[\mathbf{u}(\mathbf{x}), u(\mathbf{x}), n(\mathbf{x})\right] = \frac{\exp\left(S_T[\mathbf{u}, u, n]\right)}{Z} = \frac{1}{Z} \times \exp\left[-\frac{1}{2k_b} \int \left(K_* |\delta \mathbf{u}(\mathbf{x})|^2 + \frac{c_T^{*\,2}}{\rho_m^* \theta^*} \delta \rho_m(\mathbf{x})^2 - \lambda |\nabla \delta \rho_m(\mathbf{x})|^2 + \frac{\rho_m^* c_v^*}{\theta^{*2}} \delta\theta(\mathbf{x})^2 \right) \mathrm{d}V_{\mathbf{x}} \right], \quad (25)$$

leading to the following equations for the correlation functions[2,3]

---

[2] For a field $v(x)$ with quadratic pdf $\mathrm{p} = \exp\left(-1/2 \int v(x) k(x,y) v(y) dx dy\right)$ the correlation is such that $\int C(x,y) k(y,z) dx = \delta(x-z)$. In general $k(x,y)$ may involve operators but, in the simple case $k(x,y) = k_* \delta(x,y)$, the equation for the correlation reduces to $k_* \int C(x,y)\delta(y-z) dx = \delta(x,z)$ whose straightforward solution is $C(x,y) = 1/k_* \delta(x-y)$.

[3] By repeated integration by parts, a functional of the form $\exp\left(\int \left(Av(x)^2 - B|\nabla v(x)|^2\right) dx\right)$ can be also written as $\exp\left(\int \left(Av(y)\delta(y-x)v(x) + Bv(y)\delta(y-x)\nabla^2 v(x)\right) dx dy\right) = \exp\left(\int v(y) \left(\left(A + B\nabla^2\right)\delta(y-x)\right) v(x) dx dy\right)$. The result then follows by the technique explained in the footnote 2.

$$\frac{1}{k_b}\left(\frac{c_T^{*\,2}}{\theta^* \rho_m^*} - \lambda \nabla^2\right) \langle \delta\rho_m(\mathbf{x})\delta\rho_m(\mathbf{y}))\rangle = \delta(\mathbf{y}-\mathbf{x}),$$
$$\frac{1}{k_b}\frac{\rho_m^* c_v^*}{\theta^{*2}} \langle \delta\theta(\mathbf{x})\delta\theta(\mathbf{y}))\rangle = \delta(\mathbf{y}-\mathbf{x}),$$
$$\frac{1}{k_b}\frac{\rho_m^*}{\theta^*} \langle \delta\mathbf{u}(\mathbf{x}) \otimes \delta\mathbf{u}(\mathbf{y}))\rangle = \mathcal{I}\delta(\mathbf{y}-\mathbf{x}), \tag{26}$$

where the mixed correlations are identically zero and $1/K_*$ has been set equal to θ^*/ρ_m^* to enforce the equipartition theorem on the coarse-grained kinetic energy.

It should be noted that the capillarity introduces a finite correlation length, as shown by the solution of the equation for the density-density correlation $\langle \delta\rho_m(\mathbf{x})\delta\rho_m(\mathbf{y}))\rangle = k_b\theta^*/(4\pi\lambda|\mathbf{y}-\mathbf{x}|)\exp\left(-|\mathbf{y}-\mathbf{x}|\sqrt{c_T^{*\,2}/(\rho^*\lambda)}\right)$ [6].

Given Eq. (23), the expectation values of observables follow by averaging over the field pdf. Specifically, the joint pdf of energy, $\hat{e}$, and particle density, $\hat{n}$, at $\hat{\mathbf{x}}$ is

$$\mathrm{p}(\hat{e},\hat{n};\hat{\mathbf{x}}) = \int \mathcal{D}e\mathcal{D}n\, \delta\left(\hat{e}-e(\hat{\mathbf{x}})\right)\, \delta\left(\hat{n}-n(\hat{\mathbf{x}})\right) \times$$
$$\frac{1}{\mathcal{Z}}\exp\left\{\frac{1}{k_b}\int s(u(\mathbf{x}),n(\mathbf{x})) - s_0 - k_1(e(\mathbf{x})-e_0) - k_2(n(\mathbf{x})-n_0)dV\right\}, \tag{27}$$

where $\int \mathcal{D}e\mathcal{D}n$ denotes functional integration—the so-called path integral see, e.g., the introductory booklet by Feynman, Hibbs and Styler [4]—over the possible configurations of the energy and density fields, respectively, and $u = e - 1/2\rho|\mathbf{u}|^2$. The result of the functional integration,[4]

$$\mathrm{p}(\hat{e},\hat{n};\hat{\mathbf{x}}) = \frac{1}{Z}\exp\left\{\frac{1}{k_b}s(\hat{u},\hat{n})) - k_1(\hat{e}-e_0) - k_2(\hat{n}-n_0)\right\}, \tag{28}$$

is shown in Fig. 5 which provides the pdf of the fluctuations occurring at a given point of a homogeneous fluid in metastable conditions for a subcritical Van der Waals fluid. The left panel shows the well-known pressure-volume diagram. The fluctuation pdf plotted on the right as a function of local temperature and particle density has two distinct maxima. Since the high-density state is a local maximum, the liquid, initially prepared in the metastable state, will sooner or later transition to the vapor.

[4] The easiest way to realize that Eq. (27) holds is to take the limit $\Delta V \to 0$ of a discretized version of the path integral (27), $\int \Pi_{k=1}^M de_k dn_k \delta(\hat{e} - e_s)\delta(\hat{n}-n_s)\exp\left\{\sum_{l=1}^M \left[s(e_l,n_l) - s_0 - k_1(e_l-e_0) - k_2(n_l-n_0)\right]\Delta V\right\}/\mathcal{Z} = \exp\left\{\left[s(\hat{e},\hat{n}) - s_0 - k_1(\hat{e}-e_0) - k_2(\hat{n}-n_0)\right]\right\}/Z$.

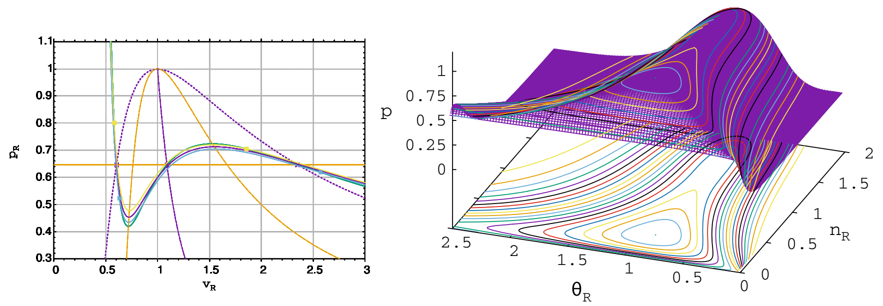

Fig. 5 Left: pressure-specific volume diagram for a Van der Waals fluid. Several lines are drawn: binodal line (purple dotted line), spinodal line (yellow dash-dotted line), a typical, subcritical isotherm (green line), the equilibrium iso-chemical potential line (purple line). Purple and green lines (purple symbols) intersect at the equilibrium pressure. Two more iso-chemical lines are plotted, in yellow and light blue, respectively, at larger and smaller chemical potential. Right: probability distribution function of equilibrium fluctuations of temperature and number density in a metastable Van der Waals liquid

4 Fluctuating Hydrodynamics of Capillary Fluids and Dynamic Nucleation Processes

The last ingredient is thermal fluctuations described by the two terms $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$ in the momentum and energy equations, Eq. (7). At a purely qualitative level, they are noise terms that force random fluctuations in the metastable fluid and trigger the transition. The noise is conservative (in divergence form), consistent with momentum and energy conservation, and is absent from the mass conservation equation.

More technically, the random contributions to stress tensor and energy flux are differential operators acting on delta-correlated in time and space, Gaussian processes with the appropriate tensorial structure [6]. After the noise terms are added, the system of partial differential equations (PDEs) for the capillary fluid is turned into a system of stochastic partial differential equations (SPDEs) which are, basically, the extension to fields of the Langevin equations for a discrete system. It is instrumental to note that the right hand side of the linearized form of mass, momentum and energy conservation, Eq. (7), rewritten in terms of mass density, velocity and temperature, $\boldsymbol{\Delta} = (\rho_m, \mathbf{u}, \theta)$, can be expressed as the functional derivative of the entropy S_T,[5] Eq. (25)

$$\frac{\partial \boldsymbol{\Delta}}{\partial t} = -\mathbf{M}\frac{\delta S_T[\boldsymbol{\Delta}]}{\delta \boldsymbol{\Delta}} + \mathbf{K}\boldsymbol{\xi}, \tag{29}$$

[5] Given a quadratic entropy functional $S[\boldsymbol{\Delta}]$, associated with the pdf $\mathrm{p}[\boldsymbol{\Delta}] = 1/\mathcal{Z}\exp(S[\boldsymbol{\Delta}]/k_b)$, its functional derivative is $\delta S/\delta\boldsymbol{\Delta} = -C^{-1}\boldsymbol{\Delta}$, where $\mathbf{C} = \langle \boldsymbol{\Delta} \otimes \boldsymbol{\Delta} \rangle$ is the correlation. A system of the form $\partial\boldsymbol{\Delta}/\partial t = \mathbf{L}\boldsymbol{\Delta} + \mathbf{f}$, can then be rewritten as $\partial\boldsymbol{\Delta}/\partial t = -\mathbf{L}\mathbf{C}\delta S/\delta\boldsymbol{\Delta} + \mathbf{f}$.

where $\mathbf{M} = \mathbf{LC}$ with $\mathbf{C} = \langle \boldsymbol{\Delta} \otimes \boldsymbol{\Delta} \rangle$ the correlation operator, see the previous section, and $\mathbf{L}$ the linearized capillary-Navier-Stokes operator.[6] In the above equation $\mathbf{f} = \mathbf{K}\boldsymbol{\xi}$ is the stochastic forcing term with $\boldsymbol{\xi}$ the delta-correlated Gaussian process with amplitude (operator) $\mathbf{K}$. The amplitude can be determined requiring that the equilibrium pdf of the fluctuating field $\mathrm{p}[\boldsymbol{\Delta}] \propto \exp\left(S_T(\boldsymbol{\Delta})\right)$ that was discussed in the previous section is a solution to the (stationary) Fokker-Planck equation associated with the Langevin equation (29) [9],

$$-\frac{\delta}{\delta \boldsymbol{\Delta}} \cdot \left(\mathbf{M} \frac{\delta S_T[\boldsymbol{\Delta}]}{\delta \boldsymbol{\Delta}} \exp\left(S_T[\boldsymbol{\Delta}]\right) \right) + \frac{\delta^2}{\delta \boldsymbol{\Delta} \otimes \delta \boldsymbol{\Delta}} : \left(\frac{\mathbf{K}\mathbf{K}^\dagger}{2} \exp\left(S_T[\boldsymbol{\Delta}]\right) \right) = 0. \quad (30)$$

After noticing that only the Onsager operator, basically the Hermitian component of $\mathbf{M}$, $k_b \mathbf{O} = -1/2 \left(\mathbf{M} + \mathbf{M}^\dagger \right)$, contributes to the first term in the Fokker-Planck equation, the noise operator is determined as $\mathbf{K} = \sqrt{2 k_b \mathbf{O}}$.

In conclusion, the noise follows from enforcing the so-called fluctuation-dissipation balance which requires the noise to drive the system to the equilibrium correlations. The procedure is, in fact, the extension of the method Einstein devised for Brownian motion, derived here from the Fokker-Planck equation of the relevant SPEDs, see [5] for a different approach.

With the appropriate noise intensity operator introduced in the balance equations (7),[7] we arrive at the extension to capillary fluids of the so-called fluctuating hydrodynamics, originally introduced by Lifschitz and Landau for Newtonian fluids.

Figure 6 provides an example of the results that are obtained by solving what we like to call the Lifschitz-Landau-Van der Waals-Navier-Stokes equations, Eq. (7), in honor of the scientists that initiated the approach that we pushed further to simulate nucleation in macroscopic systems. The numerical solution requires ad hoc numerics [2], here based on centered finite difference schemes on staggered grids, to enforce conservation, coupled with Runge-Kutta methods purposely designed for stochastic equations. The left panel shows a snapshot along the nucleation process in a bulk, metastable liquid. The large number of bubbles that are simultaneously nucleated demonstrate the unprecedented ability of the approach in describing nucleation in macroscopic systems. Comparison of the nucleation rates—namely the number of bubbles formed per unit time and volume—with state-of-the-art results available in

[6] Denoted by k the heat conduction coefficient and μ the viscosity, the linearized capillary Navie-Stokes operator is

$$\mathbf{L} = \begin{bmatrix} 0 & -\rho_m^* \nabla \cdot & 0 \\ -{c_T^*}^2/\rho_m^* \nabla + \lambda \nabla \nabla^2 & \mu/\rho^* \left(\nabla^2 - \frac{1}{3} \nabla \nabla \cdot \right) & -\, 1/\rho_m \partial p/\partial \theta|_* \nabla \\ 0 & \theta^*/(\rho_m^* c_v)\ 1/\rho_m \partial p/\partial \theta|_* \nabla \cdot & k/(\rho_m^* c_v \nabla^2) \end{bmatrix}.$$

[7] The divergence operators appearing in Eq. (7) are obtained from the fluctuation-dissipation balance as part of the noise intensity operator $\mathbf{K}$.

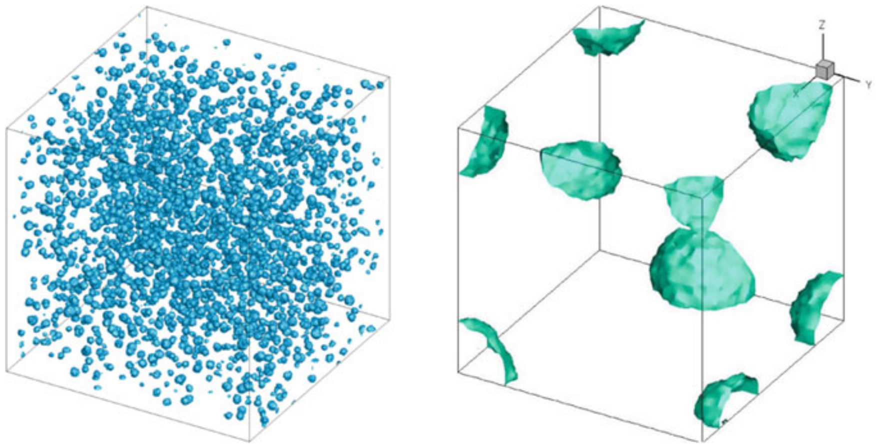

Fig. 6 Left: Bubbles dynamically nucleated in an initially homogeneous liquid in tensile conditions. The large size of the sample allows the simultaneous nucleation of a large number of bubbles and their dynamic interactions. Right: Nucleation in a cubic cavity. The favored nucleation sites are the eight vertices of the box, followed by the edges and finally the bulk region

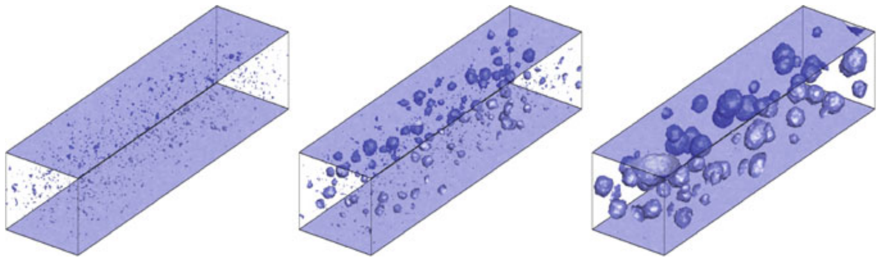

Fig. 7 Bubble nucleation process in a flowing, metastable liquid. From left to right, three successive snapshots showing the nucleating bubbles in a Couette flow

the literature confirm the accuracy of the approach. An important feature is that once nucleated, the bubbles can be followed into their highly non-linear stage [8].

The right panel in the figure provides the bubble configuration for nucleation in a highly confined environment. Here a cubic cavity containing a liquid is pushed into metastable conditions by decreasing the pressure. Nucleation is seen to take place at the solid boundary, particularly at the vertices and edges formed by the solid walls. This is a clear example of heterogeneous nucleation in a macroscopic system, something that has longly been out of reach of any other available model.

A unique feature of the theory we have set up is the direct coupling of the nucleation process with the macroscopic flow dynamics. Figure 7 shows three successive configurations of the nucleated bubbles in a Couette flow, i.e. the flow between plane parallel walls, one of which is fixed while the other moves at a constant speed. Nucleated bubbles are transported by the flow while they grow. Their presence influences the flow and the drag exerted on the wall. More importantly, this flow configuration allows tackling the effect of the shear on the bubble nucleation rate in different conditions of metastability, wall wettability, and shear level.

5 Conclusions and Perspectives

In the present chapter, we have reviewed our recent results concerning the nucleation of a new phase in an otherwise homogeneous fluid. The potential of this new approach was illustrated using as an example the nucleation of bubbles in a metastable liquid. Different conditions were addressed, ranging from bulk, homogeneous nucleation to heterogeneous nucleation on solid walls with different wettability, either hydrophobic or hydrophilic (more generally lyophobic/lyophilic) and including nucleation in flowing liquids. The problem is extremely relevant for issues related to cavitation and boiling, e.g. for heat transfer. The reverse condition of condensation in a metastable vapor can be dealt with along the very same lines. A unique feature of the approach we have been pursuing is the capability to bridge the gap between the microscopic (atomistic) scales where nucleation starts and the macroscopic scales where bubbles/drops couple to the macroscopic flow dynamics. We are aware that, for the moment, we have just initiated an approach that may, and should, be extended in many respects, like when considering droplet condensation/evaporation over surfaces in contact within a gaseous environment made of moist air. Within the obvious space limitations, we have tried to provide a clear-cut illustration of the basic principles, addressing the reader to the published literature for the many technical and physical details. We have left aside the interesting problem of how rare events can fit the present framework. They are discussed in [12] where, for the first time to our knowledge, the cavitation limit of pure water is given predictions that perfectly match the best available experimental data.

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Homogenized Boundary Conditions for Micro-Textured Surfaces

Alessandro Bottaro

Abstract Multi-scale homogenization is a theory that permits to derive boundary conditions which model the effect of microscopically patterned surfaces on the macroscale flow of a fluid. The procedure revolves around the separation of spatial scales, and comprises two steps. The first consists in solving auxiliary problems in a representative volume element, to identify appropriate tensorial coefficients. In the second step, the coefficients are used in effective conditions of the macroscale problem at a smooth virtual boundary, thus avoiding the need of numerically solving fine-grained details near the textured wall. The approach can be applied to describe the effect of wall riblets on the skin friction drag for a turbulent boundary layer flow, to treat the motion of a fluid near a porous layer, or to assess the macroscopic behavior of a liquid near regularly patterned superhydrophobic surfaces. This chapter summarizes the main recent results obtained by this powerful modeling technique.

Keywords Multiscale homogenization · Upscaling · Beavers-Joseph-Saffman-Jones condition

1 Micro-Patterned Surfaces and Upscaling

The no-slip boundary condition at a solid, impermeable surface is one of the basic tenets of the continuum mechanics approximation of fluids, although problems arise when enforcing it, for example, at a moving triple line or at the interface between an aqueous solution and a superhydrophobic surface.

Most surfaces in nature and technology are not smooth, but textured in a more or less regular way, eventually porous and/or compliant. Some examples are displayed in Fig. 1, illustrating the rich variety of possible patterns.

It is clear that in computer simulations of fluid flows over and through micro-textured surfaces, such as the ones shown, extremely fine grids are required in the proximity of the surface, to capture details of the motion. If knowledge of such

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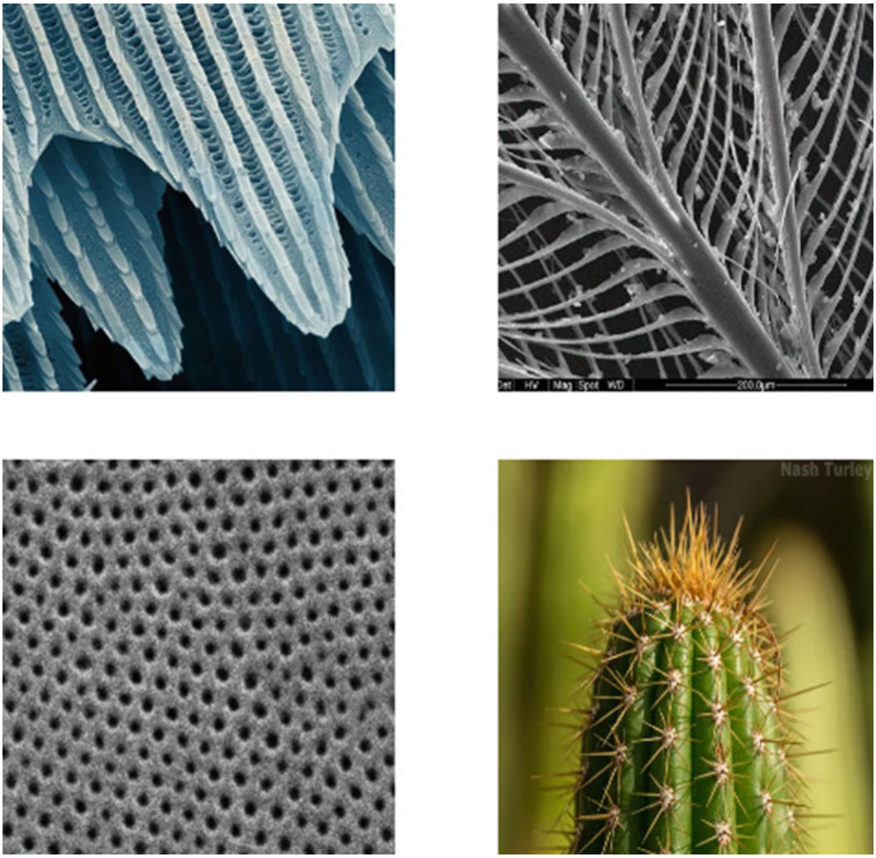

Fig. 1 Some regularly patterned surfaces. Left to right, top to bottom: SEM image of overlapping porous scales on the wing of a butterfly; bird's feather, with rachis, barbs and barbules; polysterene membrane for water purification; spines on a cactus

fine-grained details is not needed, multiscale homogenization can be used to obtain *effective* boundary conditions, to yield accurate numerical solutions of the macro-scale behavior at a fraction of the cost of fully-feature-resolving simulations. The most well-known effective condition goes by the name of *Navier slip*, and was originally proposed by Claude-Louis Navier as a boundary condition at a solid wall when he derived, for the first time, the differential equations of motion which now bear his name [25].

Homogenization (or upscaling) applies to a large variety of problems in physics and *effective* properties arising from this theory range, for example, from the thermal conductivity of a heterogeneous medium to the permeability of a porous matrix. The word homogenization was probably first used by Babuška [1] who described it as the approach which studies the macro-behaviour of a medium by its micro-properties, by replacing the rapidly varying properties of a heterogeneous material with those of an equivalent homogeneous one. A heterogeneous material is one composed by regions made by different materials or phases, and the equivalent properties stem from the solution of auxiliary problems defined in microscopic domains. Note that the prefix micro used throughout this paper does not refer to the actual size of 1 μm, but to the length scale difference between the internal, small scale of the medium or interface of interest and the external, macroscopic one. The existence of (at least) two well-separated length scales, l and L, renders the governing differential equations amenable to a solution via a formal asymptotic expansion in terms of a parameter $\epsilon = l/L \ll 1$, so that results can in principle be searched up to any order of accuracy in ϵ. The multiscale expansion lies at the heart of the homogenization approach described by Mei and Vernescu [23]. Another widely used technique in fluid mechanics is the so-called volume-averaging approach, pioneered by Whitaker [31]. Both approaches rely (i) on the definition of a periodic microscopic volume, called the *unit cell* or the *representative volume element (RVE)*, to capture fine-scale details, (ii) on scaling the equations, (ii) on approximating them neglecting small terms and, finally, (iv) on suitably averaging the fields over such a representative cell.

In fluid mechanics, upscaling has been successfully employed in particular for the case of flows through fluid-saturated porous media. The procedure leading to the equation first given empirically by Darcy [7] has been formally derived a number of times on the basis, for example, of asymptotic homogenization [10] or volume averaging [30]. A comparison between these two upscaling procedures is described by Davit et al. [8]. The crux of the matter, as anticipated, is the definition of an *RVE*, whose existence requires the existence of scale separation of spatial medium fluctuations. For systems which exhibit small scale structural disorder the *RVE* should have characteristic length scale l larger than l_{corr}, where the correlation length l_{corr} represents the smallest distance for which statistical independence occurs between two material sections. On the other hand, the size of the *RVE* must be small compared to the system size L, since otherwise the determination of the effective behavior would be just as time-consuming as the simulation of the entire system with all microscopic details. In practice, the volume of the *RVE* is the minimal volume adequate to provide desired effective properties of the heterogeneous material, i.e. porosity or permeability. Above this minimal volume the macroscopic properties obtained via suitable averaging over the *RVE* itself do not vary any longer. If the porous matrix is formed by a periodic repetition of solid inclusions and voids, the *RVE* typically coincides with the unit cell, i.e. the smallest unit over which periodicity can be enforced, at least as long as inertia within the porous medium is not significant [5].

The averaging operations to be used in upscaling are the so-called *superficial* or *phase averaging* and the *intrinsic averaging*. The former is defined as

$$\langle h \rangle = \frac{1}{\Omega} \int_{\Omega_f} h \, \mathrm{d}\Omega, \tag{1}$$

with the symbol Ω used to denote the total volume of the *RVE*, Ω_f being its fluid-filled portion; h is a generic real function of microscale variables. The porosity θ of the medium is thus defined by

$$\theta = \frac{\Omega_f}{\Omega} = \langle 1 \rangle. \tag{2}$$

The intrinsic averaging is defined as

$$\langle h \rangle^f = \frac{1}{\Omega_f} \int_{\Omega_f} h \, \mathrm{d}\Omega = \frac{\langle h \rangle}{\theta}. \tag{3}$$

Eventually, the upscaling procedure for the flow in a porous medium leads to the following macroscopic equation for the components of the seepage velocity, $\langle \hat{u}_i \rangle$, valid in the bulk of the medium:

$$\langle \hat{u}_i \rangle = -\frac{\hat{\mathcal{K}}_{ij}}{\mu} \frac{\partial \langle \hat{p} \rangle^f}{\partial \hat{x}_j}, \tag{4}$$

with $\hat{\mathcal{K}}_{ij}$ the tensorial permeability of the medium, μ the dynamic viscosity of the fluid, and $\langle \hat{p} \rangle^f$ the intrinsic averaged pressure field. For the simple case of a homogeneous and isotropic porous medium it is $\hat{\mathcal{K}}_{ij} = \hat{\mathcal{K}} \, \delta_{ij}$, with $\hat{\mathcal{K}}$ a scalar, θ-dependent variable, and δ_{ij} the Kronecker delta; Darcy's equation reduces to

$$\langle \hat{u}_i \rangle = -\frac{\hat{\mathcal{K}}}{\mu} \frac{\partial \langle \hat{p} \rangle^f}{\partial \hat{x}_i}. \tag{5}$$

Away from the bulk of the porous medium matters become more complex, mostly because the (convenient) assumption of periodicity at the boundaries of the *RVE* must be relaxed. Darcy's equation is thus not formally valid near boundaries. In the past, efforts have been devoted to address the issues of the appropriate conditions which should be enforced at the interface between porous and impermeable media [11] or when fractures are present within a porous medium [29]. Another example of *interface condition* of interest is that between a clear fluid and a porous medium, described next.

2 Upscaling at the Dividing Surface Between a Free-Fluid Region and a Porous Medium

The study of the conditions to apply at the dividing surface between a free-fluid region and a fluid-saturated porous medium has been pursued by several investigators; a short summary of the main contributions until 2009, relative to the case of a pressure-

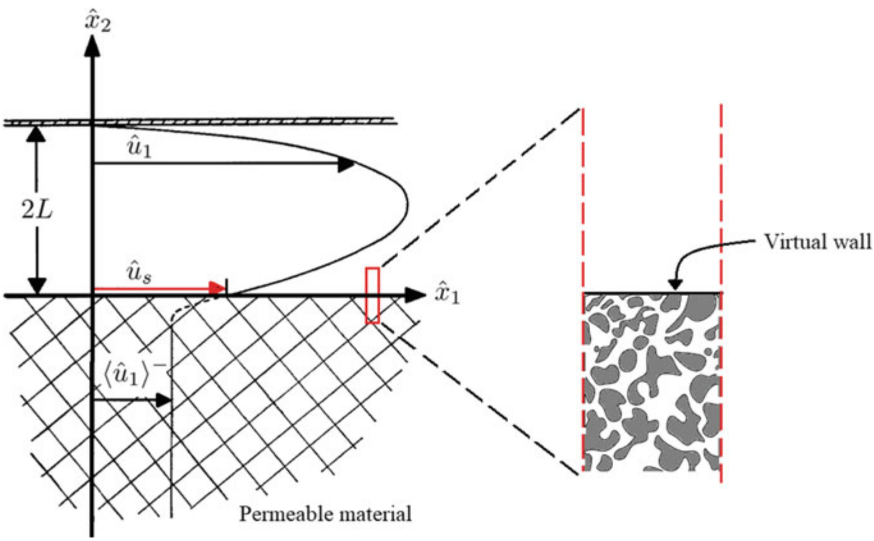

Fig. 2 Sketch of channel flow bounded by a permeable layer. On the right, between dashed red lines, a close up of a microscopic, near-interface, *RVE* is shown

driven channel flow (cf. Fig. 2 for a sketch of the configuration and for main notations employed) bounded by a permeable medium, has been published by Nield [26]. The first researchers to realized that a slip velocity existed at the dividing surface were Beavers and Joseph [2], when they observed a volume flux enhancement in the presence of a porous boundary. They expressed the macroscopic tangential slip velocity $\hat{u}_s = \hat{u}_1\big|_{x_2=0^+}$ as

$$\hat{u}_s = \langle\hat{u}_1\rangle^- + \frac{\hat{\mathcal{K}}^{1/2}}{\alpha_{BJ}} \frac{d\hat{u}_1}{d\hat{x}_2}\bigg|_{\hat{x}_2=0^+}, \tag{6}$$

where $\langle\hat{u}_1\rangle^-$ is the seepage velocity in the porous medium evaluated a sufficient distance below the dividing surface, given by Darcy's law as

$$\langle\hat{u}_1\rangle^- = -\frac{\hat{\mathcal{K}}}{\mu} \frac{d\langle\hat{p}\rangle^f}{d\hat{x}_1}\bigg|_{\hat{x}_2=0^-}. \tag{7}$$

The dimensionless empirical constant α_{BJ} in (6), a function of the structure of the permeable matrix, was fitted by Beavers and Joseph to the average pore size of the different media tested. In today's terminology the parameter $\hat{\mathcal{K}}^{1/2}/\alpha_{BJ}$ is a *slip length* and is usually denoted by $\hat{\lambda}$.

The condition by Beavers and Joseph was put on firm footing by Saffman [28] who introduced an intermediate layer between the clear fluid region (described by Stokes equations) and the isotropic bulk porous region (described by Darcy's law), and expressed the one-dimensional velocity in such an intermediate region in terms of delta function derivatives. Saffman carried out asymptotic matching at the outer boundaries of the intermediate layer to demonstrate that the slip velocity had the form

$$\hat{u}_s = \hat{\lambda}\,\frac{d\hat{u}_1}{d\hat{x}_2}\bigg|_{\hat{x}_2=0^+} + \mathcal{O}(\hat{\mathcal{K}}). \tag{8}$$

The interface condition above was later confirmed by Jäger and Mikelić [12] and many others. A little-noticed extension of Saffman's development reveals that the $\mathcal{O}(\hat{\mathcal{K}})$ term is equal to $-\frac{B\,\hat{\mathcal{K}}}{\mu}\frac{d\langle\hat{p}\rangle^f}{d\hat{x}_1}\Big|_{\hat{x}_2=0^-}$, with B an order one parameter. This suggests a modification of the permeability $\hat{\mathcal{K}}$ in the proximity of the dividing surface, an expected result on account of the fact that solid grains there are not surrounded throughout by identical grains, i.e. they are not as densely packed as in the bulk of the porous medium. Later, Jones [13] extended Saffman's result to account also for a vertical velocity at the dividing line, for the slip speed to eventually read

$$\hat{u}_s = \hat{\lambda}\left[\frac{\partial\hat{u}_1}{\partial\hat{x}_2} + \frac{\partial\hat{u}_2}{\partial\hat{x}_1}\right]\bigg|_{\hat{x}_2=0^+} - \frac{\hat{\mathcal{K}}^{itf}}{\mu}\frac{\partial\langle\hat{p}\rangle^f}{\partial\hat{x}_1}\bigg|_{\hat{x}_2=0^-}, \tag{9}$$

with $\hat{\mathcal{K}}^{itf} = B\,\hat{\mathcal{K}}$; the superscript itf is short for *interface*. Equation (9) is called the Beavers-Joseph-Saffman-Jones condition. If the constant B is equal to one, the result by Beavers and Joseph (with Jones correction) is recovered. However, on the one hand B is in general different from one and, on the other, until recently no clear indications were given to relate the parameters $\hat{\lambda}$ and $\hat{\mathcal{K}}^{itf}$ to the porosity and geometrical structure of the permeable medium. Furthermore, in a case not as simple as the one-dimensional laminar channel flow it should not be surprising to find a non-zero velocity across the dividing surface. The issue of the interface-normal component of the fluid speed had not been addressed until a few years ago.

Recently, two groups [16, 24] have employed multiscale homogenization, by expanding the microscopic variables in powers of the small parameter $\epsilon = l/L$ and carrying out asymptotic matching at the outer edges of the *RVE*, to formally derive the conditions to be enforced at the dividing surface; the full auxiliary problems necessary to compute slip lengths and interface permeability coefficients have also been given. If, away from the domain boundaries, the porous medium is isotropic, the three-dimensional conditions at $\hat{x}_2 = 0^+$ are:

$$\hat{u}_1\big|_{0^+} = \underbrace{\hat{\lambda}\left(\frac{\partial\hat{u}_1}{\partial\hat{x}_2} + \frac{\partial\hat{u}_2}{\partial\hat{x}_1}\right)\bigg|_{0^+}}_{first-order\ term} + \underbrace{\frac{\hat{\mathcal{K}}^{itf}}{\mu}\frac{\partial}{\partial\hat{x}_1}\left(-\hat{p} + 2\mu\frac{\partial\hat{u}_2}{\partial\hat{x}_2}\right)\bigg|_{0^+}}_{second-order\ term}, \tag{10}$$

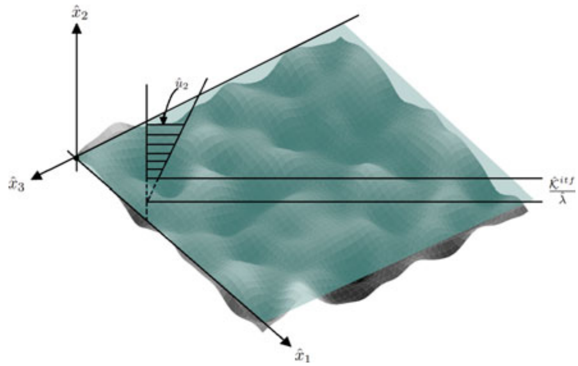

Fig. 3 Sketch of the penetration distance for the vertical velocity over a rough, impermeable surface, isotropic in the $(\hat{x}_1, \hat{x}_3)$ plane

$$\hat{u}_2\big|_{0^+} = \underbrace{\frac{\hat{\mathcal{K}}^{itf}}{\hat{\lambda}}\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\bigg|_{0^+} + \frac{\hat{\mathcal{K}}}{\mu}\frac{\partial}{\partial \hat{x}_2}\left(-\hat{p} + 2\mu\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\right)\bigg|_{0^+}}_{second-order\ terms}, \tag{11}$$

$$\hat{u}_3\big|_{0^+} = \underbrace{\hat{\lambda}\left(\frac{\partial \hat{u}_3}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_3}\right)\bigg|_{0^+}}_{first-order\ term} + \underbrace{\frac{\hat{\mathcal{K}}^{itf}}{\mu}\frac{\partial}{\partial \hat{x}_3}\left(-\hat{p} + 2\mu\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\right)\bigg|_{0^+}}_{second-order\ term}. \tag{12}$$

The order of the different terms contributing to the dividing-surface conditions is indicated below each one of them in terms of the small parameter ϵ. It is important to observe that the expressions above do not require the solution of Darcy's system in the permeable medium; the effect of the porous matrix appears only via $\hat{\lambda}$, $\hat{\mathcal{K}}$ and $\hat{\mathcal{K}}^{itf}$, available by solving auxiliary systems in the *RVE* [24]. Thus, on the one hand empirical coefficients are absent from the *effective* dividing-surface conditions and, on the other, a coupled Darcy-Stokes solution procedure, such as that described by Discacciati et al. [9], is not needed. Equation (10) coincides with the Beavers-Joseph-Saffman-Jones condition (9), once the balance of normal forces at the interface is accounted for, i.e.

$$\langle \hat{p} \rangle^f\big|_{0^-} = \hat{p} - 2\mu\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\bigg|_{0^+}. \tag{13}$$

When using (9), the coupling with the solution in the porous medium is re-established. Equation (11) shows that the vertical speed at the dividing surface is the sum of a Darcy-like term and a transpiration term, the latter of Navier-slip type. Clearly, it is not appropriate to speak about *slip* for the wall-normal component of the velocity; if the permeability of the medium is vanishingly small, and the origin of the $\hat{x}_2$ axis is placed at the upper rim of the roughness elements, the vertical velocity goes locally to zero a penetration distance equal to $\hat{\mathcal{K}}^{itf}/\hat{\lambda}$ below the $\hat{x}_2 = 0$ surface (cf. Fig. 3).

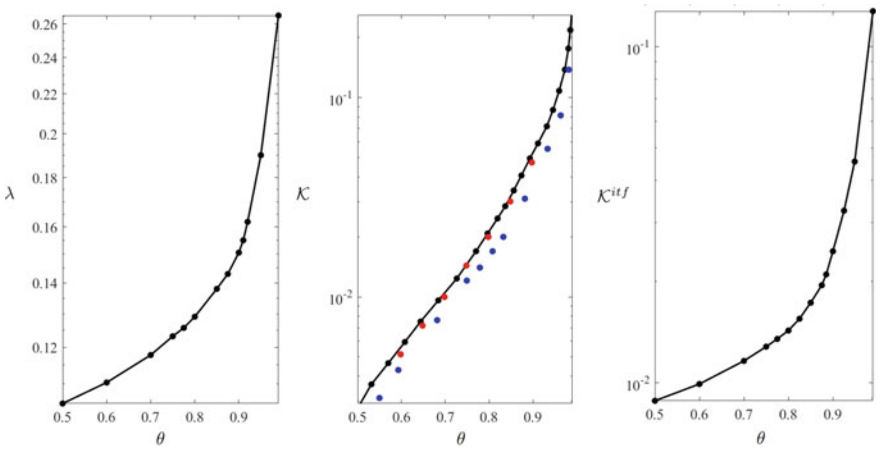

Fig. 4 Dimensionless slip coefficient (normalized by l) and permeability coefficients (normalised by l^2) for an isotropic porous bed composed by identical spheres, regularly arranged in a cubic grid. In the central frame, in red dots the points computed by Zampogna and Bottaro [32], in blue dots the permeability coefficient of a regular arrangement of Wigner-Seitz grains [23]. Since $\mathcal{K}^{itf} = B\mathcal{K}$ the results in the two right frames can be used to show the variation with θ of Saffman's parameter, B

For the case of a porous medium formed by regularly spaced spheres, with periodicity l, assuming that the dividing surface is tangent to the upper row of spherical grains, the coefficients of interest are plotted in Fig. 4. All coefficients have an increasing trend with the medium porosity θ, an expected behavior since the motion of the fluid is made easier by the growth of the inter-pore space. As the porosity is reduced, the medium permeability $\hat{\mathcal{K}}$ decreases much faster than the interface permeability, $\hat{\mathcal{K}}^{itf}$; when $\hat{\mathcal{K}}$ tends to zero there is a non-vanishing interface permeability, related to the *breathing* of fluid at the virtual interface.

If the permeable medium is anisotropic, the scalar coefficients become rank-2 and rank-3 tensors and the general form of the condition at the dividing surface in $\hat{x}_2 = 0^+$, formally valid up to order ϵ^2, is

$$\hat{u}_i = \frac{\hat{\lambda}_{ij}}{\mu}\,\hat{S}_{j2} + \frac{\hat{\mathcal{K}}^{\dagger}_{ijk}}{\mu}\frac{\partial \hat{S}_{k2}}{\partial \hat{x}_j}, \tag{14}$$

with

$$\hat{S}_{j2} = -\hat{p}\,\delta_{j2} + \mu\left(\frac{\partial \hat{u}_j}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_j}\right). \tag{15}$$

The tensor $\hat{\mathcal{K}}^{\dagger}_{ijk}$ is a property of the porous medium that measures its capacity to let fluid through, both in the bulk of the porous medium and near its boundary with the free fluid region. Both $\hat{\lambda}_{ij}$ and $\hat{\mathcal{K}}^{\dagger}_{ijk}$ are found by solving Stokes-like systems in the *RVE* [6, 16, 24].

3 The Case of a Rough, Impermeable Wall

If the boundary cannot be permeated by the fluid, the effective condition in $\hat{x}_2 = 0^+$ remains that given by equation (14); however, now, $\hat{\mathcal{K}}^{\dagger}_{ijk}$ includes only interface permeability terms, i.e. those which account for the ability of the fluid in the physical domain to move in and out across the virtual surface and around the roughness elements. All terms which depend on the solid medium permeability, $\hat{\mathcal{K}}_{ij}$, simply vanish.

If we consider a rough, solid wall with a well defined texture, it is easy to isolate an *RVE* and compute the coefficients of interest. For example, if the wall is formed by spanwise-periodic (or streamwise-periodic) grooves aligned along the longitudinal (or the spanwise) direction it results:

$$\hat{u}_1\Big|_{\hat{x}_2=0^+} = \hat{\lambda}_x \left[\frac{\partial \hat{u}_1}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_1}\right]_{\hat{x}_2=0^+} + \frac{\hat{\mathcal{K}}^{itf}_{xy}}{\mu}\frac{\partial}{\partial \hat{x}_1}\left[-\hat{p} + 2\mu\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\right]_{\hat{x}_2=0^+}, \tag{16}$$

$$\hat{u}_2\Big|_{\hat{x}_2=0^+} = -\hat{\mathcal{K}}^{itf}_{xy}\frac{\partial}{\partial \hat{x}_1}\left[\frac{\partial \hat{u}_1}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_1}\right]_{\hat{x}_2=0^+} - \hat{\mathcal{K}}^{itf}_{yz}\frac{\partial}{\partial \hat{x}_3}\left[\frac{\partial \hat{u}_3}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_3}\right]_{\hat{x}_2=0^+}, \tag{17}$$

$$\hat{u}_3\Big|_{\hat{x}_2=0^+} = \hat{\lambda}_z \left[\frac{\partial \hat{u}_3}{\partial \hat{x}_2} + \frac{\partial \hat{u}_2}{\partial \hat{x}_3}\right]_{\hat{x}_2=0^+} + \frac{\hat{\mathcal{K}}^{itf}_{yz}}{\mu}\frac{\partial}{\partial \hat{x}_3}\left[-\hat{p} + 2\,\mu\frac{\partial \hat{u}_2}{\partial \hat{x}_2}\right]_{\hat{x}_2=0^+}. \tag{18}$$

Notice in particular the presence of a non-vanishing local transpiration velocity of order ϵ^2 (the whole expression on the right-hand-side of Eq. 17), linked to the gradient of the shear stress components at $\hat{x}_2 = 0^+$. Clearly there can be no net mass flux through the fictitious wall, i.e. $\hat{u}_2$ integrated over the whole virtual wall must vanish. However, locally there can be a second order (in ϵ) wall-normal component. This should come as no surprise and has been found to occur also in other cases. The most notable example of this is probably the expression of the mean velocity in a porous medium in contact with an impermeable wall (of normal vector parallel to $\hat{x}_2$). For isotropic porous media of permeability $\hat{\mathcal{K}}$ it has been reported that the leading term of the velocity $\hat{u}_2$ at the impermeable boundary in $\hat{x}_2 = 0$ is proportional to $\sqrt{\hat{\mathcal{K}}}\ \partial \hat{u}_2/\partial \hat{x}_2$ [11], with a penetration distance of order $\sqrt{\hat{\mathcal{K}}}$ (cf. Fig. 3).

3.1 Superhydrophobic Surfaces in the Cassie-Baxter State

Superhydrophobic materials constitute an interesting test-bed for the application of homogenization theory. Surfaces made with such materials can significantly reduce flow resistance in the manipulation of small volumes of aqueous fluids [17, 18]. The key to such a reduction lies in the presence of a layer of gas or water vapor (the so-called *plastron*) in between the micro- or nano-structured solid surface. Such a plastron yields an effective slip which entails appreciable drag reduction. The amount of slip, usually quantified by a slip length, is mainly associated to the structural features of the superhydrophobic surface (i.e. the periodicity of the pattern, the solid fraction, the pattern type) and affected by the state of the liquid gas interface.

For the approximation of flat, non-deformable water-gas interface, assumed to be positioned in $\hat{x}_2 = 0$, it is easy to show that the effective conditions for streamwise- or spanwise-aligned microgrooves are

$$\hat{u}_1\big|_{\hat{x}_2=0^+} = \hat{\lambda}_x \left.\frac{\partial \hat{u}_1}{\partial \hat{x}_2}\right|_{\hat{x}_2=0^+}, \quad \hat{u}_2\big|_{\hat{x}_2=0^+} = 0, \quad \hat{u}_3\big|_{\hat{x}_2=0^+} = \hat{\lambda}_z \left.\frac{\partial \hat{u}_3}{\partial \hat{x}_2}\right|_{\hat{x}_2=0^+}, \qquad (19)$$

correct up to *second order* in ϵ. Different types of solid patterns have been considered in the literature. For example, in the case of periodic, longitudinal ridges (cf. the top right image of Fig. 5) Philip [27] has shown that $\hat{\lambda}_x = 2\,\hat{\lambda}_z = \frac{l}{\pi}\ln\left\{\sec\left[\frac{\pi}{2}(1-\phi_s)\right]\right\}$, with ϕ_s the solid area fraction, defined as the ratio between the width of the solid ridge to the pattern's periodicity, l. When the flow is turbulent, it has been argued for the case of riblets [22] that if $\hat{\lambda}_x$ is larger than $\hat{\lambda}_z$ "secondary cross-flow will experience a higher viscous dissipation, just as if it flowed in a narrower duct, than the main longitudinal flow", thus reducing near-wall turbulence. The same holds in the case of superhydrophic surfaces and leads to the conclusion that, to leading order, only the difference between the two virtual origins, $\Delta\hat{\lambda} = \hat{\lambda}_x - \hat{\lambda}_z$, plays a role in quantitatively characterizing the amount of drag reduction. This is addressed next.

3.1.1 Luchini's Analytical Approximation

We provide a few details here of an argument that permits to derive an expression for the variation ΔC_f of the skin friction coefficient of a fully developed turbulent channel flow (the argument holds in a similar way also for a turbulent boundary layer or a pipe flow). Prandtl's friction law in a turbulent channel flow reads

$$\left(\frac{C_f}{2}\right)^{-1/2} = \frac{1}{\kappa}\ln\left[Re_b\left(\frac{C_f}{2}\right)^{1/2}\right] + B + C - D, \qquad (20)$$

with $\kappa = 0.392$, $B = 4.48$, $C = 0.43$ and $D = 2.55$ [21]; the constant C arises from the law of the wake, and D stems from relating the bulk speed U_b (which, together with the half-channel thickness L, defines the bulk Reynolds number, Re_b) to the centerline value of the velocity. Neglecting possible variations in C and D, it is well know that a shift $\Delta U^+ = \Delta B$ in the position of the logarithmic velocity law, provoked for example by surface roughness, causes a variation ΔC_f of the skin friction coefficient with respect to C_{f_0}, coefficient of a smooth wall. The superscript

$+$ is employed here to denote variables scaled in wall units, as introduced by von Karman [15]. It is easy to find from (20) that

$$\frac{\Delta C_f}{C_{f0}}\left[\frac{1}{(2C_{f0})^{1/2}}+\frac{1}{2\kappa}\right]+\left(\frac{\Delta C_f}{C_{f0}}\right)^2\left[\frac{-3/4}{(2C_{f0})^{1/2}}-\frac{1}{4\kappa}\right]+\mathcal{O}\left(\frac{\Delta C_f}{C_{f0}}\right)^3=-\Delta U^+. \tag{21}$$

If the conditions at the wall are such that two different virtual origins appear, one for the longitudinal mean flow and one for the turbulent fluctuations, a relative displacement $\Delta\lambda^+$ along the wall-normal coordinate Y^+ (i.e. $\hat{x}_2$ normalized in wall units) entails a *rigid translation* of the whole velocity profile U^+, according to its slope near the wall (where $\partial U^+/\partial Y^+ = 1$). The velocity difference with respect to the smooth wall case extends to the logarithmic layer, which is then shifted by $\Delta U^+ = \Delta\lambda^+$, so that to first order in ΔC_f it is

$$\frac{\Delta C_f}{C_{f0}} = -\frac{\Delta\lambda^+}{\left[(2C_{f_0})^{-1/2}+(2\kappa)^{-1}\right]}. \tag{22}$$

The linearized expression (22), first given by Luchini [19], is well suited to describe the initial drag decrease for the turbulent flow of water in a channel with longitudinal micro-ridges capable to capture and hold a plastron of vapor within. When $\Delta\lambda^+$ exceeds some threshold value (close to 5, i.e. the thickness of the viscous sublayer) a quadratic law, neglecting terms of order $\Delta {C_f}^3$ in equation (21), may better serve the purpose of highlighting the behavior of the skin friction coefficient. The analytical approximations plotted in Fig. 5 employ the linear relation between $\Delta\lambda^+$ and s^+ found by Philip [27], i.e.

$$\Delta\lambda^+ = \frac{-\ln(1/\sqrt{2})}{2\pi}s^+ = 0.05516\, s^+. \tag{23}$$

A possibly even better result than that displayed in the figure might have been obtained by expanding $\Delta\lambda^+$ nonlinearly, as done by Kamrin et al. [14] for the case of undulated surfaces. This might, however, not even be necessary since, when the periodicity of the wall pattern is large, the fragile plastron would collapse under the effect of hydrostatic pressure and shear forces, provoking the so-called Wenzel transition. Thus, even the full direct numerical simulation results obtained for large values of s^+ by carefully alternating no-slip and no-shear conditions at the virtual wall in $Y^+ = 0$ must be looked upon with skepticism. Such results, in fact, have been computed for flat non-deformable water-gas interfaces, in the limit of infinite surface tension.

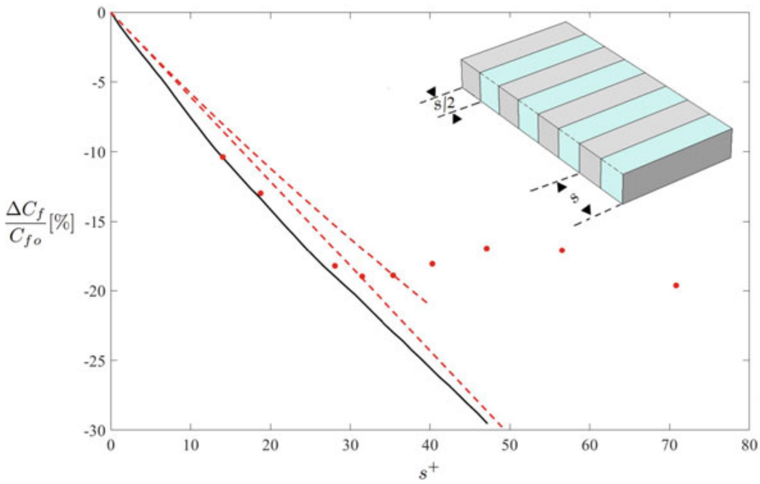

Fig. 5 Drag reduction in a turbulent channel flow with superhydrophobic walls at $Re_\tau \approx 180$ (corresponding to $Re_b \approx 2800$). The top right image shows the longitudinally micro-patterned surface, with the gas pockets in light blue color. The black solid line in the plot is obtained by direct numerical simulations with the Navier slip condition, while filled red dots indicate results obtained by modelling the boundary with alternating no-shear and no-slip patches. The red dashed lines correspond to linear and quadratic analytical approximations. Adapted from Luchini [20]

3.2 Riblets

When the working liquid permeates the whole surface, for example when the vapor plastron collapses completely or has never been present to start with, we are in a situation in which, under turbulent flow conditions, surface roughness typically produces an increase in skin friction drag with respect to the smooth-wall case. If this happens, the term ΔU^+ (sometimes termed the roughness function) becomes negative, and there is a downward shift of the velocity distribution in the Clauser plot. However, there are particular microscopic wall patterns (for example the longitudinal wall micro-grooves known as 'riblets') which yield an upward shift of U^+, associated to drag reduction.

The most important contributions to our current understanding of the effect of riblets upon the near-wall flow are due to the efforts of Dietrich Wolfgang Bechert in Berlin and Paolo Luchini in Naples. More specifically, Bechert's group demonstrated experimentally the effectiveness of a large variety of riblets' shapes upon the skin friction, and introduced the concept of protrusions heights [3, 4], and Luchini's group demonstrated and rigorously quantified this same effect in the so-called *viscous regime*, when $\Delta\lambda^+$ (and thus s^+) is sufficiently small [22]. Although not mentioned explicitly in their paper, Luchini and colleagues applied homogenization theory to first order to compute the virtual origins of mean longitudinal flow and turbulence. Some experimental results by Bechert et al. [3] are reported in Fig. 6, together with

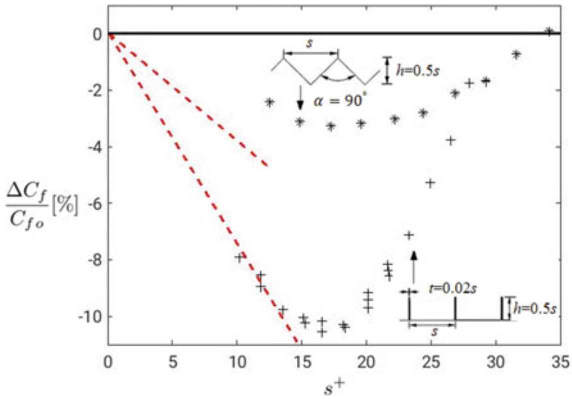

Fig. 6 Experimental results [3, 4] for the skin friction coefficients of triangular and blade riblets (symbols). The red dashed lines arise, for each case, from the linear analytical approximation by Luchini [19]

the viscous approximation (Eq. 22) by Luchini [19]. For the case of triangular riblets the linear analytical approximation does not seem to match the experimental points accurately in the low s^+ region. It is possible that a higher order approximation is needed, to account for the virtual-wall-normal velocity component, because of fluid going in and out of the riblets' valleys. This is not the case in the superhydrophobic configuration discussed previously (with a non-deformable, flat water-gas interface) for which the $\hat{x}_2$ component of the velocity at the virtual wall is zero to all orders of approximation (cf. Eq. 19).

4 Concluding Remarks

Permeable or impermeable surfaces, with regularly arranged microscopic roughness elements, are common in natural environments and technological applications, and the fluid flow over and through such surfaces can be studied effectively by the use of homogenization theory. The theory has witnessed many positive developments in these last few years and it is now possible to study the turbulent motion over rough and/or porous boundaries by employing homogenized conditions, subject to a limitation on the height of the roughness elements, which should not exceed a few viscous ('plus') units for the theory to be tenable. Despite this restriction, it is numerically advantageous to employ homogenized boundary conditions to describe a microstructured wall, for example as an initial approximation in an inverse design approach aimed at identifying those surface patterns more likely to yield a desired effect.

Acknowledgements Prof. Paolo Luchini is acknowledged for several discussions over many years on the subject of flows over rough, porous and micro-patterned surfaces. The results exposed in this chapter stem from joint work with former and current Ph.D. students including, in particular, Giuseppe A. Zampogna, Edoardo Alinovi, Sahrish B. Naqvi and Essam Nabil Ahmed. The financial support provided by the Italian Ministry of University and Research (project LUBRI-SMOOTH, PRIN2017, grant 2017X7Z8S3) is gratefully acknowledged.

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Homogenization of Periodic Architected Materials

Luigi Gambarotta, Andrea Bacigalupo, and Marco Lepidi

Abstract New avant-garde architected materials endowed with extreme stiffness, strength and lightness may be conceived through appropriate choices of the microstructural topology, mostly aimed at optimizing the periodic distribution between the solid phases and the voids. Moreover, microstructure topologies may be designed to maximize exotic mechanical properties, such as auxeticity and chirality. The tailored design of these materials is also fueled by the recent extraordinary developments in the technological fields of high-precision micro-engineering and high-fidelity additive manufacturing. Several periodic architected materials, namely lattice-like materials and rigid blocky materials, may be accurately modeled through discrete Lagrangian systems. The periodicity of the microstructure determines considerable scale effects, implying boundary layer effects and dispersive propagation of elastic waves. The need to derive synthetic descriptions of the mechanical properties and to reduce the computational burdens may motivate the formulation of non-conventional non-local homogenization techniques able of accurately describing the static and dynamic response of these materials. Within this challenging research area, the present Chapter synthesizes the most recent theoretical contributions by the Authors to the mechanical modelling of architected materials with periodic microstructure, with a methodological focus on enhanced non-local homogenization schemes, as well as on innovative surrogate optimization techniques for the spectral design of a new generation of metafilters.

Keywords Lattices · Blocky materials · Metamaterials · Continualization · Optimal design · Auxeticity · Wave propagation

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1 Introduction

The design of technologically innovative materials with enhanced or exotic mechanical properties constitutes an area of significant interest in Mechanics. Important results have been obtained with the development of composite materials, a combination of different ingredient materials spatially organized according to elaborate and optimized topologies. Artificial heterogeneous light materials with remarkable mechanical performances can be obtained with architected microstructures with empty spaces and cavities properly designed. Developments in this open research field still appear to be very promising, as evidenced by the current extensive literature on the subject and the new challenges that continuously arise in such a rapidly evolving scenario.

Lattice materials like cellular, porous, reticulated systems with regular or stochastic structure are a class of functional-structural materials whose physical and mechanical overall properties can be tailored designed by controlling the morphology of the microstructure and its constitutive properties (see for instance [1, 2]). This circumstance allows for large-scale production of truss and beam periodic reticulated lattices endowed with remarkable static and dynamic properties, by virtue of the continuously growing possibilities offered by advanced three-dimensional printing and additive manufacturing technologies (Fig. 1). Different microstructure topologies may be considered to get exotic mechanical properties such as auxeticity and chirality (see the seminal paper [4]). Recently, innovative applications of periodic lattice materials are focused on the active and passive control of the mechanical behavior through different techniques including time modulated, auxetic tunable lattices and others devices for the frequency spectral tuning (see for instance [5–9], among the others).

Periodic materials characterized by a rigid phase with dominant volumetric fraction and a soft phase with a vanishing volume fraction are common in blocky rock

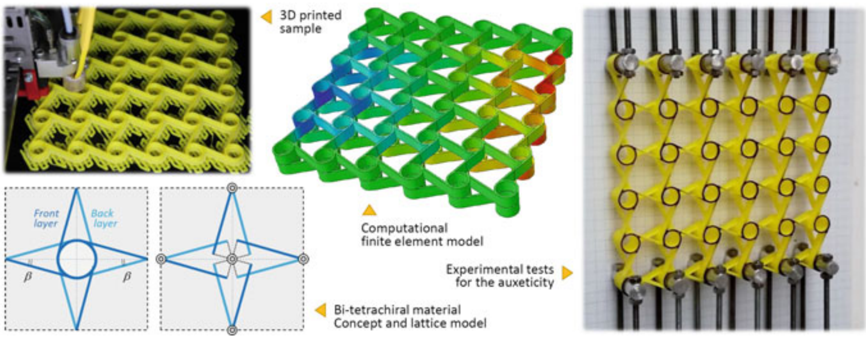

Fig. 1 A novel bi-layered topology of auxetic materials based on the tetrachiral microstructure: 3D printed sample, analytical lattice model, computational solid response, tension testing (see [3])

systems, granular materials, masonry, masonry-like biological and nacreous bio-inspired heterogeneous composites [10, 11]. Recent studies on nacreous-like materials with a microstructure similar to the running bond masonry have unveiled interesting spectral properties, in particular the existence of band gaps [12, 13] and the possibility to obtain devices for vibration reduction and isolation [14].

From the mechanical viewpoint, *lattice-like materials* with lumped mass as well as *rigid blocky materials* with elastic interfaces may be effectively modeled as discrete Lagrangian systems, in which a periodic cell with characteristic size ε and proper periodicity vectors can be identified. As well known, the periodic microstructure of these materials implies considerable scale effects, with boundary layer effects and dispersion phenomena in the propagation of elastic waves as a consequence of Mie and Bragg scattering.

Although the discrete Lagrangian modelling allows to achieve an accurate description of the mechanical behavior of the materials here considered, nevertheless the related computational burden rapidly grows up for large-dimension systems. This circumstance suggests the adoption of equivalent continuum models derived from homogenization schemes that turn out to be particularly effective for periodic assemblages. In general, these schemes provide a synthetic and rather accurate characterization of the static and dynamic behavior of the mechanical systems having a periodic microstructure in condition of "large but not too large" scale separation between the structural scale and the microstructural one (see for instance [15]). Furthermore, in order to include the scale effects due to the heterogeneity of the microstructure, it is necessary to resort to non-local homogenization schemes that introduce characteristic lengths into the continuum model allowing the description of both boundary layer effects and dispersive propagation of elastic waves.

The present chapter is devoted to a survey of the most recent contributions by the Authors to the mechanical modelling of periodic architected materials, with particular methodological emphasis on the non-conventional non-local homogenization schemes, as well as on the innovative parametric optimization techniques targeted at the spectral design of a new generation of meta-filters.

2 Homogenization of Periodic Lattice-Like Materials

Lattice-like materials with elastic massless ligaments connecting configurational nodes equipped with lumped mass are considered. The lattice periodicity vectors allow the identification of a repetitive cell characterized by the length ε (Fig. 2). Three classes of lattice-like materials are distinguished in the following and referred to as *rod-lattices*, *beam-lattices* and *cable nets*. Specifically, the kinematics of configurational nodes is described by nodal translational degrees of freedom (dofs) for the rod-lattice, by roto-translation dofs for the beam-lattice and finally by out-of-plane translational dofs for the cable net. Due to the kinematic assumptions, the mechanical properties of these materials, namely *i)* stiffness and strength, *ii)* characteristic lengths, *iii)* dispersive elastic wave propagation and others, can accurately be

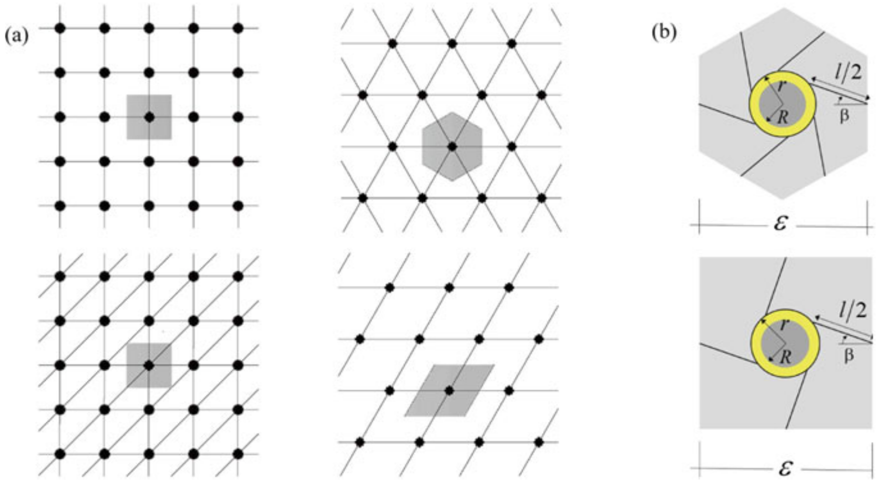

Fig. 2 Periodic lattice materials and metamaterials with lumped masses: **a** achiral materials characterized by quadrangular and triangular topologies with periodic cells, **b** periodic cell of locally resonant chiral metamaterials

described by a discrete Lagrangian model with the possibility, in the simplest systems, to achieve analytical results. A peculiar aspect of these models is the frequency band structure characterized by a finite number of branches. When complex systems with a large number of degrees of freedom in the discrete Lagrangian models have to be analyzed, the solution turns out to be computationally hard-working, so that equivalent continuum models may be preferred. These homogenized models allow the characterization of the static and dynamic behavior in a synthetic and accurate way in terms of generalized macro-displacement and its higher order gradients.

Among the different homogenization techniques of periodic lattices, also known as *standard continualization methods*, the first-order homogenization is the simplest one. A local equivalent continuum is identified by replacing the discrete equations of motion with differential governing equations. In this case, the nodal displacements in the governing equation of the discrete model are replaced by the first order Taylor polynomial expansion in ε of a continuum displacement field defined on an equivalent domain. On the other hand, it is known that these classical models are not able to catch the size effects associated to the microstructural length scale as well as to describe the elastic wave dispersion occurring in the discrete periodic material.

These drawbacks may be circumvented by exploiting *standard non-local continualization techniques* to identify equivalent continua characterized by local inertial terms and non-local constitutive tensors. More specifically, the difference of displacements of adjacent nodes in the lattice involved in the equation of motion is approximated by a properly truncated series in the characteristic length ε of a macro-displacement field that plays the role of down-scaling law. In [16] and [17] this technique has been applied to chiral and non-centrosymmetric periodic beam-lattice materials, respectively, to obtain a micropolar homogenized model. An alternative approach based on a more rigorous mathematical formalism involving the shift and pseudo-differential operators as well as their power series approximations has been developed by Bacigalupo and Gambarotta in [18], with regards to both rod-lattice and beam-lattices, and by Bacigalupo and Gambarotta in [19], with regards to both rod-lattices and cable nets. However, these papers proved in general that the resulting overall non-local elastic tensors are not unconditionally positive defined, confirming a preliminary result obtained by Bazant and Christensen in [20], for rectangular multi-storey frames, and Metrikine and Askes in [21], for rod-lattices, among the others. The resulting equivalent continua are, in general, thermodynamically inconsistent with instabilities in the acoustic branch in the short wavelengths limit. Nevertheless, for beam-lattices materials the corresponding homogenized continua present optical branches in good agreement with the Lagrangian one.

Other approaches, here conventionally called *energy based standard continualization*, consider the Lagrangian functional of the lattice-like material together with an approximation of the difference of displacements of adjacent nodes of the discrete model through a properly truncated Taylor expansion in ε of the continuum macro-displacement field. Consequently, the Lagrangian functional expressed in terms of the generalized macro-displacement, as well as its higher order gradients, are approximated to a desired order of ε and suitably manipulated through an application of the divergence theorem. This is allows considering in the energy form all the terms of the same order in ε to obtain a structure of the Lagrangian functional formally equivalent to that of a non-local continuum of the same order. Finally, the Euler–Lagrange equation is derived as well as the overall constitutive equation of the homogenized non-local continuum. In particular, non-centrosymmetric square and triangular beam-lattices have been studied in detail in [17] and the overall constitutive tensors of the micropolar continuum as well as its frequency band structure have been proved to be the same obtained by the *standard non-local continualization techniques*. Moreover, it has been shown that the same procedure in which the transformation of the Lagrangian functional based on the divergence theorem is ignored, leads to positive defined non-local constitutive tensors. These tensors coincide with those derived from the application of generalized macro-homogeneity condition expressed in terms of the Lagrangian density at the two scales. This result generalizes the observation of Bazant and Christensen in [20] regarding rectangular frames. Despite the thermodynamical consistency, the frequency band structure of the these homogenized materials shows a trend of the optical branch that is not qualitatively in agreement with that of the Lagrangian model. In fact, in the Lagrangian model the frequency of the optical branch is generally decreasing as the wave number increases while the energetically consistent micropolar models show an opposite trend.

An effort to detail the mathematical formulation underlying the most common continualization schemes and to catch the basic aspects of the physical problem has been carried by Bacigalupo and Gambarotta in [19]. Here, the spatial discrete Fourier transforms or the bilateral Z-transform of the equations of motion of discrete Lagrangian systems have been proved to provide consistent integro-differential equations of the homogenized non-local continuum in terms of the continuum macro-displacement field. The resulting integro-differential equation can be approximated with higher order differential equations by expanding the integral kernel in power series or through Padè approximants. According to this approach, what was already obtained by standard non-local continualization technique or by Padé-based continualization, thermodynamical inconsistency is found again, namely in some cases the resulting Lagrangian functional of the continuum model turns out to be equipped with elastic potential and kinetic energy density not-positive defined. In this framework, this formulation is focused on the dynamical identification of equivalent non-local continua representative of mono and two-dimensional lattice-like systems with lumped mass at the nodes. Firstly, the mono-dimensional continualization procedure is considered. This allows studying the convergence of the response of non-local higher order models, as their order increases, to that of the discrete system that plays the role of reference model (Fig. 3). Secondly, this procedure is consistently generalized to two-dimensional problems, in order to extend its use to a wide variety of periodic lattice-like materials. To this aim, the frequency band structure of the homogenized system is required to coincide with the one of the effective discrete model, namely the spatial bilateral Z-transform of the nodal displacements, appropriately mapped on the unit circle, and is matched to the Fourier transform of the macroscale displacement field. In the case of one-dimensional monoatomic lattice, the derived integro-differential equation turns out to coincide with the pseudo-differential equation by Bacigalupo and Gambarotta in [18].

To obtain a thermodynamically consistent formulation, the *enhanced non-local continualization technique* has been proposed. In particular, a proper continuous regularized auxiliary displacement field is introduced. This field is linked to the discrete displacement through a pseudo-differential down-scaling law having zeros

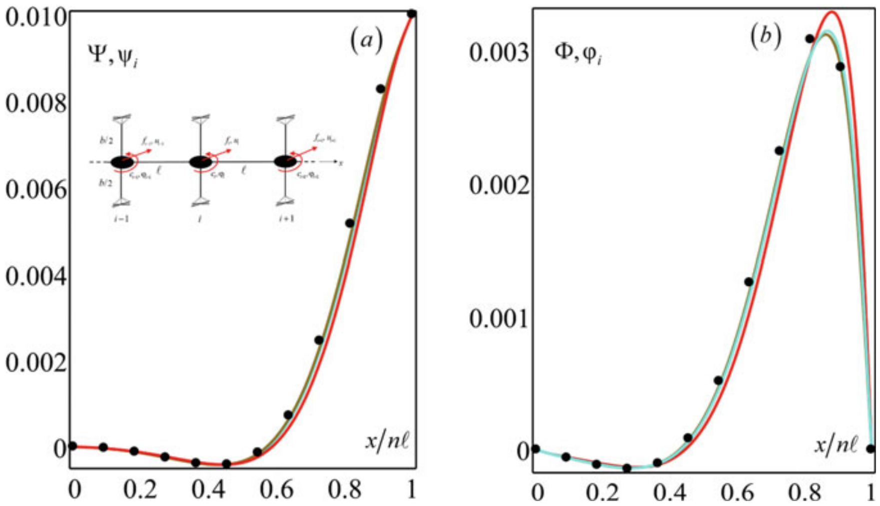

Fig. 3 Periodic continuous beam with elastic supports: **a** transverse displacements; **b** rotations (see [18]). Comparison of different models: Lagrangian model (black dots); 4th, 6th, 8th order continualization (red, cyan, brown lines)

at the edge of the first Brillouin zone, a law that is obtained via first order regularization approach. Consequently, the constitutive and inertial kernels of the integro-differential equation exhibit polar singularities at the edge of the first Brillouin zone. Moreover, the corresponding differential equations, obtained via a proper series expansion and relative truncation of higher order constitutive and inertial kernels, are consistently derived. It is worth noting that the governing equations of higher order non-local continua are characterized by constitutive and inertial non-locality. In addition, in this case the convergence of the response of non-local higher order models, as their order increases, to that of the discrete Lagrangian system has been proved. This approach has been extended to bidimensional systems such as cable nets undergoing transverse motion in a linearized framework characterized by a single nodal degree of freedom. Here, a multi-dimensional bilateral Z- and Fourier transforms have been introduced together with a suitable generalization of a pseudo-differential down-scaling law expressed in terms of the continuous regularized displacement field, a process that may be easily extended to three-dimensional systems. Examples of lattice-like systems consisting of periodic pre-stressed cable-nets with massive points have been analyzed (Fig. 4). The resulting higher order continua characterized by differential equations obtained as approximations of the integro-differential ones show dispersion surfaces in very good agreement with those of the Lagrangian reference model.

The above-described enhanced homogenization has been applied to two-dimensional beam-lattices in [22]. Equivalent generalized higher-order micropolar models, having positive defined non-local overall elasticity tensor as well as non-local inertial terms, are identified. Despite the positive defined potential energy density

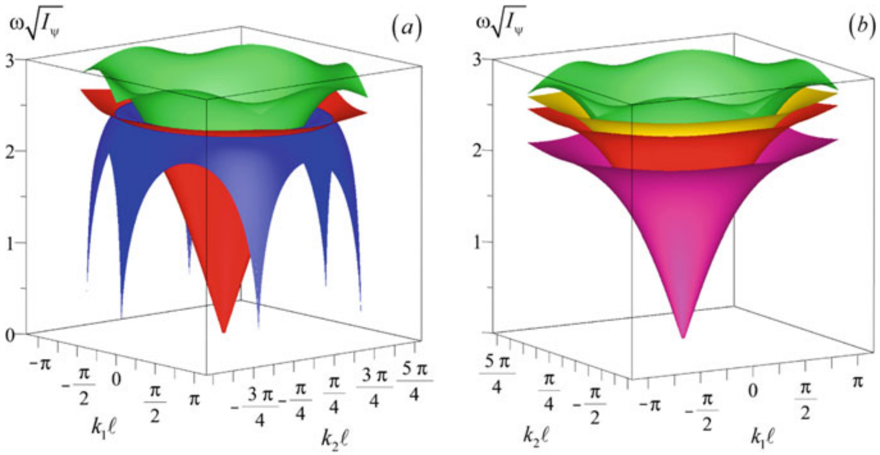

Fig. 4 Dispersion surfaces of the triangular pre-stressed cable net (see [19]): **a** Lagrangian model (green) versus 4th order standard continualization (blue), 4th order enhanced continualization (red), **b** convergence analysis: Lagrangian model (green), 2nd order enhanced continualization (violet), 4th order enhanced continualization (red), 6th order enhanced continualization (gold)

of the micropolar homogenized models, the presence of inertial terms guarantees an accurate description of the optical branch, solving in this way the problem emphasized by Bacigalupo and Gambarotta in [17]. In fact, the convergence of the frequency band structure of the higher order micropolar models to that of the discrete Lagrangian system is proved as the continualization order increases (Fig. 4b). Therefore, these homogenized models appears to be able to describe synthetically and accurately both the dynamic and static behavior of the beam-lattice materials.

It is well known that some lattice-like materials may exhibit exotic mechanical properties such as auxetic behavior. This is the case of chiral or anti-chiral beam lattices with rolling-up mechanisms realized by a periodic tessellation of circular rings properly connected by elastic ligaments. The case of rigid massive rings connected by massless ligaments has been analysed in [23]. Specifically, the overall constitutive tensors of the micropolar equivalent continua have been analytically determined for hexachiral and tetrachiral topologies, respectively, in terms of the chirality parameter β measuring the inclination of the ligaments (Fig. 2b). As major result, the auxetic isotropy of the hexachiral model and the auxetic orthotropy of the tetrachiral model have been shown in terms of the material symmetries of the overall constitutive tensors.

In general, high spectral density has been detected for these topologies and local resonators allow enriching the frequency band structure with further branches due to the degrees of freedom of the resonator so inducing low-frequency band gaps by virtue of the local resonance mechanism. In [16], the chiral topologies previously described have been enriched with cylindrical resonators embedded in the rigid ring and composed of a rigid heavy disk surrounded by a soft elastic matrix. The important effect of the local resonators on the formation of the low frequency band gaps is observed, while the effects of the chirality seem to be more limited. The enriched micropolar continuum has been derived from standard continualization and the overall equations of motion as well as the overall constitutive equation are given in closed form. Moreover, the resulting dispersion functions appear in good agreement with those corresponding to the discrete Lagrangian model for wave-lengths $\lambda \geq 3\varepsilon$. Considering the remarkable effects of the local resonances on the dynamic response of mechanical metamaterials, further investigations in [24] have been focused on the free and forced Bloch waves propagating through architected achiral topologies ($\beta = 0$). Furthermore, the so-called *meta-damping* effect, introduced by a linear viscoelastic coupling between the beam lattice microstructure and the local resonators, has been taken into account. From the theoretical viewpoint, the free propagation of damped waves is governed by a linear homogeneous system of integro-differential equations of motion, whose kernels can be approximated by a Prony series. From the methodological viewpoint, differential equations of motion with frequency-dependent viscoelastic coefficients have been obtained by applying the in-space Z-transform and in-time bilateral Laplace transform. As notable finding, the complex-valued branches of the dispersion spectrum may exceed the model dimension, due to the occurrence of pure-damping spectral components corresponding to standing non-propagating waves. The forced response to a harmonic mono-frequent external point excitation has been also investigated, in the relevant cases of non-resonant, resonant and quasi-resonant external forces. In this research line, open issues regard alternative treatment of the viscoelastic coupling and computationally efficient solutions to the forced problem.

The mechanical energy transport related to dispersive waves propagating through non-dissipative beam lattice materials has been studied in [25]. To this end, the complete eigensolution of the linear eigenproblem governing the free wave propagation–including both the dispersion functions and waveforms–has to be considered. Nondimensional *polarization factors* have been proposed to quantify the linear polarization or quasi-polarization of the waveforms, according to a proper energetic criterion. Furthermore, a vector variable related to the periodic cell has been introduced to assess the directional flux of mechanical energy in analogy to the Umov-Poynting vector related to the material point in Solid Mechanics. The physical–mathematical relation between the energy flux and the velocity of the energy transport has been recognized. The formal equivalence between the energy velocity and the group velocity has been also demonstrated, according to the mechanical assumptions. These studies pave the way for forthcoming technological applications targeted at the localization, harvesting and management of mechanical energy.

3 Homogenization of Periodic Blocky Materials

Periodic material microstructures, having a rigid heavy phase with dominant volumetric fraction and a soft phase with a vanishing volume fraction are commonly known as blocky materials. A description of these materials can be obtained by assuming a rigid behavior for the dominant phase and deformations localized at the soft interfaces connecting the rigid phases. If the interfaces are modeled as elastic with vanishing thickness, the plane blocky system may be described by a linear Lagrangian model in which the motion of each block is characterized by a roto-translation. The discrete governing equation of the reference block depends on the generalized displacement of the reference block as well as on the difference of generalized displacements of the connected blocks through the normal and the tangential stiffnesses of the interfaces. If the characteristic size of the blocks ε is negligible compared to the whole body size, then the discrete system may be approximated by a homogeneous equivalent continuum.

In [26], periodic centro-symmetric blocky materials have been considered and the discrete equations of motion have been derived. The influence of the model parameters on the frequency band structure has been analysed. By applying a standard non-local continualization technique, the overall constitutive tensors of the micropolar continuum have been analytically determined in closed form. In agreement with the outcomes of [17], referred in Sect. 2, the overall non-local elasticity tensor results, in general, negative defined. This result is also achieved by applying the energy based standard continualization previously mentioned. Conversely, by applying the generalized macro-homogeneity condition expressed in terms of the Lagrangian density at the two scales a positive-defined non-local constitutive tensor is retrieved.

Three examples of blocky materials are examined: rhombic and hexagonal patterns and running bond masonry (see Fig. 5a–d). Some numerical examples are analyzed to appreciate both the influence of the model parameters on the frequency band structure and to assess the validity limits of the micropolar continuum. This investigation was also aimed at understanding the criticism by Merkel et al. in [27] regarding the capability of the Cosserat model to approximate the spectral optical branch of hexagonally packed granular materials in the neighborhood of long waves. In fact, the inability of the thermodynamically consistent micropolar homogenized model to simulate the downward concavity of the optical branch obtained from the discrete Lagrangian model is shown in Fig. 6 in comparison to the dispersive relations derived from the standard non-local continualization technique. An interesting result concerns the hexagonal tiling, for which it has been proved that an auxetic behavior may be obtained when the tangential stiffness of the interfaces prevails over the normal one.

In this framework, two novel chiral block lattice topologies have been proposed in [28], having interesting auxetic and acoustic behavior (see Fig. 5e,f). The periodic material is obtained as a regular repetition of square or hexagonal rigid and heavy blocks connected by elastic interfaces. The blocks are rotated in the periodic pattern to obtain chiral configurations. The standard non-local continualization approach has been applied to derive the overall constitutive tensors and an approximation of the frequency spectrum. Hexachiral patterns have been analysed and from a tuning of the chirality angle and of the ratio between the tangent and normal stiffnesses of the interface it has been proved that it is possible to obtain different elastic behaviors, from strong auxeticity to zero expansion response up to not-auxeticity. Tetrachiral patterns are characterized by a cubic material symmetry behavior with the minimum

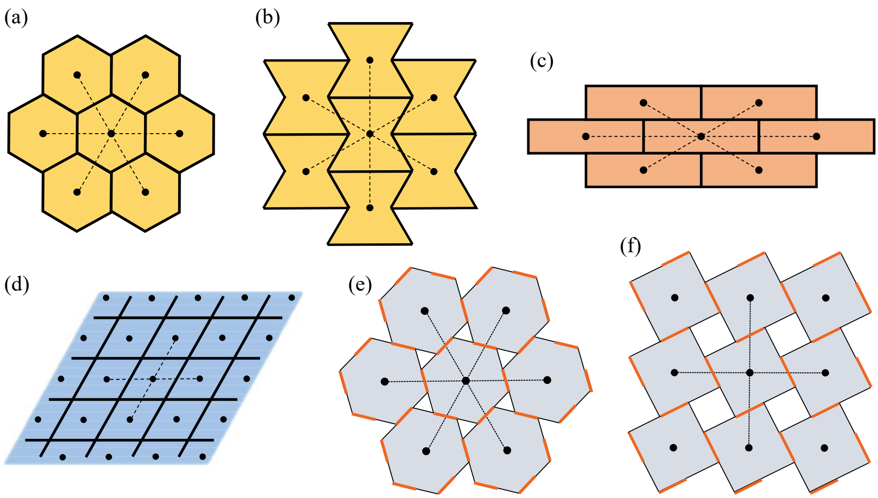

Fig. 5 Periodic blocky materials with linear elastic interfaces: **a**–**d** achiral topologies, **e**, **f** chiral topologies

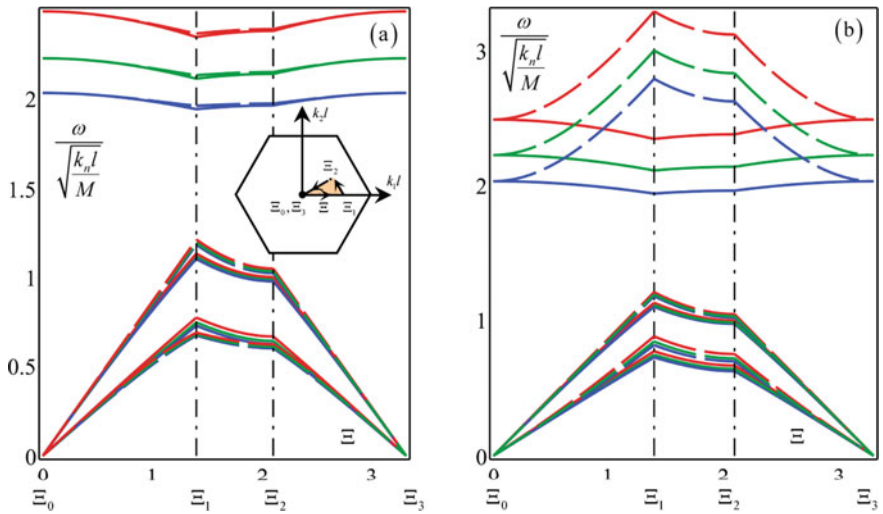

Fig. 6 Dispersive functions for hexagonal tiling (Fig. 5a). Comparison between the discrete model (continuous line) and the micropolar continuum model (dashed line) in a subdomain of the reduced Brillouin zone. **a** standard non-local continualization having non-positive defined non-local elasticity tensor; **b** generalized macro-homogeneity condition having positive defined non-local elasticity tensor. (see [26])

value of the overall Poisson's ratio approaching values close to -1 when decreasing the interface stiffness ratio and increasing the chirality angle.

The problems shown in [26] have been solved in [29] through an extension of the *enhanced non-local continualization technique* to two-dimensional and multi-degrees-of-freedom masonry systems, made up of rigid heavy blocks and elastic thin interfaces. Specifically, a novel micropolar continuum model has been formulated for running bond masonry, able of both accurately simulating the frequency band structure of the reference discrete Lagrangian model and satisfying the fundamental requirement of thermodynamical consistency. By virtue of the proposed approach, thermodynamically consistent micropolar models with constitutive and inertial non-localities have been obtained at different orders. The proposed procedure allows obtaining a sequence of higher-order micropolar continuum models, whose solution of the static problems and the dynamic problems in terms of frequency band structure are shown to be convergent to the corresponding solutions of the discrete Lagrangian reference model. It is worth to note that each higher order model may be complemented with consistent boundary conditions directly derived from the pseudo-differential down-scaling law. These enhancements allow to increase the general accuracy in the equivalent description of both the static and dynamic behavior of the periodic masonry-like systems, with excellent agreement in the solution of the benchmark tests related to (*i*) the quasi-static shear problem of a constrained two-dimensional strip involving rather complex boundary layer effects (see Fig. 7); (*ii*)

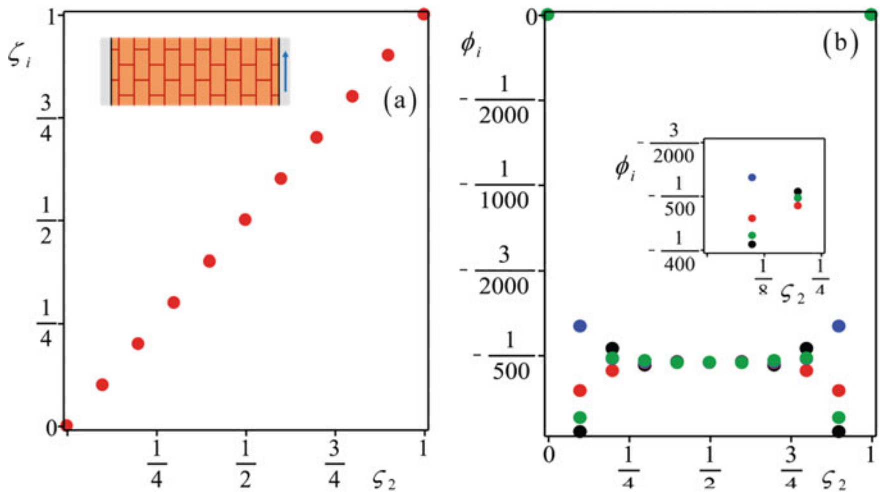

Fig. 7 Quasi-static simple shear problem for the constrained two-dimensional strip of masonry-like. Solutions obtained via down-scaling law from the continuum models (black for discrete Lagrangian model, blue for 2nd order enhanced continualization, red for 4th order enhanced continualization, green for 8th order enhanced continualization): **a** micro-displacements versus the vertical coordinate: **b** micro-rotations versus the vertical coordinate (see [29])

the dispersion properties of free undamped waves propagating through an infinite domain.

The enhanced non-local continualization approach has been applied to hexagonally packed granular materials in [30] and thermodynamically consistent higher order micropolar continua have been derived. Here, the material has been described as a pattern of hexagonal rigid heavy circles equipped with translational and rotational degrees of freedom and connected by pointwise elastic interfaces. Also in this case it has been shown that the homogenized models are able to accurately describe both the static and the dynamic behavior of the discrete granular model. Moreover, the convergence to the response of the discrete system has been demonstrated when increasing the order of the higher order continuum.

A stack of rigid heavy rectangular blocks connected through elastic interfaces has been analyzed in [31] in order to design a hyper-performance waveguide. The waveforms are described in terms of transverse displacement and rotation of the blocks so that the frequency band structure acoustic spectrum is characterized by an acoustic low-frequency branch and an optical high-frequency branch, corresponding respectively to waveforms dominated by the shear and moment components at the limit of long wavelengths. Considering an optimal design perspective, the frequency amplitudes of the spectral pass and stop bands have been conveniently assessed in the bi-dimensional space of the independent mechanical parameters. In order to achieve ultra-low frequency stop bands without the introduction of super-massive and/or extra-flexible local resonators, the elastic coupling with an elastic half-space modelled as a Winkler support is considered. As major achievement, the dispersion spectrum turns out to be characterized by two optical branches due to the systematic upshift of the entire band structure. Furthermore, the lower-frequency branch develops a critical point (corresponding to vanishing group velocity) with not-null frequency at the limit of long wavelengths. The consequent stop band opened in the ultra-low frequency range has been described in the enlarged (three-dimensional) space of the independent mechanical parameters. Finally, the equivalent micropolar governing equation derived through the enhanced non-local continualization approach turns out to have the same structure of the equation of motion of the Timoshenko beam enriched with non-local inertial terms.

4 High-Frequency Homogenization of Beam Lattice Materials

Perturbation methods represent an efficient mathematical tool, suited to determine explicit–though asymptotically approximate–analytical expressions of the dispersion relation for periodic microstructured media. In [32], a multi-parameter perturbation strategy has been outlined to build up asymptotic approximations for the linear dispersion functions of beam lattice materials. Starting from the classic single-parameter perturbation method, also known as *high-frequency homogenization*, the novel strategy is based on including both the wavevector $\mathbf{b}$ and the set $\mathbf{p}$ of the mechanical parameters in the small amplitude perturbation vector $\boldsymbol{\mu} = (\mathbf{p}, \mathbf{b})$, spanning a small-radius multi-dimensional hyper-sphere centered at a suited reference point $\boldsymbol{\mu}_0$ of the multi-parameter space. By virtue of a recursive formulation, the perturbation equations governing the direct spectral problem up to the desired approximation order have been derived for a generic beam lattice, independently of the dimensions of the discrete Lagrangian model and the parameter space. The equation solutions, consisting of the multi-parametric sensitivities of the dispersion functions, have been verified to well-approximate the spectrum of the anti-tetrachiral material, along specific directions of the Brillouin domain. Good qualitative and quantitative agreement has been recognized in [33] from the comparison between the approximated spectra obtainable by asymptotic methods and the spectra of homogenized micropolar continuum models obtained via standard non-local continualization. The actual effectiveness of low-frequency metafilters designed according to these formulations has been successfully verified by virtue of pseudo-experimental tests carried out within a multi-physical environment of numerical simulation. The multi-parameter perturbation strategy has been successfully applied in [34] to parametrically solve an inverse spectral problem targeted at functionally designing different anti-tetrachiral materials (featuring dissimilar mass density), characterized by the same assigned spectral frequency at a certain wavevector. The same multiparametric perturbation technique has been successfully employed to achieve analytical asymptotic approximations and perform sensitivity analyses of the dispersion spectra for mechanical beam lattice metamaterials in [35]. According to this general methodology, the parametric conditions for the existence of full band gaps have been established for tetrachiral topologies. Furthermore, the band gap amplitude can be analytically assessed in the admissible parameter range. As major technical result for passive control applications, target stop bands in the metamaterial spectrum can be analytically designed through the asymptotic solution of inverse spectral problems.

Architected metamaterials offering superior dynamic performances can be conceived by inducing local mechanisms of geometric stiffness or inertia amplification in the periodic microstructure. Within a finite kinematic formulation, these mechanisms may determine relevant nonlinear behaviors at relatively high amplitudes of oscillations. In [36] a one-dimensional diatomic lattice with cubic inter-atomic elasto-geometric coupling has been considered as minimal archetypical Lagrangian system simulating the essential undamped dynamics of nonlinear mechanical metamaterials (Fig. 8a). The linear spectrum, composed by a low-frequency acoustic and a high-frequency optical branch, has been analytically determined by solving the linearized

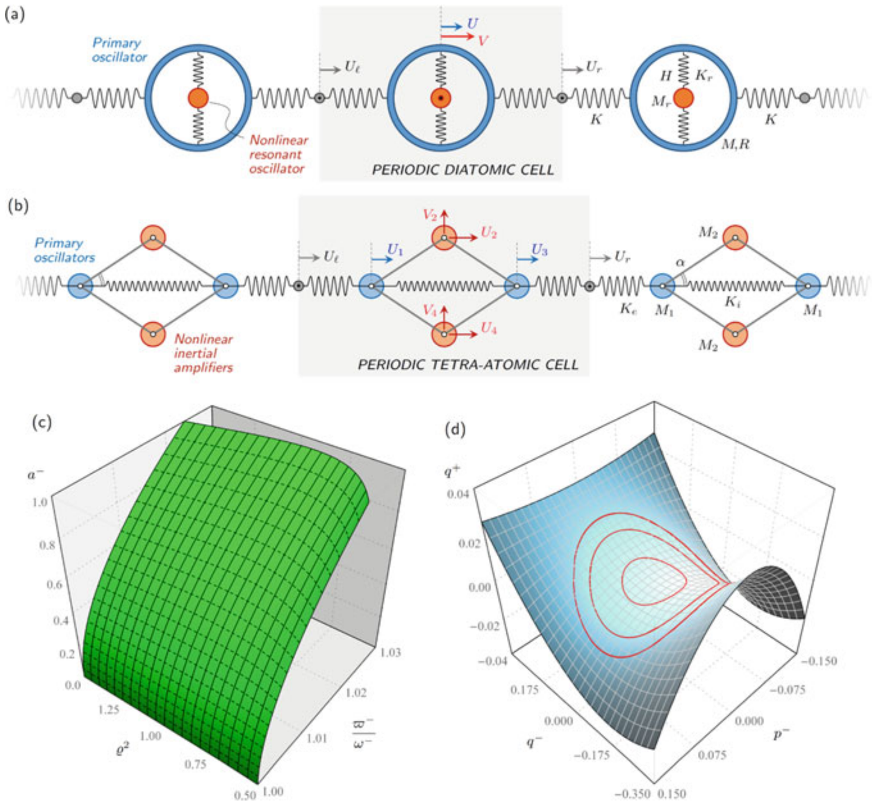

Fig. 8 One-dimensional waveguides realizing minimal mechanical metamaterials with **a** geometrically-nonlinear local resonators, **b** inertially-nonlinear pantographic mechanisms. Nonlinear dispersion properties: **c** amplitude dependent frequency backbones (black curves) for different mass ratios of the diatomic cell (see [36]), **d** invariant manifolds of the nonlinear waveforms, compared with numerical orbits (red curves), in the space of principal coordinates (see [37])

eigenproblem governing the free wave propagation in the small-amplitude oscillation range. Superharmonic 3:1 internal resonances have been recognized to occur within a wavenumber-dependent locus defined in the mechanical parameter space. A general asymptotic approach, based on the multiple scale method, has been employed to determine the nonlinear dispersion properties (Fig. 8c). Specifically, the nonlinear frequencies and nonlinear waveforms have been obtained for the two fundamental cases of non-resonant and superharmonically 3:1 resonant or nearly resonant lattices. Moreover, the invariant manifolds associated with the nonlinear waveforms have been parametrically determined in the space of the two principal coordinates. Differently, in [37] a lattice characterized by a pantograph mechanism in the tetra-atomic cell has been proposed as minimal physical realization of inertially amplified mechanical metamaterial (Fig. 8b). A Lagrangian discrete model has been formulated to describe the undamped free dynamics of the periodic cell. The ordinary differential equations of motion feature quadratic and cubic inertial nonlinearities, induced by the indeformability of the pantograph arms connecting the principal atoms with the secondary atoms, serving as inertial amplifiers. The linear and nonlinear dispersion properties have been analytically determined, in the framework of the multiple scale method. At the lowest perturbation order, an invariant parametric form has been achieved for the pass and stop bands of the linear spectrum, corresponding to propagation and attenuation branches in the plane of complex wavenumbers. The major effects due to the mass ratio of the inertial amplifiers have been discussed. At the higher perturbation orders, the nonlinear frequencies and waveforms have been obtained. Analytical, although asymptotically approximate, functions have been achieved for the nonlinear frequencies and waveforms, which show quadratic dependence on the oscillation amplitudes (Fig. 8d). The mechanical conditions for the softening/hardening behaviour of the nonlinear frequencies have been discussed and the different synclastic/anticlastic properties of the invariant manifolds associated to the nonlinear waveforms have been recognized. In this research line, open issues regard alternative Hamiltonian asymptotic treatments of the inertial, stiffness and damping nonlinearities, with focus on the particular case of superharmonic internal resonances among the acoustic and optical waveforms.

5 Optimal Design of High Performance Architected Materials

Architected materials and metamaterials represent also a challenging frontier for the development of optimal design strategies targeted at the active and passive control of elastic wave propagation. Within this research field, the microstructural optimization of mechanical metamaterials for achieving desired spectral functionalities may require considerable computational resources. Based on this motivating background, the inverse design problem concerning the optimization of the dispersion properties characterizing architected materials and mechanical metamaterials with different topologies has been tackled. The mechanical formulations have been primarily based on discrete Lagrangian models in the linear field, while the methodologies range from the asymptotic techniques [34] and computational approaches [33] commonly used in structural and solid dynamics to the numerical algorithms typically employed in machine learning, non-linear programming and passive control theory. Concerning the second, multidisciplinary approach, the optimization problem has been systematically formulated as a non-linear maximization problem by defining a non-concave multi-objective function, targeted at achieving the largest stop bandwidth at the lowest center frequency. The numerical search for the optimal solution has been properly constrained to focus on the admissible range of the design parameters, including a few mechanical properties, as well as the number, placement and properties of local resonators.

A summary of the adopted solution strategies, based on an iterative globally convergent algorithm and a random or quasi-random initialization has been summarized in [38], where some optimal results concerning the dispersion spectrum of architected materials and mechanical metamaterials characterized by hexachiral [39], tetrachiral [40] and anti-tetrachiral [41] topologies are reviewed. The most recent developments tend to attack the optimization problem by virtue of advanced techniques for surrogating the objective function. This approach has been actually proved to increase the computational efficiency in searching the optimal solutions.

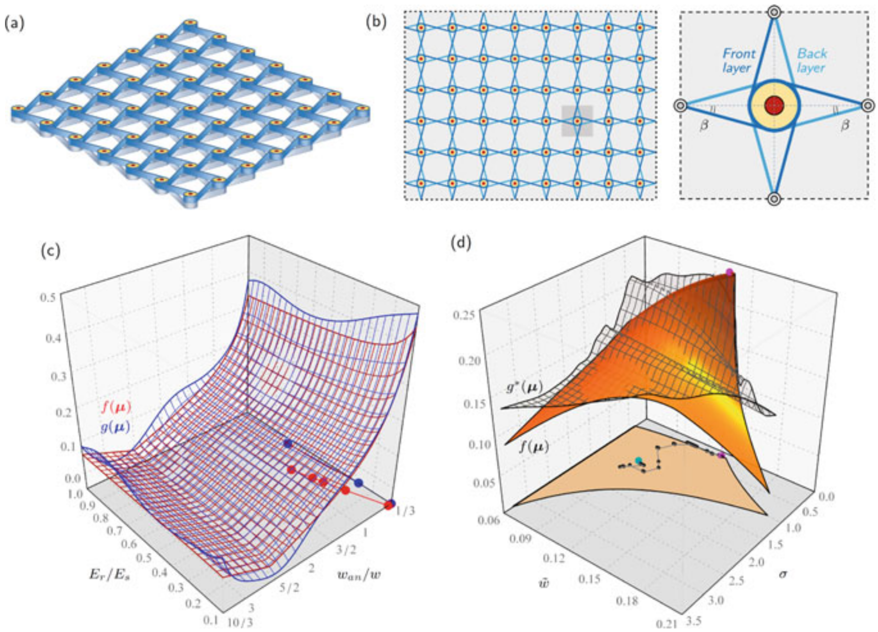

Fig. 9 Optimally designed meta-filters based on the tetrachiral topology: **a**, **b** layered by-tetrachiral metamaterial. Comparison between the exact and surrogate constrained objective functions $f(\boldsymbol{\mu})$ and $g(\boldsymbol{\mu})$ in the space of two selected mechanical parameters: **c** tetrachiral metamaterial (see [42]), **d** bi-tetrachiral metamaterial (see [43])

The surrogate strategy has been successfully employed to design optimal meta-filters based on the tetrachiral topology in [42] (Fig. 9c), as well as—in a refined version based on adaptive scheme for the local surrogation—the novel bi-tetrachiral topology in [43] (Fig. 9a, b, d). With all the progressive improvements, the machine learning methodology represents a general mathematical tool suited to attack other optimal material design problems, even characterized by higher dimension of the governing dynamic models and/or larger parameter spaces. Future developments under investigation regard the design of smart metamaterials with active resonators and micromechanisms, targeted at engineering applications in wave propagation control and mechanical energy harvesting.

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Mechanics of Microsystems: A Recent Journey in a Fascinating Branch of Mechanics

Alberto Corigliano, Aldo Ghisi, Stefano Mariani, and Valentina Zega

Abstract Microsystems or Micro Electro Mechanical Systems (MEMS) are very small machines that over the last thirty years had an impressive development in terms of potentialities and diffusion. MEMS are now widespread as micro sensors and/or micro actuators and can be found in many objects of common use. The purpose of the present Chapter is to give an overview of the importance of Mechanics in the study, design and fabrication of MEMS. Inertial and piezoelectrically actuated MEMS are first described. Key issues concerning Microsystems reliability are then discussed, such as fracture, fatigue and consequences of impacts due to accidental drop, along with other uncertainty-related issues at the device scale. The content of this chapter is based on the activity carried out in our research group at the Politecnico di Milano, along the last 20 years.

Keywords Microsystems · MEMS · Multi-physics · Modelling and simulation · Reliability

1 Introduction

1.1 Microsystems

The lecture given by R.P. Feynman on December 1959, reprinted in [1], is usually recognized as the moment in which the history of microsystems started. The paper by

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Nathanson et al. [2], considered as the first in the Microsystems literature, appeared few years after that inspiring lecture; in this paper, the Authors discussed about the creation of the resonant gate transistor.

Sixty years after those early ideas concerning micro-machines, we can now affirm that the world of Microsystems transformed in a new industry of micro devices, which produces millions of pieces every day around the world. Microsystems are nowadays complex products of modern engineering with a large number of applications in various fields such as consumer market, automotive, health monitoring and biomedical engineering.

This impressive and fast growth originated from a combination of factors: the great versatility and the reduced unit cost, the advancements in miniaturization technologies, the diffusion of smartphones, the necessity of monitoring various devices, the trend toward self-driving cars, the advent of the Internet of Things (IOT).

We are now in a new era in which augmented capabilities are being added to Microsystems; e.g. in the so-called multi-axis sensor units containing accelerometers, gyroscopes, pressure and magnetic field sensors all combined. In the near future, Microsystems included in wireless sensor networks will have extra performances, as energy autonomy, computational abilities and high reliability also in extremely aggressive environments.

A better understanding of Microsystems can be obtained focusing on the keywords *Micro*, *Electro*, *Mechanical* and *System* and recalling basic information on the *fabrication* process ([3–5]).

Micro means that we are dealing with small devices, in which the single smallest dimension can be at the sub-micrometer scale. A complete MEMS can have dimensions in the order of millimeters. These dimensions tell us that we are not yet in the field of nano world, but that we are very close to it. A real device with a small volume of 10 mm^3 and a footprint of 9 mm^2, can easily contain three sensors with three sensing axes each, like e.g. a three-axes accelerometer, a three-axes gyroscope and a three-axes magnetometer. This *nine-axes* device can be placed almost everywhere and from this we have an idea of the potentialities behind Microsystems technology.

Electro means that inside these small devices there are electric/electronic components, needed to create connections between the MEMS and the external world, to transform physical information (e.g. inertia forces, pressure…) into electric signals and to also activate movements inside the device. In addition, it is always necessary to add read-out electronic circuits.

Mechanical means that Microsystems have some portion of their architecture working thanks to mechanical principles, like small beams or plates loaded by inertia forces, caused by the overall acceleration of the device. These can be considered as structural components in the same way as beams and plates are structural components of large structures.

System means that the micro devices are not simple, they are complex entities combining various components: the electric/electronic parts, the mechanical parts, possibly other parts like optical ones. Moreover, they can be produced by means of complex fabrication processes and complex integration of different portions.

The *fabrication* of Microsystems is a complex process mainly adapted from those used in the fabrication of integrated circuits (see e.g. [5]). The starting point is usually a wafer of mono-crystalline silicon used as a support, on top of which various materials can be subsequently deposited. Each added layer has a pre-specified thickness and role in the design and fabrication. Another fundamental step is the selective elimination of portions of one or more deposited layers. This is the so-called *litography* used in the production of electronic circuits. During a litographic phase, a thin film, called *resist*, is first exposed to light on some portions of the surface, depending on the pattern or drawings which must at the end be reproduced on the wafer. The exposure of thin films to light modifies them in such a way that when chemical substances are subsequently deposited on the resist, portions of it are eliminated and therefore they can be used as a *mask* which covers the wafer surface to reproduce the desired drawing. The mask can then be used on top of the wafer and additional chemical substances can selectively eliminate portions of the pre-deposited layers on the wafer surface, again depending on the desired pattern. Additional phases are then added to eliminate portions of materials also below some pre-deposited layers. In this way, not only there is the possibility to dig but also to create suspended structures, beams and plates. With a sequence of deposition and selective elimination, which can imply various litographic phases, the whole wafer is patterned as desired.

Usually, a second wafer must be prepared, with much simpler patterning than the first one, to be used as a cap for the whole set of devices patterned on the first wafer. A procedure called *wafer-to-wafer bonding* is thus put in place: basically, some kind of glue is deposited on strips of the wafers that separate each device, then the two wafers are pressed one toward each other with a machine that also controls the process temperature. At the end of the thermo-compressive bonding process, the two wafers are glued in correspondence of the separation strips.

Finally, depending on the device, a suitable *package* is used to cover the MEMS and the dedicated electronic circuits and to protect it from the external environment.

1.2 Microsystems and Mechanics

The importance of Mechanics and correlated phenomena in Microsystems emerges starting from the fabrication process and accompanies the whole life of MEMS. We can say that mechanics accompanies MEMS from cradle to grave.

The microsystems world offers an impressive variety of mechanical and multi-physics phenomena that practically include every aspect of the modern theoretical, applied, experimental and computational mechanics.

Material and structural mechanics is necessary to interpret the response of small structures on board of microsystems in static and dynamic, linear or highly nonlinear regimes, in the presence of multi-physics coupling originating from non-mechanical loads like e.g. electrostatic actuation.

Fluid mechanics allows to study and design micro-fluidic devices, like micropumps, and to study the effect of varying gas pressure inside MEMS cavities which governs the damping of vibrating structures.

Coupled problems are intrinsic in Microsystems, where thermo-electro-magneto-chemo-mechanical coupling can be often found. Meaningful examples can be found also during various phases of fabrication, when the wafer and the single components are stressed in many ways by the thermo-mechanical processes.

Fracture, fatigue, and material damage are also originated by chemo-mechanical interactions, to set many reliability issues. Tribology, adhesion and stiction and the importance of surface phenomena is amplified in MEMS where the surface to volume ratio increases due to their geometrical dimensions. Without good mechanical interpretations it is impossible to obtain a complete control of the device reliability.

1.3 Microsystems at Polimi and Italy

Research activities on the mechanics of Microsystems started in the Department of Structural Engineering of Politecnico di Milano in 2001, within the framework of an initial collaboration with STMicroelectronics, an important industry of semiconductors. The first goal concerned the study and design of Microsystems for the mechanical characterization of polysilicon used in MEMS [6]. Since then, the activity progressively intensified and a whole *MEMS modelling and design group* was created.

The experiences of the group in various fields related to mechanical problems in MEMS have been partially collected in the book *Mechanics of Microsystems* [7]. The activity in this sector is still intense and now covers features linked to reliability, to the design of new prototypes and to the formulation of modelling and simulation strategies: recent meaningful publications are [8–12], some of which also published on the journal *Meccanica*. To be mentioned also an incoming book, mainly written by expert of Microsystems belonging to the Industry [13].

In parallel, in the Italian community of Mechanics the role of microsystems and relevant mechanical problems has grown in its importance, as witnessed from the contributions presented at various national and thematic congresses of AIMETA. Various groups started working on Microsystems under various perspectives. Notable examples are the contribution dealing with the non-linear dynamics of MEMS, as [14, 15], the material characterization at the micro scale [16, 17], the design of innovative prototypes [18], and the use of new concepts and materials at the micro and nano scale [19, 20].

The papers mentioned in this Introductory Session are by no means exhaustive of the Italian contribution in the field of mechanics of Microsystems; they have been selected to show that the field is very active and promising for future developments. In our opinion, the exciting journey in MEMS mechanics and related fields will continue for a long time, posing to researchers and designers new challenging goals.

1.4 Chapter Contents

In Sect. 2 inertial MEMS are described as representative of a very large class of Microsystems, including the widespread accelerometers and gyroscopes. The discussion on inertial MEMS is accompanied by remarks on possible nonlinear phenomena mainly related to the nonlinear dynamics of oscillating parts.

Section 3 contains a brief presentation of piezo-MEMS, a new class of Microsystems that is acquiring more and more importance in which piezoelectric materials are used to sense and/or actuate. These include micro-mirrors, micro-speakers and piezo-micro-ultrasonic transducers (PMUT), applied in video-projection, distance measurements and innovative interactions between men and machines.

Microsystems reliability is dealt with in Sect. 4, where reference is made to fracture, fatigue, accidental impact phenomena and to peculiar reliability issues linked to the fabrication process, recently addressed also via data-driven strategies.

The final Sect. 5 is devoted to closing remarks and to an outlook on future perspective of the Microsystems world.

2 Inertial MEMS

2.1 Accelerometers and Related Mechanical Challenges

MEMS accelerometers are devices designed to measure linear accelerations. They have many applications, from consumer electronics to drones and structural monitoring. Capacitive accelerometers dominate the market so far thanks to their good performances and relative low footprint. The schematic view of an x-axis capacitive accelerometer is reported in Fig. 1a. It consists in a proof mass (red) suspended through flexible springs (yellow) that allow its motion along the x-axis. When an external acceleration acts on the accelerometer, the proof mass moves thanks to the inertial force, thus changing the gap between fixed and moving electrodes kept at different voltages. The external acceleration is proportional to the displacement of the mass and sensing is achieved by measuring the displacement via the capacitance variation.

The schematic view of an out-of-plane accelerometer is shown in Fig. 1b. It consists in a proof mass suspended through a torsional spring that allows rotation around the x-(or y-) axis. When an external out-of-plane acceleration acts on the accelerometer, the proof mass tilts due to the inertia force acting on it, thus changing its distance from the underlying electrodes. As for the in-plane accelerometer, the external acceleration is proportional to the displacement of the mass and sensing is achieved by measuring the displacement via the capacitance variation. Differential readout is usually desired in capacitive accelerometers since it allows to double the sensitivity and to reject common mode unwanted displacements, e.g. induced by thermal stresses.

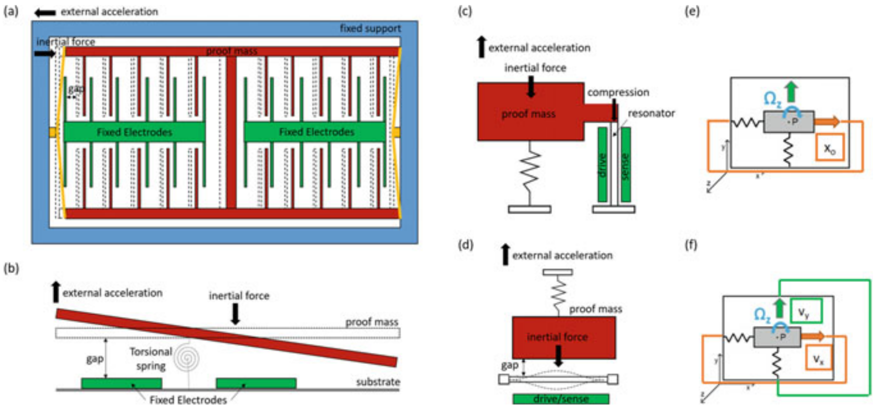

Fig. 1 Schematic view of a MEMS capacitive **a** *x*-axis and **b** *z*-axis accelerometer. Working principle of a MEMS resonant accelerometer based on **c** mechanical and **d** electrostatic stiffness variation. Schematic view of the working principle of an **e** Amplitude Modulated and a **f** Frequency Modulated Coriolis Vibrating yaw gyroscope

Examples of uniaxial capacitive MEMS accelerometers for in-plane and out-of-plane sensing are available in the literature [21, 22]. The main design challenge is to maintain high sensitivity with reduced footprints. The easiest solution to this trade-off is to lower the mechanical stiffness of the suspension springs, but it often entails the risk of pull-in instability or adhesion. An alternative is represented by the design of monolithic three-axial capacitive accelerometers that allow for the simultaneous detection of the three components of the external acceleration with a footprint smaller than the sum of the three uniaxial ones [23, 24].

Recently, Frequency Modulation (FM) accelerometers that measure the external acceleration by detecting the frequency variation of their resonant components, appeared as valid counterparts for high-end applications where the FM sensing principle is mandatory to achieve high performances in terms of sensitivity, Full Scale Range (FSR), linearity and stress rejection. Compared to capacitive ones, FM accelerometers show indeed a larger dynamic range, higher sensitivity, a quasi-digital nature of the output signal and a good integrability with other sensors (e.g. gyroscopes), since they are encapsulated in vacuum and are stable at low pressures.

The frequency shifts can be caused in many ways [7], two meaningful examples are the variation of the mechanical stiffness (Fig. 1c) and the variation of the electrostatic stiffness (Fig. 1d). The working principle of the first case is schematized in Fig. 1c: in presence of an external acceleration, the inertial force induces a displacement of the proof mass that leads to a tensile/compressive force on the resonator, thus changing its mechanical stiffness and consequently its resonant frequency. The working principle of the second class of resonant accelerometers is reported in Fig. 1d. In this case, the displacement of the proof mass induced by the inertial force, causes a variation of the gap between the mass itself and the resonator thus changing the electrostatic stiffness and consequently the frequency of the resonator.

Several innovative designs of FM accelerometers were recently proposed at Politecnico di Milano [25] for the detection of in-plane [26, 27] and out-of-plane [28] external accelerations.

The promising FM accelerometers are attracting increasing interest within the MEMS community for their high performances and challenges in the design phase. The dynamic behavior of resonators must indeed be optimized against unwanted frequency shifts sources like e.g. temperature effects [29] and nonlinearities [30, 31].

2.2 Gyroscopes and Related Mechanical Challenges

MEMS gyroscopes are devices designed to measure the angular rate. They have a variety of possible applications from consumer electronics to drones to the automotive industry and for this reason improved performances and reduced footprints are always required to fulfill the market requirements.

Among others [32], Coriolis vibratory gyroscopes constitute a significant group of MEMS gyroscopes. The underlying physical principle is that a vibrating object tends to continue to vibrate in the same plane as its support rotates. Two classes of MEMS Coriolis vibratory gyroscopes have been proposed so far in the literature. They differ in terms of the detection principle, and are referred to as Amplitude Modulated (AM) and Frequency Modulated (FM) gyroscopes. In Fig. 1e, f, a schematic view of the two working principles is reported with reference to a yaw gyroscope. In AM Coriolis MEMS yaw gyroscopes, an inertial mass is kept in oscillation with a certain displacement amplitude x_0 along one direction at its natural frequency (drive mode). When an external angular rate Ω_z acts on the gyroscope, the Coriolis apparent force (green arrow in Fig. 1e) activates the displacement of the proof mass along the sense axis. The angular rate is then measured in the form of a capacitance variation at the sensing electrodes. A variety of structures can be used to implement AM MEMS Coriolis vibratory gyroscopes. Broadly, these are grouped into lumped proof mass–spring structures, which involve one or more rigid masses translating or rotating, and ring, disc or shell structures, which involve coupled flexural deformation modes of the whole structure. The majority of devices commercially available so far are AM Coriolis gyroscopes. FM gyroscopes emerged very recently [33] as a promising solution in terms of stability against environmental fluctuations for high-end applications. Instead of controlling the motion of one mode (drive) and measuring the Coriolis-induced displacement amplitude variations along the sense axes as done in AM solutions, FM gyroscopes rely on controlling the velocities of the main orthogonal modes (drive + sense axes) of the proof mass and in measuring the resonance frequency variations induced by the external angular rate on the considered axes. In the yaw FM gyroscope schematized in Fig. 1f, the two in-plane velocities are controlled through a close-loop architecture and the frequency variation induced along the two axes is detected when an external angular rate is applied to the sensor. In Politecnico di Milano, innovative designs of pitch and three-axes [34] FM gyroscopes have been proposed for the first time in the literature. The proposed designs implement a differential mechanical structure able to reject any unwanted common mode effect induced by linear accelerations and provide a smart solution to the implementation of an out-of-plane driven motion usually not available in MEMS for the strict fabrication constraints.

Apart from the continuous request of size reduction and performances improvement that is pushing the design to the fabrication limits and is asking for innovative coupling mechanisms and working principles, the main challenge for MEMS gyroscopes designers is the efficient management of their nonlinear dynamic behavior. Independently of the AM or FM working principle, indeed, MEMS gyroscopes exhibit complex mechanical structures containing highly deformable components. Moreover, large displacements of the proof mass often end up in higher sensitivity and smaller noise and are for these reasons strongly required in operation. Finally, electrostatic readout is usually performed through parallel plates electrodes that are intrinsically nonlinear. By combining all these characteristics, it is evident that geometric, damping and electrostatic nonlinearities are usually present simultaneously in MEMS gyroscopes. MEMS designers must then try to exploit them when desired and avoid them when harmful.

Our group studied in depth several nonlinear phenomena in MEMS gyroscopes like the internal resonance in a beating-heart quad-mass structure or the self-induced parametric resonance in a disk resonator gyroscope and developed advanced analytical models and numerical model order reduction techniques able to efficiently reproduce the nonlinear dynamics of complex MEMS structures like gyroscopes [35–37].

3 Piezo-Actuated MEMS

Piezoelectricity is a material capacity to create an electric charge when subject to a mechanical stress, as it was discovered by Pierre and Marie Curie in 1880 [38], and, vice-versa, to create a deformation if a difference of potential is applied, as Lippman found in 1881 [39]. Piezoelectric MEMS, or piezo-MEMS, are microsystems exploiting a piezoelectric film, for sensing or actuation. A piezoelectric material embodies the synergy between electrical and mechanical phenomena typical of the MEMS world; therefore, one could naively consider it as an optimal choice for designing microstructures. Actually, for many years the manufacturing process of piezoelectric materials at the micro-scale remained challenging for physical and chemical reasons, in addition to production issues (including costs) deriving from possible incompatible materials in silicon-based processes [40]. As a consequence, neglecting pioneering ideas and contributions, piezo-MEMS experienced a significant increase and diffusion only in the last decade, when low cost and high quality deposition processes became available. In particular, ferroelectric lead zirconate titanate (PZT) proved the most effective piezoelectric material for industrial applications; however, nowadays research and development moved to lead-free materials, such as aluminum nitride (AlN), ultra-flexible polyvinylidene fluoride (PVDF), lithium niobate-doped materials and KNaNbO3 (KNN). The drawback of the latter mentioned piezoelectric materials is a lower efficiency in energy conversion, for which reason some of them work better as sensors rather than as actuators. Henceforth, in general, a balance between environmental reasons and performance requirements has to be carefully considered when piezoelectric materials are used for MEMS applications.

Industrially, piezo-MEMS are diffused as microphones, micro-mirrors, micro-speakers, gyroscopes or resonators, energy harvesters, fingerprint sensors [40]. Moreover, the piezoelectric effect can naturally be useful for scientific experiments at the micro-scale, since it directly provides a mechanical actuation when a suitable electric signal is applied. The structural typologies adopted have usually the intent to amplify the displacement, such as single cantilevers (e.g. biosensors, harvesters, AFM tips), clamped–clamped beams (e.g. RF switches), membranes (e.g. micro-pumps, micro-mirrors, pressure sensors, piezoelectric micro-machined ultrasonic transducers a.k.a. PMUTs, accelerometers, micro-motors).

An important characteristic of a piezoelectric material is poling, i.e. a preferred orientation of the dipole domains (also called piezoelectric domains) induced by an external source, such as an electric field. When the material is in a virgin state, dipoles are randomly oriented so that their global effect is null; therefore, when an external field (electrical or mechanical) is applied in a virgin state, the response remains linear till a given threshold is reached (this behavior is also referred in literature as *intrinsic* effect [41]). For this reason, the poling in a polycrystal will appear lower than in a monocrystal, because only the domains whose polarization is aligned with the external field will fully contribute, while the remaining domains will contribute only in terms of the projection of the polarization vector along the external field. When instead the mentioned threshold is reached, a new way of deformation is activated, because the domains rotate to align with the external field: adjacent domains are forced to accommodate the strain, either with elastic deformation or with rotation (hence the name *switching* associated to the phenomenon), and a large deformation (no more linear) is typically observed (*extrinsic* effect, see [41]). More importantly, this is an irreversible process, because a polarization is left in the material, i.e. the *poling*: when the same external field is applied again, since the domains were aligned during the poling, the observed deformation will be larger than the one observed in the virgin state. From the application point of view, this is an advantage: for example, thinking of an actuator, a larger displacement along the desired direction will occur with lower voltage with respect to an unpoled material. If the new threshold is again overcome, another switching could occur, leading again to a response in the nonlinear regime; therefore, conditions about how and when poling is induced are to be investigated with care. Upon unloading, an irreversible strain will remain in the material: this behavior is similar to elasto-plasticity, hence some ideas to phenomenologically model piezoelectricity are borrowed from that mechanical framework, see e.g. [42].

In Fig. 2a, an example of a testing device, exploiting piezoelectric activation, is depicted [43]. Each piezo-MEMS replicated on the ceramic base consists of two slender plates clamped at one end and connected in the middle by a narrow bridge (see also the cross-section sketch in Fig. 2b). In the layer stack, a PZT layer is placed on

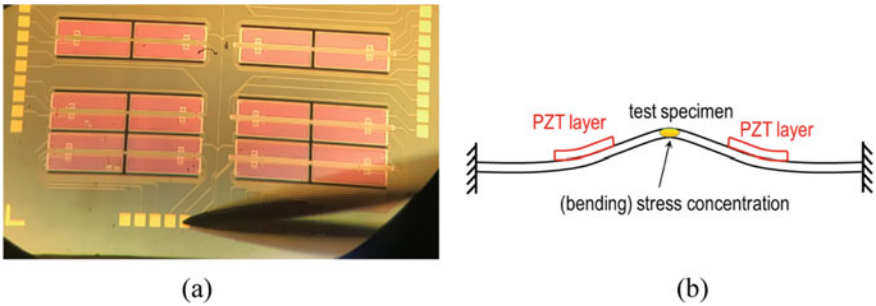

Fig. 2 Piezo-MEMS structure for fatigue testing of a metal strip: **a** plan view of multiple devices placed on a ceramic base, and **b** working principle in cross-section

the top of a polysilicon substrate and between other intermediate layers (top/bottom electrodes), so that when a difference of potential is applied to the electrodes, the PZT layer stretches or elongates. Because of its offset with respect to the plate mid-plane, see Fig. 2b, the in-plane deformation of the PZT layer induces a bending deformation of the overall system (in the piezoelectric material jargon, the so-called 31 coupling is exploited, where 1 is related to the in-plane direction aligned with the plate length and 3 is related to the out-of-plane direction): the two plates and the narrow bridge in between are accordingly moved out-of-plane. By applying a sinusoidal voltage to the device, the stress distribution in the narrow bridge cross section undergoes alternating signs, i.e. a loading condition suitable for fatigue testing. For this reason, a thin metal layer of interest is deposited at the narrow bridge top surface: the material degradation during the fatigue loading can be therefore observed during the test by monitoring the variation of the out-of-plane displacement, after the test with optical inspection by looking at the degraded areas that typically show a color modification. In a suitable ceramic holder, several piezo-MEMS specimens can be easily placed and tested at the same time, see Fig. 2a: this is a big advantage during fatigue testing, where often the scattering between the results can be significant and many tests have to be carried out to assure stochastic significance.

4 Reliability of Microsystems

4.1 Fracture, Fatigue, Impacts

It has been already highlighted that MEMS are massively used for mobile applications and are therefore exposed to extreme loading conditions. Often, even if protected by the package, drops and shocks caused by mishandling can lead to catastrophic events.

At the device level, failures are deeply linked to the brittleness of silicon, which is an orthotropic material with a face-centered cubic crystal structure. As far as its elastic behavior is concerned, the crystal lattice leads to a slight deviation from isotropy, see [44, 45]. Even if small, such anisotropy can induce remarkable effects on the device response at such length-scales. As far as the cracking is instead concerned, an agreement in terms of the orientation-dependent strength and toughness properties of silicon has still to be attained, due to the mentioned brittleness always leading to catastrophic failures.

Failures of MEMS caused by (accidental) drops are recognized of importance during the design stage. In accordance with standards, the device response to a shock loading is assessed through vibration tests featuring high acceleration peaks, or through drop tests. A drift in the working features can then be exploited as an evidence of a possible failure mechanism. By handling the information gathered during these tests, the type of failure cannot be easily identified. To cope with such a major issue, in [46–48] numerical simulations were adopted to pinpoint the physics of failure in different situations. To also avoid resting on simplified interpretations of the device response to the dynamic loading at the length-scales characterizing package, movable structure and polysilicon microstructure, a multi-scale approach was proposed by our research group. The main characteristics of this approach are as follows: at the package scale, analyses are run to link drop height and falling orientation to the way waves propagate inside the package itself, and finally hit the anchor points of the movable structure; at the movable structure level, analyses are run to link the drop-induced waves in the package to the dynamics of the mechanical structure; finally, at the polysilicon film level, analyses are run to link the dynamics of the movable structure to stress intensification at some critical spots and to the possible failure mechanism.

A sketch of the set of analyses involved in the proposed approach is depicted in Fig. 3, where reference is made to the z-axis accelerometer mentioned in Sect. 2.1, subjected to a drop leading to the failure mechanism depicted in top-right image. The suspension spring gets shaken by the waves travelling inside the entire package,

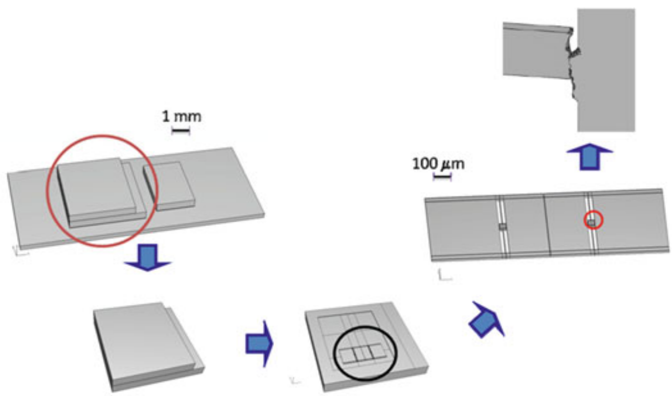

Fig. 3 Example of length-scales and models involved in failure modeling of a z-axis accelerometer (see Fig. 1b), ranging from the package level down to the polycrystal level [49]

that hit the anchor points and induce the vibrations of the proof masses. Due to the reentrant corners where the suspension springs are connected to the substrate, the failure mechanism turns out to consist of a through-crack almost independent of the film morphology. Results are obviously affected by the geometry of the movable structure and by the drop features but, in all the cases, a main crack gets triggered if drop height is large enough, and rapidly grows to finally lead to a device failure.

Another issue to cope with at the micro-scale is the fatigue of silicon. The reasons inducing fatigue are still under debate: while the crack propagation mechanism is essentially the same experienced under monotonic loading, shielding effects like crack bridging take a role and the driving force at the crack tip results larger than in the monotonic case. There seem to be two different explanations for fatigue crack propagation in polysilicon, as proposed in the literature [7]: the environmentally-assisted crack propagation in superficial silicon oxide layers; the subcritical crack growth of micro-cracks due to asperities of the crack faces that get into contact during the loading cycles, and provide a local stress intensification.

With the first class of models, fatigue is assumed to be incepted only if micro-cracks reach a dimension critical for the structural film; this multiple length-scale reasoning could explain why fatigue is not observed for silicon at the macro-scale. The second class of models seems to be supported by experimental data, testifying that results are strongly influenced by the load ratio (defined as the ratio between the minimum and maximum stress level in a cycle) and are independent of the load frequency. Regarding crack inception, a possible explanation implies a brittle to ductile transition mechanism for silicon, which can be size dependent.

4.2 Other Reliability Issues and Sources of Uncertainty at the Micro-Scale

A number of additional phenomena may take place at the polysilicon scale and affect the device reliability. Among them, we list here the spontaneous adhesion or stiction, which deeply linked to surface-related effects, see [7]. Uncertainties linked to the microfabrication can have a huge impact on the device performance indices and reliability. Such uncertainties are not only linked to the geometry of the movable structure, but also to the morphology of the film constituting the structure itself. Results discussed next were obtained with a model-based approach to the problem. Alternate multi-scale, data-driven approaches were also recently proposed in [50].

We refer now to a uni-axial resonant MEMS magnetometer, see [51, 52]. Its movable structure is a slender, clamped–clamped beam carrying a couple of plates connected at the mid-span for sensing purposes. The device can sense an out-of-plane magnetic field due to an in-plane motion induced by a time varying driving voltage. Because of its geometry and due to Joule effect and electrostatic softening, the beam behaves like a Duffing oscillator. When the system is driven into resonance, the amplitude θ_{max} of oscillations depends on the stiffness of the film, which is in turn

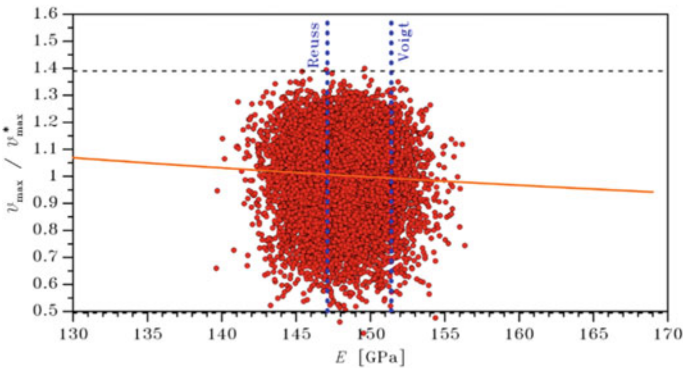

Fig. 4 Uncertainty-related scattering in the amplitude of oscillations of the uni-axial resonant MEMS magnetometer of [51]

affected by uncertainties in the elastic moduli of the polysilicon film and in the width of the film itself due to etch defects. Some exemplary results are reported in Fig. 4, in terms of dimensionless values of $\theta_{\max}$: in the graph, the orange line represents the expected trend according to the uncertainty-free physical model, while the region between the black dashed lines can be considered as a confidence interval on the basis of the scattering in the microfabrication parameters. Each red point provides a solution obtained by a Monte Carlo simulation, wherein both the elastic moduli and the etch defects were sampled out of ad-hoc set probability distributions. The blue vertical dotted lines are exploited to show that, in case of films constituted by a few grains only, the asymptotic Reuss and Voigt solutions are not valid to bound the value of the polysilicon Young's modulus E (reported along the horizontal axis) and, therefore, to estimate the spreading of the oscillation-related device performance.

5 Discussion and Outlook

The present chapter is intended to give an overview on the role of Mechanics in Microsystems, as deeply explored in more than twenty years of experience by the research group of Politecnico di Milano and to create a bridge with the impressive activity carried out in the framework of national and international AIMETA initiatives.

Three major examples were selected: the importance of mechanics in the design of inertial MEMS, the strict correlation between mechanics and other physical phenomena in piezoelectric MEMS and its major role in reliability issues in Microsystems.

Many other examples could have been selected, our goal was to rise attention on a discipline that had an impressive growth in the last twenty years and that we believe will continue to grow in the future years due to many new trends and applications, as briefly commented here below.

5.1 Microsystems as a New Engineering at the Micro-Scale

Today Microsystems and related production represent a new branch of Engineering that developed fast and obtained an important role in many applications.

Microsystems and related technology are still seen as belonging to the world of semiconductors. They share with it the production in clean rooms, the reduced unit cost and the necessity of very high production yields to compete in the market.

Nevertheless, the additional complexity given to microsystems by their multi-physics character transforms them in peculiar devices and was the reason why a new approach in research and development, specifically devoted to Microsystems, emerged in the last years.

Mechanics and interconnected disciplines play an important role in the study, design and fabrication of MEMS, this is why our group and many other groups related to AIMETA in Italy devoted a lot of energies to Microsystems.

5.2 Outlook: Future Prospects

The world of microsystems is rapidly evolving; products like inertial or pressure sensors are now considered as *classic* in the MEMS world and are reaching high performances, new products like PMUT, micro-speakers, micro-pumps, micro-mirrors, resonators are ready for widespread applications.

The development of high performance sensors and innovative devices with complex actuation systems will ask for improvement in the mastering of nonlinear and multi-physics phenomena and probably will allow for the discovery of new phenomena coming from multi-physics interactions which have not been completely explored.

In the present scenario of a more and more connected world, the use of micro sensors and actuators will become pervasive; we are already used to have sensors in many common places and objects like public offices, cars, mobile phones.

We mention here four areas in which Microsystems will evolve and in which research is of paramount importance.

The first is the area of more and more smart MEMS in which sensing on physical basis and actuation are accompanied by a pre-treatment of acquired data, also with the use of Machine Learning approaches.

The second is the area of bio-MEMS, Microsystems devoted to biomedical applications. Many products are already available, at least at a prototype stage, like micro-pumps for drug delivery, pace-makers assisted by micro-sensors, micro-robots floating in blood, micro-ecographic scanners. The massive diffusion is mainly hindered by the high level of reliability necessary for application to humans.

A third trend can be recognized in the application of ideas coming from the study of metamaterials to MEMS. Meta-MEMS are now becoming an intense area of research that will give fruitful results in few years.

As a fourth area of promising development, we mention here the use of additive manufacturing techniques at the micro-scale as a useful additional possibility for the obtainment of ad hoc designed MEMS and also for the simplification of some production phases in industrial products.

We tried in this chapter to give a flavour of our experience in the fascinating world of Mechanics of Microsystems. We believe that the journey initiated many years ago can still offer new and stimulating experiences and challenges to young researchers looking for multi-disciplinary and non-traditional research environments.

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Additive Manufacturing: Challenges and Opportunities for Structural Mechanics

Stefania Marconi, Massimo Carraturo, Gianluca Alaimo, Simone Morganti, Giulia Scalet, Michele Conti, Alessandro Reali, and Ferdinando Auricchio

Abstract Additive Manufacturing (AM) is an innovative manufacturing technology which has known over the recent years a widespread adoption among many key industrial sectors. However, the potentiality of such a technology is nowadays far away to be fully exploited in industries, making AM still a very active and multidisciplinary field of research which includes material science, metrology, computational mechanics, mathematics, chemistry, biology, medicine, and many other disciplines. In the present contribution, we overview recent progresses on AM research, with a special focus on the work carried out in the last decade by CompMech (Computational Mechanics and Advanced Materials team) at the University of Pavia.

Keywords Additive manufacturing · Computational mechanics · Numerical modeling · Bio-printing · Structural optimization · 4D printing

1 Introduction

Additive Manufacturing (AM), mostly known as three-dimensional (3D) printing, is an innovative manufacturing process that allows the creation of objects starting from a digital 3D model. The underlying approach of every AM technology is based on the same general concept (Fig. 1): the selected object is manufactured through a *layer-by-layer* process. Initially, the virtual model is divided into thin layers of equal width. Then, each layer is sent to the 3D printer which deploys the material in the specified sequence. This approach allows the creation of geometrically complex and highly customizable products with lower costs and times than traditional production technologies. AM paves the way to the creation of components even impossible to

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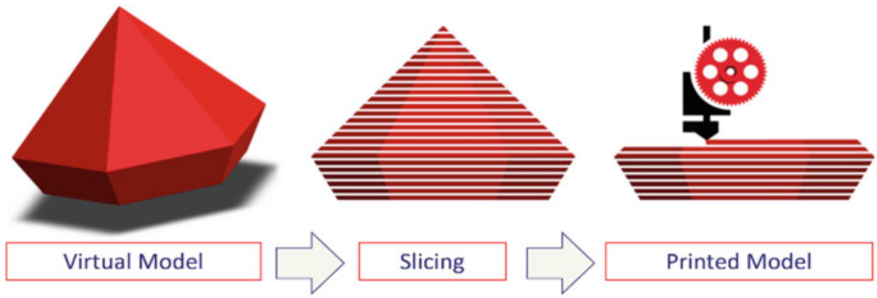

Fig. 1 The AM approach: the 3D virtual is sliced into layers and each layer is deposited and cured by the machine

produce with classic subtractive technologies—such as milling, turning, or numerical control machining—or injection molding techniques.

Furthermore, AM technologies offer the possibility of creating objects made with different materials, which therefore have different physical and mechanical characteristics, within a single production process. The panorama of AM technologies is very broad and heterogeneous and there is a wide variety of 3D printer machines available on the market, which differ mainly in the way layers are deposited and cured, according to the type of employed material. The American Society for Testing and Materials (ASTM) recognizes seven main categories of 3D printers [1], which differ mainly in the adopted **materials** and on the material **deposition and treatment** technique. Among the most commonly used materials, we can mention thermoplastic polymers, photopolymer resins, and metal alloys.

Thanks to the possibility of creating geometrically complex objects with low costs compared to other manufacturing solutions and thanks to the wide range of materials available on the global market today, AM and its applications have spread widely in the last decade in many sectors: from architecture to aeronautics, from automotive to art, from healthcare to industrial production. Recent market analyzes, carried out on the main world economic powers (USA, Europe, China, and Japan), show a significant increase in investments and revenues in the AM sector in relation to the most interesting application fields, including automotive ($\simeq$32% of total applications), consumer products ($\simeq$18%), medical ($\simeq$12%), academic ($\simeq$8%) and aerospace ($\simeq$8%). From an industrial perspective, among the main benefits of AM technologies, we can mention the production of complex geometries not achievable with other production systems, the reduction of production times and costs, the reduction of the assembly phases, the possibility of high customization, the chance to remote production and the optimization of the supply chain.

The content of the present chapter stems from the experience gained by the Computational Mechanics and Advanced Materials team (https://compmech.unipv.it/) starting from 2011. First approaches to AM technologies were carried out in the medical field, being AM the only chance to transform a complex virtual anatomical model into a physical object. From that experience, the interest in AM technologies grew and involved many applications and fields of study, such as computational modeling, process simulation, mechanical testing, and the development of innovative AM-based approaches. Ten years later, the group can count on three established laboratories:

- **Protolab**, devoted to the industrial application of plastic AM technologies and to the study of mechanical behavior of 3D printed materials. It can deal with all the thermoplastic materials, including techno-polymers and long fiber-reinforced polymers (Machines: HP MJF 580 Color, Stratasys J750 Digital Anatomy, Markforged X7, APIUM P220, 3NTR A4v3, 3NTR A4v2, Leapfrog Creatr, Leapfrog Creatr HS).
- **3DMetal**, devoted to metal AM technologies and their applications, especially to the industrial field. It can process metal powders such as Stainless-Steel, Aluminum, Titanium, Cobalt Chromium (Machines: Renishaw AM400, Nabertherm oven).
- **3D4Med**, devoted to the clinical application of AM technologies, including surgical planning, training, and devices manufacturing. The laboratory is in the Fondazione Policlinico San Matteo of Pavia, being the first Italian Clinical 3D Printing Laboratory (Machines: Stratasys Objet 260 Connex 3, Formlabs Form 2, 3DSystems Projet 460 Plus). Moreover, 3D4Med laboratory is listed as HP Multi Jet Fusion Tech Center (https://reinvent.hp.com/it-it-3dprint-servicebureau) for medical related manufacturing. The group has built strong industrial partnerships on AM related topics, including worldwide leaders in AM technology as Stratasys.

2 Computational Modeling

Numerical methods can be used to predict temperature field and residual stresses in AM processes, optimizing process parameters and part geometry. Even if a holistic computational framework able to compute a truly multi-physics and multi-scale analysis of AM processes is far from being achieved, there have been several attempts in the literature to couple results obtained solving thermal and thermo-mechanical problems at different scales.

2.1 High-Fidelity Thermo-Mechanical Model

Thermo-mechanical models including a heat source that moves following the laser scan path are called high-fidelity or mesoscale models. Such a group of models usually includes powder in the computational domain and can be used to predict temperature distribution and residual stresses in a small portion of the building domain.

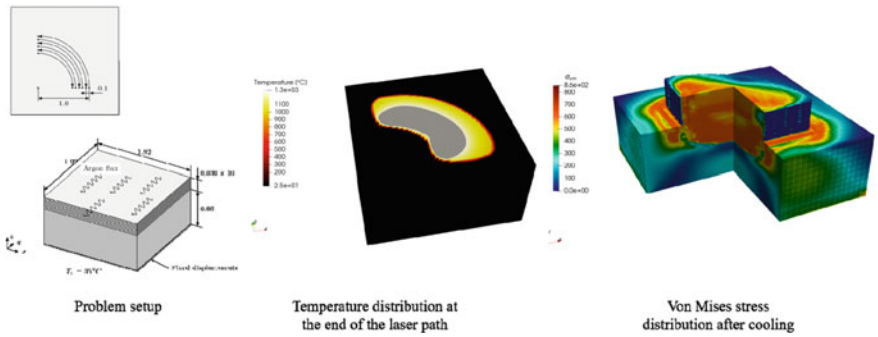

Fig. 2 An example of high-fidelity thermo-mechanical results. From left to right: A multi-layer problem setup, the resulting temperature distribution at the end of the scan path, and the von Mises residual stresses after cooling (Reprinted from [2] with permission from Elsevier)

Figure 2, for instance, reports a problem where the laser follows an arc path on ten powder layers. The simulated temperature field allows capturing melt pool morphology and the corresponding mechanical solution allowing to estimate residual stress distribution in the solidified domain [2].

2.2 Part-Scale Thermo-Mechanical Model

When we are instead interested in predicting deformations and residual stresses at the scale of the full part, further simplifications are required in the computational model. Therefore, part-scale models usually do not include powders (with exceptions) and do not model the laser heat source, whilst a constant thermal load or a fixed activation temperature is applied when either a single or a set of agglomerated physical layers is activated following a process similar to the one depicted in Fig. 3 and presented in [3].

2.3 Product Simulations

In the context of AM modeling, another computational challenge is surely the simulation of AM products with complex/small geometrical features (e.g., lattice structures). In fact, the material model of lattices components such as the one depicted in Fig. 4, shows an anisotropic behavior that strongly differs from the nominal model of bulk material components. Moreover, the process-induced geometric flaws—highlighted in Fig. 4—require to compute directly on the as-built geometry of the part as acquired, e.g., by means of Computed Tomography (CT), since it has been widely

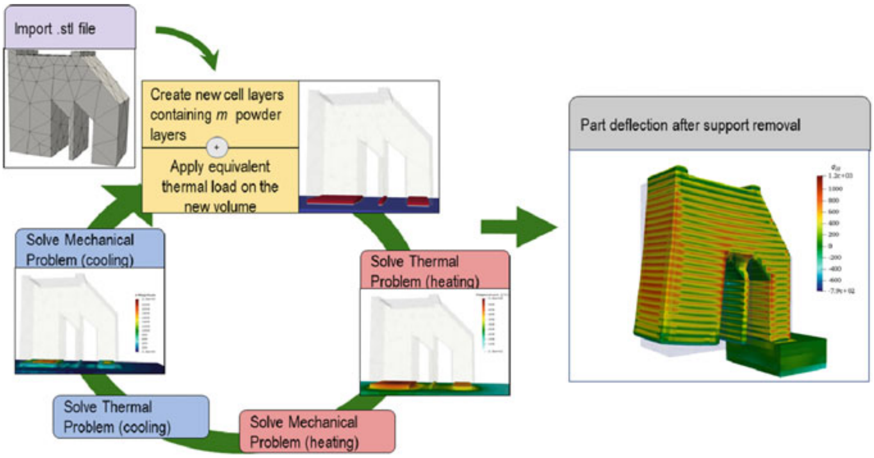

Fig. 3 Part-scale thermo-mechanical model (Reprinted from [3] with permission from Elsevier)

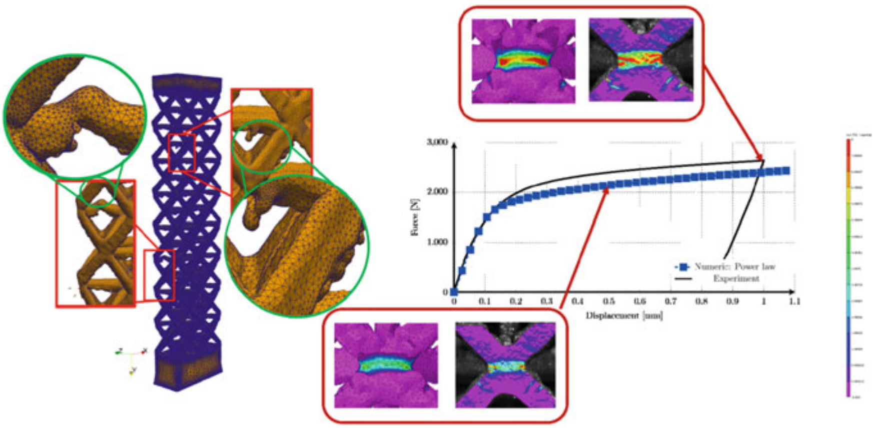

Fig. 4 AM lattice product analysis: On the left, the mesh was obtained on the CT-scan image of the specimen and on the right experimental vs. numerical Force–Displacement curves together with a comparison of numerical results with Direct Image Correlation

demonstrated in the literature that results obtained on the nominal, as-designed geometry are quite inaccurate compared to experimental measurements [4–6].

3 Structural Optimization

Structural optimization techniques aim at obtaining optimized performance from a structure, satisfying several functional constraints. The need for optimized components in structural applications has increased over the years and has become nowadays fundamental, due to the limited availability of commodities, the market competition, and new emerging manufacturing processes such as AM technologies.

3.1 Topology Optimization and Additive Manufacturing

In the context of the design for AM, a fundamental role is played by structural Topology Optimization (TO) which was firstly introduced in the seminal work by Bendsøe and Kikuchi [7]. TO aims at obtaining, inside an initial design domain, an optimal distribution of material that minimizes a given objective function such as compliance or mass and is subject to functional (e.g., stress or displacement) or manufacturing (e.g., overhang, member size) constraints. TO has been successfully extended to deal, among others, with buckling problems [8], resonance problems [9], thermal problems [10].

Several TO approaches have been proposed, as documented by the huge literature available on this topic. Nearly all such approaches are iterative, meaning that the optimized material distribution is found by repeatedly performing a finite element analysis (FEA) that involves the solution of the governing equations of the problem under study. The most common approach present in the literature is the so-called Solid Isotropic Material with Penalization (SIMP) method where intermediate values of the density function between 0 (voids) and 1 (bulk material) are penalized. Other widespread approaches are the Bi-directional Evolutionary Structural Optimization (BESO) method [11], the level-set method [12], and the phase-field method [13–15].

Structures designed by TO usually present complex shapes with geometrical features that cannot be produced through classical manufacturing techniques such as molding, casting, or even CNC machining. Conversely, AM processes allow the production of close-to-freeform components with few or no manufacturing constraints and limitations, depending on the adopted technology. The design freedom given by AM technologies is fully exploited by TO-based structures, where the focus of the design is mainly on the component functionality and not on its manufacturability.

As an illustrative example, a three-dimensional Messerschmitt–Bölkow–Blohm (MBB) beam problem is presented, where it assumed a load $P = 25$ N and a linear elastic material with Young's modulus $E = 2300$ MPa and a Poisson ratio $\nu = 0.3$ corresponding to a polyamide 12 (PA12) polymer processed by an HP 580 MultiJet Fusion® 3D printer. The goal of the TO is compliance minimization under a volume constraint of 0.35. Figure 5a shows the 3D representation of the optimized solution obtained by means of a phase-field based isogeometric TO routine [16]. The MBB

(a) (b)

Fig. 5 From numerical results to 3D printed structure: **a** three-dimensional virtual model of the optimized geometry as obtained from TO solution, **b** experimental test of the 3D printed structure

optimized design has been manufactured with an HP 580 MultiJet Fusion® 3D printer, which is a powder-based AM system equipped with an electric lamp as a power source for heating. The virtual model of the part to be manufactured is firstly sliced in layers which are subsequently processed by the printing software to obtain all the necessary process data to be sent to the 3D printer. The process starts with the deposition of a 0.1-mm thick powder layer of PA12 polymer and continues with the printing head that selectively deposits chemical agents in the sections to be melted; finally, the electric lamp moves on the top of the layer transferring heat to the powder by thermal irradiation. The obtained design has been tested through a three-point bending experimental setup, as shown in Fig. 5b. Experimental evidence showed the effectiveness of the adopted TO approach in minimizing the compliance of the proposed structure, characterized by a volume equal to 35% of the design domain.

4 The 4D Printing

The continuing advancements in materials science promote the use of active materials in 3D printing to create structures capable of evolving their shape, properties, and/or functionalities over time under the application of an external stimulus (e.g., heat, light, water, or pH). This type of 3D printing is usually referred to as "4D printing" since time represents the fourth dimension [17].

Since its introduction in 2013 [18, 19], 4D printing has been at the center of numerous fundamental and applied research works. In fact, the self-(dis)assembly of 4D printed structures under an applied stimulus enables the manufacturing of architectures impossible to realize with standard 3D printing or the simplification of the manufacturing process by lowering the number of components and/or materials needed. This feature is particularly relevant for packaging, deployment, and transportation purposes in remote applications such as in the medical, pharmaceutical, or space field.

In this context, numerical methods can be used to address the design of 4D printed structures where complex multi-physics take place. Generally, we can distinguish between the *forward* and the *inverse* design problem. The *forward* problem consists in determining the final shape of the 4D structure, given its material(s) properties

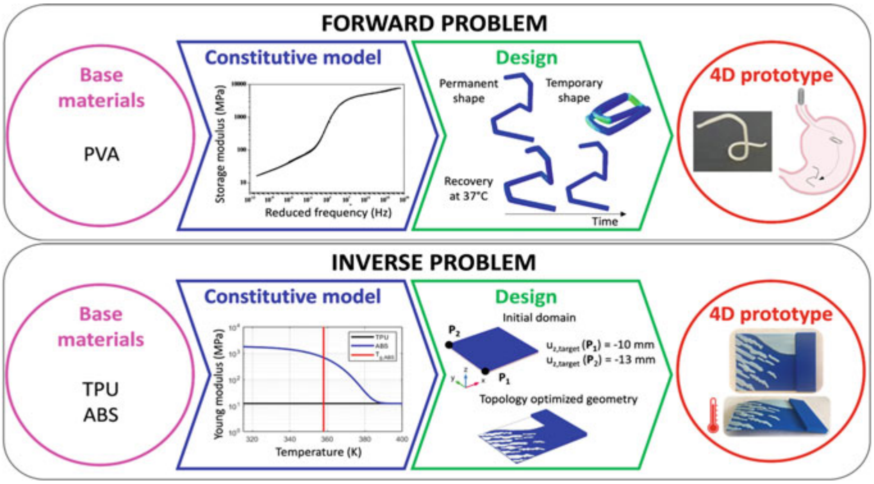

Fig. 6 Computational design of 4D systems: schematic examples showing how to use mathematical models and numerical methods to solve the *forward* and *inverse* problem. TPU and ABS are represented by white and blue colors in the 4D structure, respectively (Reprinted from [21] with permission from Elsevier)

and disposition as well as the applied stimulus and loading history. An example of *forward* problem is shown in Fig. 6, where finite element analysis is used to predict the shape change of a self-expandable drug delivery system for gastric retention based on a thermo-responsive poly vinyl alcohol (PVA) shape memory polymer. The system is first manufactured in a "permanent" configuration via hot melt extrusion, subsequently deformed in a "temporary" configuration suitable for administration, and finally able to self-expand and recover the permanent configuration upon interaction with gastric fluid at body temperature (37 °C), thus preventing passage through the pylorus [20, 21]. The model employed to describe the shape memory response of the polymer is based on the generalized Maxwell viscoelastic model, calibrated on multi-frequency dynamic-mechanical analysis. The *inverse* problem consists in determining the material(s) disposition within the 4D structure, given the target shape, material(s) properties, as well as the applied stimulus and loading history. An example of an *inverse* problem is shown in Fig. 6 where a finite element model is combined with a density-based topology optimization method to find the placement of the optimal materials in a prescribed design domain [22]. The bilayer system, manufactured via fused filament fabrication 3D printing technology, consists of thermoplastic polyurethane (TPU) and acrylonitrile butadiene styrene (ABS). The former is prestressed by means of the 3D printing process itself, while the latter undergoes a glass transition transformation close to $T_g = 85$ °C. Upon heating above 85 °C, the decrease of ABS stiffness causes the release of the prestress and, consequently, the deformation of the structure from its initial printed flat, square configuration towards the target twisted configuration.

5 New Technologies

5.1 *Exploring New Ways for 3D Printing Advanced Materials*

Innovative AM processes are currently explored to produce metallic/ceramic components following a new production paradigm, which combines a printing technology on water-based colloids and a thermal debinding/sintering based also on innovative fast and pressure-assisted sintering techniques.

The proposed, very promising, approach overcomes the major limitations of standard additive manufacturing technologies being based on:

1. environmentally friendly water-based formulations, including only non-toxic polymers as binding components;
2. extrusion of colloids, easy to prepare and print, applicable to a wide range of materials, including metals, alloys, ceramics, metal-based or ceramic-based composites, allowing great flexibility with minimal adjustment in the process;
3. very low-cost printing apparatus, making this approach particularly appealing for applications requiring rapid and cheap prototyping of complex structures;
4. drastic reduction of the time required to obtain a fully densified object.

5.2 *Advanced Simulation Technologies for the Design of Sintering Processes*

Sintering is a manufacturing process in which a powder is heated and transformed into a solid mass without melting the grains. This process allows an easier production of finite objects made of materials with a high melting point, such as metal, ceramic or composite materials. Different sintering techniques can be adopted: Solid-State Sintering, Hot Isostatic Pressing, Hot Pressing, and Spark Plasma Sintering (SPS) represent the most common ones. In brief, the SPS equipment is composed of a die-punch assembly made of graphite where the powders are placed, a hydraulic jack, and a control system. During the process, the applied electric current flows through the assembly and heats the sample by the Joule effect, meanwhile, the jack applies a vertical load to increase the product final density. Both working temperatures and current fields affect the microstructure of the resulting material. Consequently, their distribution during the process must be as homogeneous as possible. In this context, numerical simulations cover a key role in the design and optimization of the SPS process, allowing a continuous description in space and time of the temperature and current fields. Relevant data and information (that cannot be achieved experimentally) can be obtained by means of advanced computational technologies taking into account conduction, radiation, contact, thermal expansion, as well as thermo-electrical and thermo-mechanical coupling (Fig. 7). For example, both the identification of critical zones (i.e., zones characterized by localized high thermal/electrical gradients) and the prediction of the object volumetric shrinkage

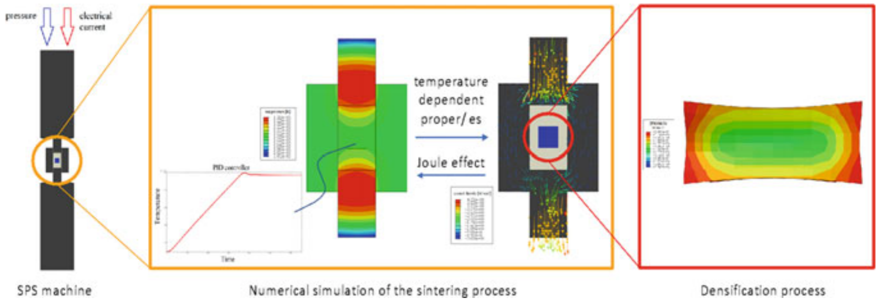

Fig. 7 Simulation of the sintering process

can drive the design of the pre-process shape. Furthermore, numerical parametric studies are relatively inexpensive and can give an improved understanding of the main parameters regulating the entire manufacturing process.

6 Support for Surgery

Among the sectors that have welcomed the emerging AM technologies with greater enthusiasm, the medical and biomedical ones stand out: geometric complexity and the possibility of producing a limited number of highly customized pieces at low costs, open the doors to multiple applications, thus responding to growing attention to the personalization of the therapeutic and care plan.

Today, AM technologies play a relevant role in the medical field with a number of destinations of use: the production of anatomical models for surgical planning [23], training [24], teaching [25, 26], consulting with the patient and his/her family to retrieve an informed consent [23], development of patient-specific intra-operative tools and instruments [27], as well as personalized prostheses and aids [28]. Many clinical specialties have already benefited from AM, with a major use from orthopedic and maxillo-facial surgical specialties for surgical planning, being the first clinical users of the technology thanks to an easier image processing to retrieve the bony structures of interest.

The success of AM technologies in the medical field stems from the anatomical understanding that derives from the use of a 3D physical object that reproduces the anatomy/pathology of interest. The manufacturing of patient-specific anatomical models starts from the acquisition of medical images with volumetric content—such as Multi-Detector Computed Tomography (MDCT) or Magnetic Resonance—saved in the Digital Imaging and Communications in Medicine (DICOM) as shown in Fig. 8. It is important to use high-quality contrast images with a slice thickness of at least 2 mm and to acquire all the contrastographic phases required to have clear visibility of all the structures of interest. Each anatomical structure is selectively identified in the phase in which it is mostly visible, thanks to a so-called *image*

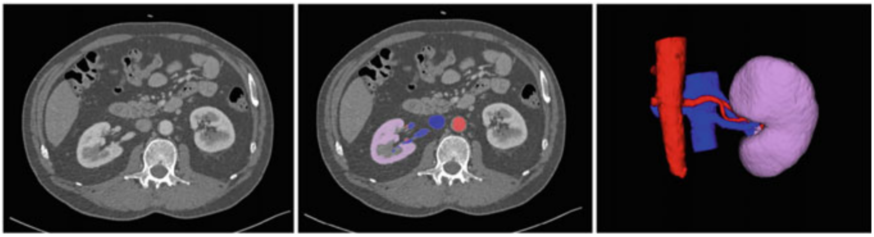

Fig. 8 From left to right: the process of image segmentation. The original MDCT, the output of the segmentation process performed on the anatomical structures of interest (kidney, renal arteries, and veins), the interpolation of the labels, and the 3D rendering of the model

segmentation process, which is based on the *labeling* of each structure of interest in each medical image slice. The segmentation process can be carried out by means of several commercial programs, implementing different approaches. The segmentation commonly starts from the definition of a gray level window including all the gray shades of the structure of interest. According to the specific approach, it can be necessary to define one or more starting points on the images from which the segmentation algorithm will start its evolution. The algorithm grows within each slice and throughout the slices, controlled by various parameters. When the algorithm has completed its evolution, the resulting label set is rendered to produce a 3D surface, which can be then exported as one or more STL files, suitable for AM production.

The 3D printed replica contains all the information displayed in the original image dataset but is able to display them in a more intuitive, easy access, and clear fashion. This is the reason why 3D printed anatomical models can also be very useful in seeking informed consent from the patient, who is usually not aware of fine anatomical details, and in teaching for students and postgraduates. AM is also used in the design and production of realistic simulators of surgical and interventional procedures, including robotic and laparoscopic ones (see Fig. 9). In this context, major efforts are made in replicating the mechanical behavior of biological tissues.

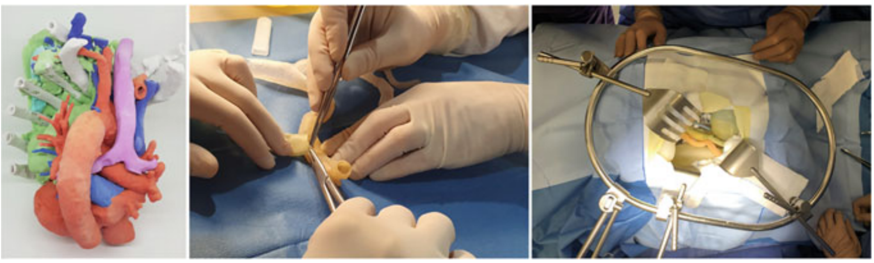

Fig. 9 From left to right: an example of a patient-specific model of a thoracic pathology for surgical planning, an optimized material for suture training on small vessels, a complete simulation platform for training on donor living kidney transplantation

A reliable replica of mechanical properties of bony structures can be obtained, while the biomechanics of soft tissues are more challenging to reproduce: nowadays available AM materials are still not capable of perfectly reproducing the behavior of soft tissues. Rubber-like photopolymers and hydrogels are doing the best job, but further research is still required on this topic. With that said, it becomes clear why fields such as abdominal and pelvic surgery are less involved in the use of 3D-printed models for training purposes [29].

Another widely used application concerns the production of surgical instruments and guides, prostheses, and orthoses. The high customization of devices, as well as the possibility of optimizing the materials used and their mechanical properties, represents the key to the success of AM in this field.

The use of AM for the production of drugs and poly pills is also included in the perspective of personalized medicine, capable of bringing together a patient's drug therapy in a single pill or in pills with customized dosages. AM also finds application in the field of regenerative medicine and tissue engineering for the production of biodegradable scaffolds, seeded with cells after the printing, and for the production of cellularized biological material constructs. The great versatility and speed of production guaranteed by the use of 3D printing allow it to be used even in emergency situations; examples come from medical and biomedical applications, created with AM techniques, born in the context of the Covid19 emergency. In fact, various companies, laboratories, and hospitals relied on AM for the production of masks, protective devices, and specific components and fittings for lung ventilators and Continuous Positive Airway Pressure (CPAP) systems, to make up for the lack of stocks due to the emergency situation [30].

7 Bio-Printing and New Materials

Bioprinting (BioP) consists of the 3D printing of a bioink, i.e., the combination of biological products (e.g., cells) and a biomaterial (e.g., hydrogel) that mimics the extracellular matrix physiological features providing support for cell growth [31].

To date, Extrusion-Based BioP (EBB) is one of the most used techniques. Bari et al. [32] used EBB to produce scaffolds, enriched with lyosecretome, a freeze-dried formulation of mesenchymal stem cells secretome, containing proteins and extracellular vesicles (EVs) for improving the speed and quality of new bone formation. Specifically, by changing scaffold parameters and tuning scaffold geometry, it is possible to control proteins and EVs release kinetics.

BioP usually involves in vitro cell culture, so it is important to ensure the sterility of printing products. In this context, Scocozza et al. [33] proposed a framework to sterilize PCL pellets using a 3D commercial bioprinter before starting the printing process. In general, BioP process (see, e.g., Fig. 10) can be divided into three phases: pre-printing (bioink formulation); 3D printing (construct fabrication); post-printing (cross-linking, incubation, and construct characterization). Nowadays, trial-and-error procedures are employed in the laboratory practice, so computational

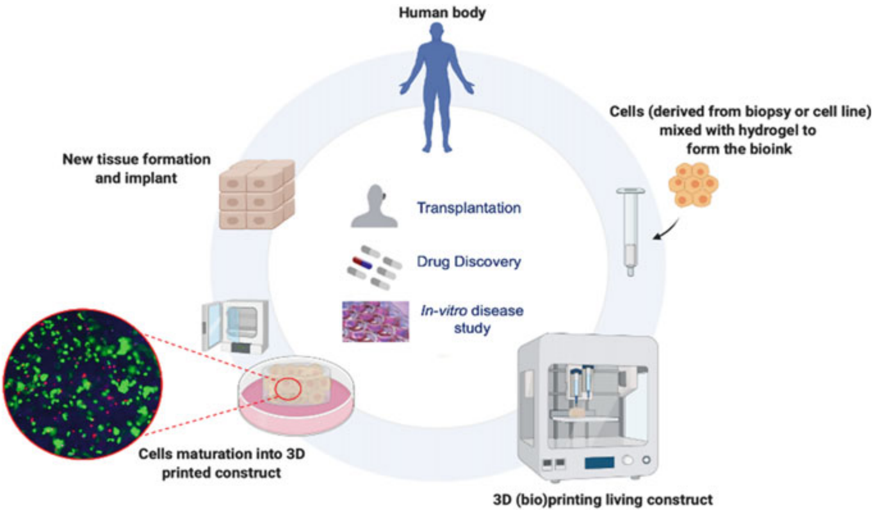

Fig. 10 Bio-printing process workflow

approaches have been developed to provide in silico answers to BioP process variables decisions. For example, Hajikhani et al. [34] implemented a reaction–diffusion model to describe the crosslinking process of an alginate and gelatin-based hydrogel with calcium chloride. Experimental results are fitted within the proposed modeling framework, which is thereby calibrated and validated. The present study addressed the evolution of the gel front with time. This information might guide the design of optimal post-printing protocols.

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Biomechanics in AIMETA

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Abstract This chapter aims at presenting a concise picture of the research in Biomechanics developed by researchers of various disciplinary areas that refer to AIMETA. The research collectors are the international journal Meccanica, published by Springer, and the AIMETA congresses including publications of studies presented therein. This chapter is devoted to studies related to AIMETA activity and mainly refers to the topics from the above-mentioned sources. Final comments and future developments are outlined.

Keywords Biomechanics · Biological tissues · Articular biomechanics · Medical devices · Biological fluid mechanics · Cardiovascular mechanics

1 Introduction, from Mechanics to Biomechanics

Biomechanics is the science that studies the structure and function of biological systems using methods and knowledge of Mechanics.

The birth of Biomechanics can be dated back to the first studies of Aristotle (384–322 BC) and later developed by Galen (129–210), Leonardo da Vinci (1452–1519), Galileo Galilei (1564–1642), and Giovanni Alfonso Borelli (1608–1679), up to a few further advancements through the nineteenth and twentieth centuries. It found in the last 50 years a rebirth of interests that became an exponential growth in the last 20 years. In the 50's of the past century, it was in fact considered a subject for

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technicians rather than a science worthy of the scientific research attention, as it is today in its own right.

The extraordinary development of Biomechanics during the recent decades is due to the combination of various factors, such as the development of both computational tools and 3D graphic representation, the improvement of measurement techniques and instruments. It also benefited of the greater attention that today is paid to the health and wellness of the human being, which led to the convergence on Biomechanics of various areas of knowledge, from Medicine to the different branches of Engineering, passing through Physics, Materials Science, advanced technologies (Vision, Magnetic Resonance Imaging (MRI), Computed Tomography (CT), Fluoroscopy, 3D printing, etc.), highlighting the strongly interdisciplinary aspect of Biomechanics.

This convergence of knowledge has allowed on the one side to build increasingly refined mathematical models of physical and biological phenomena, and on the other side to develop advanced analysis and synthesis tools, which allow prompt and preventive diagnoses, as well as efficient design methods of medical devices and instruments.

Theoretical and applied Biomechanics developed naturally as a sub-specialty of Mechanics. They started from theoretical and applied interests in various branches of Mechanics holding competencies needed to study problems of biomechanical relevance.

In order to give wide visibility to the activities of Biomechanics in the AIMETA community, the AIMETA Biomechanics Group (GMBA) was established, with the aim of aggregating skills of the different sections and stimulating synergies on Biomechanics topics. It is hardly necessary to emphasize that Italian research in Biomechanics is at the forefront of the international state of the art. Unfortunately, space limits prevent to give an account of all the biomechanical issues developed so we apologize to readers for the inevitable omissions. Priority was given to research originated or presented within the AIMETA activities.

2 Developments Since 70's to Nowadays

2.1 Tissue/Solid Biomechanics

The relevance of bioengineering problems to applied mechanics was recognized by the President of AIMETA in the preface to the proceedings of the II AIMETA conference (Naples 1974). However, it was after the XV AIMETA conference (Taormina 2001), hosting the first mini-symposium on Mechanics of Tissues and Implants [70], that AIMETA conferences became an important forum for discussion and exchange of ideas among different areas of Biomechanics and for identifying potential collaborations to address most challenging biomechanical problems. Contributions ranged from fundamental research to clinical applications, including theoretical, computational, and experimental work, facing problems whose physical dimensions span from the microscopic environment, at the cell-size, across the intermediate scales up to the macroscopic and organ level. Themes ranged from molecular and cell mechanics to cell motility, from mechanics of soft or mineralized biological tissues to growth and remodeling, from organ mechanics to medical devices. Such a variety reflected in a multiplicity of physical problems and required the development of a similarly wide range of methods and concepts.

The mechanical characteristics of *cells* are highly variable across phenotype and dynamically evolve in response to changes in the microenvironment. Cells behave as both passive and active materials, supporting and transmitting loads and generating forces. In turn, cells sense and respond to chemo-mechanical signals from their environment (e.g., [54]). Their mechanical properties have emerged as potential label-free biomarkers for detecting the presence of an underlying condition or disease (e.g., cell activation, degree of differentiation, or metastatic potential).

Several experimental techniques are available to study cell mechanics. Because a typical cell body is about 10 μm in diameter, to capture a complete picture of mechanical interactions and physical properties of cells, the resolution of the tools utilized in cellular biophysical studies has to be in that order of cells' size or smaller. Traditional methods, such as micropipette aspiration, atomic force microscopy or optical tweezers have limited throughput. Emerging microfluidics-based methods have enabled single-cell mechano-phenotyping at throughput of thousands of cells per second [40]. Information gained from these studies is utilized in computational models that address cell mechanics as a collection of biomechanical and biochemical processes (e.g., [54, 62]). These models are advantageous in explaining experimental observations by providing a framework of underlying cellular mechanisms. They also enable predictive, in silico studies, which would otherwise be difficult or impossible to perform with current experimental approaches.

Several studies investigate *nanoscale structures*, including macromolecules and their aggregates. Proteins constitute the main building blocks of biological systems, and their mechanical vibrations play a pivotal role in biological activity. Lowest-frequency vibration modes are related to protein conformational changes, which are strictly linked to their biological functionality. Scaramozzino et al. [87] present a coarse-grained finite element space truss model suitable for investigating protein vibrations. Based on modal analysis, their model turns out to be an effective tool to investigate protein dynamics, conformational changes and protein stability. Collagen is the main structural protein in the extracellular matrix. Marino and Vairo [51] propose an elasto-damage model for the mechanics of collagen fibrils. They apply a multiscale approach that allows to account for nanoscale mechanisms and to introduce model parameters with a clear biophysical/biochemical meaning. Their model is able to reproduce many well-known experimental features of fibril mechanics.

Mechanical signals received by a cell can originate in the external environment, or they can be the signals from extracellular matrix or neighboring cells. Mechanical signals are transmitted to the appropriate targets inside the cell through biochemical pathways or relayed through the *cytoskeleton*. Various signaling pathways may be activated after mechanical signal reception, depending on the type of mechanical stimulus received (whether it be tension/stretch, compression/contraction, or shear/distortion). Mechanical signals may also be transmitted to the nucleus through the cytoskeletal network and affect transcription processes. In his pioneering work, Ingber [41] surmises the cytoskeleton behaves like a tensegrity architecture, i.e., a system of isolated components under compression (microtubules) inside a network of continuous tension (microfilaments and intermediate filaments). Fraldi et al. [32] remove the standard hypothesis of rigid struts in tensegrity structures when used to idealize the cell cytoskeleton mechanical response. Accordingly, they explain some counter-intuitive mechanical behaviors actually exploited by cells for storing/releasing energy, resisting to applied loads and deforming. Focal adhesions operate at the interface between cells and extracellular matrix, as part of the cell mechano-sensing machinery. Fusco et al. [33] investigate how the dynamics of assembly and disassembly of focal adhesions is influenced by the substrate stiffness. Their approach to focal adhesion dynamics characterization is a valuable investigation tool for cell mechano-biology. Vigliotti et al. [92] analyze the response of cells on a bed of micro-posts. They use a homeostatic mechanics framework, enabling quantitative estimates of the stochastic response of cells along with the coupled analysis of cell spreading, contractility and mechano-sensitivity. Their results suggest that the increased foundation stiffness causes both the cell area and the average tractions exerted by cells to increase.

Biological tissues are ensembles of cells and extracellular matrix that together carry out a specific function. The range of mechanical properties exhibited by biological tissues is remarkable, and depends on both composition and structural organization of the constituent materials at nano- and micro-scales and the resulting tissue architecture/geometry at meso- and macro-scales. Understanding the mechanics of these complex materials is very challenging, given the multitude of intricate physical mechanisms that act over a very wide range of spatial and temporal scales. Soft tissues include muscle, tendons, ligaments, blood vessels, etc., and are characterized by abundant extracellular matrix containing collagen, elastin and ground substance. Mineralized tissues fulfill critical load-bearing functions throughout the skeleton, facilitated by hierarchically organized structures that are optimized to provide high stiffness and/or excellent resistance to fracture. Tissues have evolved over millions of years into complex and diverse shapes under the forces of natural selection. Evolution has also provided tissues with the capability to adapt to their specific environments during growth and to remodel and regenerate if they are damaged [20, 71].

Bone is a mineralized heterogeneous material with microstructural features. Fatemi et al. [28] use generalized continuum mechanics theories to account for the influence of microstructure-related scale effects on the macroscopic properties of bone. Falcinelli et al. [27] describe healthy bone and metastatic tissue using a linearly poroelastic approach, proposing a strategy for the quantification of fracture risk in metastatic femurs.

The anisotropic, non-linear elastic behavior of *soft biological tissues* may be accounted for by the hypothesis of hyperelasticity, using Fung-type potentials. Federico et al. [29] derive a necessary and sufficient condition for the strict convexity of such potentials, providing a clear physical meaning for the involved parameters and their relationship with the small-strain elastic moduli. Maceri et al. [50] study the mechanical response of soft collagenous tissues with regular fiber arrangement, using a nanoscale model and a two-step micro–macro homogenization technique. Entropic mechanisms and stretching effects occurring in collagen molecules are accounted for at the nanoscale. The model is based on few parameters, directly related to histological and morphological evidences. It is applied to tendon, periodontal ligament and aortic media, and is used to simulate some physio-pathological mechanisms.

The constitutive behavior of biological tissues is generally *time-dependent*. Vena et al. [91] present a constitutive model of the nonlinear viscoelastic behavior of ligaments, as a generalization of the quasi-linear viscoelastic theory. The time-dependent constitutive law assumes that a constituent-based relaxation behavior may be defined through different stress relaxation functions for the isotropic matrix and for the collagen fibers. The model is able to predict the time-dependent response of ligaments described in experimental works. Deseri et al. [24] introduce a hierarchic fractal model to describe bone hereditariness. The rheological behavior of the material is obtained using the Boltzmann–Volterra superposition principle. The power laws describing creep/relaxation of bone tissue are obtained by introducing a fractal description of bone cross-section, with the Hausdorff dimension of the fractal geometry related to the exponent of the power law. A discretization scheme is proposed by Di Paola et al. [25].

Tissues are organized into *organs*. Among the biomechanical studies of different organs, the biomechanics of the *eye* has received significant attention. The human cornea has the shape of a thin shell, originated by the organization of collagen lamellae parallel to the middle surface of the shell. The lamellae, composed of bundles of collagen fibrils, are responsible for the anisotropy of the cornea. Anomalies in the fibril structure may explain the changes in the mechanical behavior of the tissue observed in pathologies such as keratoconus. Pandolfi and Manganiello [63] employ a fiber-matrix constitutive model and propose a numerical model for the cornea that is able to account for its mechanical behavior in healthy conditions or in the presence of keratoconus, opening a promising perspective for the simulation of refractive surgery on anomalous corneas. Romano et al. [79] experimentally assess the differences between highly myopic eyes and emmetropic eyes in the biomechanical response to ex vivo uniaxial tests of the human sclera.

Biomechanical modeling of the *head* is crucial to analysis and simulation of traumatic brain injuries under impact loads, virtual reality and robotic techniques in neurosurgery, design and assessment of helmets and other protective tools. Velardi et al. [90] perform an experimental analysis and present a transversely isotropic hyperelastic model of tensile behavior of brain soft tissue. They adopt a transversely isotropic hyperelastic model and obtain material parameter estimates through tensile tests, accounting for regional and directional differences.

Muscles have the function of producing force and motion, and are responsible for posture, locomotion, as well as movement of internal organs. Phenomena causing muscle contraction range from the subcellular ion dynamics up to the macroscopic excitation–contraction coupling. The multi-physics behavior of muscle tissues fostered a continuous forefront research in Biomechanics. Cherubini et al. [13] present an electromechanical model of myocardium tissue coupling a modified FitzHugh-Nagumo type system with finite elasticity, endowed with the capability of describing muscle contractions. The diffusion process is set in a moving domain, thereby producing a direct influence of the deformation on the electrical activity, thus explaining various mechano-electric effects. Pandolfi et al. [64] develop a constitutive model for stochastically distributed fiber reinforced visco-active tissues, where the behavior of the reinforcement depends on the relative orientation of the electric field. They use their electro-viscous-mechanical material model to simulate peristaltic contractions on a portion of human intestine.

In addition to mechanical function, biological tissues and organs are living objects and present the capability of functional adaptation in response to diverse chemo-mechanical stimuli. Their behavior is governed by *growth and remodeling* responses on time scales from hours to months. Mechano-regulated growth and remodeling plays important roles in morphogenesis, homeostasis, and pathogenesis, including disease progression wherein normal tissue is altered (e.g., aneurysms), organs adapt (e.g., cardiac hypertrophy or dilatation) or abnormal tissue develops (e.g., tumors). Experimental methods and theoretical frameworks provide an increasingly detailed understanding of molecular and cellular mechanisms of growth and remodeling as well as tissue-to-organism level manifestations [2, 3, 21].

Grillo et al. [38] represent a biological tissue by a multi-constituent, fiber-reinforced material, in which two phases are present: fluid and a fiber-reinforced solid. They study growth, mass transfer, and remodeling. Sacco et al. [82] propose a mathematical description of biomass growth that combines poroelastic theory of mixtures and cellular population models. The formulation, potentially applicable to general mechano-biological processes, is used to study the engineered cultivation in bioreactors of articular chondrocytes.

Preziosi and Tosin [72] develop a multiphase modelling framework for the description of mechanical interactions of growing tumors with the host tissue. They account for the interaction forces between cells and a remodeling extracellular matrix, and for the diffusion of nutrients and chemicals relevant for growth, describing the formation of fibrotic tissue. Carotenuto et al. [12] take into account residual stresses that develop to make compatible elastic and inelastic growth-induced deformations. The residual stresses directly influence tumor aggressiveness, nutrients walkway, necrosis and angiogenesis.

Activity and autonomous motion are fundamental in living and engineering systems. The field of *active matter* focuses on the physical aspects of propulsion mechanisms, and on motility-induced emergent collective behavior of a large number of identical agents, whose scale range from nanomotors and microswimmers, to cells, fish, birds, and people. This is an interdisciplinary topic that involves different aspects related to the mechanics of machines, of solids and their interaction with the surrounding fluids.

Inspired by biological microswimmers, various designs of autonomous synthetic nano- and micromachines have been proposed [37]. Swimming, i.e., being able to advance in the absence of external forces by performing cyclic shape changes, is particularly demanding at low Reynolds numbers. This is the regime of interest for micro-organisms and micro- or nano-robots. Alouges et al. [1] present a theory for low-Reynolds-number axisymmetric swimmers and a general strategy for the computation of strokes of maximal efficiency. Arroyo et al. [4] study euglenoids, exhibiting an unconventional motility strategy amongst unicellular eukaryotes. That strategy consists of large-amplitude highly concerted deformations of the entire body, mediated by a plastic cell envelope called pellicle. A theory for the pellicle kinematics is devised, providing an understanding of the link between local actuation by pellicle shear and shape control, and suggesting that the pellicle may serve as a model for engineered active surfaces with applications in microfluidics. Gidoni and DeSimone [36] formulate and solve the locomotion problem for a bio-inspired crawler consisting of two active elastic segments, resting on three supports providing directional frictional interactions.

Another example of amazing mechanics taken from Nature is given by spiders' weight lifting. Pugno [73] discusses the smart technique they use, allowing a single spider to lift weights, in principle of any entity, just using a tiny pre-stress of the silk. Such a pre-stress occurs naturally with the weight of the spider itself when it is suspended from a thread. The related mechanism could be of inspiration for engineering solutions of related problems, and may have inspired ancient populations for dragging and lifting weights.

Several pathologic conditions can be effectively treated using *biomedical devices*. As an example, atherosclerosis is characterized by the presence of lesions (called plaques) on the innermost layer of the wall of large and medium-sized arteries. The plaques contain lipids, collagen, inflammatory cells, etc., and can rupture and impede blood flow downstream, leading to life-threatening problems such as heart attack or stroke. Stent therapy is widely adopted to treat atherosclerotic vessel diseases. Intravascular stents are small tube-like structures expanded into stenotic arteries to restore blood flow perfusion to the downstream tissues. The stent is mounted on a catheter and delivered to the site of blockage. The stent expansion and the stress state induced on the vascular wall are crucial for the outcome of the surgical procedure. Indeed, modified mechanical stress state may be in part responsible for the restenosis process. The outcome of artery stenting depends on a proper selection of patients and devices, requiring dedicated tools able to relate the device features with the target vessel. Migliavacca et al. [55] simulate the implantation of a coronary stent by means of a finite element analysis, showing the influence of the geometry on the stent behavior, and, more generally, how finite element analyses could help in stent design to ensure ideal expansion and structural integrity. Auricchio et al. [5] use finite element analysis to evaluate the performance of three self-expanding carotid stent designs, as a first step towards a quantitative assessment of the relation between device geometry and patient-specific carotid artery anatomy. Popliteal artery stenting is used for the endovascular management in peripheral deep artery diseases. The complex kinematics of the artery during leg flexion leads to severe loading conditions, favoring the mechanical failure of the stent. Conti et al. [17] reconstruct by medical image analysis the patient-specific popliteal kinematics during leg flexion, which is exploited to compute the mechanical response of a stent model, virtually implanted in the artery by structural finite element analysis.

Among other cardiac pathologies, aortic stenosis is the narrowing of the exit of the left ventricle of the heart. It may occur at the aortic valve as well as above or below this level, and typically gets worse over time. Percutaneous aortic valve replacement is a minimally invasive procedure introduced to replace the aortic valve through the blood vessels, as opposed to valve replacement by open heart surgery. Its clinical outcomes are related to patient selection, operator skills, and dedicated pre-procedural planning based on accurate medical imaging analysis. Morganti et al. [60] investigate a balloon-expandable valve and propose a simulation strategy to reproduce its implantation using computational tools. They simulate both stent crimping and deployment through balloon inflation. The developed procedure enables to obtain the entire prosthetic device virtually implanted in a patient-specific aortic root created by processing medical images. It allows the evaluation of postoperative prosthesis performance depending on different factors (e.g., device size and prosthesis placement site), in terms of coaptation area, average stress on valve leaflets as well as impact on the aortic root wall.

Three-dimensional (3D) printing is a disruptive technology quickly spreading to healthcare. On one hand, it allows the creation of patient-specific models generated from medical images, which can facilitate the understanding of anatomical details, ease patient counseling and contribute to the education and training of residents [69]. On the other hand, 3D bioprinting, allowing to print engineered 3D scaffold prototypes and to control the distribution of cells, can be used to create realistic in vitro models of tissues and organs, to be used for research purposes or in regenerative medicine.

2.2 Biological Fluid Mechanics

The fluid mechanics in biological systems played an important role in the scientific activity of AIMETA during last decades. Contributions belonging to this subject were present since the first AIMETA conferences in the 70's. Such pioneering studies were still limited to few presentations included in sessions of fluid dynamics. Those making explicit reference to biological flows were principally centered on the non-Newtonian behaviors of blood, whereas several others addressed fundamental aspects, like numerical methods, unsteady flows or irregular geometries, which had a later impact on the advancements of the subjects.

The first mini-symposium dedicated to Biomechanics, in 2001 AIMETA conference, hosted the first few contributions with modern approaches to the study of biological flows in situations of medical interest. A leap forward can be traced then to 2005 conference, in Firenze, which featured a dedicated session within the others in fluid dynamics. Since then, the contributions in biological fluid dynamics were constantly grown. The XIX AIMETA conference, Ancona 2009, represented a further step forward with a thematic lecture in cardiac fluid dynamics [66] and starting from that year, mini-symposia on Biomechanics were present in all following conferences where scientists could find an opportunity to meet and share knowledge and ideas on all aspects of Biomechanics including biological fluid mechanics.

The analysis of *blood flow* in the human circulation represents the principal subject around which the scientific activity centered its focus. Nevertheless, this was not the only one and other important topics, like the fluids inside the eye, gained special attention for their relevance in medical applications.

The circulatory system fulfills the task of carrying blood across the body; in this respect, fluid flow represents a principal actor for many mechanical phenomena that occur therein. Initially, a series of studies were dedicated to understanding how the non-Newtonian behavior of blood alters the solution for flow in regular vessels [22]. At the same time, blood is transported inside a domain surrounded by biological tissues, with both active and passive behaviors; therefore, cardiovascular fluid dynamics cannot be tackled without ensuring a proper account for the dynamics of soft tissues surrounding the vessel. This is a general rule for many applications of biological fluid dynamics, which involve the wider, slightly interdisciplinary topic of fluid–structure-interaction (FSI). FSI introduces a series of complexities both in the experimental settings and in numerical modelling that can be dealt with different approaches. The management of FSI is relatively straightforward when dealing with prosthetic elements whose geometrical and mechanical properties are known and well defined. Differently, biological tissues are subjected to alteration during time and represent a challenge for laboratory experiments; on the other side, the feasibility of numerical modelling critically depends on the availability of information about the mechanical properties of the biological tissue. In fact, native tissues are typically non-accessible and their properties can be only estimated. In an alternative numerical approach, characteristics on the moving geometry of the surrounding tissues can be recorded by non-invasive medical imaging (CT, MRI, Echocardiography) and these moving boundaries are implemented in the flow dynamical equation in a one-way interaction.

The *fluid dynamics inside the human heart* captured the attention of numerous studies [66]. The diagnosis of heart diseases represents a critical element of clinical cardiology because most cardiac dysfunctions are progressive and present clear symptoms only after the heart has undergone detectable pathological alteration. The recognition of a pathology at its early stage would permit its treatment by means of a non-invasive therapy such as lifestyle changes or by a light pharmacological therapy that can be effective only before the occurrence of irreversible modifications. In such a situation, the dynamics of blood flow takes special relevance as it immediately responds to minor alterations of the surrounding conditions; indeed, there are indications that a careful inspection of cardiac fluid dynamics can be informative to predict the risk of pathology progression [67].

On the methodological side, models for the fluid dynamics inside the heart chambers present the challenges of dealing with the boundaries undergoing large displacements. A straightforward numerical approach is based on the solution of the equation of motion (Navier–Stokes equations) inside a geometry assigned with a prescribed motion; this approach can be appropriate for patient-specific studies when the dynamics of the boundaries can be extracted, for example, from medical images [67]. On the other hand, this method does not face explicitly the physical phenomena associated with the reciprocal interaction between flow and tissues. More advanced computational techniques have been presented to accurately include FSI behaviors of tissues that, the other way round, can be less reliable for patient-specific studies as the mechanical properties of the tissues cannot be measured in vivo. This shortcoming can be particularly critical for the muscular ventricular walls (the myocardium), which presents a phase with active contraction driven by electrical stimulation. The description of these active phenomena requires a definition of a more complete electro-mechanical model and of the related physiological parameters [93]. Studies in cardiac fluid dynamics performed with different techniques demonstrated that the flow inside the left ventricle is characterized by the formation of a vortex structure that dominates the phenomena of blood transit and energetic balances (Fig. 1, left picture). Numerical methods were accompanied by experimental studies. They allow a validation of the numerical approaches and of the findings; more than that, experimental approaches permit to reproduce complex conditions associated with specific geometries, material and interactions between elements, whose detailed mathematical description may be difficult.

The modelling of *blood flow across cardiac valves* deserved special attention in cardiac fluid dynamics because valvular function is intimately involved in different types of cardiac dysfunctions, from valvular insufficiency, to stenosis, to the alterations induced by prosthetic valves and blood mixing [7]. The interaction between

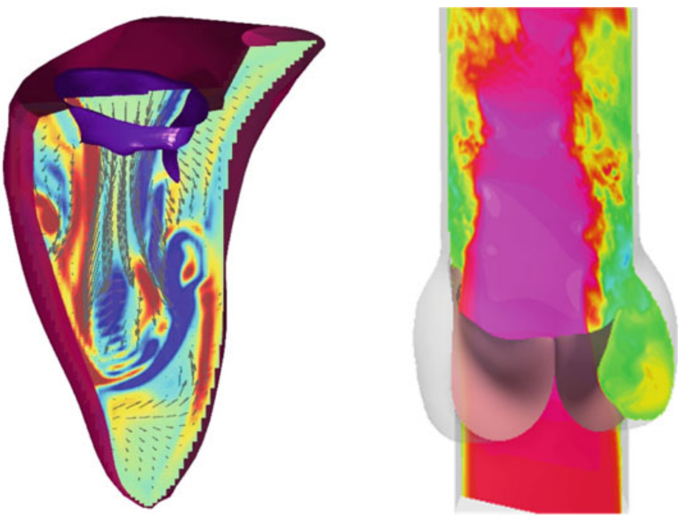

Fig. 1 Left picture: fluid flow in the left ventricle during filling, blood enters through the mitral valve and develops a circulatory pattern inside the chamber. Right picture: flow across the aortic valve at peak systole, velocity is high and although the size of the aorta is relatively small the jet develops a weak level of turbulence. (Credit: Dario Collia, left picture, and Marco Donato de Tullio, right picture, own work, visualizations from computational studies)

blood flow and valvular tissues represents a prototypal challenge of strong FSI because the movement of valvular elements is very rapid, and it is entirely driven by the flow. Fluid velocities across the aortic valve reach values above 1 m/s; such figures correspond to a fluid flowing in a transient turbulent regime (Fig. 1, right picture). Although such turbulence is relatively weak with respect to many industrial or environmental fields (Reynolds number is of the order of 10^4), flow is very unsteady, the systolic impulse grows from zero to its peak values and back to zero in about 300 ms. Transient turbulence is regularized during acceleration while it presents a more unstable character during deceleration. This behavior can be dramatically affected by pathologies or after the surgical replacement with prosthetic valves [23]. The mitral valve, at the inlet of the left ventricle, presents a peculiar asymmetric geometry characterized by a large leaflet on one side (anterior side, next to the aortic outlet) and a smaller leaflet on the other. This asymmetry was found to be important for the vortex formation process during ventricular filling; it ensures the development of a circulatory pattern inside the ventricle which is beneficial for an efficient transit of blood [95]. In both valves, the role of physiological FSI is an important topic of research; excessive stresses on the valvular elements induce a mechanical worn out the tissue, while an absence of shear can become prone to calcifications.

Blood flow in large blood vessels represents another central topic of fluid dynamics research because most life-threatening cardiovascular diseases occur in large arteries. The most common pathology is the development of atherosclerosis, which is the deposition of material on the internal vessel wall leading to the progressive narrowing of the lumen (stenosis). As discussed above, stenosis can obstruct the blood from flowing downstream, this induces a reduction/lack of oxygen to tissues supplied by those blood vessels; a phenomenon that can lead to myocardial infarction when the vessels are supplying the heart muscle or to an ictus when supplying a brain region. The interaction between blood flow and arterial walls, principally described in terms of wall shear stress pattern, plays a fundamental role in the genesis and progression of atherosclerosis. Several studies have developed during the years to identify the relationship between geometry and risk of atherosclerosis in sites of clinical relevance. Further applied insights were suggested by the observation that therapeutic procedures are often accompanied by the development of stenosis in neighboring areas due to the alteration of the blood flow therein [56]. Recent years have witnessed significant advances in computational method, which ensure a higher reliability in effective clinical conditions. Such methodological progresses are opening possibilities to achieve, in the next few years, effective definitions of interdisciplinary procedures for personalized cardiovascular care [11, 17].

A different pathology that is common to large arteries is the aneurysm: an excessive, local bulging of the vessel. The arterial wall in the dilated region becomes thinner and weaker, it is then exposed to the risk of rupture and to provoke an internal hemorrhage. The genesis and development of an aneurysm is mainly associated to tissue degeneration, that in many cases can be imputable to genetic predisposition, with the role of fluid dynamics limited to a few specific situations when abnormal high speed flow jet may weaken the tissue in region of impact. Sometimes, fluid dynamics plays a role in its progression or its stability, depending on whether the flow impinges onto the boundary, increases stresses on the weak walls or washes-out inside the dilated region [96].

Congenital cardiac diseases cover a prominent role in cardiovascular fluid dynamics literature, for the importance of addressing details of restorative procedures that are commonly performed at the early phases after birth. These children undergo a series of surgical corrections aiming to rearrange the circulation to overcome the congenital alteration. On some occasions, for the sake of example, the left ventricle is underdeveloped, here surgeries eventually transform the right ventricle in the systemic ventricle and reconstruct surgically a direct connection of the cavae veins to the pulmonary arteries without the right heart in between. Commonly the redesigned circulation requires a careful verification and optimization in several aspects. The pioneering studies developed in the early 2000' [68] underwent continuous improvements following the increasing potential of computational techniques, which are becoming effective for defining optimal therapeutic strategies. Computational techniques in this field featured the introduction of multiscale models. Multiscale models integrate three-dimensional models, that reproduce the site under analysis with high detail, with simpler models of the entire circulation made by one-dimensional and zero-dimensional, lumped-parameter models This approach combines the benefit of accurate simulations with that of taking into account how changes in the site under analysis reflect in the entire circulation [57].

Lumped-parameter *models of the entire circulation* were known since long time and they, too, underwent progressive refinements during years. In such models, individual elements of the circulation are represented by simple element with analogy to electric circuits integrated with one-dimensional fluid dynamic models of vessel elements [61]. The model sophistication arises here by the complexity of the extended network made of a large number of simple elements which reciprocal influence one on the others. These models permit to verify how acute changes in a specific region of the circulation can have an impact in other, even far, regions and alter global physiological parameters. Such models have the potential to provide clinical information that are otherwise only available through invasive measurements [39].

Biological fluid dynamics extends beyond the cardiovascular systems, although studies of effective theoretical and applied relevance remain limited when compared to cardiovascular fluid mechanics. An exception is the *fluid dynamics inside the human eye*, which had a dedicated special session during AIMETA conference 2011. The ocular chamber contains the vitreous humor: an aqueous viscous fluid that regulates the functions of the eye biomechanics. Its main roles are that of providing nutrients to cornea and lens, and of regulating the intraocular pressure (IOP) through a balance between aqueous production and drainage resistance. It also presents an intrinsic dynamics induced by eye rotations [78]. Models of the vitreous humor have been developed with increasingly reliability during the years [45]. Theoretical results have created a firm ground for the development of studies of applied interest and for supporting actual clinical applications.

2.3 From Articular Joints to Rehabilitation

In the Italian panorama, Mechanics of Machines (MoM) is a vast area that gathers different disciplines ranging from Mechanics Applied to Machines (MaM) to Machine Design (MD) through Mechanical Technology (MT), having as a common language Industrial Drawing (ID).

In the AIMETA community, MoM was represented mainly by both MaM and MD but from 1970 by MaM to an increasing extent, since in 1971 AIAS, the Italian Association for Stress Analysis, was established that attracted most of the MD and ID activities (in 2015 it became the Italian Scientific Society of Mechanical Design and Machine Construction, still maintaining the acronym AIAS). In 1986 the GMA, the Group of MaM, was also established, that collected the activities of MaM, which, however, maintained strong connections with AIMETA too. Indeed, the majority of Biomechanics papers from MoM in AIMETA Congresses and Meccanica Journal come from MaM. Most of them were presented in AIMETA Congresses rather than on Meccanica journal. This result is mainly due to the birth of specific journals in Biomechanics that have significantly attracted the activities of MaM in this specific field. The first papers of Biomechanics from MoM appeared in the XV AIMETA Congress 2001 in Taormina and in the XVI AIMETA Congress 2003 in Ferrara. In the Meccanica journal, from 1996 up to now, 14 papers on Biomechanics appeared from the MoM section, almost all coming from MaM. The first biomechanical paper appeared in 1997 [75], followed by others in 2002, while the first from the MoM section appeared in 2010 [83]. Papers in Biomechanics from MoM are mainly focused on Articular, Computational, Biotribology, Sports, Rehabilitation, Exoskeletons, Medical Devices, Instruments, and Experimental Biomechanics.

Joint Biomechanics studies the *musculoskeletal system* (MSS) consisting of bones, cartilages, muscles, tendons, ligaments, etc. Studies are conducted by both experimental and in silico methods. The basic tools are mathematical models and techniques of analysis of three-dimensional motion, imaging techniques, advanced notions of continuum mechanics [77]. The MSS system is the result of an evolutionary process of millions of years and is a very efficient mechanical system to its purpose. Many studies on the functionality of the MSS of both humans and animals currently inspire the design of mechanical systems that emulate their functional structure to obtain efficient machines [19, 77]. The measurement of movement is based on optical instruments (cameras), which underwent high technological advancements although still often inadequate to detect with high precision the movement of the bones due to the skin movement artifacts and to the problem known as marker occlusion. In [16] the problem is critically analyzed and a solution is proposed that mitigates this drawback. Inertial and magnetometric wearable sensors are now commonly used for measurements that require lower accuracy [77]. The combined use of force platforms and optical systems improves the study of motion (capture motion).

Musculoskeletal models of the human body with rigid elements have been presented in [84] and by the authors of the paper [19] with a few degrees of freedom (DoF), up to 23 DoF [77]. Most of them adopt a muscle Hill-type model and solve both the dynamic analysis and the kinetostatic analysis (i.e., given the motion law, find the driving actions to produce it). Several software packages of musculoskeletal models have been presented recently, for instance SIMM, OpenSim, AnyBody, MSMS, etc., to cite a few of the most known. Methods based on continuum mechanics, i.e. on Finite Element (FE) methods, have been developed that take into account elasticity, nonlinear material properties and complex boundary conditions [77]. Despite the great success of these solutions, several unsolved important problems still exist. For instance, the development of predictive musculoskeletal models and, very importantly, a realistic representation of most biological joints still deserve great attention.

In particular, the study of joint Biomechanics is focused on diarthrodial joints, with particular emphasis on the knee [83], hip joint [53, 80, 81], elbow, ankle, shoulder, foot and spine. The problems also concern the phenomena of friction, lubrication [53], wear and roughness of articular surfaces [81], the determination of articular surfaces, their contact areas and deformation, the study through the finite element technique of the infinitesimal and finite deformative states of biphasic materials as well as the distribution of stresses [77, 80]. The literature on the knee, perhaps one of the most important joints for its function and complexity, is impressive. The studies focus on kinematic and kinetostatic analyses, on the etiology and treatment of injuries and diseases (dislocation, arthritis and ligamentous rupture). Here, the key point is the development of mathematical models that analyze femur-tibia and femur-patella-tibia motion [83] as well as the forces transmitted between them. To validate the models, in vivo and in vitro experimental tests of great complexity are required [31]. A further difficulty arises from the fact that mathematical models have singularities whose correct handling requires special attention and experience.

In the last 20 years, specialized books have highlighted the anatomical and theoretical physical mathematical bases necessary for the development of *articular models* [46]. Moreover, numerous papers on robotics and theory of mechanisms [42], that developed precise and efficient techniques to model spatial parallel mechanisms, together with the pioneering papers of O'Connor on joint equivalent mechanisms [97], provided the essential background for more accurate and efficient 3D kinematic and dynamic models of the knee, the ankle and the human lower-limb [15, 65, 83]. These models allowed a deeper understanding of the joints in both natural/passive and loaded motion, and provide a fundamental tool to design prostheses, ortheses and exoskeletons, overcoming the limitations of the trial-and-error approach. An important concept, that is often underemphasized, is that the joints are similar to overconstrained spatial mechanisms: Nature is conservative and has equipped us with redundant constraints to facilitate a failure recovery in case of damage. This concept plays a fundamental role in the design of prostheses and above all in the definition of their positioning with respect to the bones. This is a determining factor for the restoration of the natural motion and balancing of the residual anatomical structures that reflects on the long-term outcome of the intervention/implant [83] to avoid or prolong over time the knee revision arthroplasty.

The limits of measurement techniques and ethical motivations make the accurate survey of the quantities necessary for the models complicated, if not impossible. This imposes the use of indirect techniques and optimization processes that lead to results that are not always satisfactory. New concepts based on the congruence/conformity of articular surfaces [98] (which correlate the shape of surfaces to the temporal history of the load and therefore allow the definition of a relationship between form and movement) integrated with the definition of the three-dimensional models mentioned above, have made it possible to simplify previous patient-specific models reducing the number of parameters to be detected but slightly decreasing the accuracy to the benefit of a much lower computational load [15]. In this context, modern imaging techniques (CT, MRI, dynamic MRI, fluoroscopy, stereophotogrammetry, etc.) play a fundamental role and 3D printing, once the articular surfaces have been defined, allows the construction of patient-specific prostheses, a need that is strongly felt today.

Multibody approach can also be efficiently used to model other complex biological systems. For instance, in [94] a 3D rigid body model of the ossicular chain of the human middle ear is presented that describes the middle ear behavior very well. The model showed to be computationally more efficient than FEM models, and usable also for prosthesis design.

The study of joints also involves *biotribology issues* such that friction, wear, and lubrication. Indeed, tribological principles are of enormous importance in understanding how synovial joints function and fail [80, 81] then, consequently, how important they are in the design of prostheses.

Advanced models can take into account lubrication and wear problems that directly affect the durability of the prosthesis and the definition of the materials. Total knee arthroplasty (TKA) failure is believed to be due from 10 to 18% to wear [43]. In particular, in [80] a hip FEM model was presented that allows a significant reduction of wear tests. The same authors have also studied the influence of the surface roughness on the knee and hip prostheses lifetime. In [81] a detailed hip tribological model under certain kinematic and non-Newtonian unsteady mixed elasto-hydrodynamic conditions is presented and validated by experimental tests. In [53] a hip joint wear mathematical model based on the Cross Shear effect is developed that outlines the influence of the selected wear factor law on the wear prediction. It also highlights, in particular, that the joint geometry influences wear volume more than load.

Analytical methods used within *Sports Biomechanics*, frequently defined as technique analyses sometimes devided into qualitative, quantitative and predictive components, investigate the biomechanical principles of motion and also aim at improving performances. Special attention has been focused on different parts of sport technique. Specifically, pedaling optimal movements and force in cycling have been addressed extensively also for rehabilitation purposes of the lower limbs and the definition of quantitative indicators of both the rehabilitation degree and the quality of the movement [58]. Reliable data collection is of the utmost importance in Sports Biomechanics [47]. In [34] the Biomechanics of the double poling (DP) gesture in cross-country disabled sit-skiers in the field during competition is analyzed. Data were recorded with a high-speed markerless stereophotogrammetric camera system. This study demonstrates the feasibility of a markerless kinematic analysis of sporty gestures. The same authors also presented a review showing that human postures can be efficiently analyzed through inertial sensors (IMU) directly worn on the subject body. The recent trend is to move from imagine data collection to markerless systems (IMU, etc.) whose accuracy is, however, still to be clearly established [14].

Rehabilitation engineering (RE) aims at improving the quality of life of people with disabilities and reduces the care burden of families and society. With the increase of the society well-being and the development of technologies, especially those associated with robotics, RE will have a huge development in the coming years. The aim of the RE is to design devices (orthoses, prosthetic limbs, haptic mechanisms, exoskeletons) for rehabilitation dedicated to the restoration of lost functions as well as systems to assist workers to perform heavy duty works. It therefore becomes essential to know the biomechanics of the human body and the movement and forces transmitted between the human body and the devices. Human–machine interfaces are also of great importance for the success of the devices themselves.

The synthesis of the mechanisms (prostheses, orthoses, exoskeletons), the core of the devices, becomes fundamental and it is therefore very useful to resort to the most advanced synthesis techniques, available today and efficiently usable by the great computation power at relative low cost. Motor rehabilitation of upper limbs of patients that suffered from stroke showed that early robotic training, i.e., during acute or subacute phase, can improve motor learning more than chronic-phase training. In [52] a new subacute-phase randomized controlled trial by means of a cable driven robot arm is proposed. The new protocol showed to be as efficient as other ones from the literature, then it can be used in addition and in substitution of them. In [86] a passive exoskeleton for upper-limb rehabilitation of patients who suffered from a stroke is presented. The mechanism is simple, low cost and can be used by the patient at home. Despite being not actuated the system shows remarkable back-drivability within the upper limb workspace. A very ambitious target is to realize upper limb prostheses controlled by brain computer interface (BCI) signals [48]. In addition to the complexity of the mechanical design, whose models are non-linear systems, also complex advanced control techniques are required to get patient comfort.

Since the 60s of the last century, but especially in the last twenty years there has been a remarkable development of exoskeletons, more of lower-limb devices than of the upper-limb ones since the former are easier to design and control. In [30] techniques are proposed for the ankle motion analysis and the more precise definition of the instantaneous axis of motion, based on stereophotogrammetry and markers properly attached to the skin that limit the effect of the skin artifact. The proposed methodology showed to be an effective tool to customize hinged ankle foot orthoses and other devices interacting with the human joint functionality. A pneumatic interactive lower-limb rehabilitation orthosis is proposed in [8]. In [44] a lower leg exoskeleton with 10 DoF controlled by pneumatic actuators for gait rehabilitation of patients with gait disfunctions is presented. An adaptive fuzzy controller compensates for the dry friction influence in real time.

Hand exoskeletons represent a complementary (distal) part of the upper-arm exoskeletons. Mechanical design and actuation are the most challenging problems [26]. Selection of DoF vs design and control complexity is one basic issue. How to assist the finger motion is again challenging. Indeed, different concepts can be adopted [89] to guide the fingers and to control the motion according to whether the finger are included into the mechanism links or are left apart, or only the last phalanges are moved to drive the fingers to perform the desired trajectory. Electromyography (EMG) signals can also be used to control the hand exoskeleton [48]. Very advanced hand exoskeletons are presented in [9, 18, 49, 85]. Artificial muscles that mimic the muscle human behavior are a very interesting topic, which still deserves attention for the promising features it can provide to actuated rehabilitation mechanisms [88],

Improving *medical device* efficiency and designing new devices is vital for the patients and the healthcare system. The impressive development of new technologies and fabrication processes (for instance 3D printing) gave a tremendous impetus to the design of safer, more usable and cheaper medical devices. Important areas involved in this evolution are surgery, cardiovascular devices and technology, sensor devices, and testing machines [10]. A test bench for measurements on cardiac valves by taking into account the blood characteristics is reported in [59]. A soft robotic gripper for manipulation in minimally invasive surgery (MIS) in reported in [74], while in [35] the overall architecture of a modular soft mechatronic manipulator for MIS is presented.

Experimental Biomechanics aims at collecting data to find relationships between variables and parameters in order to validate mathematical models or find empirical correspondence between input and output with the purpose to understand the biomechanical problem under investigation/study. Research on new materials, their physical and mechanical properties, as well as their compatibility with biological materials is a vast and growing area of interest that involves experiments. An original experimental approach to classify children with cerebral palsy is presented in [76]. An experimental method is presented in [6] to estimate the inertial parameters of the upper limbs during handcycling. In [31] a new test rig for static and dynamic evaluation of diarthrodial joints based on a cable-driven parallel manipulator loading system is presented.

3 Discussion and Perspective

A large amount of scientific research has been produced during last few decades in the field of Biomechanics; nevertheless, further challenges were identified and research is expected to proceed with an accelerating pace during next years.

Biomechanics is about applications of mechanics to living organisms, each one with its own specific details and peculiarities. In this respect, the rapid evolution of technology makes individual information more and more accessible, and the possibility of sharing huge amounts of data, due to the enormous evolution of communication means during last years, will allow the creation of simple and reproducible protocols that can be used effectively in a clinical setting. Along this line, the development of reliable predictive Biomechanics models, easily usable at a clinical level, is still an open problem of great importance for the implications it would have on diagnostics, rehabilitation, therapy and surgery.

An important aspect of the interaction among tissues, that lies sometimes outside the strict definition of Biomechanics, stands in the reaction of living tissues to mechanical solicitations, like stress in solids or wall shear stress at the interface between flowing blood and tissue. This aspect is central in the remodeling of tissues like muscles and bones, in vascular development, and for the adaptive response of the heart chambers that can lead to heart failure. Progresses along this topic require to improve the comprehension of how large-scale metrics and processes are sensed at the cellular level (mechano-sensing), what they stimulate and how these stimuli are translated into changes (mechano-transduction) that reflect back at the organ level. This is a fascinating field that requires a cross-fertilization between researches from distant disciplines. This point is crucial to fulfil the need of developing robust predictive models that may be useful in clinical applications.

Models capable of suggesting the pathological progression can benefit the accelerating availability of data. Improving data collection/measurement techniques with safe and non-invasive tools can rapidly improve nonlinear mathematical models in solids, fluids and haptic exoskeleton devices, which require the use of advanced nonlinear control techniques. However, an increase of the amount of data requires development of physics-based techniques, including physics-guided machine learning, of synthesis and optimization, made possible by recent theoretical developments and the continuous and growing potential of computational tools. At the same time, the availability of numerical models based on individual patient's information will be able to support the development of personalized treatments by using virtual therapeutic tools capable of reproducing the outcome of different therapeutic options and select the optimal solution. Indeed, thanks to the pioneering studies of the last decades in the field of cardiac surgery, it is now possible to think about reproducing the fluid dynamics, the pressure and wall shear stress patterns in patients with several diseases and anticipate the post-operative state. This represents an embryonal stage of virtual surgery capabilities that research in Biomechanics can help to let it become mature.

A general target for the future is the creation of systems that are simple, more robust, more accurate, personalized to the patient and, possibly, cheaper. This applies to diagnostic methods fed by information extracted by imaging and biological testing and to the development of models for personalized therapeutic procedures.

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Shape Control, Morphing and Mechanobiology

Antonio DeSimone and Luciano Teresi

Abstract We review ideas emerged in the mechanics literature in the last fifty years concerning the problem of controlling shape. The central notion is the multiplicative decomposition of the gradient of the visible deformation into elastic and inelastic parts. We show that, when applied to analyze muscle contraction, this notion is useful to understand the different functions of a muscle as motor or brake, and the key difference in the way skeletal, soft, and cardiac muscles solve the push problem.

Keywords Morphing · Shape control · Phase transitions · Target metric · Muscle contraction

1 Introduction

This article focuses on ideas emerged in the research literature in the last fifty years on the problem of controlling shape: from plastic forming of ductile metals to morphogenesis and muscular contraction in biological systems. We call this the *morphing problem*.

The main idea is that materials store elastic energy thanks to deformations relative to a ground state, which can possibly evolve in time. The evolution of this ground state can be controlled, hence the natural shape of a material, namely, the (local) shape of a volume element in the absence of internal stresses can be controlled. In some cases, the ground state determines the global shape of an object made of such material in the absence of external forces. This is the case when the (possibly spatially heterogeneous) ground state is kinematically compatible, i.e., it can be obtained as the gradient of a continuous deformation map. This is trivially true if the ground state

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is spatially homogeneous, since a constant can always be obtained as the gradient of an affine map. In other cases this is not possible, and the global shape that an object exhibits in the absence of external forces is accompanied by self-equilibrated residual stresses. In both cases, however, the visible deformations exhibited by an object under loads are the composition of an elastic part, and of a (possibly trivial) inelastic distortion.

In Sect. 2 we review how ideas about controlling shape have emerged in the last few decades in the literature on the mechanics of materials (plastic deformations, energy wells in phase transforming solids, etc.), and how they have been extended to morphogenesis due to biological growth. In Sect. 3 we focus on how these same ideas can be applied to other active biological systems, such as contracting muscles.

2 Morphing and the Multiplicative Decomposition of the Deformation Gradient

There are many ways to approach the abstract problem of controlling the shape of an (inanimate/passive) object. Two of them are particularly relevant to our story, and arise in the problem of shaping a solid metal either by inducing plastic deformations in a ductile material (metal stamping, cold drawing, deep drawing of aluminum cans, etc.), or by exploiting (solid-solid) phase transformations in shape-memory alloys. At the micro-structural scale, these two examples correspond to two different deformation modes of a crystalline lattice, namely, slip versus twinning.

In large strain plasticity, the idea of a multiplicative decomposition of the gradient of the visible deformation $\mathbf{F}(x) = \nabla f(x)$ at a point x of the material into the product of an inelastic (plastic) part $\mathbf{F}_p(x)$ and an elastic part $\mathbf{F}_e(x)$ that can be recovered (at least in principle) by unloading an elementary volume element is commonly associated with the names of Kröner and Lee [51, 55] (see, however, [18, 54, 70] and the references cited therein for additional and earlier contributions such as, e.g., [17, 39, 50]). The Kröner-Lee multiplicative decomposition of large strain plasticity reads

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p, \tag{1}$$

see Fig. 6. Since $\mathbf{F}$ must be the gradient of a map f, $\mathbf{F}_p(x)$ is compatible with a situation with no build up of internal stresses (typically characterized by trivial elastic deformations such that $\mathbf{F}_e \equiv \mathbf{I}$, with $\mathbf{I}$ the identity) only if it satisfies a condition of kinematic compatibility

$$\mathbf{F}_p(x) = \nabla f(x) \quad \text{for some continuous map } f. \tag{2}$$

This requirement can be recast as a system of differential equations in the components of $\mathbf{F}_p$ (vanishing of Riemann curvature), see Sect. 3.3 and [20].

The ground state of the material may evolve in response to internal stresses and external stimuli. This is modeled with a flow rule for the internal (structure) variable $\mathbf{F}_p$. The driving forces on this remodeling process typically include the Eshelby stress arising from elastic restoring forces, viscous resistance terms satisfying the dissipation inequality, etc., see e.g. $(7)_2$. An example of such flow rule from single crystal plasticity is

$$\dot{\mathbf{F}}_p\,\mathbf{F}_p^{-1} = \sum_{\alpha=1}^{N_s} \dot{\gamma}^{\alpha} \mathbf{s}_{\alpha} \otimes \mathbf{m}_{\alpha} \tag{3}$$

where $\dot{\gamma}^{\alpha} \geq 0,\ \alpha = 1, \ldots N_s$ are the plastic strains along the crystallographic slip systems $\mathbf{s}_{\alpha} \otimes \mathbf{m}_{\alpha}$, with $\mathbf{s}_{\alpha}$ and $\mathbf{m}_{\alpha}$ unit vectors along special crystallographic directions. The $\dot{\gamma}^{\alpha}$ are activated by the components of the stress along the slip systems (Schmid law) and the model is further enriched by additional laws to describe hardening, etc.

Shape memory alloys and other 'smart' materials (active materials, in our terminology) were developed in the US in the 60s and provided a powerful stimulus to the study of large reversible deformations of crystalline solids. While modeling hysteresis effects in this framework has seen slow progress for quite some time [30], it is now possible to design alloys with extremely low hysteresis by tuning composition with the help of mathematical models based on nonlinear elasticity [71]. A pioneering role has been played by J. L. Ericksen, who recognized in his seminal papers [41–43] the interest in studying (material) instabilities in crystalline solids arising from symmetry-breaking phase transformations, and how these can be described using the variational methods of nonlinear elasticity. In fact, morphing capabilities of materials such as shape-memory alloys arise from the shifting from one energy minimizing state to another one (energy wells), an approach culminating in the landmark papers by J. M. Ball and R. D. James on crystalline microstructures [12, 13]. The application of the multiplicative decomposition $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$ of the deformation gradient into an elastic part $\mathbf{F}_e$ and an inelastic or active part $\mathbf{F}_p$ is quite natural in this setting. Here $\mathbf{F}_p$ plays the role of the distortion of the crystalline lattice (Bain strain) that deforms the unit cell of the (high-symmetry) parent phase (say, austenite) into one of the symmetry-related distorted unit cells (martensitic variants) of the product phase. These Bain strains describe internal (structure) variables that can evolve, driven by internal (Eshelby) stresses, and hindered by (viscous, frictional, …) resistance forces according to a flow rule. A Schmid flow rule associated with martensitic transformations is discussed, e.g., in [64].

The idea of breaking up the visible deformation into the product of an elastic, recoverable one, and an inelastic part associated with a phase transition can be applied to more general symmetry breaking transformation, such as the ones exhibited by Liquid Crystal Elastomers (LCEs). These polymeric materials exhibit phase transformations from an isotropic phase to a nematic or smectic phase with (continuous) lower symmetry [29, 53, 78]. The application of the multiplicative decomposition $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$ to nematic elastomers is discussed in [35]. The same model has been used to predict stress-strain response in nematic LCEs [23, 24, 31, 33, 34], shape changes in twist-LCEs [75] and to program photothermal shape-morphing of LCEs [52] or light-activated shape memory LCEs [28].

Incorporating growth in the description of the mechanical response of solids has become important due to the increasing technological relevance of additive manufacturing and to the fundamental challenges and opportunities offered by biology to mechanics (growth of tumor cells in cancer, regenerative medicine, minimally invasive robotic surgery, etc.). In fact, the very same concept of decomposing visible deformations into an elastic part and an inelastic one has been adapted to biological tissues subject to growth and remodeling, where inelastic deformations are due to growth and $\mathbf{F}_p$ becomes $\mathbf{F}_g$, a growth stretch (see, e.g., [37] and the recent review [7]). We will return to this topic in greater detail in Sect. 3, where we discuss muscles.

Morphing is an interesting problem also in the context of lower dimensional theories for thin and slender structural elements (shells, rods). Inducing bending and curvature in initially flat plates or straight beams by means of through-the-thickness variations of inelastic (e.g., thermally-induced) distortions is a well-travelled route, with seminal contributions due to Timoshenko [76], and more recent ones in [3, 25, 32, 44]. Gaussian morphing [21], which uses modulation of in-plane stretches in order to produce Gaussian curvature as predicted by Gauss's *Theorema Egregium*, is an alternative avenue to shape programming of thin sheets. This approach has been pioneered by E. Sharon and collaborators [49] and has seen applications to a wide variety of material systems such as biological tissues experiencing growth, swimming micro-organisms, hydrogels, LCEs, inflatables, martensitic films etc., see [4, 9, 10, 16, 40, 45, 56, 58, 63, 77]. Morphing of tubular structures is discussed, in particular, in [62, 65, 66].

In most of the literature exploiting Gaussian morphing, the gradients of the metric leading to Gaussian curvature are directly provided by gradients of the underlying director pattern (the direction of the nematic director in the case of LCEs). Recent work suggests an alternative, with the advantage of a greater flexibility. Instead of feeding the *Theorema Egregium* directly with the microscopic oscillations of the director field, one can obtain Gaussian curvature from slowly varying surface stretch averages, obtained by averaging at the intermediate scale of mesoscopic patches the deformations induced by microscopic textures of the director field. This strategy of hierarchical texturing of the active distortion field is useful to enlarge the palette of available active distortions. In nematic LCEs, for example, biaxial active distortions can be obtained by zig-zag textures of the nematic director which, at the microscopic scale, can only produce uniaxial active distortions as in (12), see [45].

Morphing of one-dimensional rods is interesting as well, see the recent monograph [46]. Recent emphasis has been on morphoelastic theories of growing rods used to study growing plant shoots. These are slender elastic (biological) structures where growth phenomena are self-evident. Well before they grow tall enough to become prone to buckling under their own weight (Euler buckling), they exhibit spontaneous oscillations. These movements are too slow to be detected by a casual observer, but they become apparent through time-lapse photography. They have fascinated scientists since the pioneering work of C. Darwin [27] and they are still the object of active study and debate [14, 15, 59, 73]. Among the possible explanations, one is that spontaneous oscillations in plants can arise as an overcompensatory response to perturbations due to the (slow) mechanical and bio-chemical machinery that plants use to control their vertical posture. In fact, nutations in growing shoots may be described as spontaneous oscillations caused by a flutter-like instability (a Hopf-type bifurcation) in a growing elastic system subject to gravity loading, and capable of sensing and actively responding to external stimuli. The merits of this hypothesis have been discussed in several recent studies, see [1, 2]. Here, rods are treated as elastically inextensible, so that the multiplicative decomposition of the stretch becomes trivial, and the visible stretch is entirely due to growth. In formulas, we have $\lambda = \lambda_e \lambda_g \equiv \lambda_g$, where λ_g is the growth stretch and the elastic part of the stretch λ_e is identically equal to one. Bending elasticity is modeled instead through an additive decomposition of the curvatures into the sum of elastic and inelastic parts. Through numerical computations, one finds that the qualitative features of the predicted spontaneous oscillations do not depend on the details of the growth model that is employed, and that their frequency is comparable to the one typically observed in plants [1, 2].

3 Mechanical Modeling of Muscles

For decades, the mechanical modeling of muscles has been based on the notion of active stress, that is, a muscle generates a force and this force is described by adding a term to the constitutive prescription of the stress. More recently, the introduction of the notion of active strain based on the multiplicative decomposition has opened a new approach to muscle modeling. Starting from the papers [60, 68], the notion of active strain has gained momentum, and the comparison between the two approaches has received much attention [5].

The multiplicative decomposition is now key concept for the mechanical modeling of muscle, see [7] for an extensive review, and [26, 36, 57] for time evolutive problems. Also the mechanical model of the human heart has received much attention. Therein, myocardial contractions can be modeled using the multiplicative decomposition [6, 19].

3.1 Muscle Physiology Basics

Browsing the physiological literature dealing with muscles, the following two basic descriptions are typically encountered: (1) A muscle is a *force generator*–here emphasis is put on *dynamics*, that is, on forces; (2) A muscle *generates motion*–here emphasis is on *kinematics*, that is, on displacements.

These two different points of view often receive elusive discussion. Actually, a muscle can generate a motion while sustaining some load. The paramount feature of muscles functioning is that the contractile unit of a muscle has many different rest

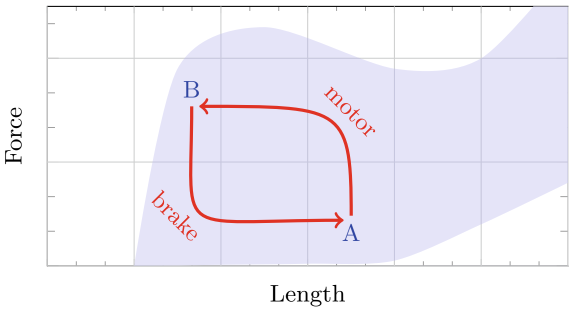

Fig. 1 In a force-length diagram a muscle must span an area. A muscle can act both as motor, when it increases its load or shortens against a load (red path A to B), and as a brake when it decreases its load or elongates resisting to a load (red path B to A)

positions: it can change its length without storing elastic energy. It follows that in a force-length diagram, see Fig. 1, each point of the shaded area is attainable by a muscle. Moreover, a muscle can be used both as a motor, when it shortens against a load, and as a brake, when it elongates resisting to a load, a key feature of animal movement [38]. Another important aspect of muscle mechanics is that muscles can only pull: no pushing is possible.

The peculiar mechanical behavior of muscles, and of many other kinds of active biological matter, can be described by using the notion of distortion introduced decades ago to model large-strain plasticity through the Kröner-Lee decomposition (1). We write $\mathbf{F} = \mathbf{F}_e\,\mathbf{F}_a$, where $\mathbf{F}_a$ refers now to active distortion.

3.2 The Contractile Unit and the Push Problem

Back to muscle physiology, and down to the microscopic level, the fundamental constituents of a contractile unit are two long proteins: a thick filament belonging to the myosin superfamily of motor proteins, and a thin filament called actin. Myosin heads might cross-bridge the two filaments and make one filament to slide over the other, thus producing a shortening, [11], see Fig. 2. The acto-myosin pair constitutes a uniaxial actuator, capable of controlling both position and force, independently from each other; the fundamental caveat is that the acto-myosin pair can only pull: softness and slenderness entail the inability to push.

This is the so-called *push problem* of soft tissues, which has to be solved by ad hoc biological design: once the pair has been actively shortened, the elongated configuration can only be recovered by other means because myosin heads cannot push against actin. The acto-myosin pair can be arranged in different ways to produce a contractile tissue, thus yielding the three main types of muscle tissue: smooth, skeletal, and cardiac. Here, we are interested in how to reproduce this diversity in muscle mechanics models. In particular, we speculate on the mechanical blueprint of different arrangements and on how the push problem is solved for the three types of muscles.

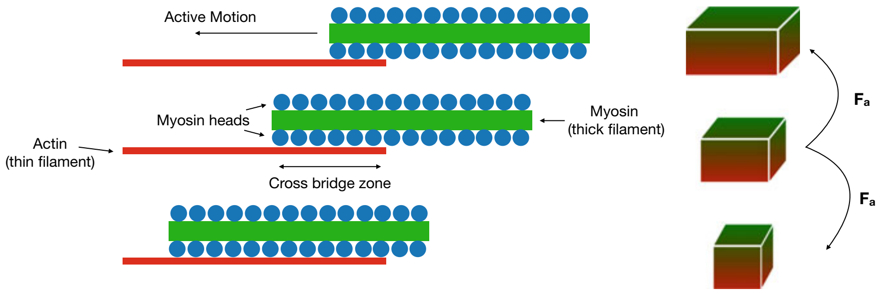

Fig. 2 Left. Cartoon of the pair acto-myosin in three different configurations. The myosin head can only pull on the actin, producing a shortening of the overall length of the pair. Active motion occurs only in one direction and the elongated configuration can only be achieved by other means. Right. The state of the filaments pair is represented by a volume element. Assuming as reference the volume element at the middle, the state of the other two is described by the active distortion $\mathbf{F}_a$

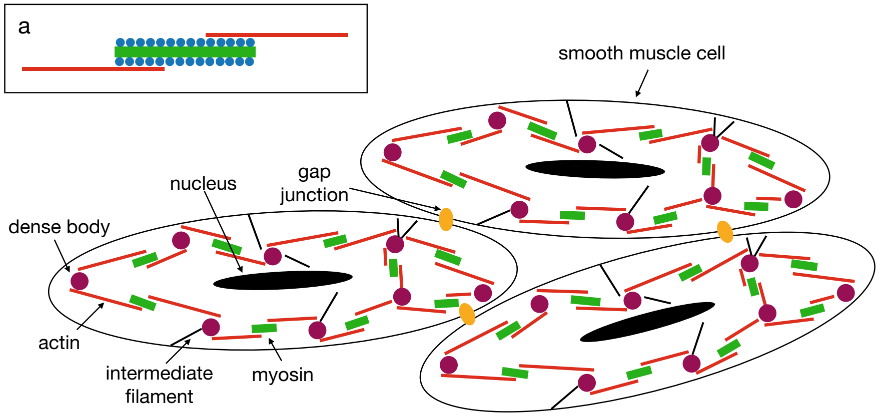

Fig. 3 Inset a) Two actin proteins and one myosin constitute a small contractile unit; such units can be connected to each other by means of a dense body, thus producing a contractile meshwork within the cell. Many cells of this type make a smooth muscle tissue.

Smooth Muscle. Smooth muscle is an involuntary muscle which is present in almost every part of the body, from the intestines to blood vessels and hair follicles. The cells are spindle shaped, and have a long axis; the pairs of actin and myosin proteins are anchored to each other by dense bodies, and form a sparse network of contractile filaments, see Fig. 3. The length of these filaments range from about 30 to 200 microns, thousands of times shorter than the skeletal muscle fibers. The effect of activation is a shortening along the long axis of the cell, accompanied by a stiffening. Many organs containing smooth muscle have multiple layers of such muscle, often with one layer oriented circumferentially and one longitudinally [11].

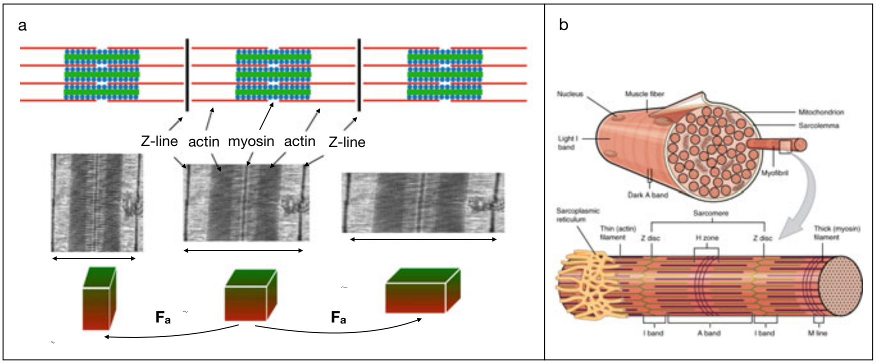

Fig. 4 Tightly packed and regularly arranged acto-myosin pairs form the sarcomere of skeletal muscle (panel a, top). A sequence of sarcomeres make a long line called myofibril, creating striped patterns which are visible under magnification (panel a, middle). The state of a sarcomere can be represented by a volume element; given the reference volume element in the middle, the state of the other two is described by the active distortion $\mathbf{F}_a$ (panel a, bottom). Myofibrils are packed together in parallel strands within the sarcolemma, forming the muscle fiber, a multinucleated cell (panel b). Right image from [11], licensed under CC BY 4.0

Because actin and myosin are not arranged in a regular fashion, the cytoplasm of a smooth muscle (which has only a single nucleus) has a uniform, nonstriated appearance, from which the name smooth muscle originates.

Skeletal Muscle. Skeletal muscle is a voluntary muscle attached to the bones via tendons, which is responsible for the body motion. In both skeletal and cardiac muscle, actin and myosin proteins are tightly packed and arranged very regularly, thus forming a contractile unit called sarcomere. A sequence of sarcomeres constitutes a long line called myofibril, and a bunch of parallel myofibrils, packed together within the sarcolemma, is the so-called *muscle fiber*, a multinucleated cell. In skeletal muscle, these muscle cells have the shape of long fibers, creating striped patterns called striations which are visible with a light microscope under high magnification [11] (see Fig. 4, panel a).

Cardiac muscle. Cardiac muscle is an involuntary muscle and is the main constituent of the heart wall; it is responsible for the heart contraction that pumps blood in the circulatory system. Cardiac muscle fibers are physically and electrically connected to each other so that the entire heart contracts as one unit (called a syncytium) [11]. Cardiac muscle has fibers, as the skeletal one, but these fibers exhibit many branches [8]. From the mechanical point of view, the main difference with respect to skeletal muscle is that cardiac contraction is multi-axial, due to the branched fibers, instead of uni-axial as the one realized by parallel fibers, see Fig. 5. This specific fiber architecture causes the heart walls to accumulate elastic energy in the systolic phase; this stored energy is then used to favour the diastole, during which the heart acts as a suction pump, and the cardiac muscle is a brake that controls the filling of the ventricles [22, 67, 72].

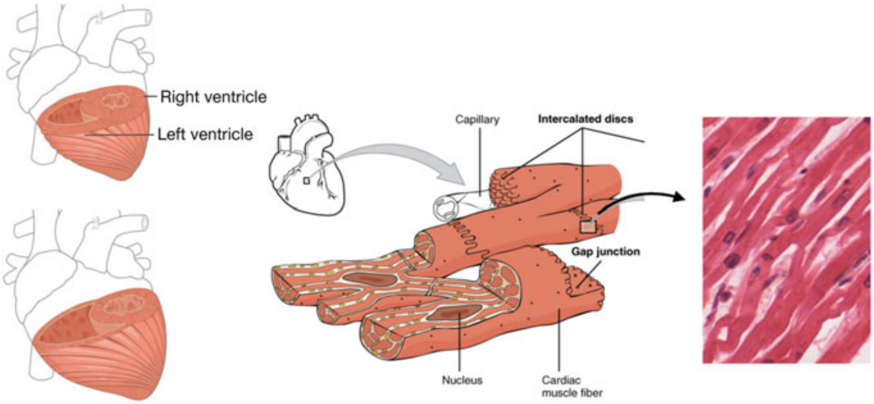

Fig. 5 The myofibrils of cardiac muscles have many branches. Left) Myocardial wall is made of cardiac muscle which is loosely arranged in fibers; here we see the contracted state (top) and the relaxed one (bottom). Center) A magnification of the cardiac muscle shows how the fibers are branched. Right) Micrograph of the cardiac muscle, LM x 1600. Image from [11], licensed under CC BY 4.0

3.3 Mechanical Modeling

Let $\mathscr{E}$ be the 3D Euclidean space, $\mathrm{V}\mathscr{E}$ its associated vector space, $\mathrm{Lin} = \mathrm{V}\mathscr{E} \otimes \mathrm{V}\mathscr{E}$ the space of double tensors on $\mathrm{V}\mathscr{E}$, and $\mathscr{T}$ the time line; finally, we denote with $\mathscr{B}$ the reference configuration of our body. In the framework of continuum mechanics, the state of muscular tissue can be described by two state variables, the motion $f : \mathscr{B} \times \mathscr{T} \to \mathscr{E}$, and the distortion $\mathbf{F}_a : \mathscr{B} \times \mathscr{T} \to \mathrm{Lin}$.

The motion f yields the position $y = f(x, t)$ of a material point x of the reference configuration, while the distortion $\mathbf{F}_a$ describes the ground state of a reference volume-element dV at x. This mechanical model has two state variables, f and $\mathbf{F}_a$, totaling $3 + 9 = 12$ degrees of freedom; each state variable has its own balance law and its source term, see Eq. (7). The sources for f are the familiar forces, while those for $\mathbf{F}_a$ are the so-called remodeling actions. A thorough presentation of the theory can be found in [37].

The main constitutive assumption is that, given $\mathbf{F} = \nabla f$ and $\mathbf{F}_a$, the elastic energy density is a function of the *elastic strain* $\mathbf{F}_e = \mathbf{F}\,\mathbf{F}_a^{-1}$, as in (1) with $\mathbf{F}_a$ instead of $\mathbf{F}_p$. This constitutive assumption is of key importance, yielding the existence of many states at zero energy, for which $\mathbf{F}_e = \mathbf{I}$.

In particular, while $\mathbf{F}$ is a purely kinematics notion, $\mathbf{F}_a$ has both a kinematical and a dynamical nature: it adds further degrees of freedom to $\mathscr{B}$, and it describes a *ground state*, that is at zero-stress, of the body element, see Fig. 6. We quote verbatim from [37]: "$\mathbf{F}_a$ cannot even be conceived without the standard notion of stress and some constitutive information on it."

Using the three tangent maps $\mathbf{F}$, $\mathbf{F}_a$, and $\mathbf{F}_e$, we may define the following metric tensors, having the role of right Cauchy-Green strain-measures:

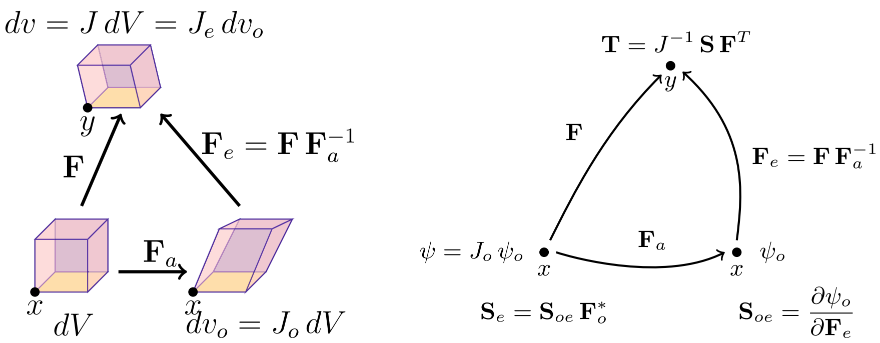

Fig. 6 Left: The volume element dV at $x \in \mathscr{B}$ of the reference configuration is mapped by $\mathbf{F}_a$ onto a distorted element dv_o, representing the ground state, at the same point x. To get the actual volume element dv, we must add a further strain to $\mathbf{F}_a$, called the elastic strain $\mathbf{F}_e$. Right: Relations between stress measures and energy densities

$$\mathbf{C}_o = \mathbf{F}_a^T\,\mathbf{F}_a, \quad \mathbf{C} = \mathbf{F}^T\,\mathbf{F}, \quad \mathbf{C}_e = \mathbf{F}_e^\top\,\mathbf{F}_e = \mathbf{F}_a^{-\top}\,\mathbf{C}\,\mathbf{F}_a^{-1} \tag{4}$$

named target, actual, and elastic metric, respectively. The name *target metric*, also known as natural or intrinsic metric, [49, 61], stems from the fact that, if the actual metric $\mathbf{C}$ is equal to $\mathbf{C}_o$, then the elastic metric is trivial:

$$\mathbf{C}_e = \mathbf{I} \quad \Leftrightarrow \quad \mathbf{C} = \mathbf{C}_o. \tag{5}$$

In general, $\mathbf{C}_o$ is not *compatible*, that is, flat Riemannian; as a consequence, the body manifold $\mathscr{B}$, endowed with such a metric, cannot be embedded in the 3D Euclidean space, and any actual configuration will have $\mathbf{C}_e \neq \mathbf{I}$. Given a target metric $\mathbf{C}_o$ on $\mathscr{B}$, a necessary condition for the existence of a deformation f_o such that $\nabla f_o^T\,\nabla f_o = \mathbf{C}_o$, is that the Riemann curvature $\mathbb{R}$ associated to $\mathbf{C}_o$ be null; is such a case, the target metric, as well the associated distortion $\mathbf{F}_o$, is called *compatible*, and the embedding is unique up to isometries of $\mathscr{E}$, see [20]. If $\mathscr{B}$ is simply-connected, the vanishing of $\mathbb{R}$ is also sufficient:

$$\mathscr{B} \text{ simply-connected } \& \ \mathbb{R}(\mathbf{C}_o) = 0 \ \Leftrightarrow \ \exists\, f_o : \mathscr{B} \to \mathscr{E} \text{ s.t. } \nabla f_o^T\,\nabla f_o = \mathbf{C}_o. \tag{6}$$

3.4 Balance Equations and the Initial Value Problem

The balance laws for time evolving distortions are provided by the principle of virtual power; details are in [37]. The mechanical model has two coupled balance equations; by neglecting inertia terms, we have the following system of equations, describing an initial-boundary value problem:

$$
\begin{aligned}
\text{Bulk: } &\operatorname{div}\mathbf{S} + \mathbf{f} = 0, && \dot{\mathbf{F}}_a\,\mathbf{F}_a^{-1} = \mathbb{M}\,(\hat{\mathbf{B}} - \hat{\mathbf{E}})\,, \text{ on } \mathscr{B}\times\mathscr{T};\\
\text{Boundary: } &\mathbf{S}\,\mathbf{m} = \mathbf{t} \quad \text{on } \partial\mathscr{B}_t\times\mathscr{T}; && \mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \partial\mathscr{B}_u\times\mathscr{T};\\
\text{Initial: } &\mathbf{u} = \mathbf{u}_o, \mathbf{F}_a = \mathbf{F}_{ao} && \text{on } \mathscr{B}\times\{0\}.
\end{aligned}
\tag{7}
$$

Here, $\mathbf{S}$ is the reference stress, aka the Piola-Kirchhoff stress, $\mathbf{f}$ and $\mathbf{t}$ are the bulk forces and the boundary forces applied at $\partial\mathscr{B}_t$, respectively; $\mathbf{u} = f(x,t) - x$ is the displacement of the point x at time t. Finally, $\bar{\mathbf{u}}$ are the kinematical constraints on $\partial\mathscr{B}_u$; $\mathbf{u}_o$ and $\mathbf{F}_{ao}$ are the initial values of the two state variables. Equation (7)$_2$, analogous to the flow rule (3), relates the remodeling velocity $\dot{\mathbf{F}}_a\,\mathbf{F}_a^{-1}$ to the difference between the remodeling actions $\hat{\mathbf{B}}$ and the Eshelby tensor $\hat{\mathbf{E}}$; finally, $\mathbb{M}$ is the mobility tensor.

To discuss the typical evolution problem described by (7), we focus on the two bulk balance equations. The state variables $\mathbf{u}$ and $\mathbf{F}_a$ are controlled by the two sources $\mathbf{f}$ and $\hat{\mathbf{B}}$; thus, by applying forces and/or remodeling actions, we can change both the shape of the body and the ground state of volume elements. In particular, we can control the shape and the stress state independently from each other; for example, we can change the shape without modifying the stress (as it is true for muscles).

These two bulk equations are coupled through the Eshelby tensor $\hat{\mathbf{E}}$, whose value depends on both the state variables $\mathbf{u}$ and $\mathbf{F}_a$. The novelty of system (7) stems from the presence of the source term $\hat{\mathbf{B}}$ that controls the evolution of $\mathbf{F}_a$; when $\hat{\mathbf{B}} = \hat{\mathbf{E}}$ the evolution stops and the system becomes steady. An example of stress-driven remodeling which produces compatible distortions can be found in [57]. We note that (7) can describe both a shortening and a lengthening that happen at zero stress and without applied forces; actually, a muscle can shorten without forces, but cannot recover its elongated configuration.

3.4.1 The Free-Energy Density

We shall briefly outline the derivation of the constitutive rules for $\mathbf{S}$ and $\hat{\mathbf{E}}$ from a free-energy. We assume the free-energy density per unit *ground volume* ψ_o to depend only on the elastic deformation $\mathbf{F}_e$, and that the elastic deformations are volume preserving: $\psi_o = \psi_o(\mathbf{F}_e(x,t))$, and $J_e = \det\mathbf{F}_e = 1$. Given ψ_o, we can represent the strain energy density per unit *reference volume* ψ as a function of $\mathbf{F} = \mathbf{F}_e\,\mathbf{F}_a$, defined by

$$
\psi = J_a(x,t)\,\psi_o(\mathbf{F}_e(x,t)),
\tag{8}
$$

with $J_a = \det(\mathbf{F}_a)$. In our case, the time rate of the free-energy (8) yields:

$$
\dot{\psi} = \dot{J}_a\,\psi_o + J_a\,\frac{\partial\psi_o}{\partial\mathbf{F}_e}\cdot\dot{\mathbf{F}}_e = \frac{\partial\psi_o}{\partial\mathbf{F}_e}\,\mathbf{F}_a^*\cdot\dot{\mathbf{F}} + J_a\,(\,\psi_o\,\mathbf{I} - \mathbf{F}_e^T\,\frac{\partial\psi_o}{\partial\mathbf{F}_e}\,)\cdot\dot{\mathbf{F}}_a\,\mathbf{F}_a^{-1}.
\tag{9}
$$

Given (9), we can define the ground stress $\mathbf{S}_{oe}$, dual to $\dot{\mathbf{F}}$, and the Eshelby stress $\hat{\mathbf{E}}_e$, dual the distortion rate $\dot{\mathbf{F}}_a\,\mathbf{F}_a^{-1}$:

$$\mathbf{S}_{oe} := \frac{\partial \psi_o}{\partial \mathbf{F}_e}, \quad \hat{\mathbf{E}}_e := \psi_o \, \mathbf{I} - \mathbf{F}_e^T \, \mathbf{S}_{oe}. \tag{10}$$

The Eshelby tensor constitutes a natural coupling between the stress and the remodeling actions. We have different stress measures, see Fig. 6 (right) and [61] for details, related to each other by a pull-back as follows:

$$\mathbf{S}_e = \mathbf{S}_{oe} \, \mathbf{F}_o^*, \quad \mathbf{S} = \mathbf{S}_e + \mathbf{S}_v, \quad \mathbf{T} = \mathbf{S} \, (\mathbf{F}^*)^{-1} = \mathbf{S}_e \, (\mathbf{F}^*)^{-1} - p \, \mathbf{I}, \tag{11}$$

here $\mathbf{S}_v = -p \, \mathbf{F}^*$, with p the pressure, is the reaction to the isochoric constraint $J_e = 1$.

3.5 The Contraction Pattern

At the continuum level, we can describe the kinematics of muscular tissue by a distortion of the form

$$\mathbf{F}_a = \lambda_\parallel \, \mathbf{n} \otimes \mathbf{n} + \lambda_\perp \, (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}), \tag{12}$$

where $\mathbf{n}$ is a unit vector, called *director*, and $\mathbf{n} \otimes \mathbf{n}$ is the associated *line field* describing the line along which a muscle can change its length; $\lambda_\parallel$ is the amount of stretching along this line; the tensor $\mathbf{I} - \mathbf{n} \otimes \mathbf{n}$ represents the projection onto the plane orthogonal to the line $\mathbf{n} \otimes \mathbf{n}$, and $\lambda_\perp$ is the stretch occurring in this plane; for volume-preserving distortion it is required that $J_a = det(F_a) = \sqrt{(\lambda_\parallel \, \lambda_\perp^2)} = 1$.

It is worth noting that distortions as in (12) have been used both to describe muscular functioning [6, 19], and phase transitions in nematic elastomer [35, 69, 75]. Given the representation formula (12), we can compute the target metric

$$\mathbf{C}_o = \mathbf{F}_a^T \, \mathbf{F}_a = \lambda_\parallel^2 \, \mathbf{n} \otimes \mathbf{n} + \lambda_\perp^2 \, (\mathbf{I} - \mathbf{n} \otimes \mathbf{n}). \tag{13}$$

In skeletal muscle, fibers run parallel to each other, thus $\mathbf{n} \otimes \mathbf{n}$ is a field of parallel lines, almost straight. It follows that, whatever be the stretches $\lambda_\parallel$ and $\lambda_\perp$, the target metric is compatible, that is

$$\mathbb{R}(\mathbf{C}_o) = 0 \;\Rightarrow\; \text{Skeletal muscle can contract without storing elastic energy.} \tag{14}$$

In cardiac muscle, fibers have many branches connecting them together, and wound around the almost ellipsoidal chambers; in such a case, it can be shown that, whenever the stretches $\lambda_\parallel$ and $\lambda_\perp$ are not trivial, the target metric cannot be compatible, that is

$$\mathbb{R}(\mathbf{C}_o) \neq 0 \;\Rightarrow\; \text{Cardiac muscle stores elastic energy while contracting.} \tag{15}$$

In smooth muscle, it is the arrangement of single cells, each with its own preferred shortening direction that yields an incompatible distortion field; thus (15) holds also for such kind of muscles.

The two results (14, 15) are of paramount importance from the physiological point of view: a skeletal muscle need an external device to recover its elongated configuration. In a pair of skeletal muscle, the one that pulls the other through a leverage is called antagonist. On the contrary, the peculiar structure of the myocardial wall produces incompatible distortions, and any contraction yields an increase of the elastic energy; then, this elastic energy is converted in mechanical work for recoil in the diastolic phase.

3.5.1 A 1D Example

The balance equation (7) simplifies by considering a 1D homogeneous bar; the state variables reduce to only two, the length x and the rest length x_a of the bar, see [36]. Denoting with x_{ref} the reference length of the bar, the three metric tensors (4) reduce to three scalar stretches:

$$\lambda_a = \frac{x_a}{x_{\text{ref}}}, \text{ target}; \quad \lambda = \frac{x}{x_{\text{ref}}}, \text{ actual}; \quad \lambda_e = \frac{\lambda}{\lambda_a}, \text{ elastic.} \tag{16}$$

The balance laws of forces and of remodeling actions simplifies to

$$f = \sigma, \quad \frac{\dot{x}_a}{x_a} = m\,(\beta - \lambda_a\, e)\,, \quad x(0) = x^{\text{init}}, \quad x_r(0) = x_a^{\text{init}}. \tag{17}$$

The *time evolution* of the muscle's state, modeled as a bar, is controlled by the force f (pulling the muscle) and the activation β (representing the mechanical effects of physiological stimuli). In (17), σ represents the stress, e the Eshelby-like term and $m > 0$ the mobility ($[f] = [\sigma] = [\beta] = [e] = F$, $[m] = F^{-1}T^{-1}$). The free-energy ψ_o is now a function of λ_e, and σ and e are given by

$$\sigma = \psi_o', \quad e = \psi_o - \lambda_e\,\psi_o' = \psi_o - \lambda_e\,\sigma, \tag{18}$$

with ψ_o' the derivative of ψ_o with respect to λ_e. It is worth noting that in the linear theory, where $\lambda_e = 1 + \varepsilon_e$, the Eshelby-like term becomes the opposite of the stress $e = -\sigma(\varepsilon_e) + o(\varepsilon_e)$. The muscle functioning relies on the term $\beta - \lambda_e\, e$, describing the competition between activation β and force f: its value determines the shortening or lengthening of x_a:

$$\begin{aligned} &\beta - \lambda_e\, e < 0 \Rightarrow \dot{x}_a < 0, && \text{shortening (motor);} \\ &\beta - \lambda_e\, e = 0 \Rightarrow \dot{x}_a = 0, && \text{purely elastic response;} \\ &\beta - \lambda_e\, e > 0 \Rightarrow \dot{x}_a > 0, && \text{lengthening (brake).} \end{aligned} \tag{19}$$

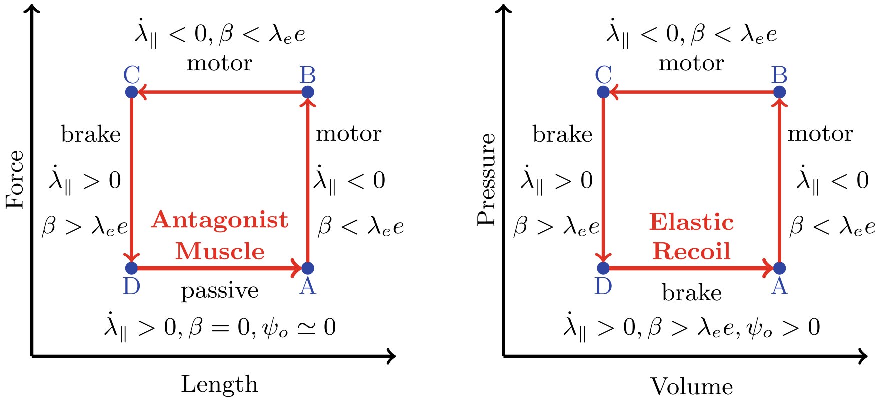

Fig. 7 Left) Force-length loop for skeletal muscle: the muscle is a motor during the path A to C, and a brake during the isometric relaxation C to D; the lengthening D to A is realized by the pulling of an antagonist muscle (no recoil). Right) Pressure-volume loop for cardiac muscle: during the systolic phase A to C the cardiac muscle is a motor and a portion of the muscular power is used to increase the elastic energy. During the systolic phase C to A the muscle is a brake working against the release of elastic energy. The filling phase D to A is realized thanks to the elastic recoil of the walls

Both skeletal muscle and cardiac muscle have a work-loop, see Fig. 7; the left and right plots are very similar, but at left we describe a force-length loop, while at right a pressure-volume loop, to emphasize that our mechanical point of view holds both at the fiber scale and at the scale of the whole organ.

The skeletal muscle (left) is a motor during the isometric contraction from A to B and the isotonic one from B to C; it is a brake during the isometric relaxation from C to D. The elongation D to A is realized by the pulling of an antagonist muscle; during this step the muscle is passive and the elastic energy might remain almost null (no recoil).

The cardiac muscle (right) is a motor during the systolic phase A to C; during this phase a portion of the muscular power is used to increase the elastic energy stored in the walls. During the systolic phase C to A the muscle is a brake working against the release of elastic energy. The filling phase D to A, a very important one, is realized thanks to the elastic recoil of the walls (not present in skeletal muscle). The reader is referred to the noteworthy experiments in [48], where the work-loop of isolated cardiomyocytes are recorded.

Smooth muscles behave very similarly to cardiac muscle and exploit elastic recoil to lengthen.

4 Conclusion and Future Directions

The idea that the ground state of materials can evolve in time, together with the notion of distortion, has brought forth an additional point of view in the mechanics of materials. This idea has been initially formulated to study ductile metals and shape memory alloys (hard materials); then, it has been successfully applied to liquid-crystal elastomers (soft materials), and eventually to biological matter to describe stress-driven growth.

Many problems of shape control, from plastic forming to morphogenesis and muscular contraction, can be formulated within the framework of non-linear elasto-plasticity with large, time evolving distortions. There are many interesting open problems that are worth investigating. Guidelines to characterize the remodeling actions that appear in the evolution law for the distortion are much sought after. We believe that it is essential that remodeling actions should come from coarse grained models of the phenomena occurring at smaller scale, similarly to what is done in plasticity, see e.g., (3) or flow rules in soil plasticity [74] accounting for the phenomenon of dilatancy in a thermodynamically consistent manner. Tools from a large body of research on configurational forces and their balance laws are also available, see [47] and the references cited therein. Moreover, the study of stress-free morphing is already very interesting, both in biological matter and in engineered structures, because compatible distortions can produce a very broad range of deformations. By including also the effect of applied loads, one often induces only small perturbations.

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Aquatic Locomotion for Self-propelled Fishlike Bodies and Different Styles of Swimming

Giorgio Graziani and Renzo Piva

Abstract Fish swimming is an intriguing subject of interest in fluid mechanics at the border with other disciplines in the field of environmental sciences. The main complexity is given by the interaction between the fish body and the unbounded fluid domain, otherwise at rest. The theoretical approach has to consider the full body-fluid system to obtain from the exchanged internal forces the whole motion, i.e. locomotion plus recoil displacements, which define, together with the prescribed body deformation, the free swimming behavior. The impulse formulation allows for an easy calculation of the potential contribution, related to the added mass, and of the vortical contribution related to bound and released vorticity. A simple two-dimensional and non-diffusive model is adopted for the numerical simulations to generate neat results able to clarify several physical phenomena. The aim is a unified procedure for both undulatory and oscillatory swimming to obtain valid answers for cruising speed and expended energy, hence for the performance in terms of the cost of transport. The paper describes the theoretical aspects of the model within the context of the relevant literature and summarizes the more significant results obtained recently by the research group of the authors.

Keywords Aquatic locomotion · Free swimming · Fish propulsion · Biological fluid dynamics

1 Introduction

After some early studies (Borrelli [5], Pettigrew [39]), fish swimming started to be investigated systematically at the beginning of the last century by experimental biologists (Gray [13, 14], Breder [6]) who analyzed the body motions constrained

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in water channels or towed in water tanks to keep them in the most suitable position for easy measurements.

The theoretical models and their computational counterpart, developed in the subsequent decades, by following the same line of reasoning considered for a quite long period of time the fishlike body with its proper deformation under a prescribed uniform stream (Lauder [23]) The idea was that, once the thrust is obtained, the body would counterbalance it by an opposite resistance to take the prescribed velocity as the locomotion speed for a cruising gait (Lighthill [26], Wu [49]). By means of this procedure, the related Froude efficiency achieved a great attention as it was the natural way to evaluate the swimming performance.

More recently, self-propulsion was recognized as a more appropriate way (Carling et al. [8], Triantafyllou et al. [44], Kern and Koumoutsakos [20], Yang et al. [51]) to investigate the optimal conditions for swimming bodies at steady state. Since in this case the total force vanishes for thrust and drag balancing each other, the Froude efficiency looses its meaning and the performance has to be measured preferably by the cost of transport, i.e. by the energy consumption per covered distance or by its inverse given by the miles per gallon (Gabrielli and von Kármán [12]).

Even though the approach could be in principle very general, different styles of swimming were usually analyzed in a different way, as it is the case for undulatory and oscillatory swimming (Smits [43]). Namely, for undulatory swimming thrust and drag are intrinsically related since the whole body is cooperating for the generation of the thrust as well as for its resistance, hence the self-propulsion is a mandatory approach for their numerical simulations. Instead for oscillatory swimming thrust and drag are in a way separable since the thrust is mostly given by the tail, taken as a flapping foil performing heave and pitch with a certain phase lag, while the remaining part of the body is somehow passive and contributes only to resistance. In this case the analysis through a prescribed stream may be still considered as a convenient approach (Floryan et al. [11]). We claim here that also for oscillatory swimming self-propulsion is the proper way to proceed to calculate correctly the performance and the optimal locomotion speed.

To this purpose, we introduce in Sect. 2 a simple theoretical approach to study free swimming and the most suitable simulation technique (Paniccia et al. [35, 36]) to obtain significant results. The impulse formulation, that we propose to use, highlights the roles of the added mass and of the vorticity release contributions to determine, beyond the fish locomotion, the recoil motions which are of major importance for the evaluation of the swimming performance. In fact, within the unbounded full system the body motion is produced by the internal forces exchanged with the fluid which generate both the main forward locomotion and the secondary motions, lateral and rotational, known as recoil motions. For the extension of this procedure to oscillatory swimming, we have to consider the presence of a virtual body in front of the flapping foil with an estimated resistance which provides the conditions to determine the locomotion speed and the energy consumption.

A few sample results are shown and discussed in Sect. 3 to proof the capability of the proposed model to capture the most subtle aspects of the problem. Particular attention is given to a form of control which may be instrumental for biomimetic applications to large scale ocean explorations. Further improvements towards real-life configurations, either by an extension to diffusive vorticity or by accounting for 3D effects, should be able to confirm the validity of the present approach.

2 Model and Simulation Approach

2.1 Mathematical Formulation

The self-propelled motion of a fish is generated by the exchange of momentum with the surrounding fluid, initially assumed at rest. By considering a single fluid-body system we impose that the momentum (linear and angular) is conserved over time, since there are no external forces applied. Hence, the internal forces and moments exchanged between body and fluid do not appear directly in the balance equations. The equations that describe the dynamics of the centre of mass of the body are obtained directly from the momentum balance expressed through the time variation of the impulse (linear and angular). This form extensively discussed by several authors (see e.g. Saffman [42], Lighthill [28], Wu et al. [48], Noca [33], Graziani and Bassanini [15], Limacher et al. [29]) allows for the separation of potential and vortical impulses so to highlight the contributions due to both added mass and release of vorticity, respectively. While the former is a non-circulatory and instantaneous contribution, the latter due to both the vorticity concentrated on the body contour and that released into the field increases gradually as the motion develops.

The forces acting on the fluid, equal and opposite to those acting on the body, are expressed through the time derivative of the impulse. This last quantity, in fact, does not show the conditional convergence properties that the momentum has in an infinite domain (see Landau and Liftschitz [22], Childress [9]) and even more it is linear with respect to the vorticity. By using classic vector identities, the linear and angular impulses are expressed by means of two terms concerning the vorticity concentrated on the contour of the body and shed within the field.

We consider an impermeable, flexible body with bounding surface S_b in an infinite dimensional volume V_∞ with zero velocity at infinity. We assume a Newtonian, incompressible fluid with density ρ. The outer surface is stationary in an absolute reference frame and the fluid velocity $\boldsymbol{u}$ is assumed to vanish on the far field boundary. In an isolated body-fluid system $(\mathcal{V}_b + \mathcal{V}_f)$, the sum of the forces (and moments) acting on the body and on the fluid is zero, as given by

$$\frac{d}{dt}\left[\int_{\mathcal{V}_b} \rho_b\, \boldsymbol{u_b}\, dV + \int_{\mathcal{V}_f} \rho\, \boldsymbol{u}\, dV\right] = 0 \tag{1}$$

The conservation of the total impulse (Saffman [41], Kanso [19], Eldredge [10]) allows to avoid the calculation of the time derivative, necessary when evaluating the forces, and the subsequent time integration for the body motion. In this way, a more accurate and simpler numerical computation is obtained. So, we remove the time derivative in Eq. (1) and, by assuming null initial condition the momentum balance gives

$$\int_{\mathcal{V}_b} \rho_b \, \boldsymbol{u_b} \, dV + \rho \, \mathbf{p} = 0 \tag{2}$$

By using a vectory identity for the unbounded fluid volume (see e.g. Wu et al. [48], Noca et al. [34]), the fluid impulse **p** is expressed as

$$\mathbf{p} = \frac{1}{N-1} \left[\int_{V_\infty} \boldsymbol{x} \times \boldsymbol{\omega} \, dV \; + \int_{S_b} \boldsymbol{x} \times (\boldsymbol{n} \times \boldsymbol{u}^+) dS \right] \tag{3}$$

where N is the dimension (below $N = 2$ will be assumed) and $\boldsymbol{x}$ is the position vector in the inertial frame. In the second term in Eq. (3), $\boldsymbol{u}^+$ stays for the limiting value of the fluid velocity on S_b and the integral over the external boundary receding to infinity has been proven to exactly vanish (Wu [46], Wu et al. [48], Noca et al. [34]). The normal $\boldsymbol{n}$ points out of the flow domain and all the vorticity is enclosed within the fluid volume V_∞ which extends to infinity. The right-hand side of Eq. (3) is independent of the choice of the reference frame origin (see e.g. [15, 34, 48]). As a brief comment, if we add and subtract the boundary condition, anyhow satisfied [7], the bound vortex sheet $\gamma = \left[\boldsymbol{n} \times (\boldsymbol{u}^+ - \boldsymbol{u_b}) \right] \delta(\boldsymbol{x} - \boldsymbol{x_b})$ would appear to identify, as shown later, the added mass term, otherwise fully embedded into the field vorticity (Limacher et al. [29]).

A similar approach for the angular momentum (positive anticlockwise) balance yields

$$\int_{\mathcal{V}_b} \rho_b \boldsymbol{x} \times \boldsymbol{u_b} \, dV - \rho \boldsymbol{\pi} = 0 \tag{4}$$

where the angular impulse $\boldsymbol{\pi}$ is

$$\boldsymbol{\pi} = \frac{1}{2} \left[\int_V |x|^2 \boldsymbol{\omega} \, dV + \int_{S_b} |x|^2 (\boldsymbol{n} \times \boldsymbol{u}^+) \, dS \right] \tag{5}$$

We consider here the moment with respect to a given pole (to be specified later either as the origin of the ground reference frame or as the body centre of mass), so $\boldsymbol{x}$ is the generic distance of the field point from the pole. Equations (2) and (4) will be used in the following to evaluate the body motion.

The velocity field $\boldsymbol{u}$ can be expressed, through the Helmholtz decomposition, as the sum of both the acyclic component and that related to circulation and vortices:

$$\boldsymbol{u}^+ = \nabla \phi + \nabla \times \Psi = \nabla \phi + \boldsymbol{u_w} \tag{6}$$

where ϕ and Ψ are referred to as scalar and vector potential, respectively, and are given by the solution of the Laplace/Poisson equation, subject to the impermeability boundary condition on S_b and to the related velocity vanishing at infinity.

The velocity decomposition (6) can be used to recast the total impulse (3) as the sum of the potential and of the vortical ones: $\mathbf{p} = \mathbf{p}_\phi + \mathbf{p}_\mathbf{v}$. The potential impulse is given by

$$\mathbf{p}_\phi = -\int_{S_b} \phi \boldsymbol{n}\, dS \tag{7}$$

where a vector identity for a scalar quantity has been used and the unit normal $\boldsymbol{n}$ to the body surface is now pointing into the fluid. The vortical impulse $\mathbf{p}_\mathbf{v}$ is given by:

$$\mathbf{p}_\mathbf{v} = \frac{1}{N-1}\left[\int_{V_\infty} \boldsymbol{x} \times \boldsymbol{\omega} dV \; + \int_{S_b} \boldsymbol{x} \times (\boldsymbol{n} \times \boldsymbol{u_w})\; dS\right] \tag{8}$$

The bound vorticity on S_b may be added to the released vorticity to adopt the concept of additional vorticity introduced by Lighthill.

The expression for the angular momentum can be similarly obtained by separating the vortical and the potential contributions using another vector identity (see Wu [48]) and the generalized Stokes' theorem. By considering Eq. (5), we can define the angular potential impulse as

$$\boldsymbol{\pi}_\phi = -\int_{S_b} \boldsymbol{x} \times \phi \boldsymbol{n}\, dS \tag{9}$$

and the angular vortical impulse

$$\boldsymbol{\pi_v} = -\frac{1}{2}\int_V |x|^2 \boldsymbol{\omega}\, dV - \frac{1}{2}\int_{S_b} |x|^2 (\boldsymbol{n} \times \boldsymbol{u_w})\, dS \tag{10}$$

2.2 Locomotion

Now we consider the two-dimensional motion of a deformable body $\mathcal{B}$ within a Cartesian inertial frame $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$. The body motion occurs in the plane $(\boldsymbol{e}_1, \boldsymbol{e}_2)$ and an angular velocity Ω about the axis $\boldsymbol{e}_3$ may be present as well. The motion of the body can be expressed as the sum of the prescribed shape deformation with velocity $\boldsymbol{u_{SH}}$ plus the centre of mass (CM) displacement with translational, $\boldsymbol{u_{CM}}$, and rotational, $\boldsymbol{\Omega}$, velocities. Thus we can split the body motion as:

$$\boldsymbol{u_b} = \boldsymbol{u_{SH}} + \boldsymbol{u_{CM}} + \boldsymbol{\Omega} \times \boldsymbol{x}' \tag{11}$$

where $\boldsymbol{x}'$ is the new position vector given by $\boldsymbol{x} = \boldsymbol{x_{CM}} + \boldsymbol{x}'$ and $\boldsymbol{u_{CM}}$ is the locomotion velocity of the body centre of mass. By integrating Eq. (11) over the body volume and by accounting for the definition of centre of mass and since $\int_{\mathcal{V}_b} \boldsymbol{\Omega} \times \boldsymbol{x}' \, dV = 0$, it follows

$$\int_{\mathcal{V}_b} \rho_b \boldsymbol{u_{SH}} \, dV = 0 \tag{12}$$

while for the angular momentum we obtain $\int_{\mathcal{V}_b} \rho_b \boldsymbol{x}' \times \boldsymbol{u_{SH}} \, dV = 0$.

Now, substituting Eq. (11) in the first term in Eq. (2) yields

$$m_b \, \boldsymbol{u_{CM}} + \rho \, \mathbf{p} = 0 \tag{13}$$

where $\boldsymbol{p}$ is now expressed in terms of $\boldsymbol{x}'$ due to the above mentioned independence of the reference system origin. Similarly, from Eqs. (4) and (11), the $\boldsymbol{e}_3$ component of the angular momentum balance, considering the impulse with respect to the body centre of mass, $\pi' = (\boldsymbol{\pi} - \boldsymbol{x}' \times \boldsymbol{p}) \cdot \boldsymbol{e}_3$, is

$$I_{zz} \, \Omega - \rho \pi' = 0 \tag{14}$$

Let us now express the locomotion Eqs. (13) and (14) in a coordinate frame attached to the body with origin in CM and with $\boldsymbol{b}_3$ parallel to $\boldsymbol{e}_3$. To the purpose we consider the ground fixed frame $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ and the body frame $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$ (see Fig. 1) where we have the angular velocity $\boldsymbol{\Omega} = \Omega \, \boldsymbol{b}_3$, the linear velocity $\boldsymbol{V_b} = V_1 \, \boldsymbol{b}_1 + V_2 \, \boldsymbol{b}_2$ and we use $\boldsymbol{P}$ and $\boldsymbol{\Pi}$ to define linear and angular impulses. Hence, in the body frame, the relevant equations are still Eqs. (13) and (14) written for the new variables. The velocity field is represented through the previously introduced Helmholtz decomposition, see Eq. (6), and the acyclic term is also split into the contribution given by shape deformation and locomotion

$$\boldsymbol{u} = \nabla \phi_{SH} + \nabla \phi_{LOC} + \nabla \times \boldsymbol{\psi} = \boldsymbol{u}_{SH} + \boldsymbol{u}_{LOC} + \boldsymbol{u}_W \tag{15}$$

The locomotion potential is expressed through the three Kirchhoff base potentials $(\phi_1, \phi_2, \phi_\Omega)$ related to unit linear motions in $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ directions as well as to unit angular rotation about $\boldsymbol{b}_3$, respectively. Then, the acyclic potential is

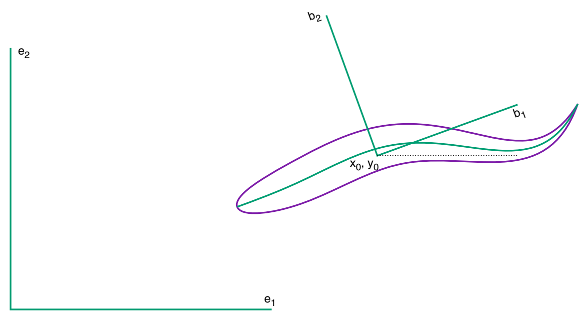

Fig. 1 Ground and body reference frames

$$\phi = \phi_{SH} + \phi_{LOC} = \phi_{SH} + V_1\phi_1 + V_2\phi_2 + \Omega\phi_\Omega \tag{16}$$

Each scalar potential is harmonic with the prescribed decay at infinity while the boundary conditions on S_b account for the impermeability condition:

$$\frac{\partial\phi}{\partial n} = [\boldsymbol{u}_{SH} + V_1\boldsymbol{b}_1 + V_2\boldsymbol{b}_2 + \Omega(\boldsymbol{b}_3 \times \boldsymbol{X})] \cdot \boldsymbol{n} \qquad \text{on } S_b \tag{17}$$

where $\boldsymbol{X}$ stays for the position vector in the body fixed frame. The vector potential in Eq. (15) satisfies $\nabla^2\boldsymbol{\psi} = -\boldsymbol{\omega}$ together with the boundary condition $(\nabla \times \boldsymbol{\psi}) \cdot \boldsymbol{n} = 0$ on $\partial\mathcal{B}$ and $\nabla \times \boldsymbol{\psi} = 0$ at infinity.

According to Kanso [19], we follow her procedure to obtain the linear and angular body motion, dividing by ρ and expressing the scalar ω as $\boldsymbol{\omega} \cdot \boldsymbol{b}_3$. By combining Eqs. (2) and (3) and using the velocity decomposition (15) we obtain

$$\int_{S_b} \phi_{LOC}\,\boldsymbol{n}\,dS - \frac{m_b}{\rho}\,\boldsymbol{V_b} = \boldsymbol{P}_{SH} + \boldsymbol{P}_W \tag{18}$$

where $\boldsymbol{P}_{SH}$ and $\boldsymbol{P}_W$ are obtained by combining Eq. (7) with Eq. (16) and Eq. (8) with Eq. (15), respectively.

Similarly, from Eqs. (4) and (5) and using another vector identity (see e.g. Wu [47]) we obtain

$$\frac{I_{zz}}{\rho}\,\Omega - \int_{S_b} (\boldsymbol{X} \times \phi_{LOC}\boldsymbol{n}) \cdot \boldsymbol{b}_3\,dS = \Pi_{SH} + \Pi_W \tag{19}$$

where Π_{SH} and Π_W are obtained by combining Eq. (9) with Eq. (16) and Eq. (10) with Eq. (15), respectively.

Let us now express the locomotion impulses in terms of the added mass coefficients reported in the classical literature (see Lamb [21], Batchelor [4], Newman [32]) by combining Eqs. (18) and (19) with the decomposition of Φ_{LOC} appearing in Eq. (16). Upon inserting the relevant added mass coefficients $m_{ij} = -\int_{S_b} \phi_j \frac{d\phi_i}{dn} dS$ in the equations, we obtain the final form of the system as

$$\begin{cases} V_1\ (m_{11} + m_b) + V_2\,m_{12} + \Omega\,m_{13} = -P_{SH\,1} - P_{W\,1} \\ V_1\,m_{21} + V_2\ (m_{22} + m_b) + \Omega\,m_{23} = -P_{SH\,2} - P_{W\,2} \\ V_1\,m_{31} + V_2\,m_{32} + \Omega\ (m_{33} + I_{zz}) = -\Pi_{SH} - \Pi_W \end{cases} \tag{20}$$

We may appreciate that the added mass terms which multiply the unknowns now appear directly in the l.h.s with a major advantage in terms of stability of the integration procedure. Let us notice that the above system of equations provides the evaluation of the body velocities without considering time derivatives as required when using the standard formulation in terms of forces and moments.

2.3 Numerical Technique

The numerical solution of the above system is obtained by considering a potential flow with concentrated vorticity on the body surface and its subsequent shedding behind the body into the vortex wake. The flow solution at each time step is obtained by using an unsteady potential panel code which is based on the Hess and Smith [18] approach while the unsteadiness of the problem, i.e. the wake release, is taken into account following the Basu and Hancock [3] procedure. The approach of Hess and Smith consists in approximating the body by a finite number of panels, each one with an individual uniform source strength. A vortex sheet of constant strength is also superposed to account for circulation. The impermeability condition on each panel plus a suitable unsteady Kutta condition are used to evaluate the source strengths as well as the uniform circulation density. Moreover, according to Kelvin's theorem, any change in the circulation about the airfoil results into vorticity release through a new wake panel attached to the trailing-edge. At each time step the released wake panel is lumped into a point vortex which is shed into the wake and advected downstream by the flow field.

As mentioned before, even if the governing equations are written in the ground frame of reference, the solution is achieved by their projection in a coordinate system attached to the body which moves according to V_1 and V_2 and rotates according to Ω. Actually, this frame is the most natural one to enforce the deformation which has to be independent of both the presence of the fluid and any rigid motion of the body itself. At each time step, the body, deforming with $\boldsymbol{V}_{SH}$, is invested by a waterspeed given by the combination of $-V_1$, $-V_2$ and $-\Omega$, which allows to update the positions of the released vortices. Moreover, the unknown waterspeed contribution, required for the unsteady Kutta condition, allows to determine in a few iterations both length and inclination of the wake panel behind the body.

Finally, it is important to notice that the linear velocity components named, from now on, forward and lateral velocity ($U = -V_1$ and $V = V_2$) respectively, change their directions at each time step, since the equations of motion are written in the body frame coordinates. After a transient acceleration phase, the body, even maintaining an oscillating pattern, reaches an asymptotic steady state, where the forward velocity has a constant mean value representing the actual locomotion speed while the mean lateral and angular velocities are equal to zero.

3 Results and Discussion

We analyze in this section the free swimming of a deformable body in an unbounded fluid domain, otherwise quiescent, with a major interest for the steady state typical of cruising gaits, namely locomotion speed and cost of transport. As anticipated in the introduction, for the sake of clarity, we will illustrate the results separately for undulatory and oscillatory swimming which have been treated traditionally in a different way, including tools and choice of parameters for their analysis. Nonetheless, we will try to follow a common path for both styles of swimming leaving partially unsolved a few points, summarized in the final remarks, to be investigated in future studies.

3.1 Undulatory Swimming

To generate the required thrust by keeping as low as possible the energy consumption, many fishes undulate a large part of their body, if not the whole body, by transmitting throughout their muscular activity a traveling wave from head to tail. As brightly explained by Gray and successively assumed by many others, the phase velocity of the wave has to be slightly larger than the forward fish locomotion velocity and the performance is increasingly better the closer the two velocities are to each other. The most famous analytical model introduced by Lighthill and Wu, based on the elongated body theory under a prescribed stream, finds nice expressions for thrust and expended energy which influenced all the subsequent research on the subject. Also in our free swimming analysis we found a great support in their theoretical results for the selection of the proper phase velocity to account for the whole motion, by including the recoil components together with the prescribed deformation. We report in Fig. 2a the asymptotic velocity reached in time for different amplitudes of the selected deformation which has been proposed in a previous paper for biomimetic applications (see Paniccia et al. [35] for details). The slope of the mid-line is defined by the following expression for a traveling wave of constant amplitude $d\beta$ and wave number k related to a wave length along s

$$\beta(s, t) = d\beta \sin(ks - \omega t) \tag{21}$$

where ω is the angular frequency. An amplitude modulation may be added to reproduce the deformations present in the literature (see e.g. Hess and Videler [17], Lauder and Tytell [24]).

We may see how the steady state value is the same for all amplitudes, as expected for inviscid flows, while the transient range shows an acceleration increasing with the deformation amplitude.

From a detailed analysis we found that the constant value of the asymptotic velocity is given by the sum of potential and vortical contributions of different amount. Namely, a larger potential component which leads to a larger vorticity release, hence to a larger acceleration, is accompanied by a lower vortical component at steady state (see Fig. 3). Since the energy consumption is essentially given by the released vorticity, as shown by its expression in terms of the excess energy

$$E = \frac{1}{2} \int_V \boldsymbol{\psi} \cdot \boldsymbol{\omega} \, dV, \tag{22}$$

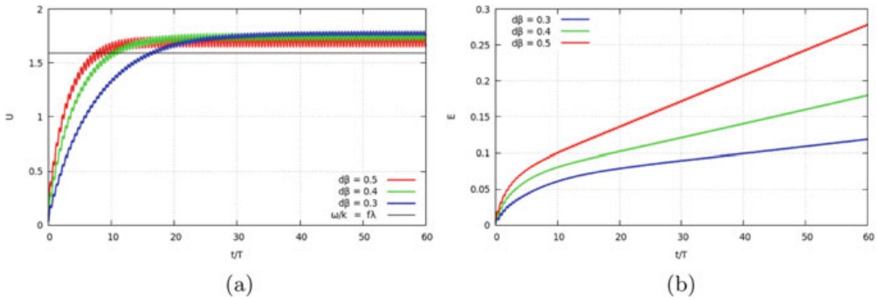

Fig. 2 Effect of the undulation amplitude: (**a**) forward swimming velocity, (**b**) time behavior of the fluid kinetic energy, from [35]

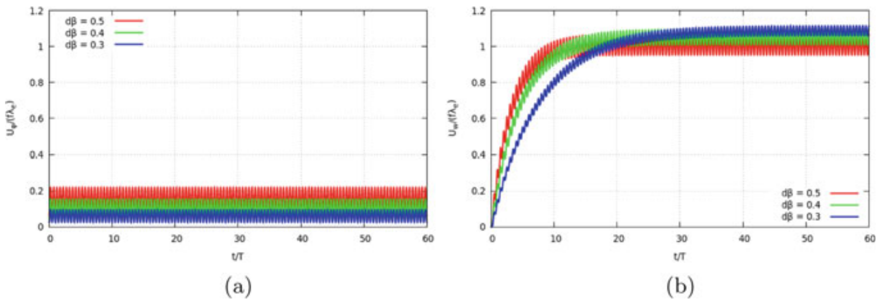

Fig. 3 Forward component of the slip velocity for different undulation amplitudes: (**a**) potential contributions U_ϕ, (**b**) vorticity contributions U_w, from [35]

the diagrams of the energy consumption reported in Fig. 2b confirm, through their larger slope, the required power for larger amplitudes, which lead for the same speed to an increase of the cost of transport, defined as the ratio between the mean rate of change of the energy $\overline{\dot{E}}$ and the mean forward velocity U_{loc} (see e.g. Bale et al. [2], Wang et al. [45], Maertens et al. [31]). As a suggestion, to reduce the cost of transport for cruising gaits we should reduce the potential contribution U_ϕ due to the shape deformation, hence the strength of the released vortices. On the contrary, larger U_ϕ is required in escape maneuvers to have an immediate response, though with a larger expended energy, not prohibitive in this case.

It is interesting to point out the importance of the recoil motion when evaluating both the asymptotic forward velocity and the energy consumption. Starting from our model, able to simulate fully free swimming, we thought reasonable to examine a series of constrained swimmers, whose degrees of freedom are partially inhibited (see Xiong and Lauder [50], Maertens et al. [30], Paniccia et al. [36]). To the purpose, we considered a fully constrained swimmer with both lateral and rotational motions canceled, but also swimmers with only one of them remaining. The results for the forward velocity in time and for the energy consumption are reported in Fig. 4a, b for a few cases which show how the fully free swimmer has the best performance, while

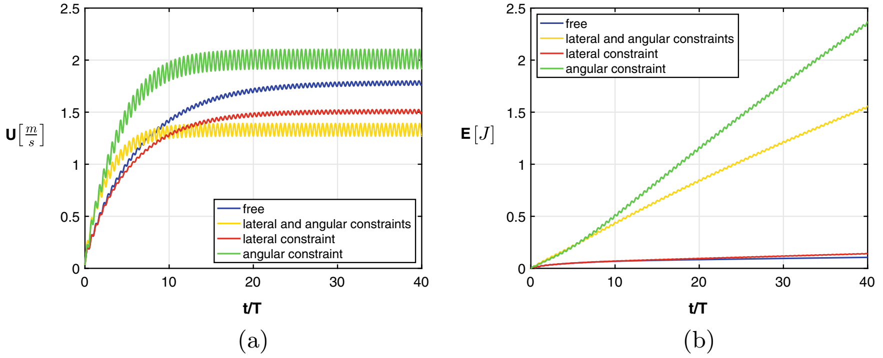

Fig. 4 Time history of (**a**) the forward velocity and (**b**) the kinetic energy for free swimming (blue) and constrained gaits: lateral and angular constraints (yellow), lateral constraint (red) and angular constraint (green), from [36]

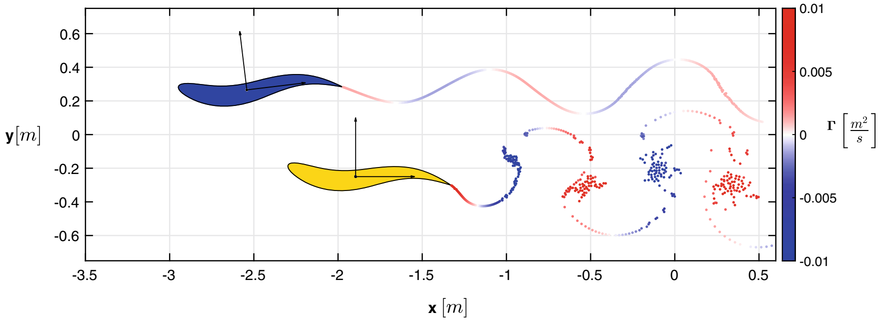

Fig. 5 Comparison at steady state of the fully constrained (yellow) and the free swimming case (blue), from [36]

the cancellation of the recoil motions are penalizing at least the cost of transport, especially if the rotational degree of freedom is prevented (see also Reid et al. [40]). A meaningful picture able to give a quick insight, see Fig. 5, is given by the frame of our simulation showing a compared locomotion of the two swimmers, i.e. the fully free and the fully constrained one. We may appreciate from the picture, where also the released wake are reported, the smooth and graceful trajectory of the free swimmer reaching a larger velocity with, at the same time, a lower vorticity release. A further case showing the importance of recoil is the analysis of a tailless fish conducted by Gray to proof the role of the tail for the evaluation of swimming gaits. His observations, both of the intact fish and of the same fish after its tail was amputated, drew to some interesting conclusions that we try to assess through our simulations. The tailless fish does not undulate any more and keeps oscillating the rear-end of its body in a symmetrical way with respect to the axial direction in the body fixed frame. When he observed the fish in its whole motion, i.e. in the inertial frame, he noticed a much longer phase pushing in the backward direction that, by

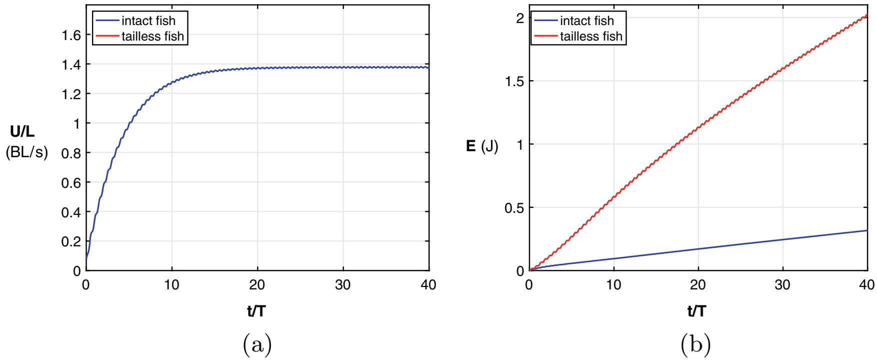

Fig. 6 (**a**) Forward velocity and (**b**) energy consumption for the intact fish (blue) and for the tailless fish (red), from [37]

breaking the symmetry experienced in the fixed frame, was producing a net thrust of the fish. It follows a locomotion speed quite comparable with the one for the intact fish, but at the same time with a much larger energy consumption. Our simulations (for more details see Paniccia et al. [37]) confirm these conclusions and the main results are reported in Fig. 6. The asymmetrical behaviour which plays a key role is figured out by comparing the free swimming of the two configurations, intact and tailless, and by accounting in both cases for the full recoil motion. As shown in Fig. 6, the asymptotic velocity of the tailless fish has a significantly lower value with respect to the intact fish while the energy consumption is much larger.

3.2 Oscillatory Swimming

For some other fish species the self-propulsion is generated, almost exclusively, by the oscillation of the caudal fin, while the other part of the body is essentially contributing to the viscous resistance. This is the reason for the procedure traditionally adopted which considered a flapping foil under a prescribed stream to find the thrust and the related Froude efficiency (Guglielmini and Blondeaux [16], Floryan et al. [11]). As previously anticipated, at steady state the flapping foil is able to propel a body with a balancing resistance which occurs at a forward speed given by the prescribed stream. By this procedure it is not an easy task to figure out for certain fish, with its shape and its mass, the values of the parameters for the optimal performance. To this purpose, let us stress that with a prescribed velocity we may obtain the maximum efficiency for the foil as a propulsor, hence the maximum thrust it can exert on a body with an opposite resistance, without any concern about the search of the best performance in terms of energy consumption. That's why, we thought reasonable to turn also in this case to a free swimming mode, very successful for undulating

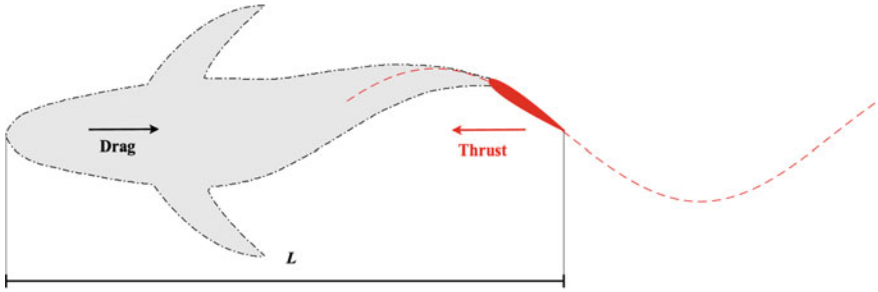

Fig. 7 A cartoon for the virtual body (gray) and the tail propulsor (red) with a sketch of the exchanged forces and the oscillatory trajectory of the tail pivot point (dashed red line)

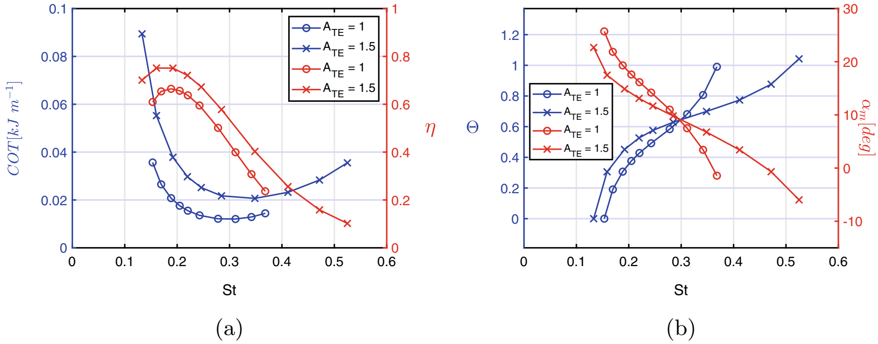

Fig. 8 (**a**) Cost of transport of the whole body (blue) and efficiency of the propulsor (red) as function of the Strouhal number St. (**b**) Feathering parameter Θ (blue) and maximum angle of attack α_m (red) for the inviscid case as function of the Strouhal number. Comparison between $A_{TE} = 1$ and 1.5 for the inviscid case, from [38]

bodies, to obtain the proper locomotion speed and the related cost of transport once a given drag coefficient for a so-called virtual body, see Fig. 7, is prescribed (see Akoz and Moored [1]).

For a complete validation of this approach, we study the free longitudinal motion of a flapping foil, with assigned values of heave and pitch, pushing the defined virtual body, in such a way to obtain for the resulting forward speed not only the cost of transport, but also the standard efficiency. In fact the assigned resistance is now known even though for the actual free swimming mode a perfect balance with the thrust is experienced. The two diagrams in Fig. 8a show the cost of transport and the efficiency as function of the Strouhal number $St = \omega l A_{TE}/(2\pi U)$, i.e. the parameter more commonly adopted in these studies, even though quite unproper when the forward velocity is the unknown as in free swimming. Actually, to generate these diagrams, we selected parameters built on assigned data like the non-dimensional peak-to-peak trailing edge amplitude A_{TE}. Figure 8a shows how the optimal values for the cost of transport and for the Froude efficiency η occur for quite different values of the

Strouhal number. The meaning is clear: the first is concerning cruising conditions and the second is more appropriate for fast escape gaits.

A deeper understanding of the two modes is given by the diagrams in Fig. 8b where the values of the maximum angle of attack α_m and of the feathering parameter Θ are reported to show that large values of α_m characterize the optimal range of efficiency, while large values of Θ are reached for the optimal range of the cost of transport. For the reader's convenience, let us recall the meaning of the parameter Θ ingeniously introduced by Lighthill [27] as the ratio between the forward velocity and the phase velocity corresponding to the heave and pitch motion in the asymptotic case of very large non dimensional wave length. These results confirm well established experimental findings reported in the literature about various cases of oscillatory swimming (Li et al. [25]).

4 Final Remarks

Aim of the present paper was to describe the main steps of a theoretical model simple enough to allow for easy simulations, but able to highlight the complex interaction of a swimming body with the unbounded surrounding fluid. The intention is to make a unified treatment for fishes adopting different styles, namely undulatory and oscillatory swimming. A collection of basic results is illustrated together with physical comments referring to previous findings starting from Gray and Lighthill up to the most recent contributions. The impulse approach provides a clear separation of added mass and vortical components which play different and specific roles in the analysis of free swimming. A special attention is given to the whole body motion, including the lateral and rotational recoil which modify substantially the overall performance with a particular effect on the energy consumption. Several aspects of the flow field, related to vorticity diffusion and to three-dimensional geometries, may have a remarkable impact on certain severe maneuvers like C-start or S-start typical of escape gaits which still deserve a further investigation.

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