# The Sound of Church Bells: Tracking Down the Secret of a Traditional Arts and Crafts Trade

# 52

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The sound of church bells is part of most people's everyday life and can easily be examined with smartphones. Similar to other experiments of this book (Chaps. 47 and 49) [1, 2], we use a suitable iOS app. The underlying physical theory of church bells proves to be difficult. A reliable prediction of their natural frequencies based on their exact dimensions is only possible using the finite element method [3]. If you ask bell founders how they calculate the rib of a bell (half longitudinal section of a bell, which completely determines the acoustic properties, Fig. 52.1) in order to get a church bell with the desired frequency spectrum, you will certainly not get an answer: the art of bell casting is based on centuries of experience and knowledge of the rib structure is only shared with direct descendants. We want to have a closer look at these well-guarded secrets, knowing full well that we cannot fully unravel them. This contribution presents simple mathematical models and a comparison with a data set of almost 700 bells.

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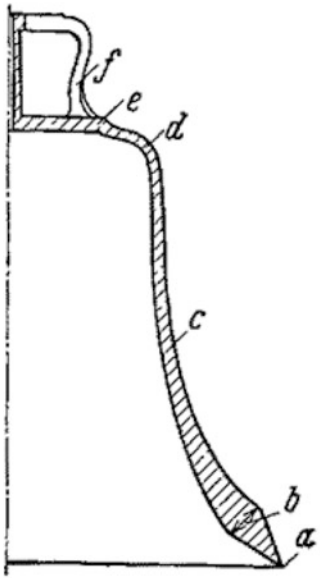
**Fig. 52.1** Half cross section (rib) of a bell [4]



## 52.1 Vibration Modes of Church Bells

When the clapper strikes against the inside rim of a bell, natural oscillations ensue. As with most musical instruments, this leads to numerous vibrational modes. However, the peculiarity is that the dominant overtones of a bell are not harmonic in the way that stretched strings are. Instead, the frequency spectrum includes pitches that go by the names hum, prime (double the frequency of hum), minor third above that, fifth, and octave (Table 52.1). These intervals, and more, characterize the full, powerful sound of a church bell. In most bells, the perceived strike note roughly corresponds to half the octave frequency and thus approximately coincides with the prime. The root of the frequency spectrum is referred to as the hum, a

**Table 52.1** Frequency ratios in the spectrum of a church bell [5]

| Name | Frequency ratio to prime | |
|---|---|---|
| | **Just scale** | **Church bells** |
| Hum | 0.5 | 0.5 |
| Prime | 1.0 | 1.0 |
| Minor third | 1.2 | 1.183 |
| Quint | 1.5 | 1.506 |
| Octave | 2.0 | 2.0 |
| Major third | 2.5 | 2.514 |
| Fourth | 2.667 | 2.662 |
| Twelfth | 3.0 | 3.011 |
| Upper octave | 4.0 | 4.166 |

"sub-harmonic" pitch that occurs at half the prime frequency. Its analytical description was an early subject of research in physics, including work by a number of outstanding scientists such as Euler, Jacques Bernoulli, Chladni, Helmholtz, and Rayleigh. Generally, they approached the problem by describing bell-like bodies with significantly simpler geometries, e.g., rings, hemispheres, or hyperboloids [6].

In the next section, we analyze how the hum frequency can be modeled taking into account the bell radius and how the bell radius can be accurately estimated based on a measurement of the hum frequency using a smartphone. In order to verify the new model, a data set with nearly 700 bells, including the hum frequency, bell radius, and the thickness of the inside rim of the bell was created based on a bell book of the Archbishopric of Cologne (Germany) [7]. The idea is to use the extensive experience of bell founders from several centuries to verify and further improve the mathematical model.

## 52.2 Frequency-Radius Relationship

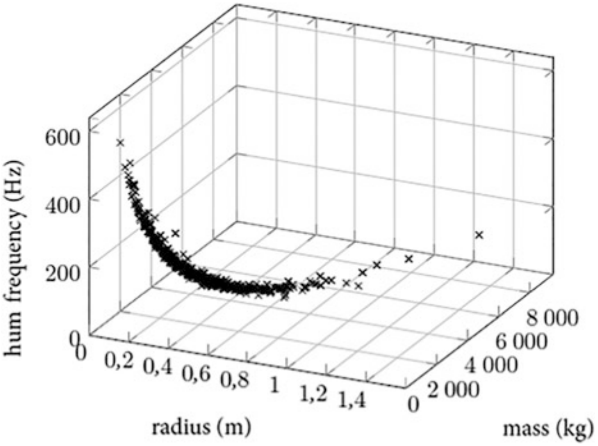
The data in Fig. 52.2 clearly show a strong relationship between the frequency of the hum tone and the radius or mass, respectively. Initially, rather than attempting to fit this data empirically, we begin the analysis by considering a physically motivated mathematical model.

The equation used by Apfel [8] to model the frequency of wine glasses is a relationship we want to apply to bells, although originally devised for two-dimensional plates bent to a cylinder:

$$f_0 = \frac{\nu_L \cdot d}{\sqrt{3}\pi \cdot R^2} \tag{52.1}$$

($f_0$ fundamental frequency, $\nu_L$ is the longitudinal or sound velocity in the material, $d$ thickness of the cylinder, $R$ radius). For this purpose, we replace the fundamental

**Fig. 52.2** Hum frequency as a function of radius and mass



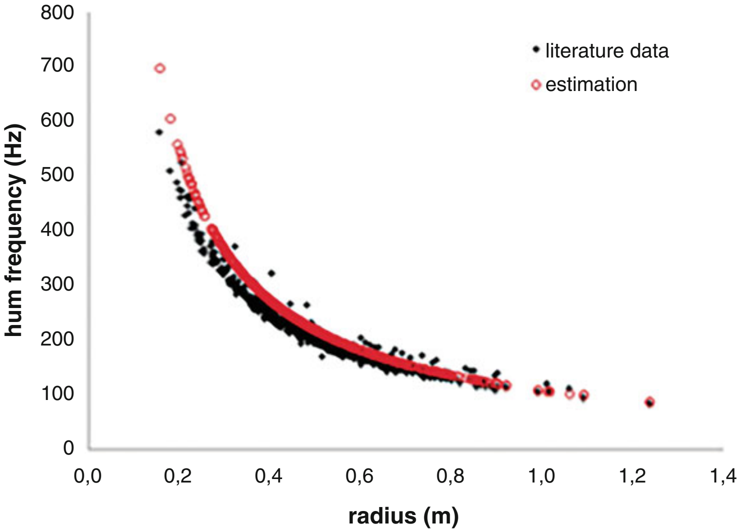


**Fig. 52.3** Frequency estimation using a hollow cylinder model vs the actual bell frequencies

frequency $f_0$ by the hum frequency $f_{\mathrm{hum}}$ and we take the speed of sound in bronze for $v_{\mathrm{L}}$ into account ($v_{\mathrm{L}} \approx 3400$ m/s). Furthermore, we also use the relation $d/R \approx 1/7$, which is a result of our own analysis of the data set described in [7]:

$$f_{\mathrm{hum}} = \frac{1}{\sqrt{3} \cdot 7 \cdot \pi} \frac{v_{\mathrm{L}}}{R}. \tag{52.2}$$

By introducing a correction factor, the deviation of the data from the model can be reduced to 3.5% on average (Fig. 52.3). We get

$$f_{\mathrm{hum}} = 0.092 \cdot \frac{v_{\mathrm{L}}}{\pi \cdot R}, \tag{52.3}$$

and as a rule of thumb we obtain

$$R \approx \frac{100\,\mathrm{Hz}}{f_{\mathrm{hum}}}\,\mathrm{m}. \tag{52.4}$$

## 52.3 Mass-Radius Relationship

In order to calculate the mass of a church bell from the measured hum frequency, we need a mass-radius relationship. In 1885 Otte [9] found that the mass $M$ of a bell is proportional to its radius cubed, as one might expect from scaling arguments alone.

Based on the data of 700 church bells and the eq. $M = c \cdot R^3$, we empirically find the relationship

$$M = 4776 \frac{\text{kg}}{\text{m}^3} \cdot R^3 \quad \text{and} \tag{52.5}$$

$$M \approx 4776 \frac{\text{kg}}{\text{m}^3} \left( \frac{100\ \text{Hz} \cdot \text{m}}{f_{\text{hum}}} \right)^3 \tag{52.6}$$

With this empirically found mass-radius relationship, the mass of a church bell can easily be estimated based on a frequency measurement with an average deviation of 11.7%.

## 52.4 Result of a Sample Measurement

With the iOS app Spektroskop [10], a measurement of the "Maria Gloriosa" bell of the Bremen Cathedral (Germany), cast in 1433, was carried out (Fig. 52.4). The bell has a radius of 0.85 m and a mass of 2500 kg [11]. The results show a good approximation of the expected frequency ratios. The hum frequency corresponds to the lowest, large frequency peak at 117 Hz, and the other peaks corresponding to the various overtones can be found in Fig. 52.4. By inserting the measured hum frequency of 117 Hz in relationships (52.4) and (52.6), we get a radius of 0.85 m and

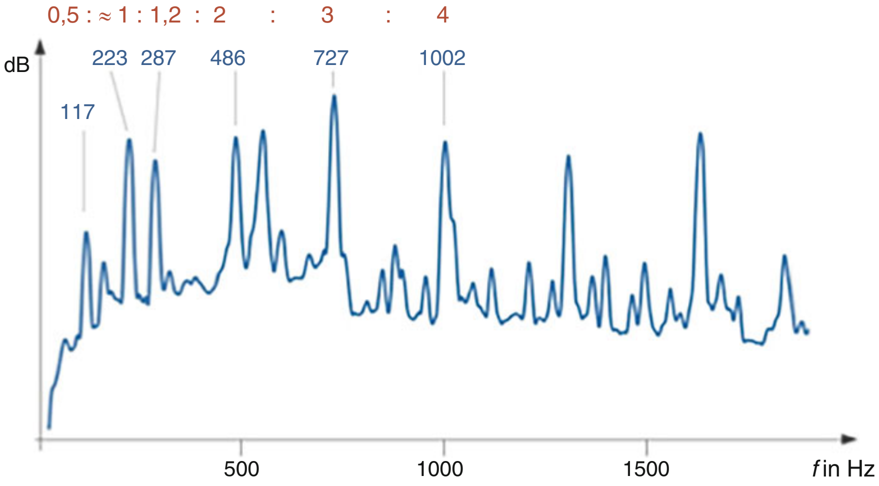


**Fig. 52.4** Frequency spectrum of the "Maria Gloriosa" bell of St. Peter's Cathedral, Bremen (Germany), recorded with the app Spektroskop [10] and visualized with a spreadsheet program

a mass of about 2900 kg. While the estimate of the bell radius agrees with the literature value, the mass is overestimated by 16%.

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