Nonlinear Denoising of Nonstationary Signals

Zoltán Germán-Salló(✉)

University of Medicine, Pharmacy, Sciences and Technology “George Emil Palade”, Targu-Mures, Romania
zoltan.german-sallo@umfst.ro

Abstract. The paper presents different time-frequency decomposition-based filtering procedures. These procedures are applied to various types of synthesized nonstationary test signals which are contaminated with different levels of Gaussian white noise. Empirical mode decomposition (EMD), discrete wavelet transform (DWT) and wavelet packet transform (WPT) based procedures are implemented in order to perform nonlinear multiscale filtering. The obtained results are compared and analyzed; conclusions summarize the comparison study.

Keywords: Wavelet transform · Time-frequency decomposition · Empirical mode decomposition · Nonlinear filtering

1 Introduction

In signal processing applications, the assumption of stationarity of the signal introduces unwanted limitations, procedures can be applied only in idealized conditions. The signals in acoustics, geophysics, biology, and biomedicine have a nonstationary behavior which demands another kind of approach. The nonstationarity of a signal may be caused by different reasons, as the signal sources, propagation channel or receivers. In all these, noise appears obviously and could have a nonpredictable and undesired effect.

Nowadays, applications of nonlinear filtering range from engineering, machine learning [1], economic science [2] and natural sciences such as geoscience [3]. Valuable applications of nonlinear filtering can be found also in biomedical engineering applications [4].

This paper presents three different nonlinear filtering procedures. A comparison study reflects the different obtained filtering results. The main parameters are the acquired signal to noise ratio and the absolute value of the reconstruction error.

Splitting the signal in components, processing the components separately and reconstruct the signal from these, is the main idea in every multiscale signal analysis and processing task. Usually, the different time-frequency analyzing methods offer various decomposition algorithms, the choice is made depending on followed or required parameters or benefits.

The paper is organized as follows. After the introductive notions, the second paragraph introduces the theoretical background of multiscale analysis, the third paragraph presents the used signals and the proposed procedures. After that, experimental and theoretical results are presented, finally, the concluding remarks end this study.

L. Moldovan and A. Gligor (Eds.): Inter-Eng 2021, LNNS 386, pp. 843–855, 2022.
https://doi.org/10.1007/978-3-030-93817-8_75

2 Time-Frequency Decomposition Techniques

2.1 Empirical Mode Decomposition

The Empirical Mode Decomposition (EMD) was proposed as the fundamental part of the Hilbert–Huang transform (HHT) [5]. The EMD decomposes a signal in the time domain through an iterative sifting process, into a set of functions, known as Intrinsic Mode Functions (IMFs). The IMFs offer a description and a reconstruction of the signal, even though they are not necessarily orthogonal as in case of other transforms.

The decomposition algorithm obtains smooth envelopes defined by local maxima and minima of a given sequence and performs a subsequent subtraction of the mean of these envelopes from the initial sequence. This procedure named sifting process needs the identification of all local extrema (maxim and minim values). These extrema are connected by cubic spline lines to create the upper and the lower envelopes. The sifting process decomposes the signal into IMFs in following way:

For a signal $s(t)$, where m_1 is the mean of its upper and lower envelopes the first component h_1 is computed as follows:

$$h_1 = s(t) - m_1 \tag{1}$$

The first step in procedure when the mean value of the two envelopes is computed, is presented on Fig. 1.

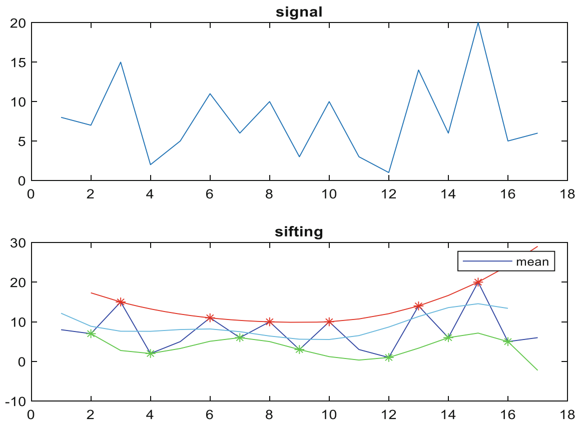

Fig. 1. The first step in sifting process.

In the second sifting process, h_1 is treated as the initial signal $s(t)$, and m_{11} is the mean of h_1's upper and lower envelopes:

$$h_{11} = h_1 - m_{11} \tag{2}$$

This sifting procedure is repeated k times, until h_{1k} is an IMF, that is:

$$h_{1k} = h_{1(\mathrm{k}-1)} - m_{1k} \tag{3}$$

Then $c_1 = h_{1k}$ will be the first IMF component from the analyzed sequence, which contains the shortest period component of the signal. After separation:

$$s(t) - c_1 = r_1 \tag{4}$$

the previously presented procedure is repeated on r_j:

$$r_1 - c_2 = r_2, \ldots r_{n-1} - c_n = r_n. \tag{5}$$

The result is a set of functions; the number of functions in the set depends on the original signal. The decomposition process stops when the residue r(t) becomes a constant or a function without extrema that no longer satisfies the requirements of an IMF.

Finally, the analyzed signal can be expressed as:

$$s(t) = c_1 + c_2 + \cdots c_k + r_k \tag{6}$$

The decomposition is made without external functions, only the signal's properties are used. In conclusion, EMD decomposes any given sequence or signal, whether it is linear, nonlinear or nonstationary, into a set of IMFs [6].

2.2 The Discrete Wavelet Transform (DWT)

In digital signal processing the Fourier Transform is probably the most popular, but many other transforms are available for engineers and mathematicians. Every transformation technique has its applicability with advantages and disadvantages in a certain research area. In Fourier analysis, the Discrete Fourier Transform (DFT) decompose a signal into sinusoidal basis functions of different frequencies and amplitudes. The signal can be completely recovered l from its DFT (FFT) components [7].

In wavelet analysis, the Discrete Wavelet Transform (DWT) decomposes a signal into a set of mutually orthogonal wavelet basis functions, which differ from sinusoidal basis functions in that they are spatially localized. Furthermore, wavelet functions are dilated, translated and scaled versions of a common function φ, known as the mother wavelet. The DWT is equivalent to a tree-structured discrete filter bank where the signal is first filtered by a lowpass and a highpass filter to yield lowpass and highpass subbands, approximation and detail components (A, D) as presented on Fig. 1.

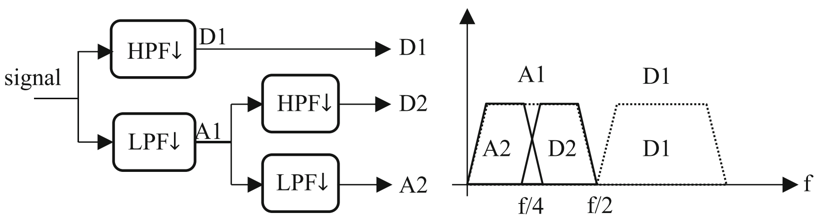

Fig. 2. Illustration of DWT implementation through filter bank.

The lowpass subband is iteratively filtered by the same scheme to yield narrower octave-band lowpass and highpass subbands [8]. In the DWT, the filter outputs are downsampled at each successive stage to eliminate the redundancy, keeping the same length of decomposition, so it returns a data vector of the same length as the input is.

Figure 2 shows that subsequent levels of the DWT operate only on the outputs of the lowpass (scaling) filter. At each level, the DWT divides the signal into octave bands [9].

2.3 Wavelet Packet Transform (WPT)

The wavelet packet transform (WPT) differ from the discrete wavelet transform (DWT) because the transform yields in equal-width subband filtering of signals which is opposed to the coarser octave band filtering obtained in the DWT. WPT is superior at time-frequency analysis because divides the frequency axis into finer intervals (Fig. 3).

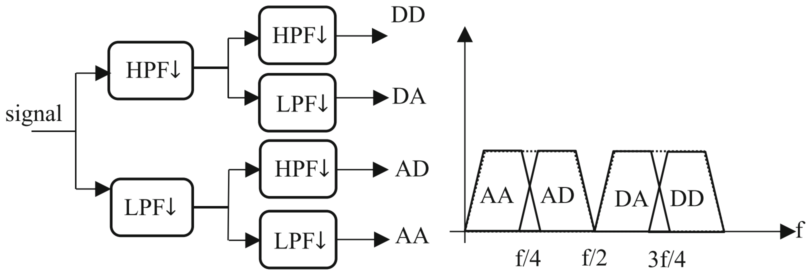

Fig. 3. Illustration of WPT.

Both of lowpass and highpass subbands are iteratively filtered by the same scheme to yield narrower octave-band lowpass and highpass subbands, obtaining detail of approximation and approximation of detail coefficients (AA, AD, DA, DD). In the non-redundant WPT the outputs of the bandpass filters are downsampled by two as in case of DWT [10].

In addition to filtering a signal into equal-width subbands at each level, the WPT partitions the signal's energy among the subbands, providing superior frequency resolution. As a result, the WPT can separate nearby frequency components which usually fall in the same octave band in case of DWT.

The advantage of DWPT is that it is possible to select the optimal representation of the signal with respect to some criterion.

The WPT best representation of a signal is of three types: it may be the DWT representation (included in the WPT scheme); it may be the input signal itself; or it may be a representation that is better than that of the input or DWT [11].

3 Materials and Methods

3.1 The Filtering Procedure

This study uses three types of test signals, three filtering procedures are compared, these can be observed on Fig. 4.

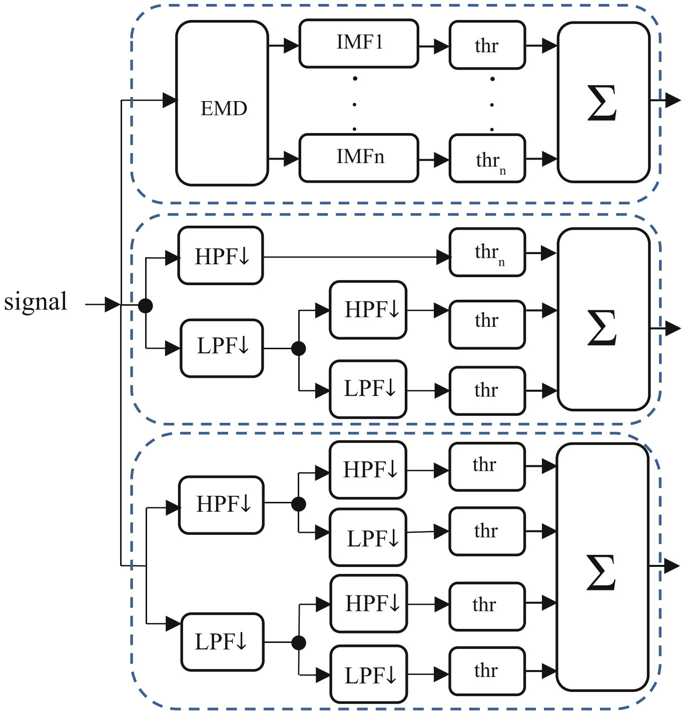

Fig. 4. The proposed filtering methods.

The all three filtering methods use threshold for components, the signal is reconstructed from the truncated components. For every filtering the same thresholds are applied to see the difference between obtained parameters.

The procedures are compared through signal to noise ratio (SNR) and reconstruction error. The power of signal $x = [x_i]$ and noise $n = [n_i]$ are defined as

$$\mathrm{P_S} = \frac{1}{\mathrm{N}} \sum\nolimits_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{x_i^2}, \qquad \mathrm{P_n} = \frac{1}{\mathrm{N}} \sum\nolimits_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{n_i^2} \tag{7}$$

The initial signal to noise ratio SNR_i (with known noise) and the obtained signal to noise ratio SNR (where x_f is the filtered signal) are:

$$SNR_i = 10 \cdot lg\left({}^{P_S}/_{P_N}\right) \tag{8}$$

$$P_{Sf} = \frac{1}{N} \sum\nolimits_{i=1}^{N} (x_{fi})^2 \tag{9}$$

$$P_{ne} = \frac{1}{N} \sum\nolimits_{i=1}^{N} (x_{ni} - x_{fi})^2 \tag{10}$$

$$SNR = 10lg\left(\frac{P_{Sf}}{P_{ne}}\right) \tag{11}$$

where N is the length of the signal and the estimated noise is the difference between filtered and nonfiltered signal $n_e = x_n - x_f$. The reconstruction error is the absolute value of the difference between the filtered and original signal.

3.2 Test Signals

This study uses three types of test signals to compare the filtering results. These signals are of different type and different lengths, all of them having special features.

The three test signals (original and noisy) are presented on Fig. 5.

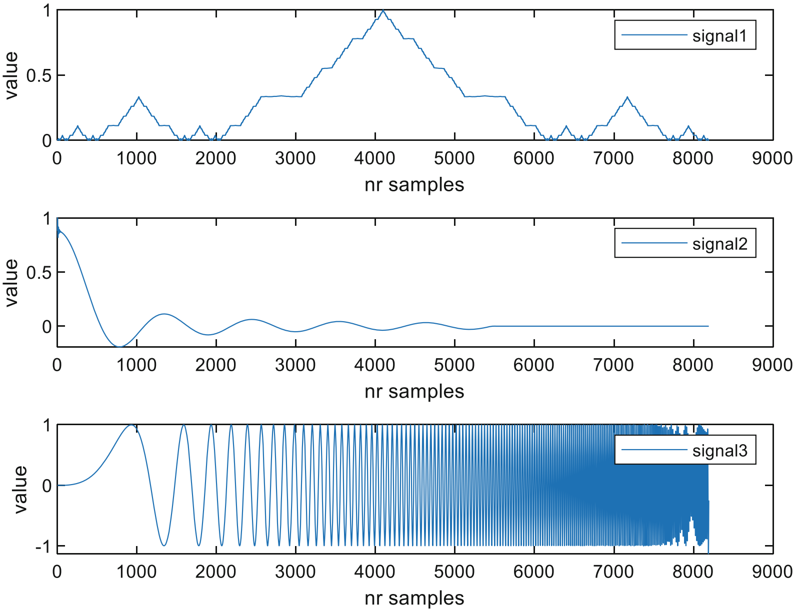

Fig. 5. The test signals.

To estimate the filtering results, different levels of Gaussian white noise were added to the signal, see Fig. 6. The noisy signal is taken as a sum between signal and noise, this study uses six level of added noise. Previously, the signals were preprocessed; resampled to have the same length and normed to have the same level (in this case the added noise level is the percentual the same). The length of the signal is 8192 (a power of two, usually, in this case the numerical procedures product better results).

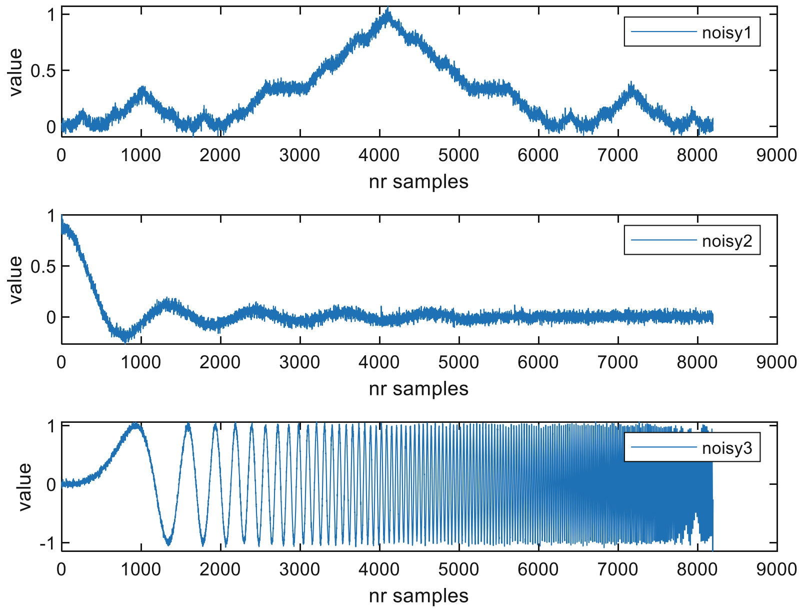

Fig. 6. The test signals with added Gaussian white noise.

4 Experimental Results

This study uses signals of equal lengths, the noise is added in six steps, the level is 1%–6% of signals maximum value (Fig. 7).

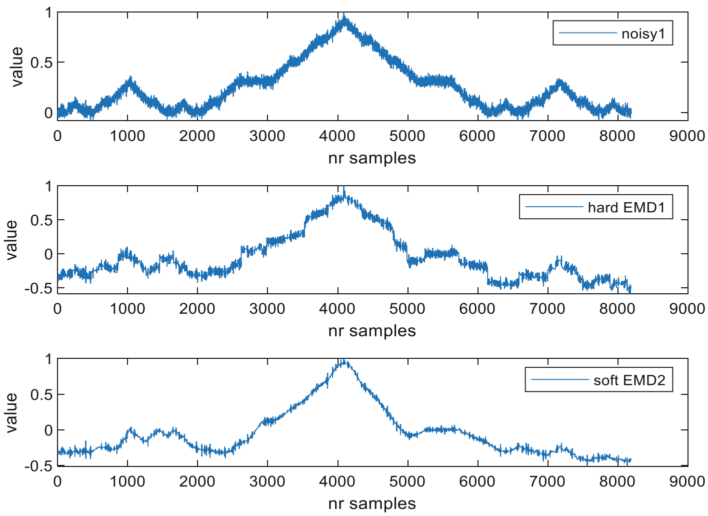

Fig. 7. The EMD filtering results on testsignal1 (level noise 5).

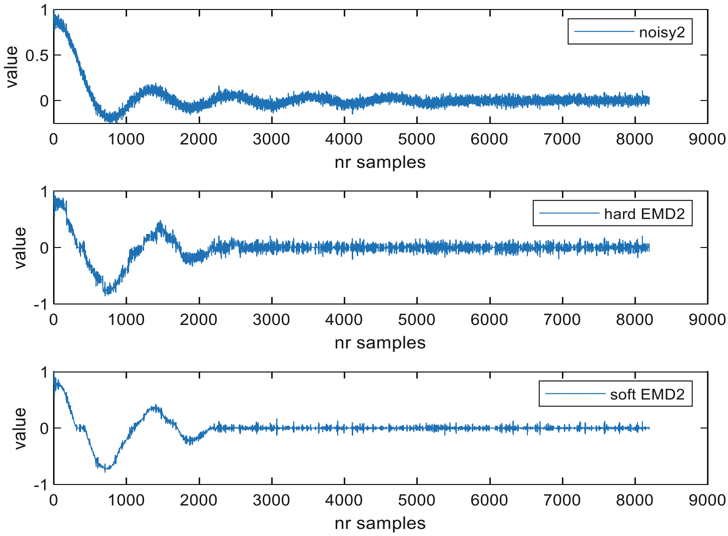

Fig. 8. The EMD filtering results on test signal 2.

The thresholding was made according to signal level in every subband, the threshold values were estimated through the same rules (based on the principle of Stein's Unbiased Risk). Both of soft and hard thresholding procedures were applied, the results can be seen on Fig. 8.

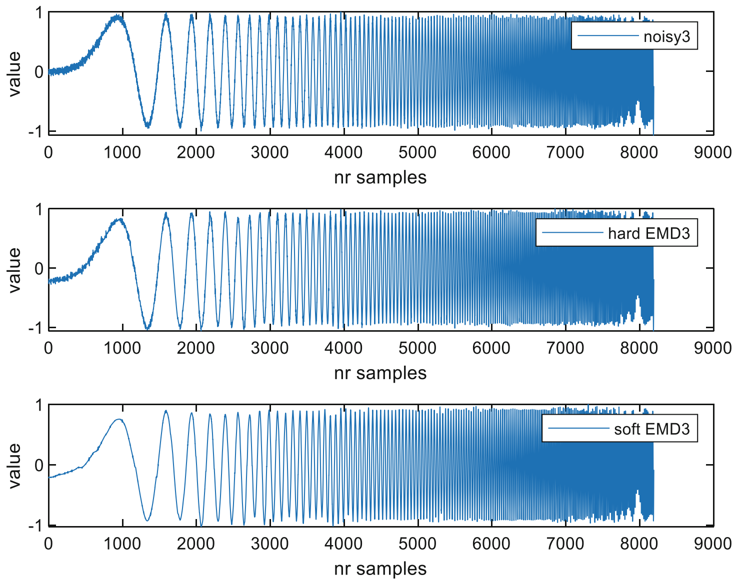

Fig. 9. The EMD filtering results on testsignal3 (noise level 5).

The next table contains the measured parameter obtained through the EMD based filtering procedure (Table 1).

Table 1. The parameters obtained for EMD based filtering results.

n	SNRi1	SNR1	e1	SNRi2	SNR2	e2	SNRi3	SNR3	e3
1	31.510	0.954	0.316	24.009	3.723	0.050	36.535	14.423	0.087
2	25.683	0.765	0.311	18.182	4.130	0.049	30.708	14.691	0.105
3	22.089	0.497	0.339	14.588	5.794	0.054	27.114	13.742	0.093
4	19.511	0.792	0.308	12.010	5.035	0.065	24.536	11.667	0.128
5	17.670	0.678	0.315	10.169	4.117	0.079	22.695	12.634	0.111
6	16.109	0.876	0.297	8.6089	4.839	0.064	21.134	13.448	0.137
7	14.759	0.234	0.368	7.2582	4.121	0.094	19.784	12.940	0.135

The noise was added in seven levels, the obtained signal to noise ratio is strongly dependent on signal features. Figure 9 presents the variation of reconstruction errors, which is taken as the absolute value of the difference between original and filtered signal. The first signal, due to his properties has the greatest reconstruction error. Figure 10 brings the values of obtained signal to noise ratios versus the initial ones.

Fig. 10. The reconstruction errors in case of EMD based filtering.

Table 2 contains the initial and the obtained signal to noise ratio values from DWT and WPT filtering and the reconstruction errors of these procedures.

The noise level in this case was set between 1% and 6% of the normed signals value.

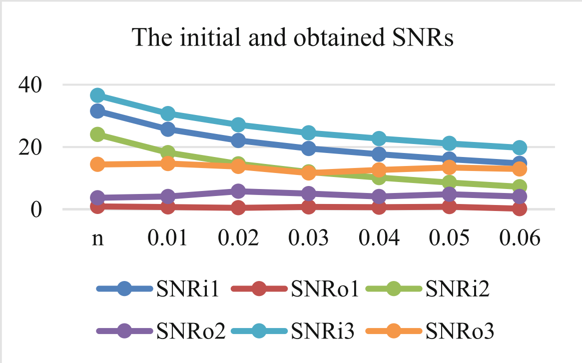

Fig. 11. The initial and obtained signal to ratios in case of EMD based filtering.

Table 2. The obtained parameters in case of DWT and WPT based filtering.

Noise [%]	1	2	3	4	5	6
SNRI1	31.5162	25.6272	21.9741	19.4939	17.6737	16.1093
SNRW1	31.6974	25.8218	22.3774	19.7786	17.751	16.3392
SNRWP1	31.6136	25.8589	22.2189	19.7273	17.9487	16.4154
eW1	0.0025	0.004	0.0055	0.0061	0.0067	0.0084
eWP1	0.0027	0.0044	0.0063	0.008	0.0101	0.012

The used wavelet function and the decomposition level were the same for both DWT and WPT transform based filtering. The threshold selection rules were the same as in case of Ifs in EMD decomposition. Figure 11 presents the DWT and WPT based filtering results for the test signals (Fig. 12).

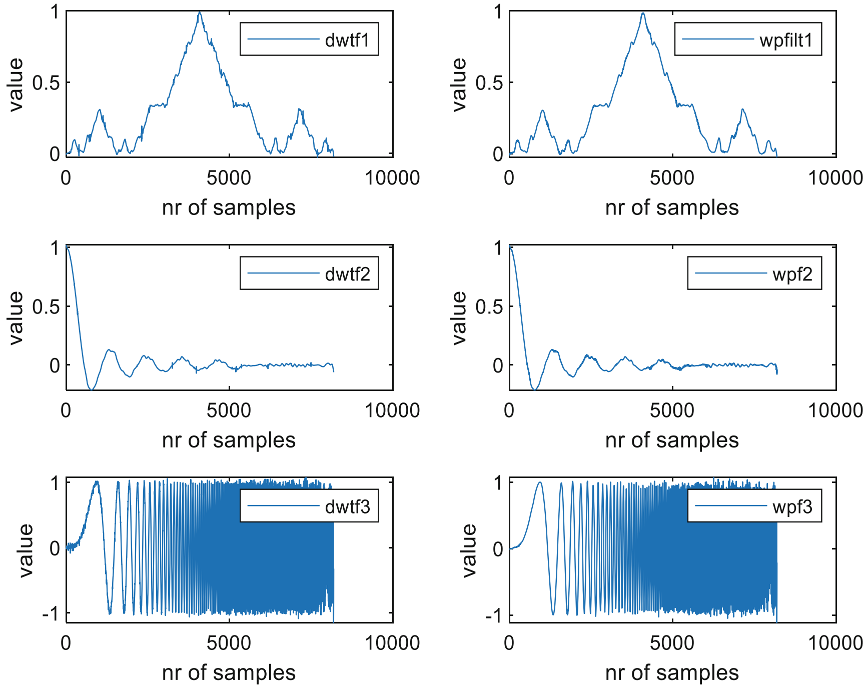

Fig. 12. The DWT and WPT based filtering results.

The resulted signal to noise ratios after the DWT and WPT filtering are illustrated on Fig. 13. The results show that this kind of filtering produces good results due to the decomposition and thresholding in subbands.

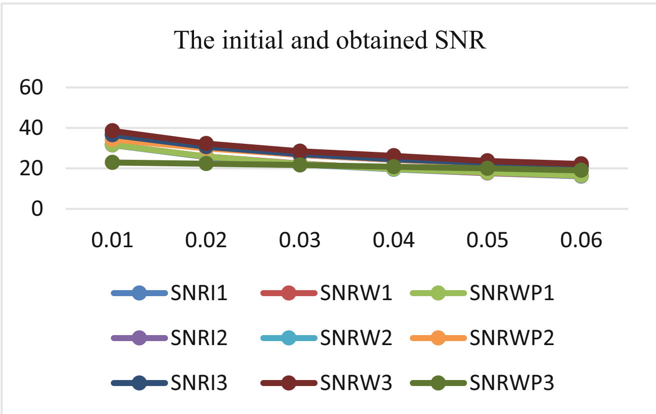

Fig. 13. The initial and obtained SNR in case of DWT and WPT filtering

The reconstruction error representation completes the filtering evaluation, good obtained SNR does not means always a good noise cancellation, the shape of filtered signal must be as close as possible to the original, other way the filtering has no sense (Fig. 14).

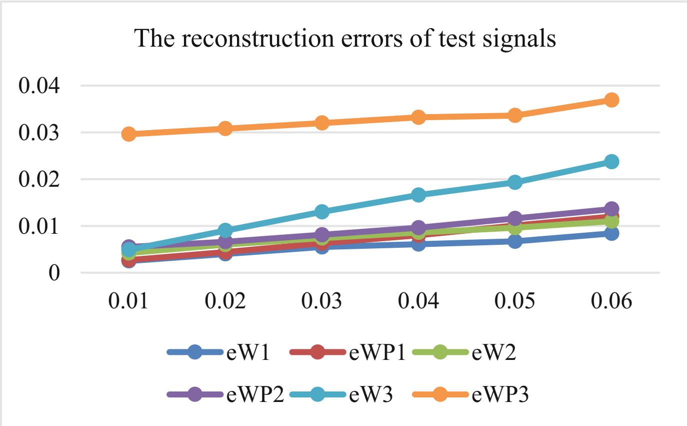

Fig. 14. The comparison between filtered and original test signals

5 Conclusions

The aim of this study is to compare different methods in nonstationary signal filtering, preserving as much as possible the given signal parameters. The main parameters in examining the methods were the obtained signal to noise ratio and the reconstruction error. Experimental results demonstrate that the DWT and WPT subband filtering were more efficient due to the accurate decomposition in subbands. The advantage of EMD is that there is no need of basic functions, the decomposition is signal driven without involving external function. To improve the quality of EMD based filtering, a better thresholding rule must be applied to the intrinsic mode functions.

References

1. Eden, U.T.: Point process adaptive filters for neural data analysis: theory and applications. In: 46th IEEE Conference on Decision and Control, pp. 5818–5825 (2007)
2. Brigo, D., Hanzon, B.: On some filtering problems arising in mathematical finance. Insurance Math. Econom. **22**, 53–64 (1998)
3. Evensen, G.: Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics. J. Geophys. Res. **99**, 10143 (1994)
4. Hassani, M., Karami, M.-R.: Improved nonlinear noise estimation in EEG signal based on Volterra filters. Biomed. Eng.: Appl. Basis Commun. **29**(04), 1750026 (2017)

5. Komaty, A., Boudraa, A.O., Augier, B., et al.: EMD-based filtering using similarity measure between probability density functions of IMFs. IEEE Trans. Instrum. Meas. **63**(1), 27–34 (2014)
6. Huang, N.E., et al.: The empirical mode decomposition and the Hilbert spectrum for non-linear and non stationary time series analysis. Proc. R. Soc. London A **454**, 903–995 (1998)
7. Wickerhauser, M.V.: Adapted Wavelet Analysis from Theory to Software. A.K. Peters, Wellesley (1994)
8. Percival, D.B., Walden, A.T.: Wavelet methods for time series analysis. In: Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge (2000)
9. Graps, A.: An introduction to wavelets. IEEE Comput. Sci. Eng. **2**(2), 50–61 (1995)
10. Bultheel, A.: Learning to swim in a sea of wavelets. Bull. Belg. Math. Soc. Simon Stevin **2**, 1–45 (1995). https://doi.org/10.36045/bbms/1103408773
11. Donoho, D.L.: De-noising by soft-thresholding. IEEE Trans. Inf. Theory **42**(3), 613–627 (1995)