Instantaneous Frequency Identification in Nonstationary Signals

Zoltán Germán-Salló(✉)

University of Medicine, Pharmacy, Sciences and Technology "George Emil Palade", Targu-Mures, Romania

zoltan.german-sallo@umfst.ro

Abstract. This paper presents a comparison study of different instantaneous frequency (IF) estimation procedures. These procedures use different types of synthesized test signals, the instantaneous frequencies of these signals are estimated through three procedures, one based on Hilbert Transform (HT) and the other two on time frequency distribution, Short Time Fourier Transform (STFT) provided power spectrum density and Fourier analysis-based spectrogram. The three different signals and the three different estimation procedures gave a comprehensive view over the instantaneous frequency interpretation as the derivative of instantaneous phase. The obtained experimental results from different procedures are compared and analyzed, conclusions that were drawn are presented.

Keywords: Instantaneous frequency · Time–frequency distribution · Analytic signal · Hilbert transformation

1 Introduction

The analysis of nonstationary signals, in order to reveal their hidden properties in time and frequency domains, became a very important task in signal processing. Neither time analysis nor frequency investigation method alone can completely describe or model the nonstationary characteristics of a given signal.

Non-stationary oscillatory signals occur in a wide range of fields, including physics, biology, medicine, electronic engineering and many others. They contain different oscillatory components, which usually reflect properties of the underlying system. In most of situations, in order to extract information from an observed or acquired signal is important to know the properties of the different constituents of the signal. Signal analysis usually means noise elimination from signal (filtering), identification and isolation of individual components in order to contour and evaluate the underlying rules that control the system.

The frequency spectrum-based analysis tools introduced by the Fourier Transform (FT), have been intensely used in systems showing periodic and quasi-periodic behaviors, including oscillations and vibrations. Spectral analysis has been applied to solve various problems in different fields, including mechanical vibrations and fault detection and monitoring, speech and music recognition, telecommunications, power systems and

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medical engineering. The nonstationary nature of the signals and nonlinear systems require the idea of instantaneous frequency (IF). The IF is the basis of the time–frequency distributions (TFD) or time–frequency-energy (TFE) representation and analysis of a signal. The IF is a practically important parameter of a signal which provide explanations for physical phenomenon in many applications.

The spectral methods were improved by D. Gábor [1], who introduced short time, or windowed Fourier transforms in order to help time–frequency analysis and eventually contribute to the development of wavelet theory. Gabor and wavelet transforms provide rich visualizations of time–frequency representations with spectrograms and scalograms respectively [2].

This paper presents instantaneous frequency determination procedures applied to synthetic test signals and a comparison study between them in order to figure out how signal properties and estimation methods are or could be correlated [3].

2 Instantaneous Frequency Estimation Fundamentals

Recently, a variety of approaches to the instantaneous frequency and phase estimation problem, have been developed. These approaches differ by estimation precision, computational resources complexity, and processing duration.

The instantaneous frequency of a nonstationary signal is a time-dependent parameter that reflects the mean value of the frequency components present in the time-varying signal. There are many procedures developed and presented in literature.

Trying to find a valuable definition of instantaneous frequency (IF), Carson and Fry in 1937 [4] arguing that the notion of IF is a generalization of the definition of constant frequency (i.e., it is the rate of change of phase angle at time t), they considered a variable frequency in an electric circuit theory context and then applied the concept to frequency modulated (FM) signals. They defined an FM wave as

$$\omega(t) = exp\left[j\left(\omega_0 t + \lambda \int_0^t m(t)dt\right)\right] \tag{1}$$

where $\omega_0 = 2\pi f_0, f_0$ is a constant carrier frequency, λ is the modulation index and $m(t)$ represents a low-frequency signal to be transmitted ($|m(t) \leq 1|$). They defined the instantaneous angular frequency as

$$\Omega(t) = \omega_0 + \lambda m(t) \tag{2}$$

where m(t) has the dimension of frequency, and the instantaneous cyclic frequency as

$$f_i(t) = f_0 + \frac{\lambda}{2\pi} m(t) \tag{3}$$

The next important step in the study of IF was made by D. Gábor [5] who proposed a method for generating a unique complex signal from a real one. His method for doing so is first to find the FT of the real signal and then to “suppress the amplitudes belonging to negative frequencies and multiply the amplitudes of positive frequencies by two." He also showed that this procedure is equivalent to the following time-domain procedure:

$$z(t) = s(t) + jH[s(t)] = a(t) \cdot e^{j\Phi(t)} \tag{4}$$

where $z(t)$ is a complex signal, introduced by D. Gábor, also named as analytic signal, $s(t)$ is the real signal and H is the Hilbert Transform (HT), defined as a convolution between the real signal $s(t)$ and $\frac{1}{\pi t}$:

$$H[s(t)] = pv \cdot \int_{-\infty}^{+\infty} \frac{s(t-\tau)}{\pi t} d\tau \tag{5}$$

where pv is the Cauchy principle value of the integral [6]. Gabor's motivation to define and work with a complex signal was that only by doing so was he able to define the central moments of frequency of the signal:

$$\langle f^n \rangle = \frac{\int_{-\infty}^{+\infty} f^n \cdot |Z(f)|^2 df}{\int_{-\infty}^{+\infty} |Z(f)|^2 df} \tag{6}$$

where $Z(f)$ is the spectrum of the complex signal [6]. Ville [7] unified the work done by Carson and Fry and Gabor and defined the IF of a signal expressed by $s(t) = a(t) \cdot cos\Phi(t)$

$$f_i(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}\left[arg(z(t))\right] \tag{7}$$

where $z(t)$ is the analytic signal defined previously in Eq. (4). Ville went further and noted that since the IF was time-varying, there should intuitively be some instantaneous spectrum associated with it-with the mean value of the frequencies in this instantaneous spectrum being the IF. Using Gabor's average measures [8], he showed that the average frequency in a signal's spectrum was equal to the time average of the IF [9]:

$$\langle f \rangle = \langle f_i \rangle \tag{8}$$

where

$$\langle f \rangle = \frac{\int_{-\infty}^{+\infty} f \cdot |Z(f)|^2 df}{\int_{-\infty}^{+\infty} |Z(f)|^2 df} \tag{9}$$

$$\langle f_i \rangle = \frac{\int_{-\infty}^{+\infty} f_i \cdot |z(t)|^2 dt}{\int_{-\infty}^{+\infty} |x(t)|^2 dt} \tag{10}$$

Time–frequency moments provide an efficient way to characterize signals whose frequencies change in time (having also a sssnonstationary spectrum).

The Fourier transform (FT) is very good at identifying frequency components present in a signal. However, the FT does not identify when the frequency components occur [10].

Only the time–frequency distributions generated by short-time Fourier transform (STFT) or other time–frequency analysis techniques can observe the time varying behavior [11]. Usually, these techniques require serious computational resources. In contrast, transforming the time–frequency distribution results into low-dimension time–frequency moments provide a method for capturing essential features of the signal [12].

3 Test Signals and the Proposed Procedure

This study uses three types of test signals to compare the instantaneous frequency estimation procedures. All these signals were selected and generated in order to have specific and various time varying frequency components.

The first is a Bessel function-based signal and it consists of a set of pulses of decreasing duration separated by regions of oscillating amplitude and fluctuating frequency with an increasing trend. The second is a quadratic chirp sampled at 1 kHz for 4 s which instantaneous frequency is 0 Hz at $t = 0$ and crosses 20 Hz at $t = 2$ s and the third is quadratic chirp modulated by a Gaussian with sample rate of 2 kHz and a duration of 4 s. The three test signals are presented on Fig. 1.

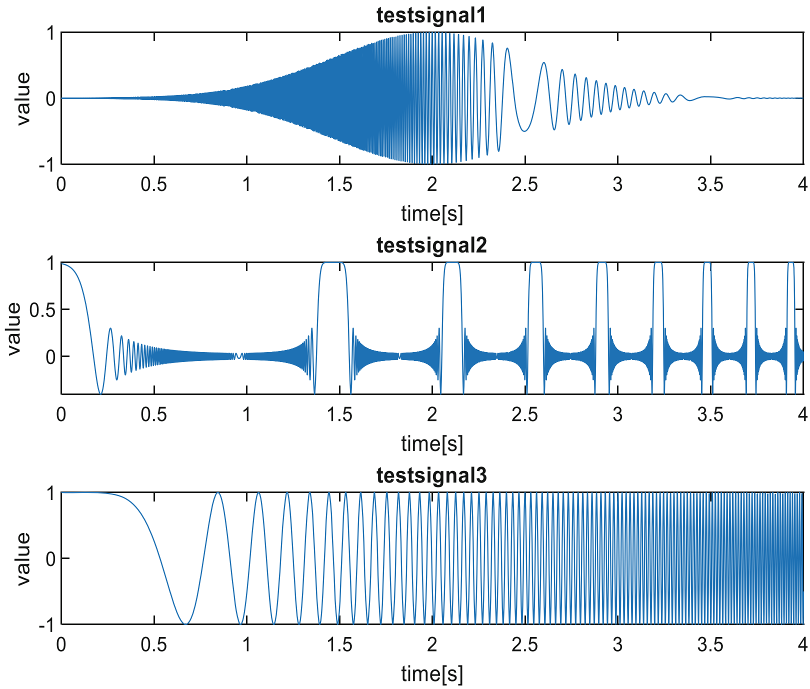

Fig. 1. The used test signals

The signals were resampled in order to have the same length.

Three IF estimation procedures are compared, their schematic representations are illustrated on Fig. 2.

The first estimation computes the instantaneous frequency as the derivative of the phase of the analytic signal using the Hilbert transform. This method accepts only uniformly sampled, real-valued signals.

The second method computes the instantaneous frequency as the first conditional spectral moment of the time–frequency distribution of the signal.

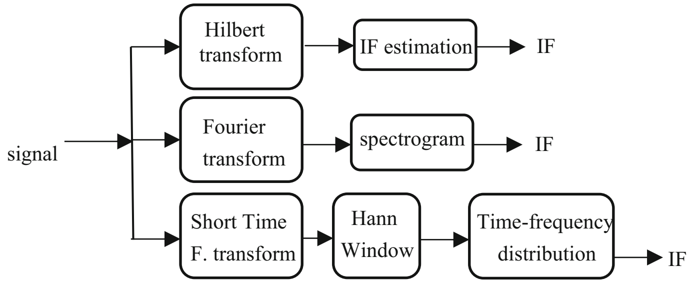

Fig. 2. The proposed IF estimation procedures

The spectrogram which is the visual representation of the spectrum of frequencies of the signal as it changes with time can be generated using several ways, which include Fourier transform, wavelet transform and band-pass filter.

The Short Time Fourier Transform (STFT) serves as time–frequency distribution. By default, the signal is divided into eight segments with 50% overlap, and each segment is windowed with a Hamming window.

The number of frequency points used to calculate the discrete Fourier transforms is 256 or the next power of two greater than the segment length. If the signal cannot be divided exactly into eight segments, it will be truncated.

The third approach uses a the synchrosqueezed Fourier transform, a post-processing method which circumvents the uncertainty relations, inherent to these linear transforms, by reassigning the coefficients in scale or frequency. A 500-sample Hann window is used to divide the signal into segments and window them. From this the spectrogram is estimated and IF computed.

Figure 3,4,5,6 present the obtained instantaneous frequencies for testsignals 1, 2, 3 through the three presented procedures.

Figure 3 allows a good visual verification of the instantaneous frequency both the first image (Hilbert method) and second image (time/frequency distribution from Fourier transform based spectrogram) presents oscillation at the beginning because the signal is truncated to the next power of two greater than the segment length.

All the applied methods produce the same variations in this case.

The estimation of IF for the second signal produces slightly individually results, the fast and irregular changes in the frequency are interpreted in different ways.

As it can be observed, in case of the first test signal, the spectrogram based method produces discontinuities in IF estimation at the final of sequence due to the coefficient truncation.

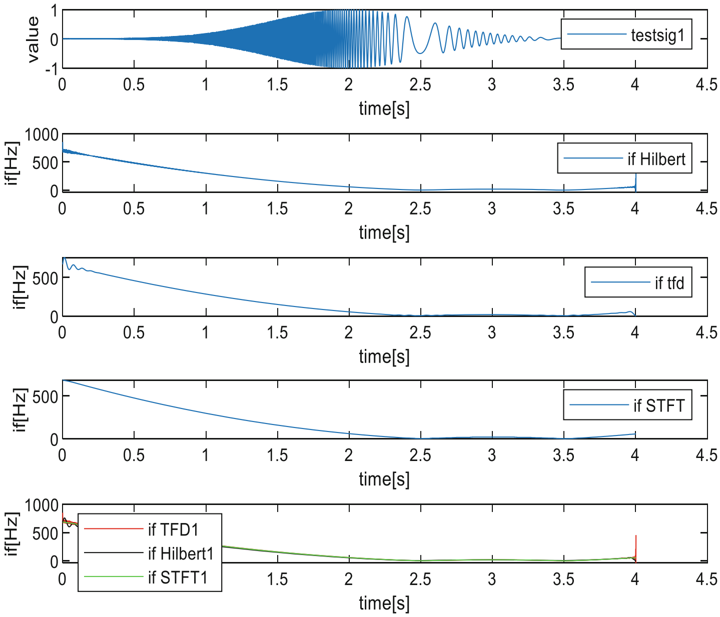

Fig. 3. The obtained Ifs for test signal 1

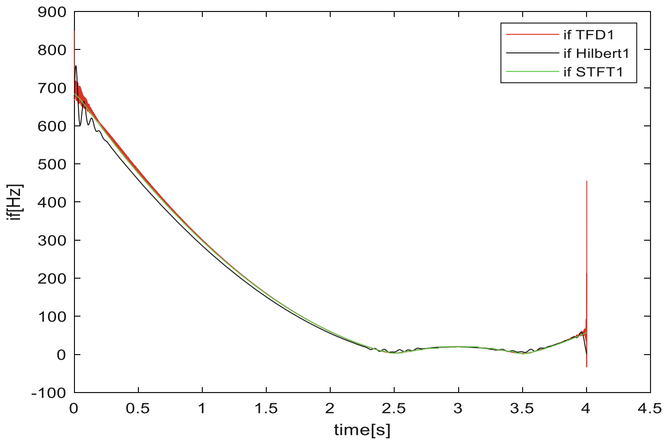

Fig. 4. The discontinuities in IF estimation

The discontinuities at the final of estimated IF are represented more accurately on Fig. 4.

As a comparison study, Fig. 7 presents the three estimations obtained for the first test signal.

In case of the second test signal the time–frequency distribution based method presents slightly differences due to the rescaling in the FFT algorithm (Fig. 8).

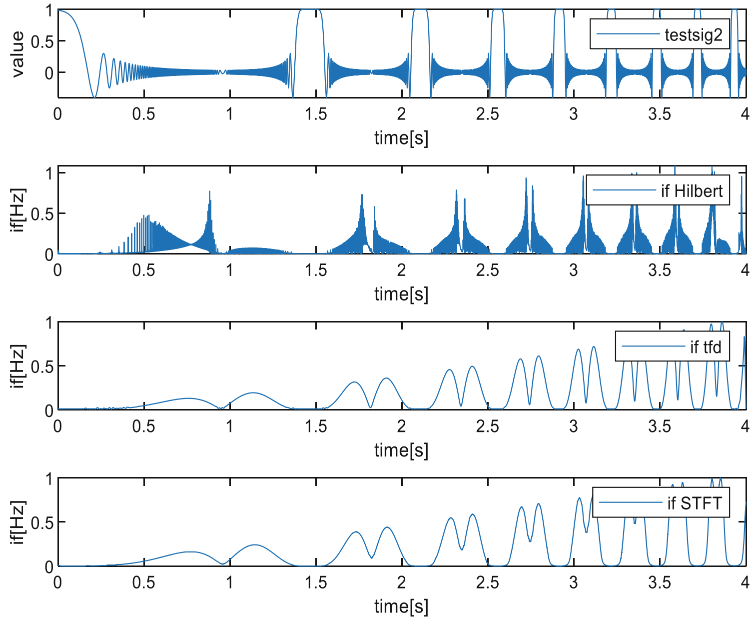

Fig. 5. The obtained Ifs for test signal 2

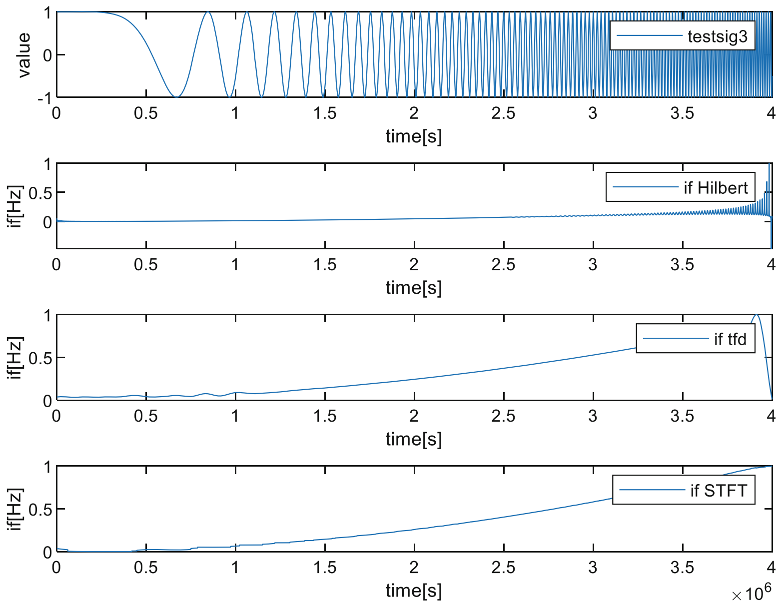

Fig. 6. The obtained Ifs for test signal 3

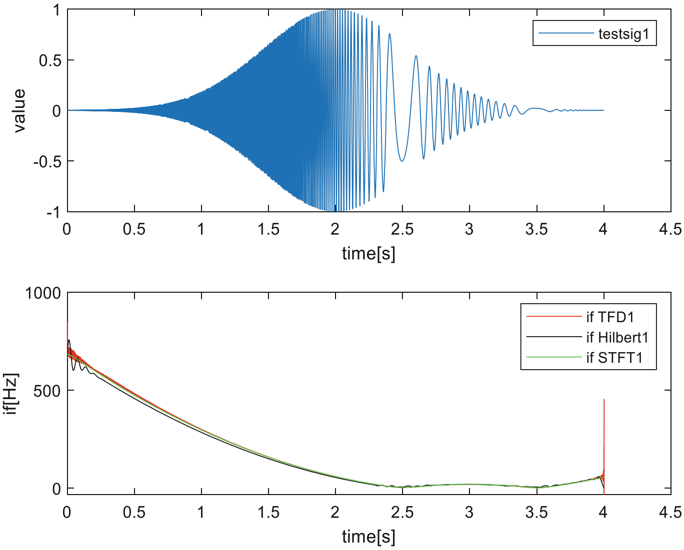

Fig. 7. The comparison study for the first test signal

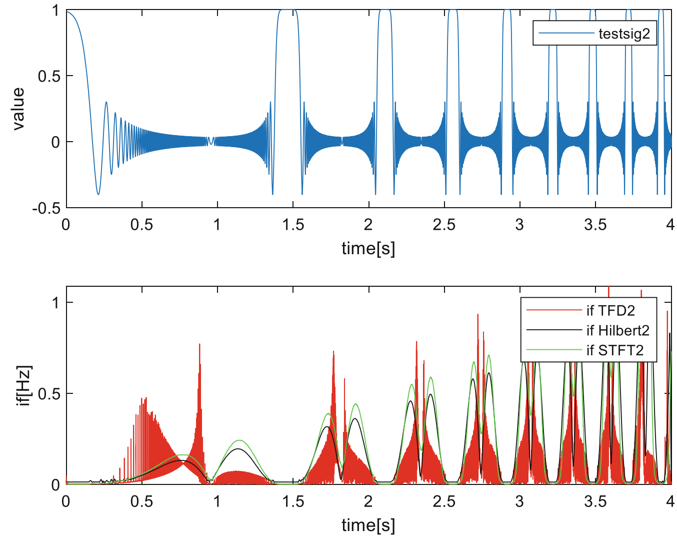

Fig. 8 The comparison study for the second test signal

The third test signal has the estimation results presented on Fig. 9.

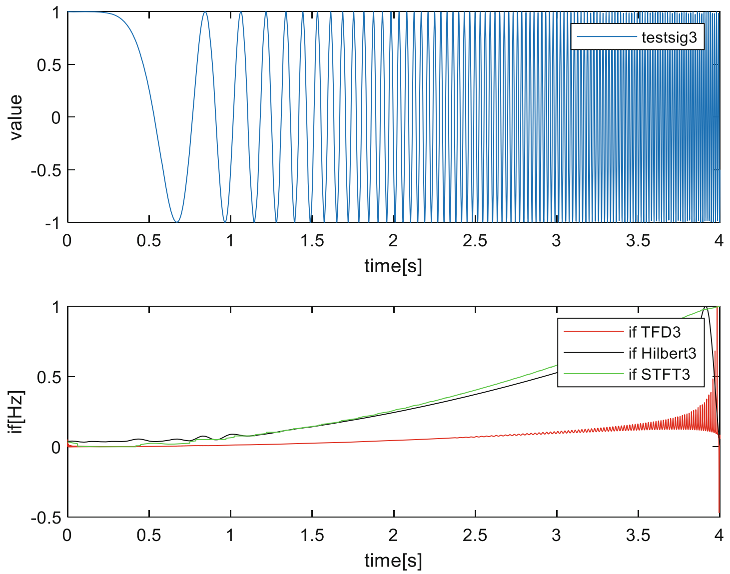

Fig. 9 The instantaneous frequency estimation of the third test signal

4 Conclusions

Instantaneous frequency, taken as the derivative of the phase of the signal, is interpreted in the time–frequency literature as the average frequency of the signal at each time.

In present study three synthesized signals were analyzed, their instantaneous frequencies were estimated in three different ways. From spectral behavior point of view, the synthetic signals have special properties, nonstationary frequency variations and amplitude changes. These test signals were generated to have specific but different properties in time and frequency domains, the applied estimation methods also were different, the results show the advantages and drawbacks of these.

The Hilbert transform estimates the instantaneous frequency of a signal for monocomponent signals only, otherwise is not possible to form the analytic signal.

The Short Time Fourier Transform based spectrogram offers a good time–frequency distribution which depends on sequence length and window type but due to the truncation can appear oscillatory errors. The length of signal is important, because on it depends the number of subbands.

The best results are obtained with the synchrosqueezed Fourier transform, where the reassignment of coefficients in frequency and the windowing smooths the spectrum. provides a sharp, concentrated representation, while remaining invertible. This method received a renewed interest and provides guarantees for the decomposition of a multicomponent signal.

It is obvious from the obtained results that different methods are producing the different time–frequency distributions, but frequencies present in the Fourier spectrum are not always present all the time in a signal. In conclusion, the time–frequency representation depends on the number of bands in which data has been divided. as the signal is decomposed into more narrow bands, true frequencies present in the signal under analysis are revealed, frequency resolution is also increasing.

As a final concluding remark, the IF estimation depends strongly on the analyzed signal, modifying the signal length through truncation to a next power of two, also can have unwanted effects.

As further work, the IF estimation methods could be extended on noisy signals. The relationship between noise level and estimation precision, could be an important task, mostly in biomedical application. The information from a heart rate variability signal spectrum could be extended with the instantaneous frequency information, a better classification of frequency domains would be obtained.

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