Optimization of Joint Power and Bandwidth Allocation for Multiple Users in a Multi-spot-Beam Satellite Communication

Heng Wang, Shijun Xie, Ganhua Ye(✉), and Bin Zhou

The 63rd Research Institute of National University of Defense Technology, Nanjing, China

Abstract. Multi-spot-beam techniques have been widely applied in modern satellite communication systems, due to the advantages of reusing the frequency of different spot beams and constructing flexible service networks. As the on-board resources of bandwidth and power in a multi-spot-system are scarce, it is important to enhance the resource utilization efficiency. To this end, this paper initially presents the formulation of the problem of joint power and bandwidth allocation for multi-users, and demonstrates that the problem is one of convex minimization. An algorithm based on the Karush-Kuhn-Tucker (KKT) conditions is then proposed to obtain an optimal solution of the problem. Compared with existing separate power or bandwidth algorithms, the proposed joint allocation algorithm improves the total system capacity and the fairness between users. A suboptimal algorithm is also proposed, to further reduce computational complexity, with a performance level much closer to that of the optimal allocation algorithm.

Keywords: Multi-spot-beam satellite communication system · Joint bandwidth and power allocation · Convex optimization · Optimal allocation algorithm · Low computational complexity allocation algorithm

1 Introduction

In recent years, as an important component of internet, satellite communication has played a key role in seamless internet access. In a modern satellite communication system, the satellite has multiple-spot-beams, each one of which covers different areas of the earth. Thus the multi-spot-beam system can reuse the frequencies of the different spot beams, to significantly increase the total system capacity. In addition, the system can provide high power density to a particular spot beam, by allocating more resources to it, thereby supporting high traffic rates to small antenna terminals [1].

However, the on-board resources of bandwidth and power are scarce and expensive in multi-spot-beam satellite systems. As a result, it is crucial for us to improve the resource utilization efficiency. To this end, dynamically allocating these resources to each user according to their traffic demands is a viable solution.

In previous works, separate optimal power or bandwidth allocation for spot beams have been investigated by [1–4] and [5]. In these works, the metric to evaluate the

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system performance is to minimize the deficit between the traffic demand and the capacity allocated, taking into account a compromise between the total system capacity and the proportional fairness among spot beams. It was proved that it is need to allocate more resource to the spot beam with higher traffic demand to get fairness between among spot beam, thus the total system capacity decreased, due to concavity of the capacity function with a fixed power or bandwidth allocation. To overcome this drawback, in this paper we propose a joint bandwidth and power allocation algorithm. Moreover, we propose solving the problem of joint bandwidth and power allocation for multiple users in each spot beam. As a result, the constraints and complexity are greater than for those problems mentioned in the above referenced works.

In [6], the joint bandwidth and power allocation of downlink transmissions were investigated. The object of the optimization problem was to maximize the system capacity and fairness between each link. Fairness was achieved by assigning different weights to different links, which were the reciprocals of the average long term rates. However, the author only solved for a maximum of two-user allocation simultaneously, based on the Concave Envelope Theorem, and ignored larger simultaneous multi-user allocations. In [7], the optimal joint bandwidth and power allocation in wireless, multi-user networks, both with and without relays, was proposed. The author focused on a scenario in which a source served multi-users with different channel conditions, simultaneously. The optimization objective was to maximize the total system capacity. However, the author failed to consider the traffic demands of each user. The results showed that for a set of users served by one source, all the power from that source was allocated only to the user having the highest channel gain. It is obvious that the conclusion is questionable, when the traffic demands of the user with the highest channel gain does not exceed the source capacity. In [8], a joint power and bandwidth allocation algorithm with Quality of Service (QoS) support in heterogeneous wireless networks was proposed, using convex optimization methodology. The terminal was supported to access different wireless networks simultaneously, and the objective of the convex optimization was to maximize the system capacity without regard for the fairness amongst the users. However, the conclusions obtained in this work cannot be applied to our system, because the users cannot access different spot beams in parallel, in multi-spot-beam satellite systems.

In this paper, the objective is to solve for the optimal joint bandwidths and power allocation for users in a multi-spot-beam Satellite System. We initially formulate the problem of joint power and bandwidth allocation for users as a nonlinear optimization problem, and demonstrate that the optimization problem is a convex optimization problem. The object of our optimization is to match the capacity allocated to each user, as closely as possible to the traffic demand, taking into account a compromise between the total system capacity and the proportional fairness between the users. Then we propose an algorithm based on the Karush-Kuhn-Tucker (KKT) conditions to achieve an optimal solution. Compared with the individual power or bandwidth optimal allocation algorithms, the proposed joint bandwidth and power allocation algorithm improves the total system capacity and the fairness between users. A suboptimal algorithm is also proposed, to reduce the computational complexity, the performance of which is much closer to the optimal algorithm.

The remainder of this paper is organized as follows: Sect. 2 formulates the optimization problem of joint bandwidth and power allocation, utilizing a compromise between system capacity maximization and proportional fairness between the users; demonstrating that the optimization problem is a convex minimization type problem. Section 3 proposes an optimal joint bandwidth and power allocation algorithm based on KKT conditions, and a suboptimal algorithm to reduce the computational complexity. Section 4 presents the simulation results and compares the performance of the low computational complexity algorithm with that of the optimal algorithm, and finally, in Sect. 5 the conclusions are presented.

2 System Model

2.1 Downlink Capacity of Users

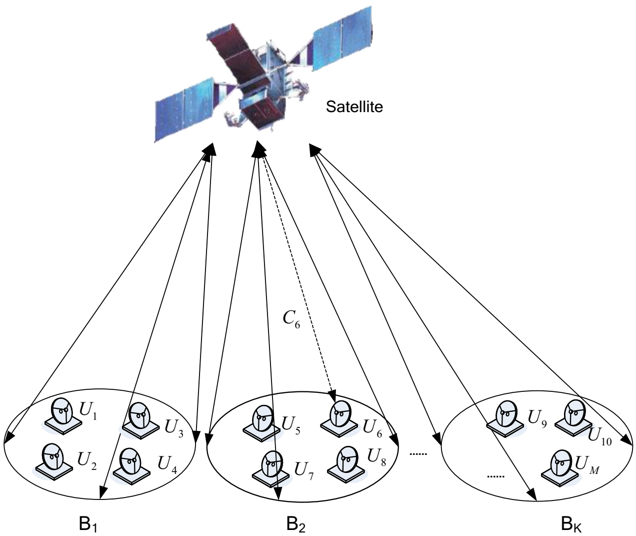

Fig. 1. System configuration of a multi-spot-beam satellite system with multiple users.

The configuration of a multi-beam satellite system with multiple users is shown in Fig. 1. The system consists of K beams B_i, $I \in \{1,\ldots, K\}$, and M users U_i, $I \in \{1,\ldots, M\}$. The set of users which are served by the beam B_i is denoted by $\mathcal{N}_{B_i}$. The traffic demand of the user U_i is T_i, the power and bandwidth allocated to the user U_i are P_i and W_i, and the signal attenuation factor of the user U_i is α_i^2. It is noted that α_i^2 consists mainly of the effects of weather conditions, free space loss and antenna gain. The total power and bandwidths of the system are P_{total} and W_{total}.

Using time sharing for Gaussian broadcast channels [9], we obtain the Shannon bounded capacity C_i for the user U_i as:

$$C_i = W_i \log_2\left(1 + \frac{\alpha_i^2 P_i}{W_i N_0}\right) \quad (1)$$

where N_0 is the noise power density of each user. It is noted that interbeam interference from the sidelobes of adjacent spot beams to degenerate the Shannon capacity. However, in this paper, we ignore interbeam interference, because we consider very narrow spot beams over a large number of spot beams [1]. It is observed from (1) that the user capacity C_i is increased as the bandwidth or power allocated to the user increases. However, the total bandwidth and power of the satellite is fixed, so the capacity of the system is limited.

In the multi-beam satellite system, there is always a power or bandwidth pre-allocation for each beam. Therefore, in this paper we analyze the following four situations: (a) No pre-allocation for any beam. (b) There is only power pre-allocation for each beam. (c) There is only bandwidth pre-allocation for each beam. (d) There are both power and bandwidth pre-allocations for each beam. Let P_{B_i} and W_{B_i} denote the pre-allocated power and bandwidth of the i-th beam.

If the total system resources of power and bandwidth are sufficient to support the traffic demand generated by all the users, it seems meaningless for us to make efforts to improve the resource utilization efficiency. Therefore, we only focus on the resource allocations for scenarios where the total traffic demand exceeded the total available system capacity.

2.2 Optimization Problem Formulation

There are many metrics to evaluate the system performance, and different metrics may lead to different allocation results. Therefore, it is very important to choose an appropriate metric. Motivated by J. P. Choi and V. W. S. Chan [1], in this paper the metric is designed to minimize the deficit between the traffic demand and the capacity allocated, taking into account a compromise between the total system capacity and the proportional fairness between the users. The problem is formulated as follows:

$$\min_{\{P_i\},\{W_i\}} \sum_{i=1}^{M} (T_i - C_i)^2 \quad (2)$$

s.t.

$$C_i = W_i \log_2\left(1 + \frac{\alpha_i^2 P_i}{W_i N_0}\right) \leq T_i, \ \forall i \quad (3)$$

$$\sum_{i=1}^{M} P_i \leq P_{total} \quad (4)$$

$$\sum_{i=1}^{M} W_i \leq W_{total} \quad (5)$$

$$\sum_{i \in \mathcal{N}_{B_j}} P_i \leq P_{B_j} \tag{6}$$

$$\sum_{i \in \mathcal{N}_{B_j}} W_i \leq W_{B_j} \tag{7}$$

The constraint (3) indicates that the allocated resources should not exceed the traffic demands of each spot beam. Conditions (4, 5, 6 and 7) imply the constraints for the total system power, total system bandwidth, and the power and bandwidth for the j-th spot beam, respectively.

As mentioned in Subsect. 2.1, in this paper we analyze four cases. Different cases result in different constraints for the optimization problem. Case (a) does not use constraint numbers (6 and 7). Case (b) does not use constraint number (7). Case (c) does not use constraint number (6). Case (d) uses all four constraints.

Without a loss of generality, we first solve the optimization problem with all four constraints, numbers (4–7). Introducing the non-negative Lagrangian multipliers μ, λ, $\boldsymbol{\rho} = [\rho_1, \rho_2, \ldots, \rho_K]$, and $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \ldots, \sigma_K]$, yielded the Lagrange function, given as:

$$\begin{aligned} L(\mathbf{P}, \mathbf{W}, \boldsymbol{\rho}, \boldsymbol{\sigma}, \lambda, \mu) &= \sum_{i=1}^{M} (T_i - C_i)^2 - \mu\left(W_{total} - \sum_{i=1}^{M} W_i\right) \\ &- \lambda\left(P_{total} - \sum_{i=1}^{M} P_i\right) - \sum_{i=1}^{K} \rho_i \left(P_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} P_j\right) - \sum_{i=1}^{K} \sigma_i \left(W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j\right) \end{aligned} \tag{8}$$

where $\mathbf{P} = [P_1, P_2, \ldots, P_M]$ and $\mathbf{W} = [W_1, W_2, \ldots, W_M]$.

According to the KKT conditions, we obtain the following equations:

$$\frac{\partial L}{\partial P_i} = \frac{2\alpha_i^2 W_i}{(W_i N_0 + \alpha_i^2 P_i)\ln 2}(T_i - C_i) - \lambda - \rho_j = 0,\ i \in \mathcal{N}_{B_j} \tag{9}$$

$$\begin{aligned} \frac{\partial L}{\partial W_i} &= 2(T_i - C_i)\left[\frac{C_i}{W_i} - \frac{W_i P_i}{\ln 2(N_0 W_i^2/\alpha_i^2 + P_i W_i)}\right] \\ &- \mu - \sigma_i = 0 \end{aligned} \tag{10}$$

It is clear from (9) that the non-negative λ and ρ_i means that $T_i \geq C_i$. As a result, constraint number (3) is satisfied.

It is known that when the optimization problem is convex, and a feasible solution satisfies the KKT conditions, then the solution is a global optimal solution to the optimization problem [10]. Fortunately, the optimization problem mentioned above is a convex type, the proof for which is shown in the appendix. Therefore, in the next section we propose an iterative algorithm based on the KKT conditions. Although the optimization problems are different for different cases, the proposed algorithm solves them well within the same architecture.

3 Proposed Joint Bandwidth and Power Allocation Algorithm

3.1 Optimal Allocation Algorithm

When the Lagrangian multiplier variables are given, the optimal P_i is obtained from (9) by numerical calculation methods, e.g., the Golden Section Method. If the optimal $P_i < 0$, then P_i is set to zero.

Substituting the optimal P_i into (8), we obtain the optimal W_i from (10) by using the Golden Section method. Similarly, if the optimal $W_i < 0$, then W_i is set to zero.

Here, we only have one problem to solve, which is how to search the Lagrangian multipliers. Motivated by W. Yu and G. Ding [11, 12], we use the sub-gradient method to update the Lagrangian multipliers, which are obtained according to the following equations:

$$\mu^{n+1} = \left[\mu^n - \Delta_\mu^n\left(W_{total} - \sum_{i=1}^{M} W_i\right)\right]^+ \tag{11}$$

$$\lambda^{n+1} = \left[\lambda^n - \Delta_\lambda^n\left(P_{total} - \sum_{i=1}^{M} P_i\right)\right]^+ \tag{12}$$

$$\rho_i^{n+1} = \left[\rho_i^n - \Delta_\rho^n\left(P_{B_i} - \sum_{j\in\mathcal{N}_{B_i}} P_j\right)\right]^+ \tag{13}$$

$$\sigma_i^{n+1} = \left[\sigma_i^n - \Delta_\sigma^n\left(W_{B_i} - \sum_{j\in\mathcal{N}_{B_i}} W_j\right)\right]^+ \tag{14}$$

where $[x]^+ = \max\{0, x\}$, n is the iteration number and Δ is the iteration step size.

The above sub-gradient update method is guaranteed to converge to the optimal as long as the iteration step chosen is sufficiently small [11–14].

The whole process of the proposed optimal joint bandwidth and power allocation algorithm is summarized as follows:

Step 1. Set appropriate initial values for the Lagrangian multipliers and the bandwidth of each user.
Step 2. Substitute the values of the bandwidth of each user and the Lagrangian multipliers into (9), and then calculate the optimal power allocated to each user.
Step 3. Substitute into (10), both the power values for each user obtained from Step 2 and the Lagrangian multipliers, and then calculate the optimal bandwidth allocated to each user.
Step 4. Substitute the values of the power and the bandwidth of each user, which are separately obtained from Steps 2 and 3, into (11)–(14), and then update the Lagrangian multipliers.

Step 5. If the conditions of $|\mu^{n+1}(W_{total} - \sum_{i=1}^{M} W_i)| < \varepsilon$, $|\lambda^{n+1}(P_{total} - \sum_{i=1}^{M} P_i)| < \varepsilon$, $|\rho_i^{n+1}(P_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} P_j)| < \varepsilon$, $\forall i \in \{1, \ldots, K\}$, and $|\sigma_i^{n+1}(W_{B_i} - \sum_{j \in \mathcal{N}_{B_i}} W_j)| < \varepsilon$, $\forall i \in \{1, \ldots, K\}$ are simultaneously satisfied, terminate the algorithm; otherwise go to Step 2.

According to the above process, it is shown that the computational complexity is $O(4SK + 2SMT)$, where M is the number of the users and K is the number of the spot beams, S is the number of iterations, and T is the computational complexity of the Golden Section method. It is noted that either the S or T are independent of K and N. Therefore, the computational complexity of the proposed algorithm is linear in the number of the spot beams and users.

As mentioned above, different cases result in different optimization problems. For different optimization problems, we only need to remove the corresponding Lagrangian multipliers in (8), for the optimal solution to be obtained by the same algorithm.

3.2 Low Computational Complexity Allocation Algorithm

In this subsection we present a suboptimal algorithm to further reduce the computational complexity. The performance of this algorithm is much closer to that of the optimal algorithm. As the coverage of each spot beam is limited, the channel conditions of the users in the same spot beam are always the same. In such circumstances, the performance is equal to that of the optimal algorithm. The low computational complexity algorithm is based on spreading the spot beam power evenly over the whole spot beam bandwidth for all the users in the same spot beam. Let P_{ai} and W_{ai} denote the powers and bandwidths allocated to the i-th spot beam. The bandwidths and powers allocated to the users in the same spot beam will thus have the following relationship:

$$\frac{P_j}{W_j} = \frac{P_k}{W_k} = \frac{P_{ai}}{W_{ai}}, \quad \forall j, k \in \mathcal{N}_{B_i} \tag{15}$$

As a result, the capacity allocated to the user is given as follows:

$$C_j = W_j \log_2\left(1 + \frac{\alpha_j^2 P_{ai}}{N_0 W_{ai}}\right), \quad \forall j \in \mathcal{N}_{B_i} \tag{16}$$

According to (9), we obtain the following equations for the users in the same spot beam.

$$\frac{T_j - C_j}{T_k - C_k} = \frac{N_0/\alpha_k^2 + P_{ai}/W_{ai}}{N_0/\alpha_j^2 + P_{ai}/W_{ai}}, \forall j, k \in \mathcal{N}_{B_i}, \ j \neq k \tag{17}$$

$$W_k = A_{jk} W_j + B_{jk}, \forall j, k \in \mathcal{N}_{B_i}, j \neq k \tag{18}$$

where: $A_{jk} = \frac{P_{ai}M_j/W_{ai} + N_0 M_j/\alpha_k^2}{P_{ai}M_k/W_{ai} + N_0 M_k/\alpha_j^2}$,
$B_{jk} = \frac{P_{ai}T_k/W_{ai} + N_0 T_k/\alpha_j^2 - P_{ai}T_j/W_{ai} - N_0 T_j/\alpha_k^2}{P_{ai}M_k/W_{ai} + N_0 M_k/\alpha_j^2}$,
and $M_k = \log_2\left(1 + \frac{\alpha_k^2 P_{ai}}{N_0 W_{ai}}\right)$, $M_j = \log_2\left(1 + \frac{\alpha_j^2 P_{ai}}{N_0 W_{ai}}\right)$.

Since the sum of the bandwidth allocated to users in the same spot beam is equal to the bandwidth allocated to the i-th spot beam, we obtain the following equation:

$$\sum_{j \in \mathcal{N}_{B_i}} W_j = W_{ai} \tag{19}$$

According to (18) and (19), we obtain the bandwidth allocated to the users in the same spot beam:

$$W_j = \frac{W_{ai} - \sum_k B_{jk}}{\sum_k A_{jk} + 1}, \quad \forall j, k \in \mathcal{N}_{B_i}, k \neq j \tag{20}$$

As a result, the power allocated to the users is given as:

$$P_j = \frac{W_j}{W_{ai}} P_{ai}, \quad \forall j \in \mathcal{N}_{B_i} \tag{21}$$

When there is a power or bandwidth pre-allocation in the i-th spot beam, $P_{ai} = P_{B_i}$ or $W_{ai} = W_{B_i}$. Otherwise, the power or bandwidth allocated to the i-th spot beam can be calculated according to J. P. Choi and U. Park [1, 5], where the traffic demand of the spot beam is the sum of the traffic demand of the users in it, and the signal attenuation factor of the spot beam is the mean value of the signal attenuation factor of the users in it. In summary, the whole process of the low computational complexity algorithm is given as follows:

Step 1. Calculate the bandwidth and power allocated to each spot beam according to Choi and Chan and Unhee Park [1, 5].
Step 2. For the users in the same spot beam, calculate the bandwidth and power allocated to each user according to (18) and (19).
Step 3. If the bandwidth and power allocated to a user is smaller than zero, then set the bandwidth and power allocated to the user equal to zero, and go to step 2 to recalculate the bandwidth and power allocated to the remaining users; otherwise terminate the algorithm.

According to the above process, it is seen that the computational complexity of step 1 is $O(K)$ [1, 5]. The computational complexity of step 2 is $O(M)$. Therefore, the total computational complexity of the suboptimal algorithm is much less than the optimal algorithm.

4 Performance Analysis and Simulation Results

For the simulation, we set up a Ka band multi-spot-beam satellite communication system. The system has four spot beams and 20 users, and each spot beam has five users. The total power of the satellite is 200 W, and the total bandwidth of the satellite is 500 MHz. The noise power spectral density parameter N_0 is e^{-6}. The traffic demand of a user in each spot beam increases from 30 Mbps to 70 Mbps, by steps of 10 Mbps.

4.1 Efficiency of the Proposed Joint Allocation Algorithm

To verify the efficiency of the proposed optimal joint allocation algorithm, we compare the algorithm with the following 3 algorithms in case (d). The signal attenuation factors α_i^2 of all the users are set to be 5.

a. Uniform bandwidth allocation and uniform power allocation (UBUP).
b. Uniform bandwidth allocation and optimal power allocation (UBOP).
c. Uniform power allocation and optimal bandwidth allocation (OBUP).

Since channel conditions of users in different beams are the same, the allocation results of users in different spot beam are the same. Therefore, we only need to plot the allocation results for the first beam.

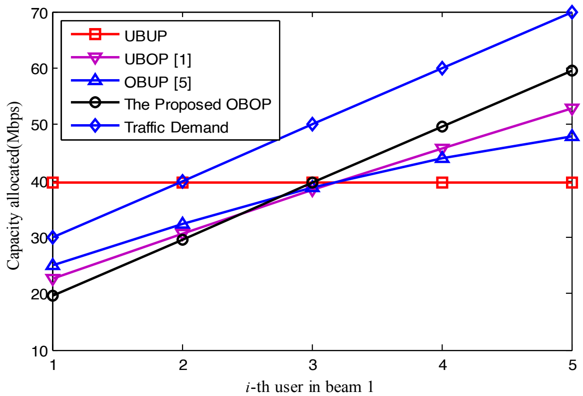

Fig. 2. Comparison of capacity allocated to the *i*-th use in beam 1 for the four algorithms.

Figure 2 shows the comparison of the capacity allocated to the *i*-th use in beam 1 for the four algorithms. Table 1 shows the comparison of the total system capacity for the four algorithms. It is known that the traffic demand of the users increases linearly, thus to obtain fairness between the beams, the separate optimal allocation algorithms (UBOP, OBUP) will provide more power or bandwidth resources to higher traffic demand users. However, due to the concavity of the capacity function with a fixed bandwidth or power allocation, the capacity allocated to each user is not linearly increased. It is clearly seen from Fig. 2 that the capacity curve is concave. The UBUP

Table 1. Comparison of total capacity for the four algorithms

Algorithm	$\sum Ci$
UBUP	753.985 Mbps
UBOP [1]	761.272 Mbps
OBUP [5]	751.822 Mbps
The Proposed OBOP	792.455 Mbps

algorithm allocates resources to each user regardless of the traffic demand, resulting in user one being allocated more resources than are needed, causing resource waste. The OBOP algorithm dynamically allocates bandwidth and power resource to each user, thus the capacity curve is almost linear, and the total system capacity is improved. This conclusion is also demonstrated by the data in Table 1.

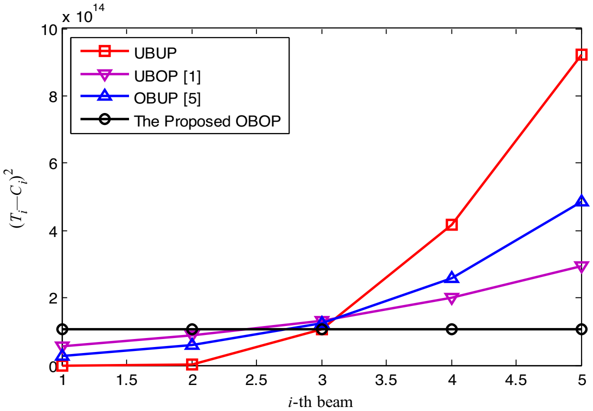

Fig. 3. Comparison of the deficit between the traffic demand and the capacity allocated to beam 1 for the four algorithms.

Figure 3 shows the deficit between the traffic demand and the capacity allocated to the *i*-th user in beam 1. Table 2 presents the sum of the deficit between the traffic demand and the capacity allocated to each user. It is seen from Fig. 3 that in the optimal joint allocation algorithm (OBOP), the deficit between traffic demand and capacity allocated is almost the same, so the fairness between the beams is much better than for that of the separate optimal allocations (UBOP and OBUP). This conclusion is also shown in Table 2. Together with the conclusion above regarding total system capacity, we can conclude that the performance of the optimal joint allocation algorithm (OBOP) is much improved compared with the individual optimal algorithms (UBOP and OBUP).

Table 2. The total sum of $(T_i - C_i)^2$ of the four algorithms

Algorithm	$\sum(Ti - Ci)2$
UBUP	6.0606E15
UBOP [1]	3.0894E15
OBUP [5]	3.8173E15
The proposed OBOP	2.1537E15

4.2 Performance of the Low Computational Complexity Allocation Algorithm

To show the performance of the low computational complexity algorithm (LBLP), we compare it with the optimal allocation algorithm (OBOP) in the following three scenarios.

Scenario 1: The channel conditions of all users are the same. The signal attenuation factors α_i^2 of all the users are set to 5 (Table 4).

Table 3. The total system capacity of two algorithms in the four cases in scenario 1.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	792.48 Mbps	775.51 Mbps	790.38 Mbps	789.48 Mbps
LBLP	792.48 Mbps	775.51 Mbps	790.38 Mbps	789.48 Mbps

Scenario 2: The channel conditions of each user in the same beam are the same, while the channel conditions of users in different beams are different. The signal attenuation factors α_i^2 of the users in four beams are set to be 10/2, 10/2.5, 10/3, and 10/3.5, respectively, and the signal attenuation factors α_i^2 for users in the same beam are set to be the same (Tables 5 and 6).

Scenario 3: We compare the performance of the two algorithms when the channel condition of each user is different. The signal attenuation factor α_i^2 of each user conforms to uniform distribution between 5 and 3.5 (Table 7).

Table 4. The objective function value of two algorithms in the four cases in scenario 1.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	2.15E15	2.83E15	2.21E15	3.14E16
LBLP	2.15E15	2.83E15	2.21E15	3.14E16

Table 5. The total system capacity of two algorithms in the four cases in the scenario 2.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	655.81 Mbps	645.76 Mbps	652.52 Mbps	645.25 Mbps
LBLP	652.40 Mbps	645.76 Mbps	652.52 Mbps	645.25 Mbps

Table 6. The objective function value of two algorithms in the four cases in scenario 2.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	6.03E15	6.28E15	6.08E15	6.33E16
LBLP	6.08E15	6.28E15	6.08E15	6.33E16

Table 7. The total system capacity of two algorithms in the four cases in scenario 3.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	652.84 Mbps	648.89 Mbps	653.41 Mbps	652.70 Mbps
LBLP	650.84 Mbps	646.56 Mbps	650.68 Mbps	651.48 Mbps

Table 8. The objective function value of two algorithms in the four cases in scenario 3.

	Case (a)	Case (b)	Case (c)	Case (d)
OBOP	6.09E15	6.86E15	6.15E15	6.92E16
LBLP	6.12E15	6.87E15	6.18E15	6.94E16

From the above tables, it is seen that the performance of the LBLP algorithm is the same as that of the OBOP algorithm, for the four cases when the channel conditions of each user are the same. When the channel conditions of each user in the same beam are the same, the performance of the LBLP algorithm is the same as that of the OBOP algorithm for cases (b)–(d). When the channel condition of each user is different, the value of the objective function of the LBLP algorithm is little more than that of the OBOP algorithm, thus the fairness between each user of the LBLP algorithm is lower, however, the total system capacity of the LBLP algorithm is improved over that of the OBOP algorithm. Therefore, the performance of LBLP algorithm is much closer to the OBOP algorithm, especially when the channel conditions of each user in the same beam are the same.

4.3 The Impact of Pre-allocations for Each Beam

From Tables 3 through 8, it is shown that in the same scenario, the value of the objective function of case (a) is lower than the other three cases. In other words, the power or bandwidth pre-allocations to each beam deteriorate the total system performance. Because when there is no power or bandwidth pre-allocation to each of the beams, the power and bandwidth allocated to each user can be more flexible.

5 Conclusion

In this paper, we sought to solve a problem of the joint power and bandwidth allocations for multiple users in a multi-beam satellite communication system. To this end, we first formulated the problem as a convex optimization problem. Then we proposed an optimal joint allocation algorithm and a low computational complexity algorithm. The optimal joint allocation algorithm was more efficient than the separate bandwidth or power allocation algorithm. The performance of the low computational complexity algorithm was very close to that of the optimal joint allocation algorithm.

Appendix

From the analysis in Sect. 3, it is shown that the constraints can indeed be ignored.

Taken together with the fact that the constraints (4, 5, 6 and 7) are linear, to prove the optimization problem is convex, we only need to prove that $\sum_{i=1}^{M}(T_i - C_i)^2$ is convex [10].

It is known that the sum of convex functions is also convex. Therefore, to prove that $\sum_{i=1}^{M}(T_i - C_i)^2$ is convex, we just need to prove the following function is convex:

$$f(P_i, W_i) = (T_i - C_i)^2 \tag{22}$$

where $C_i = W_i \log_2\left(1 + \frac{\alpha_i^2 P_i}{W_i N_0}\right)$.

It is known that is the Hessian of one function is semi-definite, thus the function is convex [10]. The Hessian of $f(P_i, W_i)$ is given as follows:

$$H_f = \begin{bmatrix} \frac{\partial^2 f(P_i, W_i)}{\partial P_i^2} & \frac{\partial^2 f(P_i, W_i)}{\partial P_i \partial W_i} \\ \frac{\partial^2 f(P_i, W_i)}{\partial P_i \partial W_i} & \frac{\partial^2 f(P_i, W_i)}{\partial W_i^2} \end{bmatrix} \tag{23}$$

To prove that H_f is positive semi-definite, we obtain the following equations:

$$\begin{aligned}\frac{\partial^2 f(P_i, W_i)}{\partial P_i^2} &= 2\left(\frac{\partial C_i}{\partial P_i}\right)^2 - 2(T_i - C_i)\frac{\partial^2 C_i}{\partial P_i^2} \\ &= 2\left(\frac{\partial C_i}{\partial P_i}\right)^2 + 2(T_i - C_i)\frac{W_i}{\ln 2(N_0 W_i/\alpha_i^2 + P_i)^2}\end{aligned} \tag{24}$$

$$\begin{aligned}|H_f| &= \frac{\partial^2 f(P_i, W_i)}{\partial P_i^2}\frac{\partial^2 f(P_i, W_i)}{\partial W_i^2} - \frac{\partial^2 f(P_i, W_i)}{\partial P_i \partial W_i}\frac{\partial^2 f(P_i, W_i)}{\partial P_i \partial W_i} \\ &= 4(T_i - C_i)\frac{C_i^2}{W_i \ln 2(N_0 W_i/\alpha_i^2 + P_i)^2}\end{aligned} \tag{25}$$

When $T_i \geq C_i$, it is obvious that (24) and (25) are non-negative. Therefore, H_f is positive semi-definite, and $\sum_{i=1}^{M}(T_i - C_i)^2$ is convex, thus the optimization problem is convex. As a result, the solution obtained from the joint bandwidth and power algorithm based on KKT conditions is the global optimal solution of the optimization problem.

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