Chapter 8
Local, Modal and Shape Control Strategies for Active Vibration Suppression of Elastic Systems: Experiment and Numerical Simulation

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Abstract The problem of active vibration suppression of the distributed elastic system is considered in the example of a slender metal beam undergoing bending vibrations. Control systems include piezoelectric sensors and actuators. Three different strategies for vibration suppression are considered: local, modal and shape control strategy. The local approach means that each feedback loop includes only one sensor–actuator pair placed at specific location on the beam, while the modal strategy implies that each feedback loop corresponds to a specific vibration mode of the object. The shape control method is based on the compensation of known distribution of the external excitation using only one feedback loop with all available sensors and actuators. First, experimental results are obtained for the local and the modal control systems using the same two sensor–actuator pairs, and then the transfer functions in feedback loops for these systems are improved as the result of numerical modeling. After that, the modal method is compared numerically with the shape control strategy. The results show that the modal method is the most effective if it is needed to suppress several vibration modes of the object.

Keywords Modal control · Shape control · Feedback control · Active vibration suppression · Piezoelectric sensors and actuators · Distributed elastic systems

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V. A. Polyanskiy and A. K. Belyaev (eds.), *Mechanics and Control of Solids and Structures*, Advanced Structured Materials 164,
https://doi.org/10.1007/978-3-030-93076-9_8

8.1 Introduction

Undesirable vibrations can be dangerous for mechanical systems and cause failure, damage and unwanted noise, while the resonance vibrations are especially harmful. Active vibration control is a modern method of vibration attenuation, which is developing rapidly due to advances in digital signal processing and sensor and actuator technology. The presence of the infinite number of eigenmodes and resonance frequencies complicates the active vibration control of the elastic systems with distributed parameters and leads to reduced accuracy and stability of the control systems.

Piezoelectric materials give wide opportunities for control and monitoring of elastic systems, since these are smart materials, which combine two physical fields: mechanical and electrical. On one hand, sensor networks can be used for health monitoring and damage detection in engineering structures [1]. On the other hand, actuators allow one to control the stress–strain state of structures by applying electrical voltage, which helps to realize not only the displacement tracking but also the stress control in order to protect the structure or the piezoelectric actuator itself from damage and destruction [2]. The joint use of sensors and actuators allows one to organize the feedback control of elastic systems. This raises the following questions: how to locate the piezoelectric elements on the object, how to process the sensor signals, how to arrange the feedback loops and which control laws in these loops to specify. There are several strategies to control the elastic objects, which give different answers to the aforementioned questions.

The most simple is the local approach [3]: sensors and actuators are placed in pairs at several locations on the object, and each feedback loop includes only one sensor–actuator pair. The second method is modal and it accounts for the dynamics of the object: each control loop corresponds to a specific vibration mode of the object and uses all sensors to measure this mode and all actuators to affect it [4]. The third approach under consideration is the shape control strategy [5]. This strategy was originally formulated as a method to fully compensate the external excitation on the elastic object with known spatial distribution provided that the disturbance is known and the appropriate control capabilities are available. In real cases, the time variation of the external excitation may be not given, which makes it necessary to use feedback control. The shape control method can be used if the spatial distribution of the disturbance is known in advance and does not vary in time. The control action is distributed on the object in the way that allows one to compensate this disturbance, while sensor and actuator systems are collocated and form a single feedback loop.

The objective of the present study is to compare the three mentioned strategies for the problem of active vibration suppression of a slender metal beam using piezoelectric sensors and actuators. The previous work of the authors [6] provides the experimental comparison of local and modal approaches, and in the present study, control systems obtained previously are improved by means of numerical modeling, and their results are compared with the numerically obtained results for the shape control strategy.

8.2 Theoretical Background

This section presents the basic theoretical information and schemes for the three control strategies under consideration.

8.2.1 *Local Method*

Figure 8.1 shows the scheme of the local control system with two sensor–actuator pairs. In the scheme, y_i are the sensor signals, and u_i are the control actions on the actuators. After attaching sensors and actuators to the control object, it is necessary to specify the transfer function for each feedback loop $R_i(s)$. The drawback of this approach is that different feedback loops are not independent, because they are connected through the elastic object and can strongly influence each other.

8.2.2 *Modal Method*

Modal control of flexible structures is also called independent modal space control (IMSC). This approach was first formulated by Gould and Murray-Lasso [4] and further developed by Meirovitch [7].

Let us consider a thin metal beam undergoing bending vibrations. Consider the equation of motion of the beam in spectral decomposition, assuming that n eigenmodes are enough to describe the motion of the beam:

$$w(x,t) = \sum_{i=1}^{n} w_i(x) q_i(t), \tag{8.1}$$

where w is the displacement, w_i is the ith bending mode of the beam, and q_i is the ith generalized coordinate. The matrix equation of motion for the vibration modes

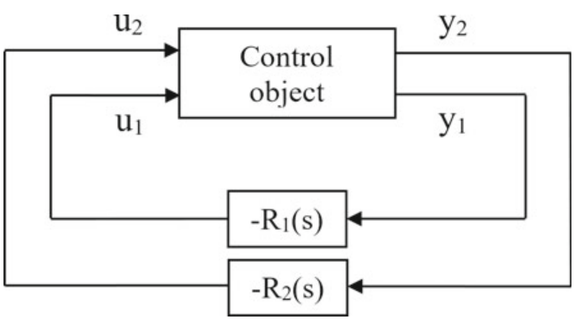

Fig. 8.1 Scheme of the local control system

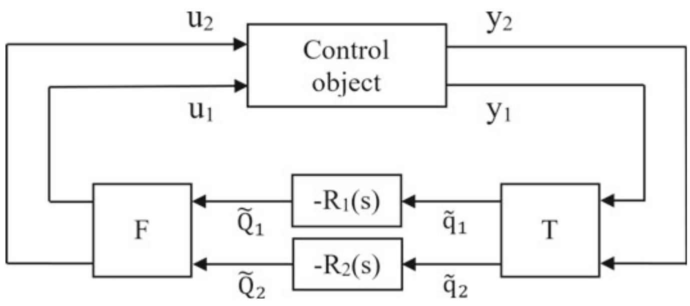

Fig. 8.2 Scheme of the modal control system

will have the following form:

$$\ddot{q} + 2\xi\Omega\dot{q} + \Omega^2 q = Q^d + Q^c, \tag{8.2}$$

where q is the vector of generalized coordinates, ξ is the diagonal damping matrix, Ω is the diagonal matrix of eigenfrequencies, Q^d is the vector of external excitation and Q^c is the vector of control actions on the modes.

The scheme of the modal control system is given in Fig. 8.2. For simplicity, we consider a system with two sensors, two actuators and two modal control loops. In the scheme $\widetilde{q}_i$ are the estimates of the first and the second generalized coordinates, $\widetilde{Q}_i$ are the desired generalized forces acting on the first and the second modes, T is the mode analyzer matrix, used to estimate the generalized coordinates, and F is the mode synthesizer matrix, used to generate proper control actions.

Let us introduce the excitation matrix θ^a and the measurement matrix θ^s. The excitation matrix θ^a shows how strong is the influence of each actuator on each eigenmode of the object, and the measurement matrix θ^s shows how strong is the influence of each eigenmode on each sensor:

$$Q^c = \theta^a u = \theta^a F \widetilde{Q}, \tag{8.3}$$

$$\widetilde{q} = Ty = T\theta^s q. \tag{8.4}$$

It is obvious that the correspondence between eigenmodes of the object and the modal control loops requires that the modal matrices T and F have the following form:

$$F = (\theta^a)^{-1}, T = (\theta^s)^{-1}. \tag{8.5}$$

Then, the single equation from the system (8.2) for ith eigenmode of the beam takes the following form:

$$\ddot{q}_i + 2\xi_i\Omega_i\dot{q}_i + \Omega_i^2 q_i = Q^d - R_i(s)q_i. \tag{8.6}$$

This means that one can control each mode individually by setting the corresponding transfer function $R_i(s)$ and thus realize the independent modal space control. Of course, all these derivations are valid only if the matrices θ^a and θ^s are square and have the size $n \times n$, which means, that there are enough sensors and actuators in the control system and no higher modes are active. In real problems, the presence of higher, uncontrolled modes always complicates the overall situation: for example, some higher modes can become unstable. This is called the spillover effect, which can be minimized by increasing the number of sensors and actuators.

8.2.3 Shape Control Method

Originally, the notion *shape control* was first mentioned in a study by Haftka and Adelman [8], where the problem of minimizing static distortion of large space structures using thermal control elements was considered. Then, the shape control method was further developed by Austrian researchers from Johannes Kepler University Linz. The review of shape control is given in [9]. Paper [5] presents the theory of dynamic shape control of beams by piezoelectric actuation and sensing. The presented method allows one to eliminate force-induced vibrations of a beam by the use of piezoelectric actuators attached to the structure.

As stated in [5], in order to fully compensate the deformations of the beam, the actuation bending moment M^t should be opposite to the statically admissible bending moment M^{q,p_z} produced by the distributed forces p_z:

$$M^{q,p_z}(x,t) + M^t(x,t) = 0. \tag{8.7}$$

Thus, in order to compensate the external disturbance by the piezoelectric actuation, the distribution of this disturbance should be known in space and time, and the needed actuation should be available. However, usually in real problems, the possibilities of actuation are limited: for example, often only finite set of rectangular piezoelectric actuators is available. In these cases, the desired distribution of the actuation moment should be approximated by discrete step functions corresponding to separate piezoelectric patches [10]. In the present study, the deflection of the beam with applied actuation is analyzed directly, and the condition of minimization of the maximum deflection is applied in order to find positions and actuation intensity for discrete piezoelectric actuators.

Classic example of feedback control within the framework of shape control strategy in application to bending vibrations of a beam is given in the book of Nader [10]. The following control scheme is used: all the actuators and all the sensors form a single control loop, which means that $\widetilde{q}$ and $\widetilde{Q}$ in the scheme in Fig. 8.3 are scalars, and vectors f^s and f^a are used instead of matrices T and F in the modal system. Moreover, the design of the sensor system completely repeats the design of the actuator system (sensors are located symmetrically to the actuators at the opposite side

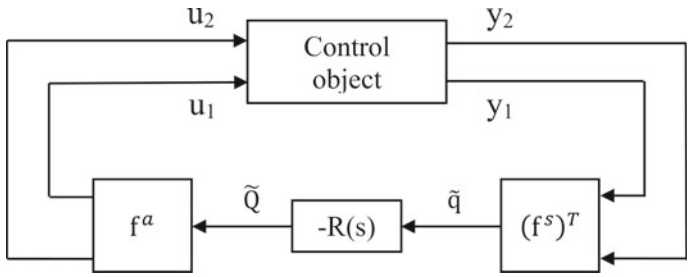

Fig. 8.3 Scheme of the shape control system

of the beam), that is, the sensor and the actuator systems are collocated. Therefore, vectors of weighting factors f^s and f^a are equal:

$$f^s = f^a. \tag{8.8}$$

8.3 Experimental Setup

The first part of the study is the experimental investigation and comparison of local and modal approaches to active vibration suppression of a metal beam, which is described in detail in the paper [6]. The second part of the study is the numerical modeling of the same system and the synthesis of more efficient control laws [11]. At the final stage of the study, the shape control systems are synthesized and compared to the previously obtained local and modal systems.

The experimental setup is shown in Fig. 8.4. The control object is an aluminum beam 70 cm long with the cross-section of 3 × 35 mm. It is disposed vertically and fixed at one point 10 cm far from the lower end. The external excitation is the base vibration. It is applied by means of a piezoelectric stack actuator, which is a part of the fixation that connects the beam to the massive basement. Axial displacement of the stack actuator causes bending vibration of the beam. The control system includes PI Ceramic DuraAct patch transducers P-876.A15, which are used as sensors and actuators. They consist of rectangular PZT plates with dimensions 50 × 30 × 0.5 mm, thin metal electrodes and the polymer coating. Actuators and sensors are connected through a digital controller dSPACE DS1103 PPC Controller Board. Apart from this, feedback loops also contain low-pass-filters (LPF) and a signal amplifier, which also gives contribution to the frequency characteristics of the object.

The purpose of the control system is to reduce forced vibrations of the beam in the frequency range containing the first and the second resonance frequencies. The first and the second bending modes are shown in Fig. 8.4. Two sensor–actuator pairs are located on the beam on both sides at the beginning of the experiment and hold the same positions for all tested local and modal control systems. In order to monitor

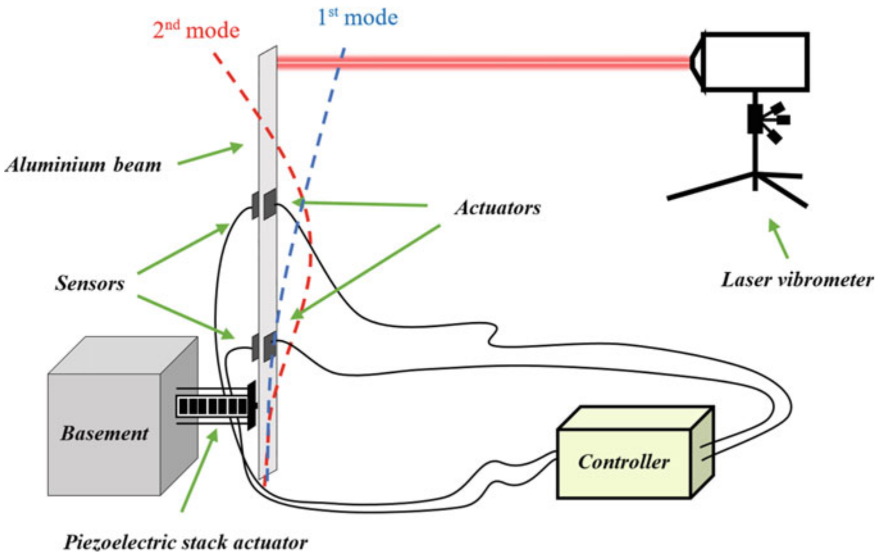

Fig. 8.4 Experimental setup

the control efficiency, the vibration amplitude of the upper endpoint of the beam is measured by the laser Polytec Scanning Vibrometer PSV-400. The choice of this point is caused by the fact that the amplitude of its vibration is the highest among all points of the beam for the first and the second vibration modes.

8.4 Sensor and Actuator Placement

In this section, we consider a problem of sensor and actuator placement on the beam for each control strategy and choosing proper matrices T and F for the modal control systems and vectors f^a and f^s for the shape control systems.

8.4.1 Local and Modal Methods

First, we need to define the positions of the piezopatches on the beam for the experimental study of local and modal methods. This process is described in the paper [6]. The positions of sensors and actuators in the framework of the numerical investigation are the same as for the experimental research. These positions are obtained as a result of analyzing the first and the second bending modes of the beam. Sensors and actuators should be placed in those locations where the curvature $w''(x)$ of these modes gets maximum values: in this case, they can excite and measure these modes

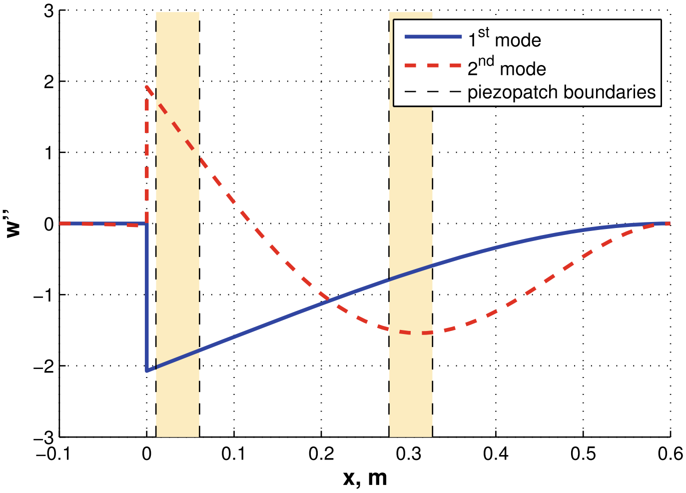

Fig. 8.5 Curvature of the first and the second bending modes of the beam with the piezopatch locations

most effectively. The curvature of the first and the second modes with the chosen positions of two sensor–actuator pairs is shown in Fig. 8.5. The coordinates of the centers of the piezopatches are the following: $x_1 = 0.0355$ m, $x_2 = 0.3025$ m.

The matrices T and F (mode analyzer and synthesizer) for the experimental study are obtained using the identification procedure described in detail in [12]. As the result, these matrices are defined in the following way:

$$T^{(exp)} = \begin{pmatrix} 0.99 & 1.03 \\ -0.49 & 1.53 \end{pmatrix}, \tag{8.9}$$

$$F^{(exp)} = \begin{pmatrix} 0.98 & -0.49 \\ 1.02 & 1.51 \end{pmatrix}. \tag{8.10}$$

8.4.2 Shape Control Method

Here, we need to specify the positions for sensors and actuators and define the vectors of weighting factors f^a and f^s for the numerical study of the shape control method. As stated before, sensor and actuator systems are collocated; therefore, $f^a = f^s$. We consider a system with either two or five sensor–actuator pairs. Thus, we need to define the positions of the piezopatches and the weighting factors, that is, the actuation moments for each actuator, from the condition of the best compensation of the external excitation.

We know the form of the external excitation: it is the transverse vibration of the support, which is equivalent to the transversal inertia force uniformly distributed

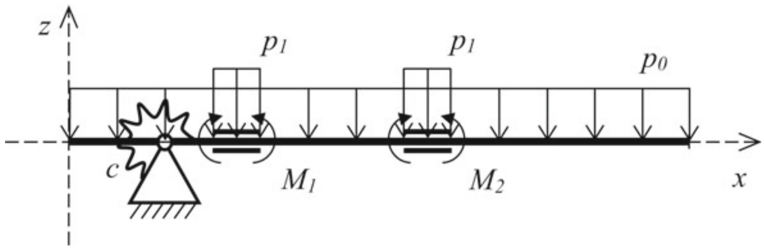

Fig. 8.6 Beam with two sensor–actuator pairs

over the whole beam as shown in Fig. 8.6. Here, the model of the beam from the experimental setup shown in Fig. 8.4 is presented. We use for calculations the value of the distributed load $p_0 = 1$ N/m. In the case of using sensors and actuators, the load is not uniform: the control elements present an additional mass, therefore, the distributed load at the corresponding locations slightly increases (the resulting load is denoted as p_1). The fixation is modeled by the torsional spring with the stiffness $c = 400\,\mathrm{N}\cdot\mathrm{m/rad}$ (this model is verified in [6] and [11]). We choose the locations of the piezopatches and actuation moments M_i from the condition of the minimization of the maximum deflection of the beam in the static case. We also require that the deflection at the right endpoint of the beam is zero, because the efficiency of the obtained control systems is defined using the deflection at this point.

As a result of the investigation, two configurations of shape control systems were obtained, differing in the number of sensor–actuator pairs. The beam deflection corresponding to these two systems in the static case with feedforward control is shown in Fig. 8.7. The black curve corresponds to the system with two sensor–actuator pairs, and the red curve corresponds to the system with five pairs of piezopatches. The maximum deflection of the beam without control (not displayed in the figure) is 3.2 mm, while for the systems with two and five sensor–actuator pairs this value equals to 0.044 mm and 0.012 mm, respectively. For the system with two pairs of piezopatches, the maximum deflection cannot be smaller because the actuators on the right side of the beam do not influence the deflection of the left side. For the system with five sensor–actuator pairs, the maximum deflection cannot be smaller because the piezopatches technically cannot be located closer than 10 mm to the fixation point due to the presence of a nut, which fixates the beam. For the first system, the coordinates of the centers of the piezopatches are the following: $x_1 = 0.048$ m, $x2 = 0.237$ m, and the actuation moments are: $M_1 = 0.835\ \mathrm{N}\cdot\mathrm{m}$, $M_2 = 0.603\ \mathrm{N}\cdot\mathrm{m}$. For the second system, the coordinates are: $x_1 = -0.035$ m, $x_2 = 0.035$ m, $x_3 = 0.1345$ m, $x_4 = 0.235$ m, $x_5 = 0.363$ m, and the moments are: $M_1 = -0.22\,\mathrm{N}\cdot\mathrm{m}$, $M_2 = 0.622\,\mathrm{N}\cdot\mathrm{m}$, $M_3 = 0.354\,\mathrm{N}\cdot\mathrm{m}$, $M_4 = 0.267\ \mathrm{N}\cdot\mathrm{m}$, $M_5 = 0.1603\ \mathrm{N}\cdot\mathrm{m}$.

Thus, the vectors of the weighting factors for the shape control systems with two or five pairs of piezopatches are defined, since they are taken equal to the actuation moments:

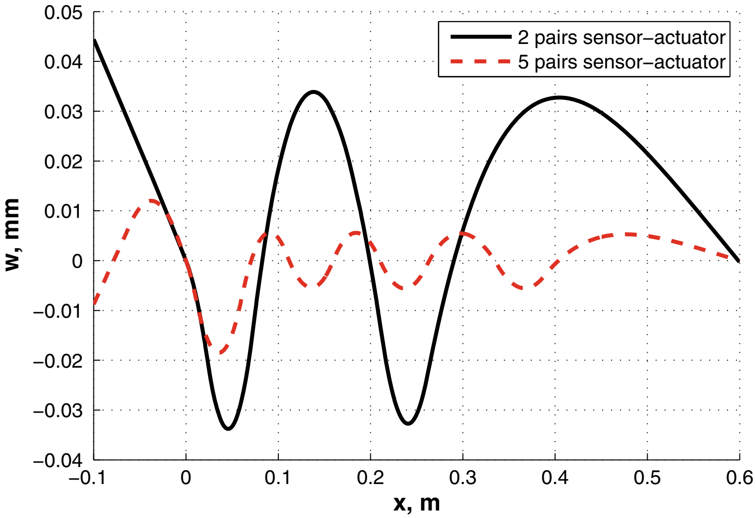

Fig. 8.7 Compensation of the external excitation with two or five sensor–actuator pairs

$$f^{a(2)} = f^{s(2)} = \left(0.835\ 0.603\right)^T, \tag{8.11}$$

$$f^{a(5)} = f^{s(5)} = \left(-0.22\ 0.622\ 0.354\ 0.267\ 0.1603\right)^T. \tag{8.12}$$

8.5 Finite Element Modeling

Before the synthesis of the transfer functions in the feedback loops of the control systems, one needs to obtain the frequency response functions (FRFs) of the beam. In order to obtain FRFs of the beams with piezopatches and without piezopatches for reference, the finite element (FE) models of these beams are created in `ANSYS` software. The choice of this software is caused by the fact that it can give precise results for a wide range of eigenmodes of the beam, while the numerical simulation of the beam dynamics in `MATLAB` software resulted in errors for the higher vibration modes. The previous numerical research in `MATLAB` for the beam with different boundary conditions (simply supported beam) has shown the importance of taking into account the influence of piezopatches on the beam eigenmodes during the modeling of beam dynamics [13, 14].

Two types of FE models of the system were created: the first one (Fig. 8.8a) constructed of three-dimensional elements (`Solid186` for ordinary materials and `Solid226` for piezoelectric materials), and the second one (Fig. 8.8b) constructed of one-dimensional elements `Beam189`. At first, both models were created for the system with two sensor–actuator pairs constructed within the experimental study, and both were verified using the experimental data [11]. These models differ greatly in

(a) Model with 3D solid elements

(b) Model with 1D beam elements

Fig. 8.8 Finite element models of the beam

complexity: the first contains 3534 elements and 21088 nodes, and the second—only 161 element and 283 nodes. Moreover, in the 3D model, the fixation construction is modeled entirely with the stack actuator and additional elements, and the piezoelectric effect is modeled directly, while in the 1D model the beam fixation is modeled by two springs (longitudinal and torsional), and the piezoelectric effect in this model is absent. Instead of this, the actuator excitation is specified by an application of corresponding forces and moments, and the sensor signal is calculated from the longitudinal deformation of the piezoelectric material.

The modeling has shown no big difference in the results for two described FE models. Therefore, for testing local and modal control systems, the 3D model was used as more precise one, and for the subsequent calculations (shape control systems and the beam without piezopatches for reference) for simplicity, the 1D model was used. Figure 8.8b shows the 1D model of the system with five sensor–actuator pairs created for the testing of the shape control strategy.

In order to obtain FRFs of the beams, the harmonic analysis is performed in the frequency range from 1 to 2000 Hz, where a harmonic excitation is applied either to the beam support or to the actuators. The measured values are the sensor signals and the deflection of the right endpoint of the beam. In all models, the same damping coefficient $\xi = 0.002$ is used for all vibration modes; this choice is justified in [15].

In order to obtain the mode analyzer and synthesizer for the modal control system in the framework of the numerical study, the FRFs of the beam with piezopatches are analyzed. The height of the resonance peaks in the FRFs allows one to determine the matrices θ^a and θ^s. It is important to mention here that the rows of matrix θ^a and the columns of matrix θ^s, therefore, the rows of matrix T and the columns of matrix F are defined up to a constant. The matrices T and F are calculated from the matrices θ^a and θ^s using Eq. (8.5). The results are the following:

$$T = F^T = \begin{pmatrix} 1.01 & 0.96 \\ -0.49 & 1.49 \end{pmatrix}. \tag{8.13}$$

8.6 Design of the Transfer Functions

Within the experimental study, the transfer functions for local and modal strategies were designed using the frequency response design method [16, 17]. In order to do this, FRFs of the beam were previously measured for two variants of excitation (each of two actuators) and two variants of measured signal (each of two sensors) in the frequency range from 1 to 2000 Hz. After that, the designed control systems were tested experimentally. The optimization criterion was the height of the resonance peaks in the FRF showing the vibration amplitude of upper endpoint of the beam under the excitation of the fixation point at the first and the second resonances of the beam. As a result, three most efficient control systems were obtained: the first local system is designed to work at the first resonance, the second local system—at the second resonance, and the modal system with two feedback loops is effective at both resonances. These results are presented in the paper [6], and they are also included in Table 8.1.

At the next stage of the investigation, the designed transfer functions for local and modal systems were improved in the framework of the numerical simulation. During this process, the FRFs of the beam were used that were obtained not experimentally, but numerically by means of FE modeling (Sect. 8.5). In order to calculate the FRF of the beam under feedback control with two loops using only existing FRFs of the beam without control and the control laws, the mathematical procedure is used described in [11, 15].

The main advantage of the numerical simulation is that it is so fast and precise, that it is possible to realize the numerical optimization of the designed transfer functions. Thus, the special optimization procedure was realized in MATLAB [11]. This algorithm optimizes parameters of different filters composing the transfer function of selected feedback loop and its gain value and ensures the stability of the closed-loop system. Let us consider this procedure in more detail.

First of all, each transfer function is constructed from the finite number of special filters. The first one is an inverse notch filter, which raises the phase of the control signal in the working frequency domain near to the resonance to be controlled so that the control action has the opposite phase with the external excitation and can effectively compensate it. The second one is a low-pass filter, which reduces the amplitude of the signal at high frequencies and thus increases the stability of the closed-loop system. The third one is optional—it is a notch filter, which reduces the amplitude of the signal at one of the higher resonances where the risk of instability is the greatest, and thus allows one to raise the overall gain value. The transfer function of the designed feedback loop is obtained by multiplying the transfer functions of the individual filters. The described procedure calculates the control results for different variants of filter parameters from the specified range, finds the optimal gain value for each variant of the control law, compares the control results and finds the best combination of parameters, which provides the most effective vibration suppression and at the same time does not cause instability in the closed-loop system.

With the help of the described algorithm, the control loops were optimized for local, modal and shape control strategies. The results are presented in Sect. 8.7. Here as an example, the control laws and the Bode diagrams are presented for the most efficient of the created control systems—namely, the modal control system with two feedback loops. Mode analyzer and synthesizer for this system are given in Sect. 8.5. The transfer functions in the feedback loops are the following:

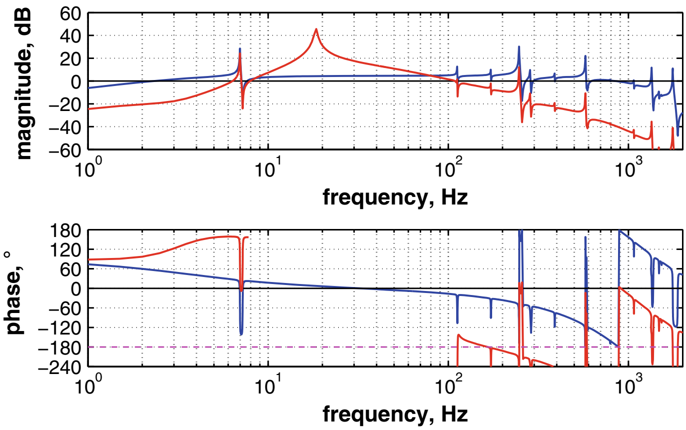

Fig. 8.9 Bode diagram for the first loop of the modal control system (blue lines—control object, red lines—open-loop system)

$$R_1(s) = \big(3.07 \cdot 10^5\, s^4 + 9.84 \cdot 10^6\, s^3 + 1.52 \cdot 10^{11} s^2 + 2.74 \cdot 10^{12} s + \\ +6.14 \cdot 10^{13}\big) / \big(s^6 + 572\, s^5 + 5.99 \cdot 10^5\, s^4 + 2.79 \cdot 10^8\, s^3 + 4.48 \cdot 10^{10} s^2 + \\ +3.72 \cdot 10^{12} s + 4.83 \cdot 10^{14}\big), \quad (8.14)$$

$$R_2(s) = \frac{2.17 \cdot 10^5\, s^2 + 3.61 \cdot 10^6\, s + 3.43 \cdot 10^7}{s^4 + 414\, s^3 + 5.57 \cdot 10^5\, s^2 + 7.65 \cdot 10^7\, s + 6.13 \cdot 10^{10}}. \quad (8.15)$$

The Bode diagrams for both loops of the modal control system are given in Figs. 8.9, 8.10. Here, the blue lines correspond to the control object, and the red lines

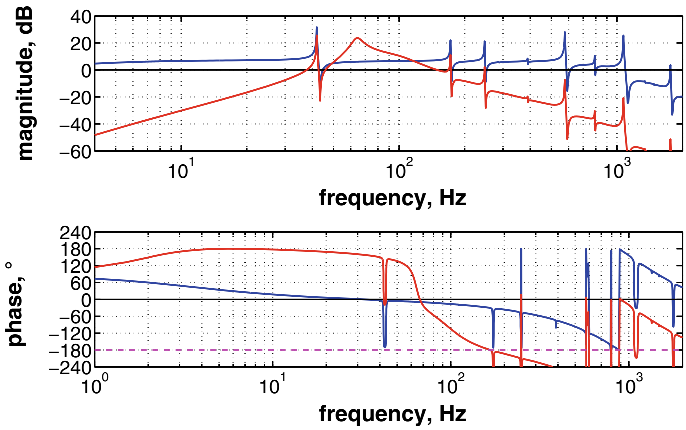

Fig. 8.10 Bode diagram for the second loop of the modal control system (blue lines—control object, red lines—open-loop system)

—to the open-loop system. In the used FRFs of the control object, the characteristics of the low-pass filters and the amplifier, which are included in the control loop, are also taken into account. These characteristics were measured during the experimental part of the investigation [6].

8.7 Comparison of the Results

In this section, the control results for all designed systems are summarized. In Figs. 8.11, 8.12, 8.13 and 8.14, the resulting FRFs of the beam with control showing the vibration amplitude of upper endpoint of the beam under the excitation of the fixation point in the vicinity of the first and the second resonances are shown in comparison with the FRF of the beam without piezopatches. Then, the difference in the level of endpoint vibrations of the beam at both resonances for each control system compared to the level of vibrations without control is given in Table 8.1. Figures show only results obtained numerically, while the experimental results for tested earlier local and modal systems are presented in the table. In the table, Δw_1 and Δw_2 are the change in the magnitude of endpoint vibrations at the first and the second resonances, respectively.

First, three local and three modal control systems were obtained (Figs. 8.11, 8.12). In the first local system, both loops were designed to work at the first resonance of the beam, and in the second system—at the second resonance. In the third local system, the first feedback loop (lower sensor–actuator pair) was designed to suppress

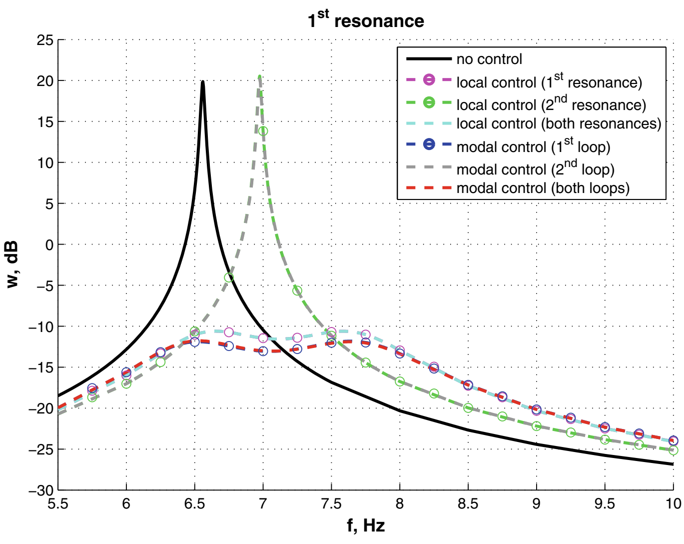

Fig. 8.11 Compensation of the first resonance of the beam with local and modal control systems

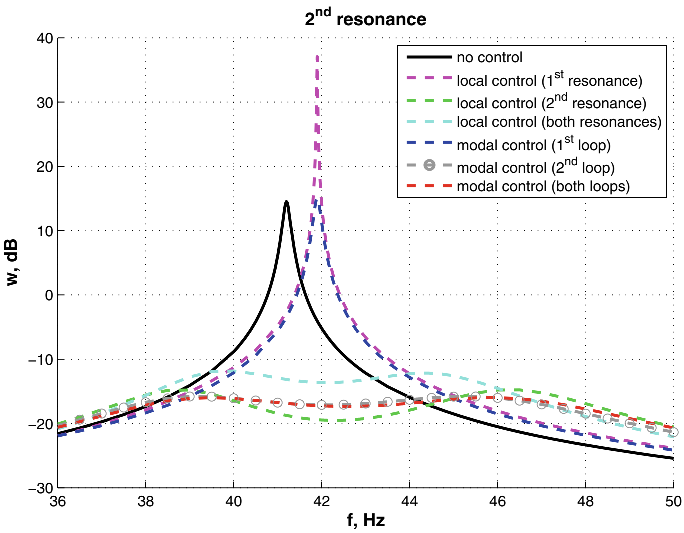

Fig. 8.12 Compensation of the second resonance of the beam with local and modal control systems

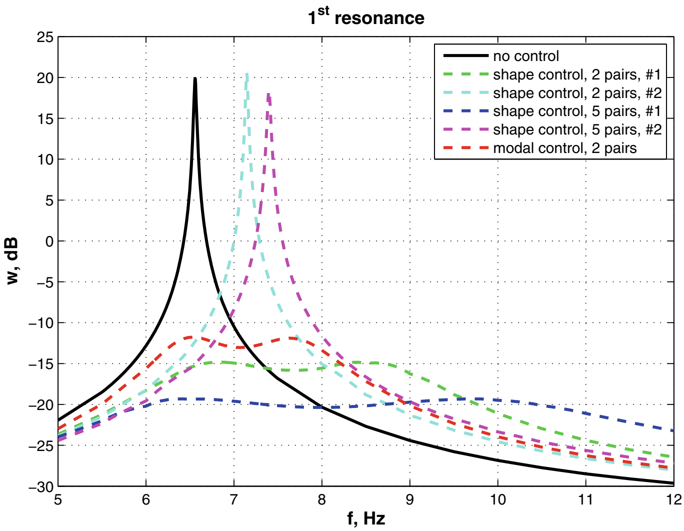

Fig. 8.13 Compensation of the first resonance of the beam with modal and shape control systems

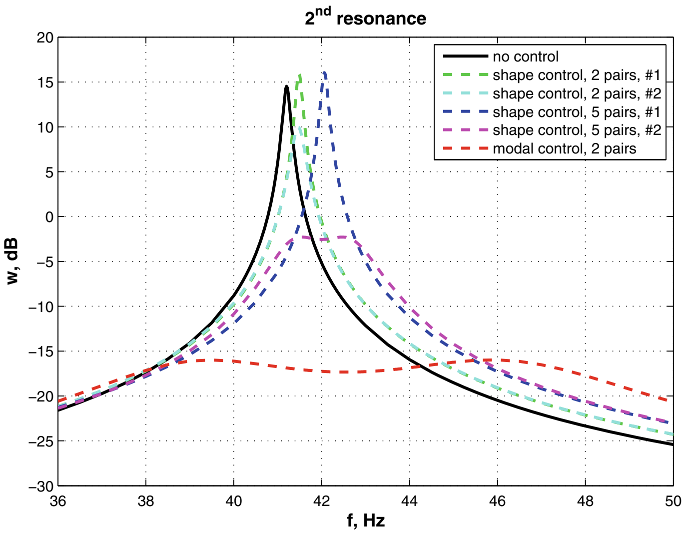

Fig. 8.14 Compensation of the second resonance of the beam with modal and shape control systems

the first resonance, and the second loop (upper sensor–actuator pair)—the second resonance. The first and the second modal systems have only one active feedback loop—either the first (corresponding to the first bending mode of the beam) or the second (corresponding to the second bending mode), while in the third system both loops are active.

After that, four shape control systems were obtained (Figs. 8.13, 8.14). The first and the second variants of the shape control system with two sensor–actuator pairs were designed to suppress the first and the second resonances of the beam, respectively, and the same is true for the systems with five sensor–actuator pairs. The results for these four systems are compared to the results for the modal control system with two feedback loops.

The results of all the obtained control systems are summarized in Table 8.1. As can be seen from the table, the local and modal systems tested experimentally are much less effective than the systems obtained numerically. This result emphasizes the effectiveness of the optimization procedure used to design the latter systems. The second conclusion is that the modal system with two loops is the most efficient at both resonances among all local and modal systems. The local system №3 is works also very well at both resonances, but its efficiency is lower compared to the modal system. The results of the local system №1 is close to the local system №3 at the first resonance, and the same holds for local systems №2 and №3 at the second resonance. Similarly, the results of the modal system with two loops practically repeat the individual results of the modal systems with only one loop at corresponding resonances. This means that two loops almost do not interfere with each other not only in the modal system, but, surprisingly, in the local system too.

Table 8.1 Change in the level of endpoint vibrations of the beam at the first and the second resonances for different control systems (experiment and simulation)

Control system	Δw_1, dB	Δw_2, dB
Local control, №1 (exp.)	−12.7	4.8
Local control, №2 (exp.)	−5.2	−18.9
Modal control, both loops (exp.)	−15.7	−17.9
Local control, №1	−30.48	22.55
Local control, №2	0.67	−29.29
Local control, №3	−30.47	−26.37
Shape control, two pairs, №1	−34.69	1.42
Shape control, two pairs, №2	0.72	−4.57
Shape control, five pairs, №1	−39.19	1.51
Shape control, five pairs, №2	−1.47	−15.82
Modal control, first loop	−31.7	0.57
Modal control, second loop	0.31	−30.33
Modal control, two loops	−31.65	−30.5

It can also be seen that the first variants of the shape control systems work only at the first resonance, while the second variants of the shape control systems suppress vibrations only at the second resonance. At the same time, the shape control systems with five sensor–actuator pairs are more efficient than the similar systems with two pairs, what is expected. At the first resonance, the shape control systems are more effective than the modal system; on the contrary, at the second resonance, the modal system with two sensors and two actuators is much more effective even than the shape control system with five sensor–actuator pairs. Therefore, the modal control strategy is preferable compared to the shape control strategy in the cases, where it is necessary to suppress vibrations at several resonance frequencies.

8.8 Conclusion

Within the present study, the problem of active suppression of forced bending vibrations of a thin metal beam was analyzed experimentally and numerically. The purpose of the designed control systems was to suppress forced vibrations in the frequency range containing the first and the second eigenfrequencies of bending vibrations of the beam. Different control systems based on each of the three strategies (local, modal and shape control strategy) were tested and compared to each other. All considered systems contained piezoelectric sensors and actuators, but the number of piezopatches and feedback loops in these systems was different. The local and modal systems included two sensor–actuator pairs and two feedback loops, while in the shape control systems either two or five pairs of piezopatches integrated into only one control loop were used.

The results of the investigation show that local and modal systems can demonstrate effective vibration suppression at both resonances together, and the efficiency of the modal system is higher than the local one (more than 30 dB resonance amplitude suppression for the systems under consideration). It should be noted that the experimental results for local and modal systems were significantly improved by the numerical optimization procedure. At the same time, the efficiency of the shape control systems at the first resonance is much greater than at the second resonance (for the system with five sensor–actuator pairs, the results are, respectively, 39 dB and 16 dB). This can be explained by the fact that the distribution of the control action of the shape control system is designed to compensate the static disturbance, which causes the deflection of the beam close to the first bending eigenmode. Therefore, the influence of the control loop of this system on the second eigenmode of the beam is rather small. On the contrary, in the modal system, each control loop corresponds to the particular bending mode of the beam. The lower result of the modal control system at the first resonance compared to the shape control systems can be explained by the fact that the distribution of the control action in the first control loop of the modal system is specified not to compensate the first mode most effectively, but it is specified from the condition of not affecting the second mode (in order to separate the modes).

In summary, the shape control strategy is the best choice if it is needed to suppress vibrations only at the first resonance of the beam, but not at the higher resonances. In the cases, where it is necessary to suppress vibrations at several resonance frequencies, the modal control strategy gives the best result.

Acknowledgements This work was supported by the Ministry of Science and Higher Education of the Russian Federation (Project Number 075-15-2021-573).

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