Euclid's Elements: Geometric Figures in the Copy of the Yuso Monastery, in San Millán de la Cogolla (Spain)

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Abstract. Euclid's treatise The Elements, first printed in the 15th century, is unanimously recognized as one of the most widely disseminated books in history as a whole, along with the Bible. The publisher of the first printing, Erhard Ratdolt (1442–1528), acknowledged in the preface that the real reason for the absence of publication of mathematical texts at the time lay in the difficulty of printing their geometric figures. And it was this master printer who contributed to transcribing geometric figures with the same ease as letters.

The mathematician Luis Javier Hernández Paricio, speaks, in his "On the Fundamental Principles of Geometry: inaugural opening lesson of the academic year 20002001", and in relation to the incunabula of the Yuso Monastery, of "the admirable geometric figures that appear in its margins". And it also talks about how well preserved it is compared to the other six copies that are kept in Spain, some of which would have their margins cut the observation of these figures and their analysis, by comparison to the text written in the volumes, is the objective of the publication presented here. And also stand out in having a copy in the small town of San Millán de la Cogolla.

Keywords: Treatise · Elements · Euclid · Geometry · Yuso

1 Introduction

The opinion held on the Elements of Euclid is universally accepted as the first fundamental mathematical work, founder of Geometry as a scientific discipline and, above all, the one that has had the most influence to this day. Whichever reference is sought, in all of them the accent is placed on it being the most published book in history as a whole, along with the Bible. Its compiling nature is undoubtedly one of its main values (not the only one), along with the fact that, the putting into crisis of some of its postulates generated various scientific controversies around geometry, and enriched it even the point of this treatise being the real origin of the appearance, in the 19th century, and through various channels, of the so-called Non-Euclidean Geometries. Whichever reference is sought, in all of them the accent is placed on it being the most published book in history as a whole, along with the Bible. Its compiling nature is undoubtedly one of its main values (not the only one), along with the fact that, the putting into crisis of some of its

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Let us think that Non-Euclidean Geometries are, in the words of Professor Hernández Paricio,

"one of the most spectacular advances that has been made during the second millennium in the world of mathematical thought."

Indeed, the controversy that generated, for centuries, the attempts to demonstrate the fifth postulate, better known as the "axiom of parallels", with its complex and intricate wording, opened a history of more or less successful attempts that led, twentyone centuries later, in the appearance of non-Euclidean Geometries, that is, those that do not satisfy the postulates of Euclid of Alexandria.

Euclid's treatise, insofar as it is a text dated around 300 BC. and is, therefore, twenty-three centuries old, it has a history rich in historical vicissitudes, which have to do with its translation into different languages and with the legacy of one generation to another.

2   Relevance of the Venetian Edition of Euclid's Treatise

The copy of the monastery of the Augustinian Recollects (of which there are only five more copies in Spain), corresponds to the edition of the master printer and typographer Erhard Ratdolt, of German origin and installed in Venice since 1476. It is one of the scientific books most beautiful prints, and their importance is advanced by himself in the preface to Euclid's Treatise. Thus, the editor describes how the impossibility of typographically representing technical drawings was an obstacle when it came to publish and disseminating mathematical texts. His contribution consisted in giving the sheets an eight-centimeter margin on the sides so that the graphics that accompanied the mathematical texts could be clearly represented with the same printing press. The resolution of this typographic problem viralized the printed edition of the Elements, to the point that it thus became the main mathematical book of the Western European schools until the 16th century.

This edition corresponds to an Arabic-Latin version commented in the middle of the 13th century by the Italian mathematician and chaplain of Pope Urban IV Campano de Novara, who in turn reworked the translation from Arabic to Latin by the English monk Adelardo de Bath, in the 12th century. In other words, we are talking about a text whose transmission throughout history corresponds to that of many other academic texts of Greek origin. These were first translated from Greek to Arabic in the 9th century; later translated into Latin in the 12th century; later, after the appearance of the printing press, these Latin versions were printed in the 15th century and later, editions of Latin versions were promoted from the Greek, thus facilitating the subsequent appearance of the "editio princeps" of the text itself Greek. The vernacular versions would come later.

The propagation of the Ratdolt edition of 1482 therefore predates even the prince edition, a fact that further highlights the significance of this edition. Despite the fact that Professor López Pellicer, in his speech at the commemorative session of the Instituto España Foundation (2003), indicated the edition corresponding to the Yuso incunabula as the princeps edition, other sources consider that the one published in Basel really is.

half a century later, in 1533 by the German theologian Simon Grynaeus the Elder and which is now written in Greek. The existence in Spain of this princeps version has been confirmed in, at least, the library of the University of Seville and in the library of the Monastery of El Escorial.

Finally, and going back to the Venetian edition, it is worth highlighting the beauty of its cover (Fig. 1), where there is a stormy three-edge engraving (upper, lower and left side) that would be used in other editions, becoming like this a kind of corporate brand. This engraving includes the title inside, in red ink, which reads: "Preclarissimus liber elementorum Euclidis perspicacissimi in artem Geometrie incipit qua [m] foelicissime".

Throughout the book (there are actually thirteen, but in a single binding), you can see how to achieve great precision and delicacy in printing the drawings, even with curved lines, especially if they are circumferences. In the elliptical arches, however, it does not seem to have reached such a perfect typeface.

Fig. 1. Handwritten and color annotations, both on the first page (left) and right on the graph of the axiom of parallels (right). Copy of Yuso.

3   Spanish Copies and the Uniqueness of Yuso

It is still an anecdotal fact that Yuso's copy rests in a place called "el infiernillo" along with other books that were banned or censored by the Holy Inquisition. Indeed, alongside these apocrypha, our treatise occupies a place. The objective of the analysis presented here is clear. Of the version that was printed for the first time in Venice in 1482, there are only six copies in Spain: three in Madrid, one in Barcelona, one in Valladolid and another in San Millán de la Cogolla, in La Rioja. According to the illustrious institutions that house a copy of such a valuable copy (see Table 1), it can be clearly said of the

San Millan copy that it is the most remote. And the presence of this incunabula in the monastery library deepens the idea that it is the owner of books of special historical singularity.

Table 1. Parties to the treaty elements

Parties to the treaty		Thematic
Basic plane geometry	Book I	Properties of triangles, theory of parallels, polygons, Pythagorean Theorem and its inverse
	Book II	Areas and geometric algebra of the Pythagorean academy. Cosine law
	Book III	Circumference, chords, tangents and angle measurement
	Book IV	Regular polygon constructions with ruler and compass
	Book V	Commensurable and incommensurable magnitudes. Irrational numbers
	Book VI	Similar triangles, proportional measures. Quadric equations. Proposition of the bisector
Prime and radical numbers, divisibility	Book VII	Prime numbers. Arithmetic Legacy of Pythagoras
	Book VIII	Series of numbers in continuous proportion and geometric progression
	Book IX	Even, odd, relationships Prime factors
	Book X	Irrationals
Solids, polyhedra, circumstantial spheres	Book XI	Solid geometry. Platonic solids
	Book XII	
	Book XIII	

It has been possible to verify on the site that Yuso's copy corresponds to that of Erhard Ratdolt, and it has also been possible to compare, thanks to the digitization of the institutions' funds, with other copies that exist not only in Spain but with some from other countries. Two characteristics of it are that it is restored, and that it presents handwritten comments, presumably in Latin, accompanying the drawings, during the first half of the book (Table 2).

Table 2. The 6 copies in Spain of the 1st printed edition of Erhard Ratdolt (1482).

Year	Edition	City	Institution	Link	Observations
1482	Ratdolt Venice	San Millán de la Cogolla	Yuso Monastery	Yuso	Latin annotations in margins in the first half of the book
		Madrid	National Library	B.Nacional	Digitization in b/w. Handwritten footnotes, different from Yuso's
			Spanish Royal Palace	Available upon request	Ex libris manuscript of Fouquier Brunet, S XVIII
			Casa de Alba	In-person consultation by appointment at Palacio de Liria	Bought in London in 1950 by the Duke of Alba
		Barcelona	Athenaeum	Barcelona	
		Valladolid	University	Valladolid	

Pérez Barriocanal and Sacristán Marín, authors of the catalog of the collection of the library of the monastery between the 15th and 18th centuries, clarify that Yuso's copy is printed with Gothic letters, with 45 lines per page and with the cover in two inks, red and black. (There are at least 7 copies in Germany with dedications in gold letters). But, unfortunately, they neither verify the possible origin of the specimen, nor do they identify the handwriting with any monk of the monastery (Tables 3 and 4).

Table 3. Some international copies of the 1st printed edition of Erhard Ratdolt (1482).

Año	Edición	Ciudad	Institución	Enlace	Observaciones
1482	Ratdolt Venecia	Munich	German National Library	Munich	Dedication in gold letters
		Berkeley, California	Bancroft Library	Berkeley	Gift of bibliophile Julius Wangenheim

Tabla 4. Copies of the princeps edition in Spain (1533).

Year	Edition	City	Institution	Link	Observations
1533	Simon Grynaeus Basilea	El Escorial	El Escorial Monastery	Escorial	In greek
		Seville	Sevilla University	Sevilla	

The monks have not dated or identified how this specimen could reach the Cárdenas valley. After being consulted about the incunabula, they also confirmed the difficulties of knowing its origin, alluding only to the interest of the friars, as forerunners of culture, in possessing books of the highest intellectual capacity; apparently it was common between monasteries the exchange of the same. It is important to note at this point that the Yuso monastery, as we know it today, was completely rebuilt in a style close to the Herrerian or Escurialense, in the 16th century. And it could be considered, therefore, that the Benedictine Order had tried to have books of notable reference in its new library. The preservation of this and other essential references in its library is also surprising, even knowing the multiple expulsions, plunder, confiscation and abandonment that the Royal Monastery has suffered throughout its history. Currently, it is a World Heritage Site in Europe, due to various cultural and historical facts of interest, the most relevant being the appearance of the codex with the Emilian Glosses, considered the first words written in Spanish (Figs. 2 and 3).
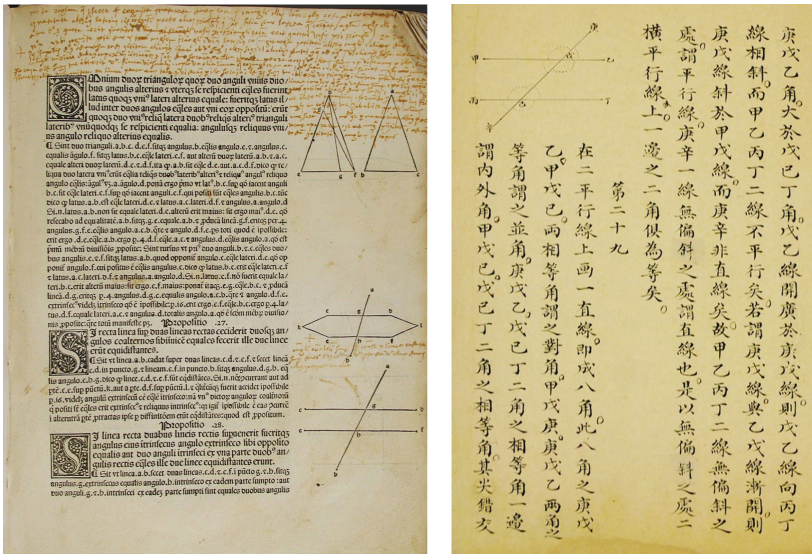

Fig. 2. Book I. Proposition 28. If a segment when incident on two lines makes the alternate angles equal to each other, the two lines will be parallel to each other. Yuso copy (left) and Chinese handwritten version of the s. XVII with the same graphic.
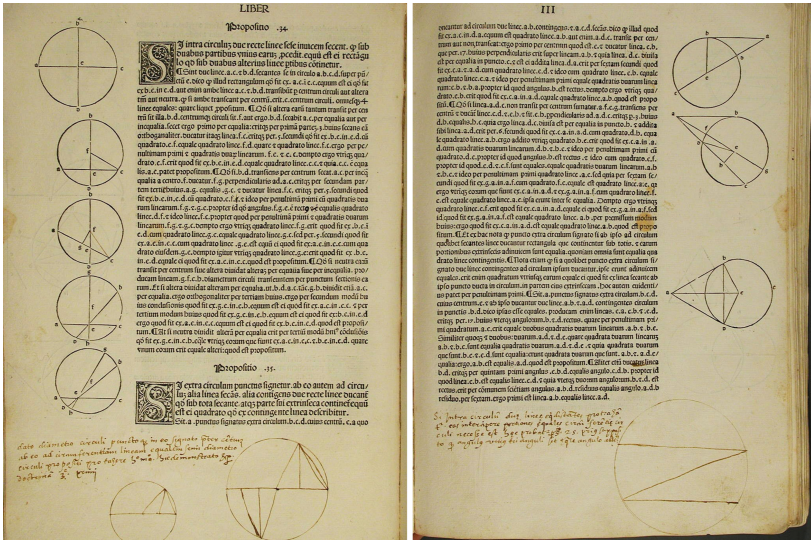
Fig. 3. Book III, proposition 35 (left and right): If two lines intersect each other in a circle, the rectangle comprised by the segments of one is equal to the rectangle comprised by the segments of the other. (Power of a point with respect to a circle). You can check the amount of handwritten annotations in red, in Yuso's copy.

Table 5. Other examples of interest

Year	Edition	City	Institution	Link	Observations
1576	Rodrigo Zamorano - Sevilla	Seville	Seville University Library	Sevilla	First edition in Spanish
1690	I.G., Pardies, J.F. Gerbillon, J. Bouvet	Beijing	Beijing National Library	Pekín	Chinese translation of two French Jesuit missionaries for the Kangxi Emperor
2ª half XVIII	Juan González Sysniega,	Logroño	Local library	Logroño	Handwritten in Spanish. All drawings at the end

Another curiosity worthy of mention is in the latest handwritten version, in Spanish, that is kept in the Public Library of La Rioja, corresponding according to the figure on its cover to Juan González de Sisniega, who died in 1805 as captain of the ship in battle from Trafalgar. On the first page of the book, you can read "This book is by Dn (sic) Juan Gonzalez de Sysniega, Royal Navy frigate ensign" (Table 5).

In this article, up to four tables are presented with the editions consulted. Due to the facilities that the consultation of funds that have been digitized presents, the link of the same is also provided, so that it is very immediate to go to the specific edition. The comparison between some editions and others is of enormous interest, taking into account their origin and their country of destination.

References

1. Clagett, M.: The medieval latin translations from the arabic of the elements of euclid with special emphasis on the versions of adelard of bath. ISIS J. **43**(1/2), 16–42 (1953)
2. García Turza, J.: El Monasterio de San Millán de la Cogolla. Una historia de santos, copistas, canteros y monjes. 1 edn. Ediciones Everest. Madrid (2013)
3. Hernández Paricio, L.J.: Sobre los principios fundamentales de la geometría: lección inaugural del curso académico 2000–2001. Universidad de La Rioja, Logroño (2001)
4. López Pellicer, M.: Vida de los trece libros de Euclides, 1st edn. Fundación del Instituto de España, Madrid (2003)
5. González Urbaneja, P.M.: Los elementos de Euclides. Biblia de la geometría griega. 1era edición. Editorial Plaza Abierta, Barcelona. (2018)
6. Pérez Barriocanal, C., Sacristán Marín, E.: Catálogo de impresos s.XV-XVIII Monasterio de Yuso. Parlamento de La Rioja. Logroño (1999)
7. www.euclides.org
8. www.euler.us.es https://euler.us.es/~libros/griegos.html. recuperado por última vez en 2021/03/27
9. www.todolibroantiguo.es   http://www.todolibroantiguo.es/libros-raros/elementoseuclides.html. recuperado por última vez el 27 Mar 2021
10. www.wdl.org https://www.wdl.org/es/item/7103/ recuperado por última vez en 27 Mar 2021
11. https://www.christies.com/en/lot/lot-6296916