# Viscoplastic Fluid Flow in a T-shaped Channel Under Given Pressure Boundary Conditions

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**Abstract.** A numerical simulation of a non-Newtonian incompressible fluid flow in a planar T-shaped channel is performed. A mathematical basis of the problem includes the momentum and continuity equations written in the dimensionless form. The rheology of the fluid is specified by the Shvedov-Bingham law. No-slip boundary conditions are assigned on the solid walls. The motion of the fluid is caused by a given pressure difference between inlet and outlet boundaries. The flow is assumed to be laminar and steady. A numerical solution to the problem is obtained using the finite volume method and the SIMPLE procedure. The rheological equation is regularized to provide both a finite value of the viscosity in the regions of low strain rates and the stability of the computational algorithm. Hydrodynamic characteristics of the viscoplastic fluid flow are studied at various dimensionless criteria and pressure values assigned at the inlet/outlet sections. The boundaries of the unyielded regions, occurring in the flow, are determined. The flow patterns with the unyielded region overlapping one of three boundary sections of the channel are presented.

**Keywords:** Laminar flow · Non-Newtonian fluid · T-shaped channel · Boundary conditions · Numerical simulation

## 1 Introduction

Pipeline systems, which are used to transport liquids and gases in different technologies, consist of many elements. One of the elements is a T-shaped channel serving for branching and mixing the flows. Studies on the flows in such structural elements are of practical importance for irrigation systems, chemical and petroleum industries, and other industries [1, 2]. Over the past decade, a large number of the results related to a fluid flow in T-microchannels have been obtained. These data are used in biomedicine for transportation of nanoparticles, bacteria, DNA molecules, as well as in a cooling technique for microelectronic devices [3, 4].

One of the main features of a T-shaped channel is that the design includes three boundaries which can serve either as inlet or as outlet sections. There is no commonly accepted opinion on the preferred conditions to be preassigned on the inflow/outflow boundaries of the channel of such geometry, neither from mathematical nor from physical standpoints [5]. By present time, many studies on the Newtonian [6–11] and non-Newtonian


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[11–15] fluid flows in T-shaped channels with a given velocity profile have been carried out. However, in many cases the velocity profile at the inflow/outflow boundaries is unknown. Therefore, it is more advantageous to specify pressure values instead of velocity profile, and afterwards to determine the flow rate and flow directions. Nowadays, a limited number of works devoted to viscous fluid flows use pressure as boundary conditions [16–22].

In this paper, a steady flow of an incompressible Shvedov-Bingham viscoplastic fluid in a planar T-shaped channel under the given pressure boundary conditions is considered. The problem is solved numerically using a self-developed software package. The computed solution is analyzed depending on main parameters of the problem.

## 2 Problem Formulation

A laminar steady flow of an incompressible viscoplastic fluid in a planar T-shaped channel is considered. Figure 1 demonstrates the flow region geometry; the origin of the Cartesian coordinate system is located at point $A$. The fluid flows into or out of the channel through boundary sections $AM$, $FE$, and $BC$ under the given pressure drop. Mathematical formulation of the problem includes the momentum and continuity equations written in the dimensionless vector form, as follows:

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nabla \cdot (2\eta \mathbf{E}), \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0. \tag{2}$$

Here, $\mathbf{u} = (u, v)$ is the dimensionless velocity vector, $p$ is the dimensionless pressure, and $\mathbf{E}$ is the dimensionless strain-rate tensor.

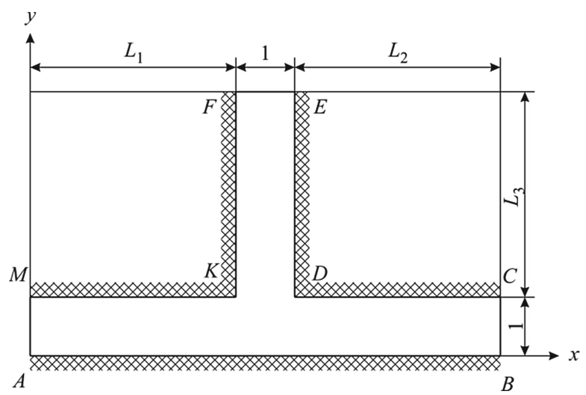


**Fig. 1.** Flow region

The rheological behavior of viscoplastic fluids is described by the Shvedov-Bingham model. The corresponding apparent viscosity in the dimensionless form is determined by the following expression:

$$\eta = \frac{Bn + A}{A}, \tag{3}$$

where $A = \left(2e_{ij}e_{ji}\right)^{0.5}$ is the dimensionless intensity of the the strain-rate tensor, $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ are the components of the strain-rate tensor, and $Bn = \frac{\tau_0 L}{\mu U_0}$ is the Bingham number. At $Bn = 0$, the Shvedov-Bingham model describes the rheological behavior of the Newtonian fluid.

The following values are used to scale the length and the velocity: $L$ (the width of boundary section $AM$) and $U_0 = \mu/\rho L$, respectively. The dimensionless pressure is calculated by formula

$$p = \frac{P - P_{FE}}{\mu^2/\rho L^2}. \tag{4}$$

Here, $\mu$ is the plastic viscosity of the fluid, $\tau_0$ is the yield stress, $\rho$ is the density of the fluid, $P$ is the dimensional pressure, $P_{FE}$ is the dimensional pressure specified at boundary $FE$.

An essential feature of the Shvedov-Bingham model is the presence of a "yield stress" [23], i.e. a critical value of the shear stress below which the fluid retains a rigid structure and moves like a solid. The region of this motion is termed "unyielded". When the yield stress is exceeded, the destruction of the solid structure immediately occurs, and the fluid moves in accordance with Newton's law of viscosity.

On the solid walls, no-slip boundary conditions are imposed:

$$\begin{aligned} u &= 0, \\ v &= 0. \end{aligned} \tag{5}$$

In boundary sections $AM$, $FE$, and $BC$, zero tangential components of the velocity vector and the given pressure values are specified. These boundary conditions can be written in terms of dimensionless variables as follows:

$$\begin{array}{ll} v = 0,\ p_{AM} = p_1,\ x = 0, & 0 \le y \le 1; \\ u = 0,\ p_{FE} = 0,\quad L_1 \le x \le L_1 + 1, & y = L_3 + 1; \\ v = 0,\ p_{BC} = p_3,\ x = L_1 + L_2 + 1, & 0 \le y \le 1. \end{array} \tag{6}$$

Here, $L_1$, $L_2$, and $L_3$ are the dimensionless geometric parameters of the flow region (Fig. 1). The considered T-shaped channel has branches of unit width and of the same length ($L_1 = L_2 = L_3 = 3$). For such a problem formulation, the flow characteristics are dependent on the Bingham number and on the pressure values $p_1$ and $p_3$ assigned at boundaries $AM$ and $BC$, respectively.

## 3 Numerical Method and Verification

A self-developed software package is used to solve the formulated problem. A finite volume method [24] is applied to rewrite the initial system of equations in the discrete differential form. The correction of the velocity and pressure fields is carried out utilizing the SIMPLE procedure [25]. Steady fields of the velocity and pressure are obtained using the false transient method [26].

The Shvedov-Bingham model has a feature of "infinite" apparent viscosity as the intensity of the strain-rate tensor approaches zero ($A \to 0$). Using a through calculation of the viscoplastic fluid flow without explicit separating of unyielded regions, the rheological model is regularized to eliminate the singularity in the regions of zero values of $A$. In this study, a modified rheological model [27] is utilized according to which the apparent viscosity is defined by formula

$$\eta = \frac{Bn + \sqrt{A^2 + \varepsilon^2}}{\sqrt{A^2 + \varepsilon^2}}, \tag{7}$$

where $\varepsilon$ is the small regularization parameter. The unyielded regions are distinguished in the flow by the following expression:

$$\eta A \leq Bn, \tag{8}$$

which represents a dimensionless analog of the condition for separation of the flow regions where the shear stress is below the yield stress.

During the software development, test calculations have been carried out. The results of the approximation convergence verification for the computational algorithm are presented in [28]. A comparison with experimental and numerical data of other authors is also shown in [28].

## 4 Results and Discussion

Figure 2 demonstrates characteristics of the flow at $p_1 = 300$, $p_3 = 160$, and $Bn = 1$. According to Fig. 2 (a), illustrating streamline distributions, the fluid flows into the channel through section $AM$ and flows out through section $FE$; a negative value of the flow rate corresponds to the case when the fluid flows out of the T-shaped channel. There is no flow in the right branch of the T-shaped channel due to a dead zone formation in this part of the region where the shear stress is below the yield stress. As a consequence, the flow rate through boundary section $BC$ is equal to zero. One-dimensional motion of the viscoplastic fluid with a fully-developed velocity profile is observed in the vicinity of boundaries $AM$ and $FE$.

The flow structure and the viscosity field at $p_1 = 300$, $p_3 = 160$, and $Bn = 1$ are shown in Fig. 3. The unyielded regions are painted in black and formed in each branch of the T-shaped channel. The unyielded regions in the left and middle branches are of the same size, and the largest one is observed in the right branch.

Further results are related to a study of the influence of the viscoplastic fluid rheology on the flow pattern in the T-shaped channel. It is found that an increase in the Bingham number leads to the growth of the unyielded regions. Figure 4 and Fig. 5 demonstrate the flow structures observed for the Bingham number varying in the range of $0 \leq Bn \leq 8$. Figure 4 (a) shows that there is no unyielded region at $Bn = 0$, when the fluid flows in accordance with Newton's law of viscosity. An unyielded region is formed in

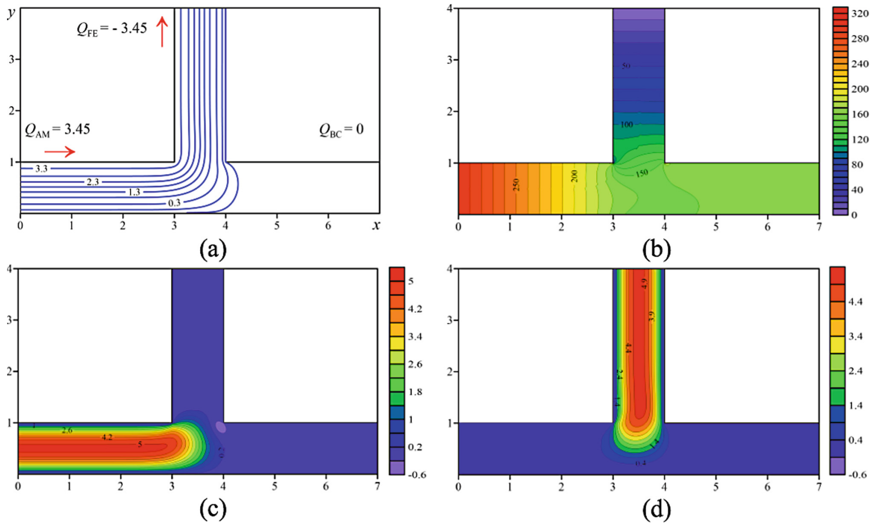


**Fig. 2.** Characteristics of the flow at $p_1 = 300$, $p_3 = 160$, $Bn = 1$: (a) – the streamlines, (b) – the field of pressure $p$, (c) – the field of velocity $u$, and (d) – the field of velocity $v$

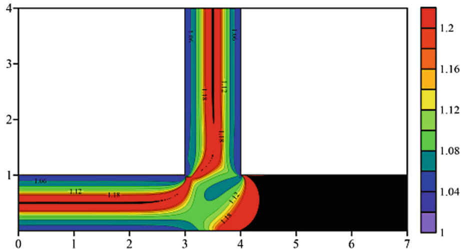

**Fig. 3.** Flow structure and a viscosity field at $p_1 = 300$, $p_3 = 160$, $Bn = 1$

the right branch of the T-shaped channel with an increase in the Bingham number, and it completely overlaps this part of the channel at $Bn = 1$. As a consequence, a viscoplastic plug is observed in the right branch of the channel.

The flow structure at $Bn > 1$ is presented in Fig. 5. Further increase in the Bingham number leads to the formation of two unyielded regions in the left and middle branches of the channel, where the one-dimensional motion of the fluid is observed. The higher the Bingham number is, the larger unyielded regions occur; while the dead zone boundaries remain the same in the right branch of the T-shaped channel.

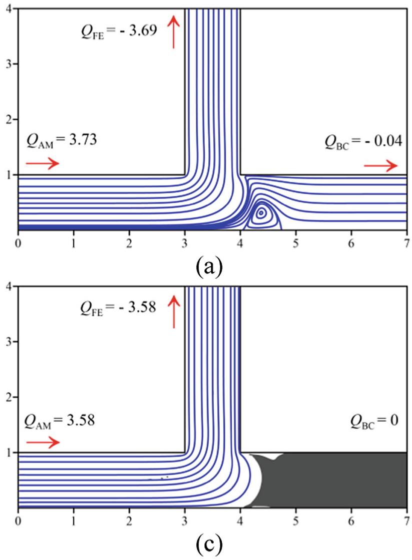


**Fig. 4.** Flow structures at $p_1 = 300$, $p_3 = 160$: (a), (b), (c), (d) – $Bn = 0$, 0.25, 0.5, and 1

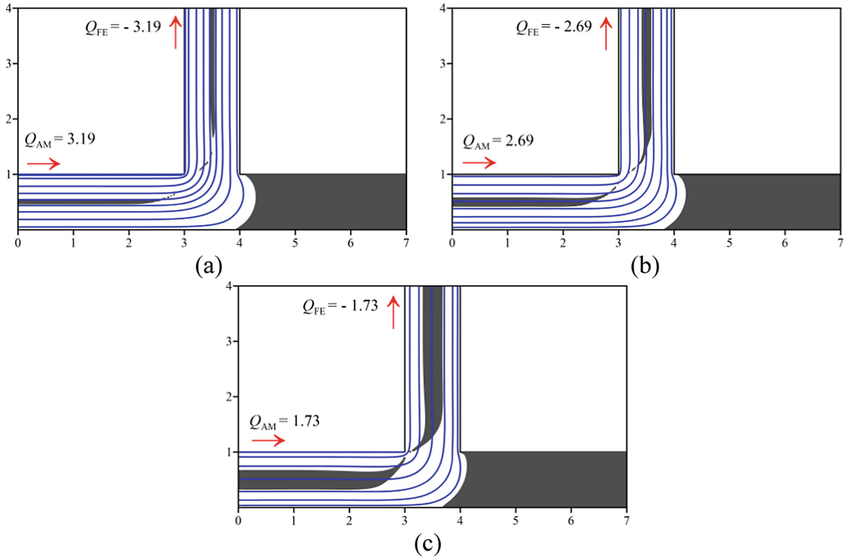


**Fig. 5.** Flow structures at $p_1 = 300$, $p_3 = 160$: (a), (b), (c) – $Bn = 2$, 4, and 8

## 5 Conclusion

In this work, a numerical simulation of a steady laminar flow of an incompressible Shvedov-Bingham viscoplastic fluid in a planar T-shaped channel under the given pressure boundary conditions has been carried out.

The effect of dimensionless parameters (the Bingham number, the pressure values $p_1$ and $p_3$ specified at boundaries $AM$ and $BC$, respectively) on the flow pattern as well as on the kinematic and dynamic characteristics of the flow has been studied. The flow patterns with the formation of unyielded regions and a dead zone in one of the channel branches have been demonstrated at the Bingham number varying from 0 to 8.

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## References

1. Grace, J.L., Priest, M.S.: Division of flow in open channel junctions. Engineering Experiment Station Auburn, Alabama (1958)
2. Neofytou, P., Housiadas, C., Tsangaris, S.G., Stubos, A.K., Fotiadis, D.I.: Newtonian and power-law fluid flow in a T-junction of rectangular ducts. Theoret. Comput. Fluid Dyn. **28**(2), 233–256 (2014)
3. Kovalev, A.V., Yagodnitsyna, A.A., Bilsky, A.V.: Flow hydrodynamics of immiscible liquids with low viscosity ratio in a rectangular microchannel with T-junction. Theor. Found. Chem. Eng. **352**, 120–132 (2018)
4. Lobasov, A.S., Minakov, A.V., Rudyak, V.Y.: Flow modes of non-Newtonian fluids with power-law rheology in a T-shaped micromixer. Theor. Found. Chem. Eng. **52**(3), 393–403 (2018)
5. Sani, R.L., Gresho, P.M.: Résumé and remarks on the open boundary condition minisymposium. Int. J. Numer. Meth. Fluids **18**(10), 983–1008 (1994)
6. Liepsch, D., Moravec, S., Rastogi, A.K., Vlachos, N.S.: Measurement and calculations of laminar flow in a ninety degree bifurcation. J. Biomech. **15**(7), 473–485 (1982)
7. Khodadadi, J.M., Nguyen, T.M., Vlachos, N.S.: Laminar forced convective heat transfer in a two-dimensional 90 bifurcation. Numer. Heat Transf. **9**, 677–695 (1986)
8. Neary, V.S., Sotiropoulos, F.: Numerical investigation of laminar flows through 90-degree diversions of rectangular cross-section. Comput. Fluids **25**(2), 95–118 (1996)
9. Shamloo, H., Pirzadeh, B.: Investigation of characteristics of separation zones in T-junctions. WSEAS Transaction on Mathematics. **7**(5), 303–312 (2008)
10. Beneš, L., Louda, P., Kozel, K., Keslerová, R., Štigler, J.: Numerical simulations of flow through channels with T-junction. Appl. Math. Comput. **219**(13), 7225–7235 (2013)
11. Miranda, A.I.P., Oliveira, P.J., Pinho, F.T.: Steady and unsteady laminar flows of Newtonian and generalized Newtonian fluids in a planar T-junction. Int. J. Numer. Meth. Fluids **57**(3), 295–328 (2008)
12. Khodadadi, J.M., Vlachos, N.S., Liepsch, D., Moravec, S.: LDA measurements and numerical prediction of pulsatile laminar flow in a plane 90-degree bifurcation. J. Biomech. Eng. **110**(2), 129–136 (1988)
13. Khodadadi, J.M.: Wall pressure and shear stress variations in a 90-deg bifurcation during pulsatile laminar flow. J. Fluids Eng. **113**(1), 111–115 (1991)
14. Matos, H.M., Oliveira, P.J.: Steady and unsteady non-Newtonian inelastic flows in a planar T-junction. Int. J. Heat Fluid Flow **39**, 102–126 (2013)
15. Khandelwal, V., Dhiman, A., Baranyi, L.: Laminar flow of non-Newtonian shear-thinning fluids in a T-channel. Comput. Fluids **108**, 79–91 (2015)
16. Kuznetsov, B.G., Moshkin, N.P., Smagulov, S.: Numerical investigation of viscous incompressible fluid flows in channels of complicated geometry with given pressure. Proc. Inst. Theor Appl. Mech. Siberian Branch USSR Acad. Sci. **14**(15), 87–99 (1983)

17. Hayes, R.E., Nandakumar, K., Nasr-El-Din, H.: Steady laminar flow in a 90 degree planar branch. Comput. Fluids **17**(4), 537–553 (1989)
18. Heywood, J.G., Rannacher, R., Turek, S.: Artificial boundaries and flux and pressure conditions for the incompressible Navier-Stokes equations. Int. J. Numer. Meth. Fluids **22**(5), 325–352 (1996)
19. Kelkar, K.M., Choudhury, D.: Numerical method for the prediction of incompressible flow and heat transfer in domains with specified pressure boundary conditions. Numer. Heat Transfer, Part B **38**(1), 15–36 (2000)
20. Fernandez-Feria, R., Sanmiguel-Rojas, E.: An explicit projection method for solving incompressible flows driven by a pressure difference. Comput. Fluids **33**(3), 463–483 (2004)
21. Barth, W.L., Carey, G.F.: On a boundary condition for pressure-driven laminar flow of incompressible fluids. Int. J. Numer. Meth. Fluids **54**(11), 1313–1325 (2007)
22. Moshkin, N., Yambangwi, D.: Steady viscous incompressible flow driven by a pressure difference in a planar T-junction channel. Int. J. Comut. Fluid Dyn. **23**(3), 259–270 (2009)
23. Balmforth, N.J., Frigaard, I.A., Overlez, G.: Yielding to stress: recent developments in viscoplastic fluid mechanics. Annu. Rev. Fluid Mech. **46**, 121–146 (2014)
24. Klapp, J., Medina, A., Cros, A., Vargas, C.A.: Fluid Dynamics in Physics, Engineering and Environmental Applications. Springer-Verlag, Berlin (2013)
25. Patankar, S.: Numerical Heat Transfer and Fluid Flow. McGraw-Hill, New York (1980)
26. Godunov, S.K., Ryabenkii, V.A.: Difference Schemes. Nauka, Moscow (1977).(in Russian)
27. Bercovier, M., Engelman, M.: A finite-element method for incompressible non-Newtonian flows. J. Comput. Phys. **36**(3), 313–326 (1980)
28. Borzenko, E.I., Dyakova, O.A., Shrager, G.R.: Laminar power-law fluid flow in a T-shaped channel at given pressure differences. Fluid Dyn. **54**(4), 501–509 (2019)