Shape Control and Modal Control Strategies for Active Vibration Suppression of a Cantilever Beam

Aleksandr V. Fedotov(✉)

Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, V.O., Bolshoj pr. 61, 199178 Saint Petersburg, Russia
alvafed@yandex.ru

Abstract. The study is devoted to the numerical simulation and comparison of two different control strategies, shape control and modal control, applied to the problem of suppression of forced bending vibrations of a thin metal cantilever beam at the first and the second resonance frequencies. The shape control strategy is based on the compensation of known distribution of the external disturbance in the static case, while the modal control strategy implies the correspondence between the control loops and the vibration modes of the object. The results show that the modal system can be efficient at both resonance frequencies. The shape control strategy provides efficient vibration suppression only at the first resonance, while at the second resonance frequency it is significantly less effective than the modal approach. Therefore, the modal method is preferable to the shape control method in the cases where it is necessary to suppress forced vibrations at several resonance frequencies of the object.

Keywords: Active vibration suppression · Shape control · Modal control · Piezoelectric sensors and actuators

1 Introduction

The present paper is devoted to the problem of vibration suppression of continuous systems, which is widespread in various fields of technology. These systems formally do not possess the properties of controllability and observability due to infinite number of degrees of freedom. They also tend to demonstrate resonance behavior, which in the case of low damping leads to high vibration amplitudes at the resonance frequencies and may cause the performance degradation and damage to the structure.

Different passive or active systems can be used to protect the mechanical structure from the undesired vibrations [1–3]. Active control systems include feedback loops, which use sensors and actuators, and can provide the influence on the structure depending on its dynamics. There are known various strategies for organizing feedback control systems. We analyze three of them: local, modal and shape control strategies.

The shape control method [4–6] is used to compensate the known distribution of the external excitation. It implies using only one feedback loop with collocated system of

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sensors and actuators. On the contrary, the modal system [7–9] allows one to control independently different vibration modes of the object regardless of the shape of the external excitation. This possibility is provided by using a separate feedback loop for each mode of the object to be controlled. More simple is a local method [8, 10], which implies using local connections sensor-actuator. In the local system multiple feedback loops could also be used.

In order to compare the control strategies mentioned the problem of vibration suppression of a thin cantilever beam is considered. The control purpose is to suppress forced bending vibrations of the beam caused by the base excitation at the first and the second resonances. All control systems use the same number of piezoelectric sensors and actuators. Our previous investigations [8] have shown that under the considered conditions the modal method is more effective than the local one if it is needed to suppress several vibration modes of the object. Therefore, the aim of the present study is to compare the modal and the shape control methods for the above stated problem.

2 Theoretical Background

2.1 Shape Control Method

The theoretical description of the shape control method for controlling the bending vibrations of Bernoulli-Euler beams using piezoelectric sensors and actuators is given in [6]. The main idea of this method is the compensation of the known distribution of the external excitation by the piezoelectric actuation: the actuation bending moment should be opposite to the statically admissible bending moment produced by the external load.

The first problem is that in real cases control possibilities are usually limited: actuators used in the control system cannot fully compensate the shape of the external excitation. Therefore, it is needed to approximate the bending moment to be compensated by available actuators. For this purpose, it is suggested to use the *equal-area-rule* or to divide the beam into sections and compensate the deflection individually in each section by a single actuator. These variants of compensation are considered in [2]. In the present study two variants of compensation are compared: the “sections method” and the method based on the minimum deflection criterion, this methods are described below in Subsect. 4.1. Of course, all these methods are valid only if the distribution of the external excitation is constant.

The second problem is that the time variation of the external load is often not known in advance, which makes necessary the use of feedback control systems. In this case, the collocated system of sensors and actuators is usually used. It means that the design of the sensor system repeats the one of the actuator system: sensors are located symmetrically with respect to actuators at the opposite side of the beam. All sensors and actuators are integrated in a single feedback loop.

2.2 Modal Control Method

The modal control method, or IMSC (independent modal space control), implies the correspondence between the feedback loops and the vibration modes of the object. First formulated in 1966 [11], it was further developed in [12]. The application of this approach to active vibration control of continuous systems is considered in [7–9, 13]. Our previous study [8] has shown that the modal method is more effective than the local one if it is needed to suppress forced vibrations of the object at more than one eigenfrequency.

The general scheme of the modal control system is shown in Fig. 1. Here y is the vector of sensor signals, which is transformed into the vector of the estimates of the generalized coordinates $\tilde{q}$ using the matrix T, which is called the mode analyzer. The length of the vector $\tilde{q}$ is equal to the number of feedback loops. In i-th feedback loop one component of this vector, $\tilde{q}_i$, is transformed into the desired generalized force $\tilde{Q}_i$ acting on the corresponding vibration mode of the beam, using the transfer function $R_i(s)$. That means that $R(s)$ is a diagonal matrix of the control laws, where the negative sign indicates the negative feedback. After that, the vector $\tilde{Q}$ is transformed into the vector of control signals to the actuators u using the matrix F, which is called the mode synthesizer.

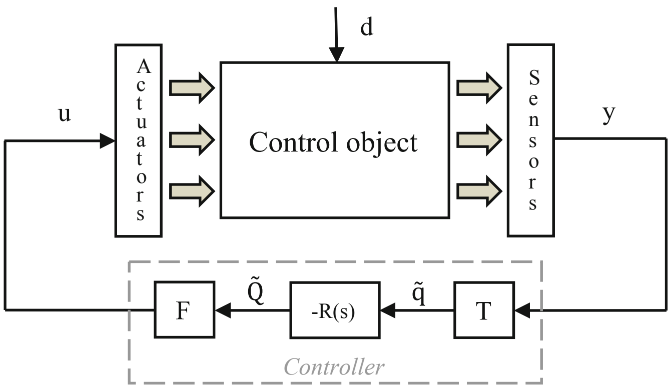

Fig. 1. Scheme of the modal control system

When creating the modal control system, one should specify the matrices T and F. To clarify this step, we need to introduce the matrices θ^a and θ^s. θ^a is the excitation matrix, which shows, how strong is the influence of each actuator on each eigenmode of the object. θ^s is the measurement matrix, which indicates, how strong is the influence of each eigenmode on each sensor. In order to provide the correspondence between the feedback loops and the vibration modes of the object, the following relations should be satisfied:

$$F = \left(\theta^a\right)^{-1}, T = \left(\theta^s\right)^{-1} \tag{1}$$

Unfortunately, the total amount of modes needed to describe the dynamics of the control object is usually greater than the number of controlled vibration modes. The problem of activation of higher, uncontrolled modes is known as a spillover effect, which can cause not only the mutual influence between the modal feedback loops, but also the instability of the closed-loop system. This effect can be minimized by enhancing the mode separation, for example, due to increasing the number of sensors and actuators.

3 The Control Problem

The scheme of the considered control problem is shown in Fig. 2. The control object is a thin cantilever beam made of aluminium with length of 50 cm and cross section of 3 × 35 mm. The beam is undergoing forced bending vibrations due to the base excitation. Small rectangular piezoelectric patches with dimensions 60 × 30 × 0.5 mm covered by electrodes on both sides are used as sensors and actuators. The sensors measure the bending deformation of the beam at specific locations, while the actuators cause this deformation.

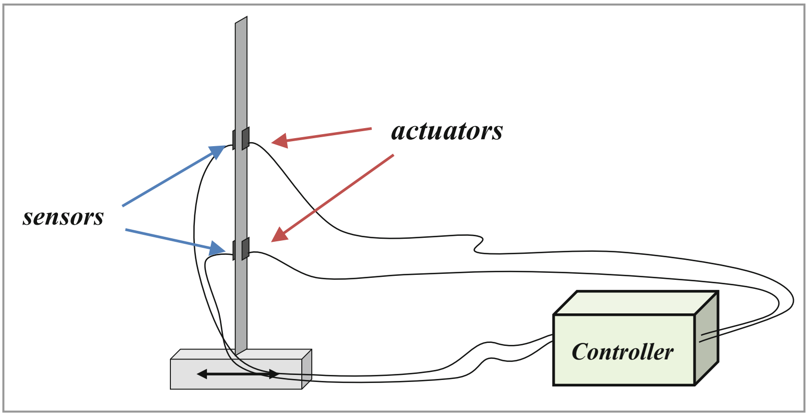

Fig. 2. Scheme of the control problem

The control purpose is to suppress forced bending vibrations of the beam in the frequency range containing the first and the second resonance frequencies. Each control system created includes two sensor-actuator pairs, where the elements of each pair are mounted to the beam symmetrically on both sides. For each control strategy the locations of sensor-actuator pairs on the beam are different.

To evaluate the efficiency of the created control systems, the vibration amplitude of the upper endpoint of the beam is analyzed. This choice is caused by the fact that the vibration amplitude of this point is the biggest among all points of the beam for the vibration modes to be controlled.

The solution of the control problem for each control system is computed from the frequency response functions (FRFs) of the beam obtained using finite element modeling.

4 Creating the Control Systems: Actuator and Sensor Placement

4.1 Shape Control Method

The first stage of creating the control system is the placement of actuators and sensors on the beam. In this subsection we will consider the shape control method.

In the shape control system, actuators should compensate the external excitation, which is equivalent to the distributed inertia load (Fig. 3). Here we consider the static case of loading with $p_0 = 1$ N/m. Obviously, two discrete actuators cannot fully compensate this load. Therefore, we consider two variants of compensation (Fig. 4).

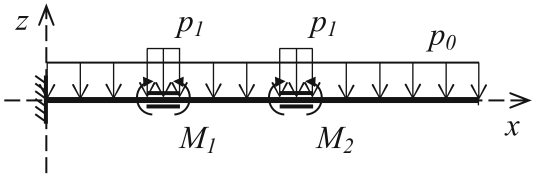

Fig. 3. Cantilever beam with two sensor-actuator pairs

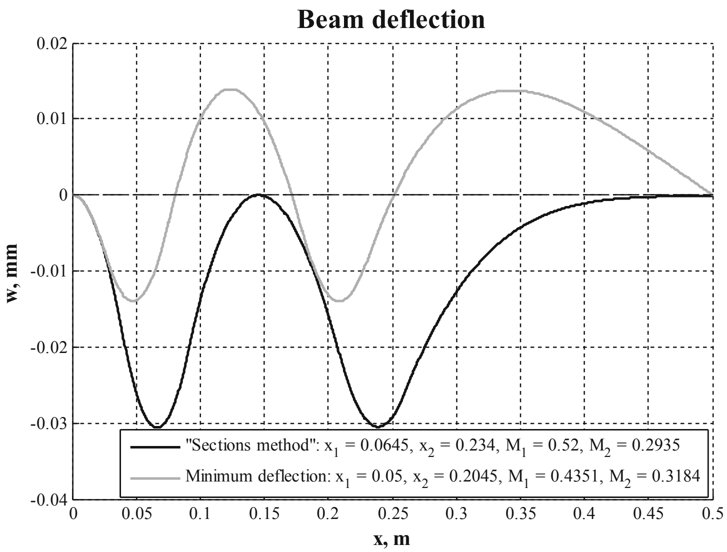

Fig. 4. Shape control: two variants of compensation of the external load

The first variant of compensation is obtained using the so-called "sections method": the beam is divided into two sections, at the ends of which deflection and slope are zero. This method of compensation of the beam vibrations is described in [2]. For the case under consideration, the coordinates of the centers of piezopatches for this method are $x_1 = 6.45$ cm, $x_2 = 23.4$ cm, and the actuation moments are $M_1 = 0.52$ N·m, $M_2 = 0.2935$ N·m. The second method of compensation is based on the minimum deflection criterion: it means the minimization of the maximum deflection of the beam. At the free end of the beam, deflection is required to be zero, since the performance of the control system is analyzed from the vibration amplitude of this endpoint. For this method, the piezopatch coordinates and the actuation moments are the following: $x_1 = 5$ cm, $x_2 = 20.45$ cm, $M_1 = 0.4351$ N·m, $M_2 = 0.3184$ N·m. The maximum deflection of the beam for the first and the second methods of compensation is respectively 0.031 mm and 0.014 mm, while the maximum deflection for the uncontrolled case (corresponding to the tip of the free end) is 2.83 mm.

The actuation moments obtained give the weighting factors for actuators and sensors used in the feedback loop of the control system. The weighting factors for the actuator and sensor systems are equal, since these systems are collocated.

4.2 Modal Control Method

In the modal control system, two sensor-actuator pairs should be attached to the beam in those locations where they can most efficiently measure and affect the first and the second bending modes of the beam. These are the locations of maximum curvature of the considered eigenmodes. This curvature obtained numerically is shown in the Fig. 5. The first sensor-actuator pair is placed at the clamped end of the beam ($x_1 = 3$ cm), where the curvature of both modes gets maximum values, while the second pair is located approximately at the center of the beam ($x_2 = 26.5$ cm), where there is a local maximum of the curvature of the second bending mode.

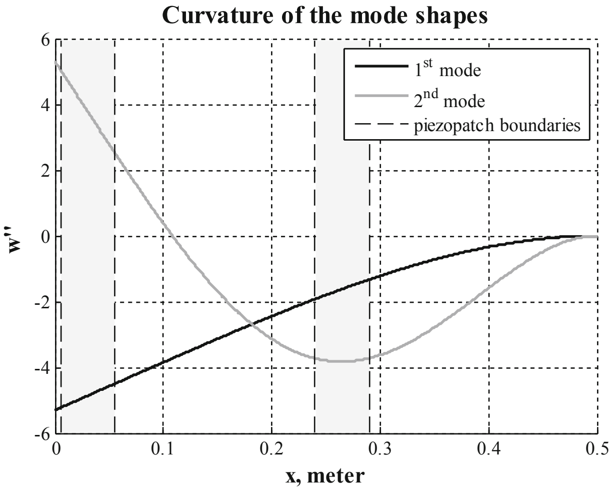

Fig. 5. Curvature of the 1st and the 2nd bending modes of the beam with the piezopatch locations

After the placement of the piezoelectric patches, the matrices T and F for the modal control system (mode analyzer and synthesizer), which define the linear transformation of the measured and control signals, are specified. These matrices are calculated from the numerically obtained matrices θ^a and θ^s (excitation and measurement matrices) according to the Eqs. (1). The obtained values are the following:

$$F = \left(\theta^a\right)^{-1} = 100 \cdot \begin{bmatrix} 6.06 & -0.41 \\ 6.79 & 1.27 \end{bmatrix} \tag{2}$$

$$T = \left(\theta^s\right)^{-1} = 10^{-5} \cdot \begin{bmatrix} 2.82 & 3.16 \\ -0.19 & 0.59 \end{bmatrix} \tag{3}$$

5 Finite Element Modeling

To compute the solution of the control problem, the frequency response functions of the beam are obtained using finite element (FE) modeling. The FE models of the beam are created using ANSYS software.

The models of the beam are constructed from 3-node one-dimensional elements Beam189. The model without piezoelectric patches contains 100 elements, while the models with piezopatches include 120 elements. Harmonic analysis is performed for each model for different variants of excitation: the base vibration or the actuator excitation. For each variant, the measured values are the sensor signals and the vibration amplitude of the point at the free end of the beam. Figure 6 shows the FE model of the beam with sensors and actuators corresponding to the shape control system #2, which is created using the minimum deflection criterion, with actuator excitation.

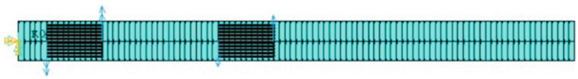

Fig. 6. FE model of the beam with piezopatches for shape control system #2

The piezoelectric effect in this FE simulation is not modeled directly. Instead of this, for simplicity, the bending moments are applied to the end sections of the actuators, and the rotation of the end sections of the sensors is analyzed to compute the sensor signals.

6 Design of the Controller and Comparison of the Results

The second stage of creating the control systems is the synthesis of the transfer functions for each feedback loop. They are designed using the loop shaping method.

In order to model the delay in the feedback loop, in each transfer function the low-pass filter with the cut-off frequency of 200 Hz is included. Therefore, the gain values in each loop are limited by the occurring of instability at high resonance frequencies due to the phase shift. The transfer functions are designed to provide the best vibration suppression at the desired resonance frequencies. As an example, Eqs. (4, 5) and Figs. 7 and 8 show the transfer functions and the Bode diagrams for both loops of the modal control system. The black curves correspond to the control object, and the gray ones – to the open-loop system.

$$R_1^m(s) = \frac{2.74 \cdot 10^8 s^2 + 5.16 \cdot 10^9 s + 1.08 \cdot 10^{10}}{s^4 + 1513 s^3 + 3.35 \cdot 10^5 s^2 + 1.89 \cdot 10^7 s + 3.6 \cdot 10^9} \quad (4)$$

$$R_2^m(s) = \frac{4.73 \cdot 10^{10} s^2 + 5.94 \cdot 10^8 s + 7.47 \cdot 10^{12}}{s^4 + 1861 s^3 + 1.26 \cdot 10^6 s^2 + 9.23 \cdot 10^8 s + 3.72 \cdot 10^{11}} \quad (5)$$

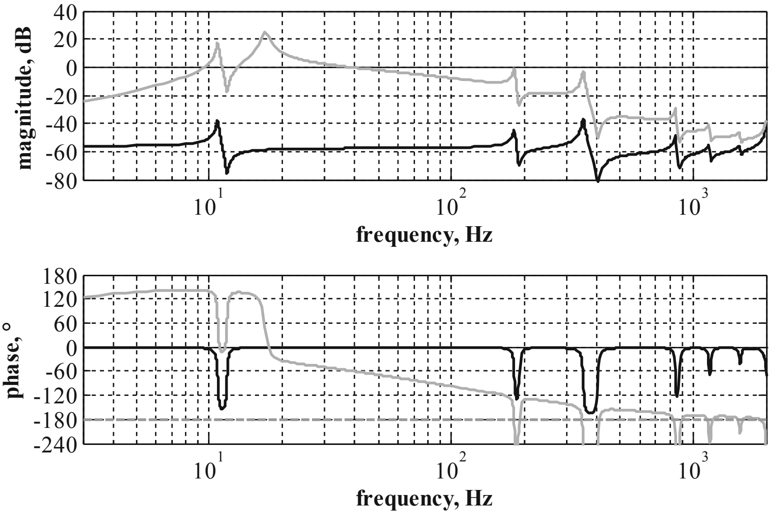

Fig. 7. Bode diagram for the 1st loop of the modal control system

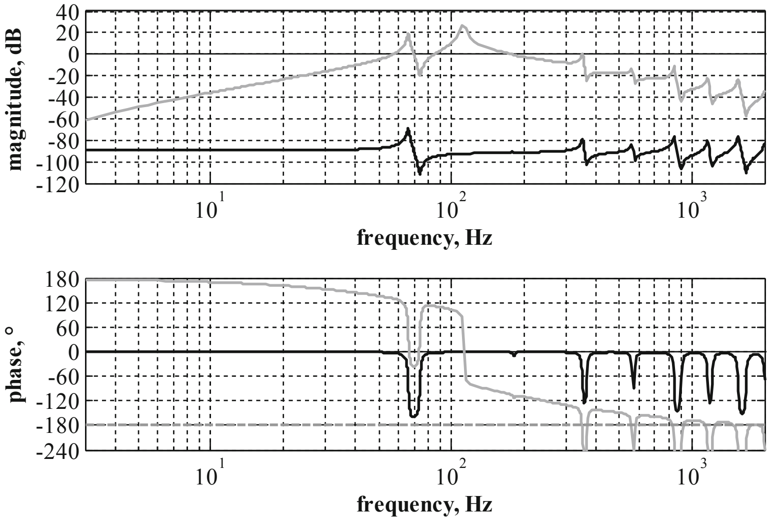

Fig. 8. Bode diagram for the 2nd loop of the modal control system

The performance of all created control systems in the vicinity of the first and the second resonance frequencies is shown in Figs. 9 and 10. These figures show the vibration amplitude of the upper endpoint of the beam with and without control, which is calculated using the FRFs of the beam obtained by the FE modeling and the transfer functions in the feedback loops.

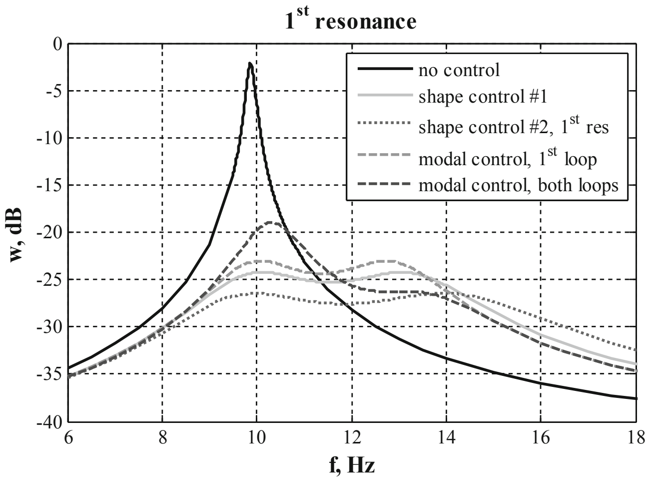

Fig. 9. Performance of the created control systems at the 1st resonance

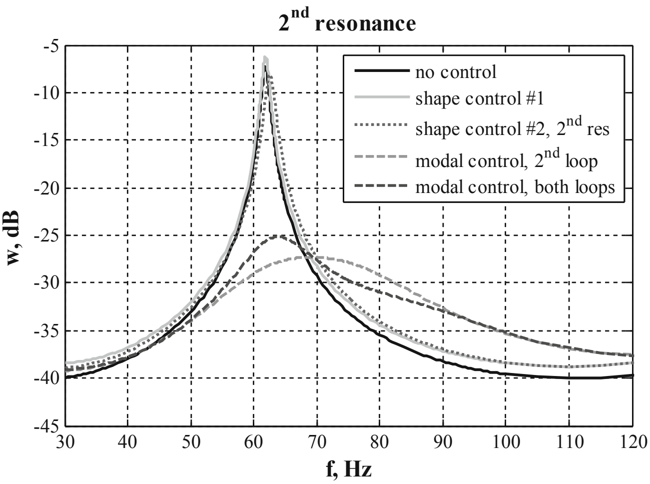

Fig. 10. Performance of the created control systems at the 2nd resonance

For the modal method the results of three control systems are used: two systems with only one active feedback loop (either the first or the second), and a system with both feedback loops active. The performance of the latter system is not as efficient as in the case of using only one loop, since the gain factors in both loops of this system are reduced compared to the other cases to avoid instability at high frequencies.

For the shape control system #2 two variants of transfer functions are designed: the first one is providing the most efficient performance of the system at the first resonance, and the second one – at the second resonance. For shape control system #1 only one transfer function is designed, because this system is unable to suppress the second vibration mode of the beam and does not work at the second resonance.

It can be seen from the figures, that the modal systems efficiently suppress forced vibrations at both resonances. Shape control systems, especially the system #2, work slightly better than the modal system at the first resonance, but at the second resonance they in fact do not work at all. Therefore, the modal method is more efficient than the shape control method when it is needed to suppress several vibration modes of the object. The second conclusion is that in the framework of shape control approach the method of compensation based on the minimum deflection criterion is more effective than the "sections method", which implies dividing the beam into sections.

The numerical data on the performance of the designed control systems is summarized in Table 1. Here the gain factors in each control loop are presented as well as the difference in the vibration amplitude of the upper endpoint of the beam with and without control at both resonances.

Table 1. Performance of the created control systems

Control system	Gain		Δw_1, dB	Δw_2, dB
Shape control #1	3.41		−22.09	1.00
Shape control #2, 1st resonance	3.08		−24.35	1.06
Shape control #2, 2nd resonance	0.763		0.10	−0.83
Modal control, 1st loop	3		−20.90	0.71
Modal control, 2nd loop	21		0.27	−20.04
Modal control, both loops	2.1	14.07	−16.85	−17.91

7 Conclusions

The present study is devoted to realization and numerical comparison of shape control and modal control strategies for the problem of active suppression of forced bending vibrations of a thin cantilever beam. The purpose of the control systems created was to suppress forced vibrations of the beam at the first and the second resonances. Each control system includes two pairs of piezoelectric patches used as sensors and actuators, located in two different positions on the beam. For the shape control strategy, two variants of placement of the piezoelectric patches on the beam were considered, realizing two variants of compensation of the external excitation.

The numerical modeling has shown that the modal control system designed can efficiently suppress vibration of the beam at both the first and the second resonances (the level of vibration amplitude is reduced by 17–20 dB). At the same time, the systems based on shape control demonstrate even better performance at the first resonance (reduction up to 24 dB), but at the second resonance they almost do not work at all (reduction less than 1 dB). In the framework of the shape control strategy, the variant of compensation based on minimum deflection criterion turned out to be more efficient at both resonances than the method based on dividing the beam into sections.

Therefore, the modal control strategy is preferable to the shape control if it is needed to suppress several vibration modes of the object. This can be explained by the fact that the modal method implies using several feedback loops designed to suppress specific vibration modes of the object. On the contrary, the shape control systems include only one feedback loop, the design of which provides efficient suppression of only one mode (the first bending mode of the beam). In other words, the modal control system is more complicated than the shape control system, which determines its greater efficiency.

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