Fast Adaptive Limit Analysis of Masonry Arch Bridges in Presence of Differential Settlements of Bridge Piles

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Abstract. In this work, we present a fast and reliable NURBS-based kinematic approach for the evaluation of the settlements-response of masonry arch bridges. This method is based on a discretization of the arch through NURBS rigid blocks. Here, the use of NURBS (Non-Uniform Rational B-Spline) approximating functions allows composing mesh through very few elements maintaining the exact representation of the three-dimensional curved shape. The main non-linear properties of masonry (almost null tensile strength, high crushing resistance, and frictional behavior in shear) are assigned at interfaces, which represent possible hinges between the curved macro-blocks. Starting from a known displacement applied at the external boundaries, a simple linear programming problem can be written to determine the discontinuous displacement field. Then, a meta-heuristic mesh-adaptation procedure is applied to exclude mesh dependency effects. The initial mesh is adjusted by modifying the shape of macro-blocks until interfaces coincide with the correct position of cracks. The settlement-response is so obtained.

Keywords: Masonry arch bridges · Settlement · Adaptive limit analysis · NURBS

1 Introduction

The study of the settlement response in masonry structures is a recent topic that is encountering growing interest. When undergoing foundation settlements, masonry structures deform like mechanisms to achieve a new equilibrium configuration accompanied by a pattern of cracks [1]. The strong nonlinearities of the masonry material, which have been summarized by Heyman [2] by means of the no-tension material assumption (null tensile strength, infinite crushing and shear resistance), render masonry constructions quite unsuited to resist differential settlements.

The problem of settlements is particularly important in the assessment of masonry bridges. Among the several typologies of bridges in Europe, masonry arches are one of the most ancient and still widespread. Especially in ancient bridges, defects or degradation are often observed even without using advanced monitoring tools, and most of the time are due to small modifications in boundary conditions. Therefore, differential

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settlements derived from local pier scour constitute one of the most frequent causes of structural collapse in masonry arch bridges [3–6].

In the last decades, the problem of settlements on masonry structures has been studied by means of nonlinear analyses within the Finite Element Method (FEM) [3, 7–9], the Discrete Element Method (DEM) [10, 11], rigid blocks analyses under the hypothesis of unilateral contact and simple linear programming techniques [12–14], and novel limit analysis-based tools aimed at defining the discontinuous displacement field that represents the settlement-induced mechanism [15, 16]. Limit analysis-based methods are particularly suited for masonry arch bridges, where the differential movement of piles generate the opening of flexural hinges in arches resulting in typical flexural mechanisms of easy evaluation.

In this contribution, we present the study of masonry arch bridges subjected to differential piles settlement through a new adaptive NURBS-based kinematic approach. A masonry arch bridge is modeled through NURBS (Non-Uniform Rational B-Spline [17]) elements, in which the actual curved geometry is exactly reproduced thanks to NURBS approximating function. Elements are assumed infinitely rigid and resistant, and dissipation is allowed only along interfaces between adjacent blocks. Given an initial mesh composed of very few NURBS elements, the displacement field is found by minimizing the total potential energy. A Genetic Algorithm mesh adaptation search scheme is then followed to find the correct position of cracks induced by a given settlement.

The arch bridge on the Trigno river (Italy), which collapsed because of the movement of a pile [3], is here analyzed through the presented numerical procedure.

2 Method

The geometry of each element is completely described by a NURBS surface. The advantage of using NURBS is the possibility to consider blocks of any curved shape without discretization. Moreover, mesh composed of very few NURBS elements in which the real geometry is still maintained can be easily realized.

Consider now a mesh obtained in this way. Within the fundamental assumption of rigid blocks, internal dissipation allowed only along interfaces, and small displacements, the potential energy associated with the rigid-body motion becomes linear and is defined as:

$$\Pi(\mathbf{x}) = \mathbf{c}^T \mathbf{p} - \mathbf{q}^T \mathbf{u} \tag{1}$$

where $\mathbf{u}$ are the displacement of centroids of each rigid block, $\mathbf{p}$ are the non-negative plastic multipliers related to the jumps of displacements according to the associated flow rule, $\mathbf{x} = [\mathbf{u}, \mathbf{p}]$ is the unknowns vector, $\mathbf{q}$ is the vector of loads multiplier for the kinematic matrix (which differs from the identity matrix when the loaded point is different from the centroid), and finally $\mathbf{c}$ is the vector of coefficient in the evaluation of the internal dissipated work. The internal dissipated work is evaluated by imposing the associated flow rule at interfaces:

$$\Delta\tilde{\mathbf{u}} = \mathbf{p}\frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{2}$$

where $\Delta\tilde{\mathbf{u}}$ are the jumps of displacement in the local reference systems, $\boldsymbol{\sigma}$ are the local stress, and f is the assumed three-dimensional failure domain (see Fig. 1). Once that f is linearized into planes in the space of local stress, it is easy to deduce that the vector $\mathbf{c}$ in Eq. (1) contains the known terms in the equation of planes.

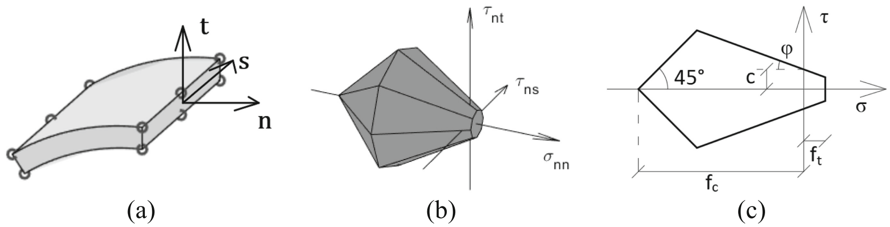

Fig. 1. NURBS shell element: (a) interface discretization and local reference system, (b) 3D Mohr-Coulomb failure domain and (c) 2D section of the failure domain.

A known settlement $\mathbf{u}_0$ can be applied at the external boundaries by writing a kinematic matrix $\mathbf{A}_{disp}$:

$$\mathbf{A}_{disp}\mathbf{x} = \mathbf{u}_0 \tag{3}$$

A simple linear programming problem can be solved to find the discontinuous displacement field that minimizes the total potential energy under the imposition of the associated flow rule (Eq. (2)), which is summarized through the compatibility matrix $\mathbf{A}_{comp}$, and the known settlements (Eq. (3)):

$$\min\{\Pi(\mathbf{x})\} \text{ such that } \begin{cases} \mathbf{A}_{comp}\mathbf{x} = \mathbf{0} \\ \mathbf{A}_{disp}\mathbf{x} = \mathbf{u}_0 \\ \mathbf{p} \geq \mathbf{0} \end{cases} \tag{4}$$

The presented optimization problem coincides with the maximization of the external work when a no-tension material is assumed (i.e. no internal dissipation is involved). In order to find the global minimum of the total potential energy, the exact position of interfaces must be found. In this way, a simple mesh adaptation scheme via Genetic Algorithm is adopted. The initial mesh is iteratively modified until interfaces coincide with the exact position of cracks, which is associated to the global minimum of the total potential energy.

The procedure here presented in brief was originally proposed for the limit analysis of masonry vaults and has been recently applied to several typologies of masonry constructions [18–22].

3 Numerical Example

The masonry arch bridge on the Trigno river is here analyzed. The bridge collapsed because of the scouring of a pier (Fig. 2a), and the phenomenon was investigated by means of several FE analyses in [3]. Considering the shape of the collapse observed, the two spans adjacent to the moved pier have been here modeled and analyzed through the presented adaptive NURBS-based kinematic approach. The geometry of the two spans is depicted in Fig. 2b, and the with of the bridge has been set equal to 9.7 m.

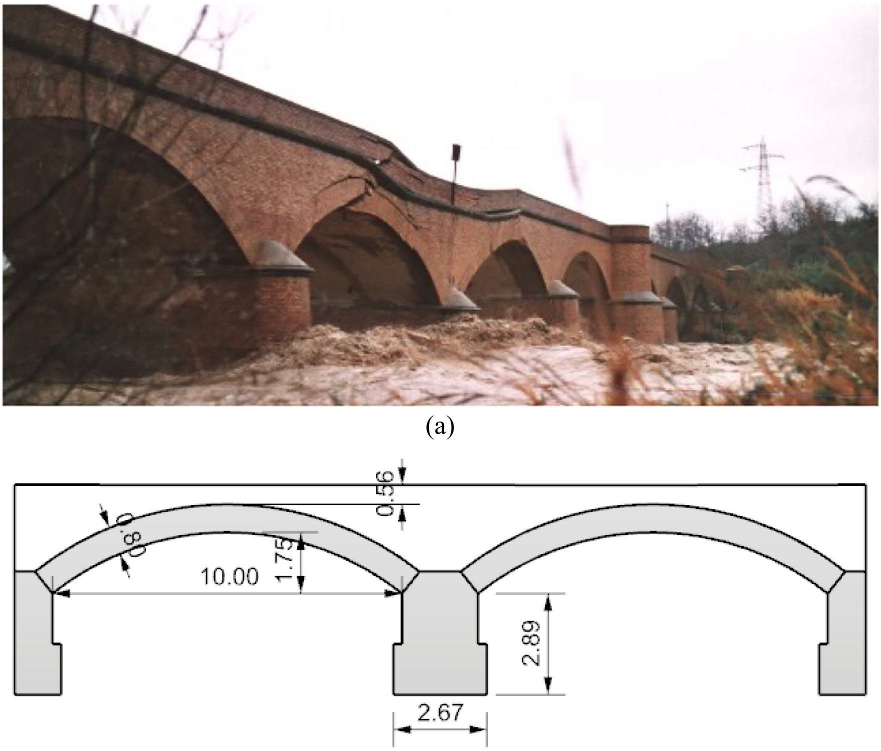

Fig. 2. The multi-span masonry arch bridge on the Trigno river (Italy) [3]: (a) collapse induced by scour of the pier, (b) geometry of the two spans modeled (width equal to 9.7 m).

The two-spans have been model with 2D elements. Initially, 5 elements were used (2 arches and 3 piers). Then, each arch has been subdivided into 4 elements with a total of 5 interfaces representing possible flexural hinges (considering also interfaces with piers at extremities). The settlement is applied to the central pier and the mesh adaptation is conducted by moving the position of hinges within the arches. Moreover, once the mechanism has been correctly identified, interfaces associated with closed hinges have been removed to reduce the number of elements to the minimum.

A Mohr-Coulomb failure domain with tension cut-off and linear cap in compression has been assigned to interfaces. 0.19 MPa has been assigned as tensile strength, 3.85 MPa as compression strength, whereas shear is governed by a cohesion equal to 0.43 MPa and a friction angle equal to 64.8°. 18 kN/m^3 has been assigned as specific weight for arches, piers and backfill. Finally, the backfill has been here applied as vertical load and contributes to the total potential energy in terms of external work only. Alternatively, the direct modeling of the backfill material could be performed: this would allow introducing possible fracture lines within the backfill to take into account its contribute in terms of both internal and external dissipation.

Two settlement configurations have been analyzed. In the first one, a vertical settlement equal to 1 cm is applied to the pier. As a result, a symmetric mechanism has been found and is depicted in Fig. 3. It is interesting to note that the lowered shape of the arches allows the development of a mechanism characterized by a lower number of hinges than that usually observed in classic limit analysis problems. A vertical reaction force at the collapse equal to 4886 kN has been found, a result which is in good agreement with [3].

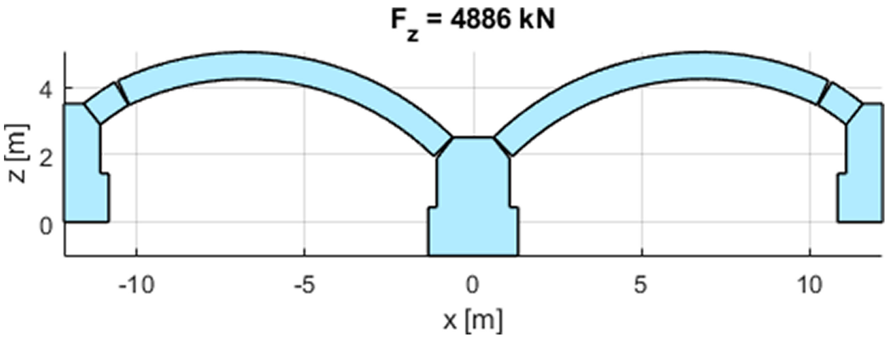

Fig. 3. Vertical settlement of the central pier: deformed structure (displacements scaled by 100) and vertical reaction at the collapse.

As second simulation, a rotational settlement is applied to the pier. The settlement is imposed by assigning a vertical displacement equal to 1 cm just to the left bottom vertex of the pier, whereas the translation of the right bottom vertex has been imposed null. The obtained result is shown in Fig. 4. Even in this case, a single hinge at the key has been found, in addition to hinges located at extremities. A specular mechanism, due to the differential movement of their supports, is observed in the arches. The vertical reaction force and the momentum at the collapse are respectively equal to 5665 kN and 6765 kNm clockwise.

In both cases, the settlement response is evaluated in very efficient way. The correct discontinuous displacement field and the position of hinges are determined by maintaining minimum the total amount of degrees of freedom and, consequently, requiring a negligible computational effort. It can be finally observed that the failures occurred through the opening of flexural hinges. This is due also to the high resistance values assigned in shear: in presence of reduced cohesion and friction angle, collapses through both flexural hinges and sliding effects would have been obtained.

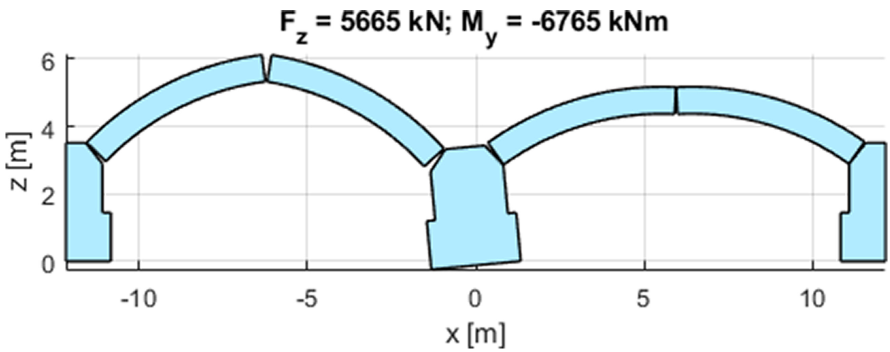

Fig. 4. Rotational settlement of the central pier: deformed structure (displacements scaled by 25), vertical reaction and momentum at the collapse.

4 Conclusions

In this paper, an adaptive NURBS-based kinematic approach for the evaluation of the settlement-response of masonry arch bridges has been proposed. A given masonry bridge is described through NURBS surfaces, which are then discretized into few elements without affecting the actual geometry. A known displacement, which comprises possible settlements, is applied at the base of one pier. Within the hypotheses of rigid elements, small displacements and associated flow rule, a linear programming problem is solved to find the discontinuous displacement field that minimizes the total potential energy. Then, the correct position of hinges in arches is determined by means of a Genetic Algorithm. The example of the Trigno river bridge allowed to present the main advantages of the adaptive NURBS approach, i.e. reliable representation of the behavior of masonry curved structures by maintaining low the computational effort.

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