Static Stiffness of the Crane Bridges Under Moving Load Distribution

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Abstract. The paper deals with issues related to the calculation and design of prestressed crane structures of span type. The problem consists of the further application and use of a refined mathematical model of a preformed crane bridge, which allows analytically investigating its deformed behavior according to actual operating conditions. This paper analyzes the mathematical models of the main girders of overhead cranes adapted for use. The most dangerous positions for them with a movable transverse load are considered. The authors provide a refined mathematical model of an overhead crane with prestressed beams based on the general theory of stability of elastic systems. In the design scheme, the resulting vertical movable load was distributed over several transverse movable loads, corresponding to the actual conditions of its loading. In this work, equations for the deflection curve of a span were obtained, which made it possible to additionally investigate its static stiffness, depending on the nature and action of a temporary moving load. The results obtained in this work can be used to modernize cranes to increase their lifting capacity, extend their service life without dismantling, and improve existing structures and engineering calculation methods under actual operating conditions.

Keywords: Overhead crane · Prestress · Main girders · Static rigidity · Flexural stiffness · Deflected mode · Deformed condition

1 Introduction

The subject of consideration is preliminarily stressing the structures: artificial creation of internal forces and stresses to obtain or increase necessary beneficial qualities before operation [1]. The concept of prestressing is based on the property of statically indeterminate mechanical systems to allow internal forces and stresses in their elements in the absence of external force effects [2].

Span cranes with prestressed main girders are widely used in mechanical engineering. Metal bridges of such cranes have a lower moment of inertia of the section. Therefore they are much lighter and cheaper than metal structures of conventional cranes operating under the same operating modes and the same load capacities [3].

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At the same time, they are more deformable [4]. Therefore, the criteria for calculating and designing such metal structures are linked inextricably with the causes of their failures [5]. One of the main reasons for malfunctioning a structure is its unacceptably large elastic deformations [6, 7]. So, for example, a significant deflection or bending of span beams can lead to distortions of the end beams, slipping of the running wheels of freight bogies, and unnecessary power consumption for overcoming the slope of the load belt [8, 9].

In this regard, a more accurate determination of the deformation value of the bridge and analysis of its behavior under load will not only improve the operating conditions of the beam and the crane as a whole but also lead to a number of positive measures such as reducing the weight of the crane's metal structure and its cost [10]. Since the crane girder is subjected to longitudinal-transverse bending, the deformed state of the crane bridge must be taken into account using the deflection arrow of the girder itself.

Thus, the purpose of this work is to further study the span beam for static stiffness. And the issues considered in it, in which the nature of the loading of the beam is put forward in the first place with the maximum approximation of the design scheme to the actual constructive form, are relevant [11].

For achieving this goal, it is necessary to solve the following tasks: consider and analyze the already known mathematical models of an overhead crane with prestressed main beams; to develop a mathematical model of an overhead crane with prestressed main beams, which allows you to study its deformed behavior according to the actual conditions of its operation; analyze the results obtained.

2 Literature Review

The analysis of publications on the topic under study shows that the moments unloading a prestressed crane bridge do not depend on the position of the external load and its value [12] (Fig. 1).

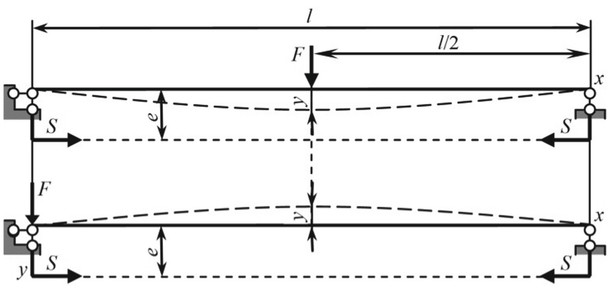

Fig. 1. Prestressed crane bridge.

As a result, one of the disadvantages is the case when a movable cart with a load is located above the support or near it [13]. In this connection, the reverse deflections of the bridge can be commensurate with the working deflections of the beam and even exceed its permissible deflections. Thus, the calculation of the prestressed main beam for the third limiting state is carried out according to the accepted design scheme for the two most unfavorable loading cases [14, 15]: when the movable vertical load F is in the middle of the span l, that is, the current coordinate of deflection determination $x = 0.5$ or l, where the calculated deflections are checked; when the load F is above the support, where the current coordinate of deflections is $x = 0$ or $x = 1$. In this case, the reverse bends of the prestressed bridge are checked.

Analysis of the design scheme for the first case of loading shows that the scheme is significantly simplified and cannot accurately reflect the actual operating conditions of the span beam. This is because the scheme does not consider distributing the concentrated vertical load F between the corresponding n number of crane wheels. The distribution of the force F between the wheels leads to an increase in the lateral forces F_1, F_2,…, F_n acting on the beam and complicates the design scheme. But neglect of this factor leads to overestimated values of the calculated deflection of the bridge, and in some cases - to unreasonably overestimated reserves of its static or dynamic stiffness.

The analysis of the second case of loading suggests that the accepted design scheme of the span beam also does not correspond to the maximum approximation of the scheme to the real structural one. So, the design scheme does not consider the permissible minimum distance from the axis of the drive wheels to the axis of the crane rail. Therefore, in a confirmed case of loading, one of the wheels of the cargo carriage at its extreme position will always be at a distance l_b from the end beam, which significantly reduces the value of the calculated bridge deflection (Fig. 2).

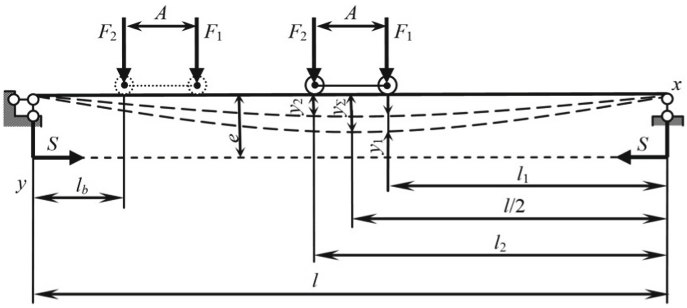

Fig. 2. The design scheme of the crane bridge.

Neglect of this factor, as in the first case of loading, can lead to overestimated reserves of the crane bridge according to the deformation criterion of performance.

3 Research Methodology

When constructing the proposed refined mathematical model, it is assumed that all the crane elements are solid, the beam operates in an elastic stage, rests on ideal hinges, and its bending is carried out in the area of the load suspension. We distribute the transverse working load (from the weight of the bogie with the rated load) between the wheels of the freight bogie and represent it by vertical forces F_1, F_2. Then, using the method of superposition of deflections known from the theory of stability of elastic systems (caused separately by each of the transverse forces F_1 and F_2, acting together with the total longitudinal eccentrically applied force *S*), we compose the differential equations of the deflection curve for the left section of the prestressed beam

$$EI \cdot \ddot{y}_{1,L} = -\frac{F_1 l_1}{l} x - P\left(y_{1,L} - e\right),$$

$$EI \cdot \ddot{y}_{2,L} = -\frac{F_2 l_2}{l} x - P\left(y_{2,L} - e\right).$$

Deflection arrow differential equations for right beam segment

$$EI \cdot \ddot{y}_{1,R} = -\frac{F_1 (l - l_1)(l - x)}{l} x - P\left(y_{1,R} - e\right),$$

$$EI \cdot \ddot{y}_{2,R} = -\frac{F_1 (l - l_2)(l - x)}{l} x - P\left(y_{2,R} - e\right).$$

where *EI* is the bending rigidity of the beam in the plane of the load suspension; *l* is the length of the beam; *x* is the current coordinate of the location of deflections (bends) y_1 and y_2, respectively, from the forces F_1 and F_2; l_1, l_2 − distances from the proper support of the beam to the place of action of transverse loads, respectively F_1 and F_2.

To simplify the subsequent mathematical calculations, we introduce the notation

$$k^2 = \frac{S}{EI}.$$

Then the above differential equations take the form

$$\ddot{y}_{1,L} + k^2 y_{1,L} = -\frac{F_1 l_1}{lEI} x + k^2 e, \quad (1)$$

$$\ddot{y}_{2,\mathrm{L}} + k^2 y_{2,L} = -\frac{F_2 l_2}{lEI} x + k^2 e, \quad (2)$$

$$\ddot{y}_{1,R} + k^2 y_{1,R} = -\frac{F_1 (l - l_1)(l - \mathrm{x})}{lEI} + k^2 e, \quad (3)$$

$$\ddot{y}_{2,R} + k^2 y_{2,R} = -\frac{F_2(l - l_2)(l - x)}{lEI} + k^2 e. \quad (4)$$

The general solutions of these equations will be, respectively, for the expression (1)

$$y_{1,\mathrm{L}} = C_1 \cos(kx) + C_2 \sin(kx) - \frac{F_1 l_1}{Sl} x + e, \quad (5)$$

for the expression (2)

$$y_{2,L} = C_3 \cos(kx) + C_4 \sin(kx) - \frac{F_2 l_2}{Sl} x + e, \quad (6)$$

for the expression (3)

$$y_{1,R} = C_5 \cos(kx) + C_6 \sin(kx) - \frac{F_1(l - l_1)(l - x)}{Sl} + e, \quad (7)$$

for the expression (4)

$$y_{2,\mathrm{R}} = \mathrm{X}_7 \cos(kx) + C_8 \sin(kx) - \frac{F_2(l - l_2)(l - x)}{Sl} + e. \quad (8)$$

Integration constants C_1, C_3, C_5, C_7 are determined from the conditions at the beam ends, where its deflections are equal to zero.

$$\left(y_{1,L}\right)_{x=0} = 0, \text{then } C_1 = -e, \quad \left(y_{2,L}\right)_{x=0} = 0, \text{ then } C_3 = -e,$$

$$\left(y_{1,R}\right)_{x=l} = 0, \text{then } C_5 = -C_6 \mathrm{tg}(kl) - e \sec(kl),$$

$$\left(y_{2,R}\right)_{x=l} = 0, \text{then } C_7 = -C_8 \mathrm{tg}(kl) - e \sec(kl).$$

Other integration constants are determined from the conditions that at the point of application of the transverse forces F_1 and $F_{2,}$ both sections of the beam deformation curve have the same deflection.

$$\left(y_{1,L}\right)_{x=l-l_1} = \left(y_{1,R}\right)_{x=l-l_1}, \quad \left(y_{2,L}\right)_{x=l-l_2} = \left(y_{2,R}\right)_{x=l-l_2}$$

and the common tangent.

$$(\dot{y}_{1,R})_{x=l-l_1} = (\dot{y}_{1,R})_{x=l-l_1}, \quad (\dot{y}_{2,L})_{x=l-\mathrm{l}_2} = (\dot{y}_{2,R})_{x=l-l_2}.$$

Then, expressions for determining the integration constants C_2, C_4, C_6, C_8 will have the following final form

$$C_2 = \frac{F_1 \sin(kl_1)}{Sk \sin(kl)} - e\,\mathrm{tg}\left(\frac{\mathrm{kl}}{2}\right), \quad C_4 = \frac{F_2 \sin(kl_2)}{Sk \sin(kl)} - e\,\mathrm{tg}\left(\frac{kl}{2}\right),$$

$$C_6 = -\frac{F_1 \sin(k(l - l_1))}{Sk \sin(kl)} - e\,\mathrm{tg}\left(\frac{kl}{2}\right), \quad \mathrm{C_8} = -\frac{\mathrm{F_2 sink(l - l_2)}}{\mathrm{Sk\,sin(kl)}} - \mathrm{e\,tg}\left(\frac{\mathrm{kl}}{2}\right),$$

Substitute the obtained expressions for the constants $C_1, \ldots, C_8$ into the corresponding initial Eqs. (5)–(8) and denote, after some transformations, the following expressions for the left and right sections of the deflection curve of the crane bridge

$$y_{1,L} = \frac{F_1}{S}\left(\frac{\sin(kx)\sin(kl_1)}{k \sin(kl)} - \frac{xl_1}{l}\right) - eU, \tag{9}$$

$$y_{2,L} = \frac{F_2}{S}\left(\frac{\sin(kx)\sin(kl_2)}{\mathrm{k} \sin(kl)} - \frac{xl_2}{l}\right) - eU, \tag{10}$$

$$y_{1,R} = \frac{F_1}{S}\left(\frac{\sin(k(l - x))\sin(k(l - l_1))}{\mathrm{k} \sin(kl)} - \frac{(\mathrm{l} - x)(l - l_1)}{l}\right) - eU, \tag{11}$$

$$y_{2,R} = \frac{F_2}{S}\left(\frac{\sin(k(l - x))\sin(k(l - l_2))}{k \sin(kl)} - \frac{(l - x)(l - l_2)}{l}\right) - eU. \tag{12}$$

Applying the method of superposition of deflections, we obtain the total deflection $y_{\sum}$ of the bridge when the forces F_1 and F_2 act simultaneously together with the eccentric longitudinal force S. Adding expressions (9) and (10), we find the equations for the deflection curve for the left section of the beam $(0 \leq x \leq (l - l_2))$

$$\begin{aligned} y_{\Sigma,L} &= y_{1,L} + y_{2,L} \\ &= \frac{1}{S}\left(\frac{\sin(kx)}{k \sin(kl)}(F_1 \sin(kl_1) + F_2 \sin(kl_2)) - \frac{x}{l}(F_1 l_1 + F_2 l_2)\right) - 2eU. \end{aligned}$$

Adding expressions (11) and (12), we find the equations of the deflection curve for the right section of the beam $(x \geq (l - l_2))$

$$\begin{aligned} y_{\Sigma,R} = y_{1,R} + y_{2,R} = \frac{1}{S}\left(\frac{\sin(k(l - x))}{k \sin(kl)}(F_1 \sin(k(l - l_1)) + F_2 \sin(k(l - l_2))\right) - \frac{(l - x)}{l} \\ \times (F_1(l - l_1) + F_2(l - l_2)) - 2eU. \end{aligned}$$

Taking into account the above, we will compose the equation of the deflection curve of the span between the wheels of the cargo trolley in the section between the vertical loads F_1 and F_2

$$y_{\Sigma} = -2eU + \frac{F_1}{S}\left(\frac{\sin(kx)\sin(kl_1)}{\mathrm{k}\sin(kl)} - \frac{xl_1}{l}\right) + \frac{F_2}{S}\left(\frac{\sin(k(l - x))\sin(k(l - l_2))}{k\sin(kl)} - \frac{(l - x)(l - l_2)}{l}\right).$$

For studying the span beam's stress state, it is necessary to differentiate twice the total equations of the deflection curves

$$\dot{y}_{\Sigma L} = \frac{\cos(kx)}{S\sin(kl)}\left(F_1\sin(kl_1) + F_2\sin(kl_2)\right) - \frac{F_1 l_1 + F_2 l_2}{Sl} - 2ek\left(\cos(kx)\mathrm{tg}\left(\frac{kl}{2}\right) - \sin(kx)\right),$$

$$\dot{y}_{\Sigma R} = \frac{\cos k(l - x)}{S\sin kl}\left(F_1\sin(k(l - l_1)) + F_2\sin(k(l - l_2))\right) - \frac{F_1(l - l_1) + F_2(l - l_2)}{Sl} - 2ek\left(\cos(kx)\mathrm{tg}\left(\frac{kl}{2}\right) - \sin(kx)\right),$$

$$\dot{y}_{\Sigma} = -2ek\left(\cos(kx)\mathrm{tg}\left(\frac{kl}{2}\right)\right) - \sin(kx) + \frac{F_1\cos(kx)\sin(kl_1)}{S\sin(kl)} - \frac{F_1 l_1 - F_2(l - l_2)}{Sl} - \frac{F_2\cos(k(l - x))\sin(k(l - l_2))}{S\sin(kl)}.$$

After the first differentiation, the expressions obtained above represent the small angles of rotation of the ends of the beam, which can be used in the design of a prestressed crane bridge. After the second differentiation, we obtained

$$\ddot{y}_{\Sigma L} = -\frac{k\sin(\mathrm{kx})}{S\sin(\mathrm{kl})}\left(F_1\sin(kl_1) + F_2\sin(kl_2)\right) + 2ek^2(\sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right) + \cos(kx)\Big),$$

$$\ddot{y}_{\Sigma R} = -\frac{k\sin(k(l - x))}{S\sin(kl)}\left(F_1\sin(k(l - l_1)) + F_2\sin(k(l - l_2))\right) + 2ek^2\left(\sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right) + \cos(kx)\right),$$

$$\ddot{y}_{\Sigma} = 2ek^2\left(\sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right) + \cos(kx)\right) - \frac{F_1\sin(kx)\sin(kl_1) + F_2\sin(k(\mathrm{l} - \mathrm{x}))\sin(k(l - l_2)}{S\sin(kl)}.$$

In the final form, the equations of bending moments are

$$M_L = -EI(\ddot{y}_{\Sigma L}) = \frac{\sin(kx)}{k\sin(kl)}(F_1\sin(kl_1) + F_2\sin(kl_2)) - 2eS\left(\cos(kx) + \sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right)\right),$$

$$M_R = -EI(\ddot{y}_{\Sigma R}) = \frac{\sin(k(l - x))}{k\sin(kl)}(F_1\sin(k(l - l_1)) + F_2\sin(k(l - l_2))) - 2eS\left(\cos(kx) + \sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right)\right).$$

$$M = -EI(\ddot{y}_{\Sigma}) = \frac{F_1\sin(kx)\sin(kl_1) + F_2\sin(k(l - x))\sin(k(l - l_2))}{k\sin(kl)} - 2eS\left(\cos(kx) + \sin(kx)\mathrm{tg}\left(\frac{kl}{2}\right)\right).$$

4 Results

Based on the obtained expressions for the deflection curves of the prestressed beam, mathematical studies of its deformed state were carried out. The base is a beam with a span of $L = 10.5$ m. For overhead cranes with a lifting capacity of $F = 1000$ kg, an I-beam No. 24M is recommended [16]. Conveniently, the weight load is distributed between the wheels of the cargo trolley and $F_1 = F_2 = 500$ kg is considered. The distance between the forces F_1 and F_2 is taken as $A = 0.560$ m. It corresponds to the distance between the wheels of the TE1–521 electric hoist. Part of the calculation results in the form of conditional deflections $y/[y]$ of the main beam for a group of operation mode 4K, with an allowable value of conditional deflections $[y/l] = 2 \cdot 10^{-3}$ is presented in Table 1. Graphic interpretation of calculations in the form of deflection curves beams are given for the ratio of transverse and longitudinal forces $F / S = 1.5$ (Fig. 3).

Table 1. Conditional deflections of the main beam.

$\frac{F_1+F_2}{S_i}$	Position carts	Calculation scheme			
		Acting			The proposed
		$e_1 = e_2$	$e_1 \neq e_2$	$e_1 = e_2 = 0$	$e_1 = e_2$
$\frac{F_1+F_2}{1.25}$	Mid-span	0.92	0.96	1.05	0.72
	Near the support	−0.17	−0.12		−0.11
$\frac{F_1+F_2}{1.5}$	Mid-span	0.88	0.94		0.68
	Near the support	−0.20	−0.16		−0.04
$\frac{F_1+F_2}{1.75}$	Mid-span	0.85	0.9		0.63
	Near the support	−0.25	−0.18		−0.06
$\frac{F_1+F_2}{2}$	Mid-span	0.8	0.85		0.51
	Near the support	−0.3	−0.23		− 0.1

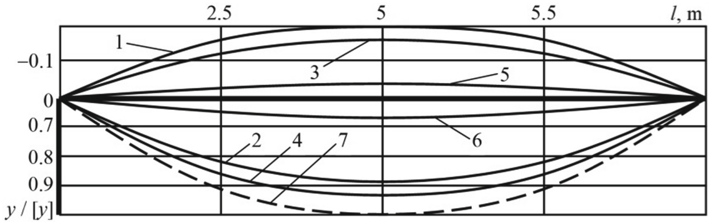

Fig. 3. Deflection curves of a prestressed beam.

Deflection arrows 2, 4, 6, 7 are shown when the crane is operating with a load in the middle of the span. And arrows of bends 1, 3, 5 − when placing one of the cart's wheels near the support at a distance of l_b − the minimum permissible safety zone for overhead cranes. Strain curves 1−4 are obtained by applying an accepted mathematical model for prestressed beams. For deflections 1, 2, the beam was subjected to the action of longitudinal forces applied on one line of action with eccentricity $e_1 = e_2$, and deflections 3 and 4 were obtained in the case of applying compressive forces with different eccentricities $e_1 \neq e_2$. Deflection curve 7 is built for a conventional crane bridge without prestress.

The analysis of the obtained results shows that the deformations of the unloaded beam do not exceed the deflections of a conventional crane bridge. The mathematical models used in the calculation and design of prestressed span beams are significantly simplified and do not always correspond to the actual conditions of its operation, and the deformations of the crane bridge are overestimated significantly. Thus, the deflection arrows 5 and 6, obtained using a new mathematical model proposed by the authors, say that the values of the deflections and deflections of the beam are, on average, 20% and 25% less, respectively, than in the currently used design scheme. The results obtained should be further used to improve existing structures and engineering calculation methods, both at the design stage and in actual operating conditions. At the same time, in the proposed new model, the use of several vertical forces F_1, F_2, …, F_n, although it corresponds to the actual loading conditions of the beam, significantly complicates its calculation according to the main criteria of performance.

5 Conclusions

In work, a more accurate mathematical model of a prestressed crane bridge was proposed and investigated. Its use makes it possible to significantly reduce the calculated deformations of the span beams compared to the previously adopted mathematical models of span beams.

The results obtained in this work can be further used for the modernization of cranes to increase their lifting capacity, increase the service life without dismantling, and improve existing structures and engineering calculation methods during design and in actual operation.

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