Experimental Investigations of the Flexible Rotor System by Introducing Parametric Excitations into Both Ends of the Rotating Shaft Axially

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Abstract. **Purpose:** It has been shown that it is possible to modify the vibration amplitude by the introduction into one end of the shaft of a rotating flexible rotor system, an axially parametric excitation. This concept drives this study and seeks to introduce excitations into both ends of the shaft theoretically. The purpose of this work is to presents an experimental investigation into the behaviour of a flexible rotor system subjected to an active vibration controller.

Design/Methodology/Approach: The equations motion was first solved using the perturbation method of multiple scales. Under principal parametric resonance, the theoretical results revealed an extra decline in the amplitude of the rotor response. Secondly, an experimental test was performed where a test machine with two piezoelectric exciters is mounted on both ends of the shaft. The steady-state feedback was investigated in the presence and absence of a double parametric excitation term.

Findings: Findings from the work show a significant decrease in amplitude of rotor response by 23.4% under principal parametric resonance, as well as a convincing agreement between theory and experiment.

Research Limitation/Implications: The study performs an analytical investigation of the behaviour of two piezoelectric exciters mounted on both ends of the shaft in a flexible rotor system. The results were verified experimentally. This work does not constrain itself to the numerical verification of the analytical solution, but rather performs experiments.

Practical Implication: This study confirms the fact that the amplitude of vibration be can be altered in the presence of an axially parametric excitation that has been introduced into one end of the shaft of a rotating flexible rotor system and establishes the behaviour in the case where excitations are introduced into both ends of the shaft. The literature will be enriched thus far with an active controller of vibration for a flexible rotor system.

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Social Implication: The knowledge advanced by this research will inform stakeholders in the mechanical engineering fraternity particularly, the manufacturers of a flexible rotor in enhancing their designs for efficiency, sustainability and value for money.

Originality/Value: This research gives an understanding of vibration control behaviour in a flexible rotor system. This work is unique as it presents readers with an innovative way of introducing excitations into both ends of the shaft as an active controller of vibration for flexible rotor systems.

Keywords: Actuator · Exciter · Flexible rotor system · Parametric excitation · Piezoelectric

1 Introduction

1.1 Rotor Dynamics Problem

Mass unbalanced forces are one of the main causes of vibration in flexible rotor systems. Forces due to the depositions the machines are subjected to and wear are also important sources of vibration. Achieving perfect balance is virtually impossible. Vibration reduction in the rotor system is critical for safe and efficient operation. It is therefore critical to collaborate with research and product development in the rotary machine industry so as to refashion shaft rotor systems and their frequencies, in the quest to reduce vibration.

1.2 Piezoelectric Actuator Solution

Piezoelectric actuators have been used for the stabilization of parametric resonance produced in a cantilever beam and also as an efficient bifurcation control device, which act to shift bifurcation set as well as expand the stable region (Palazzolo et al. 1993; Barrett et al. 1995; Yabuno et al. 2001). Sui and Shi (2012) used an active piezoelectric actuator engine mount to support a vehicle engine while reducing vibrations and force transmitted from the engine to the vehicle structure as well as road surface irregularities. Berardengo et al. (2015) also used shunted piezoelectric actuators with electric impedances consisting of a series of resistance and inductance to passively control vibration in light structures. To actively control vibration in journal bearings, Tuma et al. (2017) used a double assembled linear piezoactuators to actuate the bearing journal position to first damp the vibrations and secondly maintain the preferred position to micrometer-order accuracy. William et al. (2019) studied the possibility of attenuating the vibrations emanating from within a link of a system using active vibration controller with piezoelectric patches as actuators.

1.3 Double Piezoelectric Exciter Concept

In Atepor (2008, 2009), the author-controlled vibrations emanating from within a rotor system by designing a piezoelectric exciter together with parametric excitations and incorporated into a flexible rotor-bearing system axially. The idea was to regulate the response of the rotor's already existing mass-unbalance vibration. The author also introduced axial excitations into the shaft with the help of a piezoelectric stack actuator to enable him study the interactions between forced vibrations that are caused by rotor unbalance and parametric excitations that are caused by periodic stiffness variations which arise from the actuator's axial excitations. A workable strategy was recommended. The recommendation was to manipulate intrinsic and prevalent instabilities associated with the flexible rotor-bearing system a manner that will efficiently control the overall performance of the rotor system. To justify this work, a schedule of research was performed. Findings showed a 21.9% decrease in the resonant amplitudes for a forward whirl in the flexible rotor-bearing system. To further lower the inherent vibrations and instabilities associated with the rotor, a theoretical application of double piezoelectric exciters and an intentional insertion of parametric excitations into the flexible rotor-bearing at both ends of the shaft has been explored by a referenced study seen in Atepor (2013). An introduction of double excitation force terms into the governing equations of motion was done theoretically. The famous perturbation method of multiple scales was employed to solve the equations of motion. The steady-state responses were examined for the cases where the double parametric excitation terms were present and absent respectively. Findings suggest a further decline in the rotor response amplitude.

2 Theoretical Works

2.1 Equations of Motion

Equations (1) to (4) are the equations of motion adapted from Atepor (2008, 2011), and Fig. 1 is the reference frame for a disk on a rotating flexible shaft. A 3-D view of the rotor.

$$\ddot{q}_1 + \hat{c}\dot{q}_1 - \Omega\hat{a}_5\dot{q}_2 + \omega^2 q_1 + \hat{b}q_1^3 = \mu d\Omega^2 \sin\Omega t \tag{1}$$

$$\ddot{q}_2 + \hat{c}\dot{q}_2 + \Omega\hat{a}_5\dot{q}_1 + \omega^2 q_2 + \hat{b}q_2^3 = \mu d\Omega^2 \cos\Omega t \tag{2}$$

$$\ddot{q}_1 + \hat{c}\dot{q}_1 - \Omega\hat{a}_5\dot{q}_2 + \omega^2 q_1 + \hat{b}q_1^3 - \hat{F}_{act}q_1 = \mu d\Omega^2 \sin\Omega t \tag{3}$$

$$\ddot{q}_2 + \hat{c}\dot{q}_2 + \Omega\hat{a}_5\dot{q}_1 + \omega^2 q_2 + \hat{b}q_2^3 - \hat{F}_{act}q_2 = \mu d\Omega^2 \cos\Omega t \tag{4}$$

$$\ddot{q}_1 + \hat{c}\dot{q}_1 - \Omega\hat{a}_5\dot{q}_2 + \omega^2 q_1 + \hat{b}q_1^3 - 2\hat{F}_{act}q_1 = \mu d\Omega^2 \sin\Omega t \tag{5}$$

$$\ddot{q}_2 + \hat{c}\dot{q}_2 + \Omega\hat{a}_5\dot{q}_1 + \omega^2 q_2 + \hat{b}q_2^3 - 2\hat{F}_{act}q_2 = \mu d\Omega^2 \cos\Omega t \tag{6}$$

Equations (1)–(2) and (3)–(4) are illustrates two equations of motion. The former are without parametric excitation force terms and the latter has single parametric excitation force terms present. Equations (5) and (6) are equations with double parametric excitation force terms.

Where, $\hat{a}_5 = \frac{a_5}{m}, \omega^2 = \frac{k}{m}, \hat{b} = \frac{b}{m}, \hat{c} = \frac{c}{m}, \mu = \frac{m_u}{m}, \hat{F}_0 = \frac{F_{act}}{m}$, k–linear stiffness coefficient, c-damping coefficient, q_1, q_2-displacements, ω-natural frequency, b-nonlinear cubic stiffness coefficient, Ω-excitation frequency, m_u-mass unbalance, a_i-characteristic equation coefficient, F_{act}-external applied force, $F_{act}q_i$ denotes the axial excitation force term (Atepor 2008) and the dots denote differentiation with respect to t. Parameters used in this work were computed with data sourced from the experimental rig. k is the radial stiffness of the rotor-bearing which characterises the combined circumferentially-symmetric stiffness of the rotor bearings and shaft.

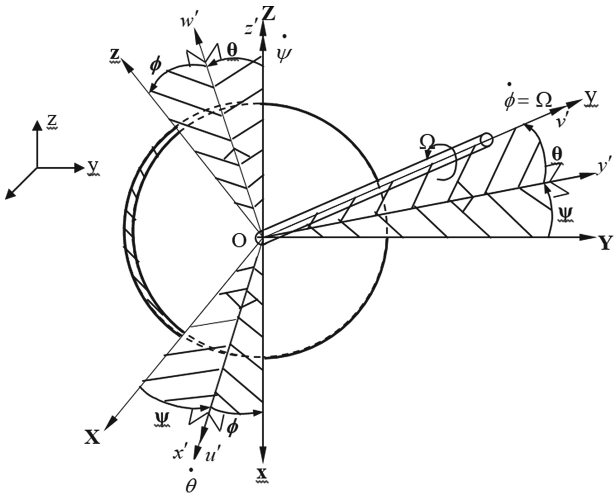

Fig. 1. A 3-D reference frame for a disk on a rotating flexible shaft.

2.2 Solutions to the Equations of Motion

As stated earlier, the method of multiple scales is employed to establish an approximate solution for flexible rotor system models with and without parametric excitation terms and given as

$$\begin{aligned}\bar{q}_1 &= 2p\cos\left(\frac{\Omega t}{2\omega}\right) - 2q\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}p^3\cos\left(\frac{3\Omega t}{2\omega}\right) \\ &-\frac{3\hat{b}}{4\omega^2}p^2q\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}pq^2\cos\left(\frac{3\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}q^3\sin\left(\frac{3\Omega t}{2\omega}\right)\end{aligned} \quad (7)$$

$$\begin{aligned}\bar{q}_2 &= 2r\cos\left(\frac{\Omega t}{2\omega}\right) - 2s\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}r^3\cos\left(\frac{3\Omega t}{2\omega}\right) \\ &-\frac{3\hat{b}}{4\omega^2}r^2s\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}rs^2\cos\left(\frac{3\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}s^3\sin\left(\frac{3\Omega t}{2\omega}\right)\end{aligned} \quad (8)$$

$$\begin{aligned}
\bar{q}_1 &= 2p\cos\left(\frac{\Omega t}{2\omega}\right) - 2q\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}p^3\cos\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}p^2q\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}pq^2\cos\left(\frac{3\Omega t}{2\omega}\right)\\
&+ \frac{\hat{b}}{4\omega^2}q^3\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{2\hat{F}_{act}p}{2\Omega_2^2+4\Omega_2\omega}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{2\hat{F}_{act}q}{2\Omega_2^2+4\Omega_2\omega}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{2\hat{F}_{act}\Omega_2\hat{c}kp}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\Omega_2\hat{c}kq}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{2\hat{F}_{act}\hat{c}kp}{2\Omega_2^2\omega+4\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&+ \frac{2\hat{F}_{act}\Omega\Omega_2\hat{a}_5kr}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\hat{c}kq}{2\Omega_2^2\omega+4\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{2\hat{F}_{act}\Omega\Omega_2\hat{a}_5ks}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&+ \frac{2\hat{F}_{act}\Omega\hat{a}_5kr}{2\Omega_2^2\omega+4\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{2\hat{F}_{act}\Omega\hat{a}_5ks}{2\Omega_2^2\omega+4\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}Qp}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{9\Omega_2 t}{4\omega}\right)\\
&+ \frac{2\hat{F}_{act}Qq}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{9\Omega_2 t}{4\omega}\right)
\end{aligned} \tag{9}$$

$$\begin{aligned}
\bar{q}_2 &= 2r\cos\left(\frac{\Omega t}{2\omega}\right) - 2s\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}r^3\cos\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}r^2s\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}rs^2\cos\left(\frac{3\Omega t}{2\omega}\right)\\
&+ \frac{\hat{b}}{4\omega^2}s^3\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{2\hat{F}_{act}r}{2\Omega_2^2+4\Omega_2\omega}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{2\hat{F}_{act}s}{2\Omega_2^2+4\Omega_2\omega}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{2\hat{F}_{act}\Omega_2\hat{c}kr}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\Omega_2\hat{c}ks}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\hat{c}kr}{2\Omega_2^2\omega+4\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{2\hat{F}_{act}\hat{c}ks}{2\Omega_2^2\omega+4\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\Omega\Omega_2\hat{a}_5kp}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\Omega\Omega_2\hat{a}_5kq}{2\Omega_2^2\omega^2+4\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{2\hat{F}_{act}\Omega\hat{a}_5kp}{2\Omega_2^2\omega+4\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}\Omega\hat{a}_5kq}{2\Omega_2^2\omega+4\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{2\hat{F}_{act}Qr}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{9\Omega_2 t}{4\omega}\right)\\
&+ \frac{2\hat{F}_{act}Qs}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{9\Omega_2 t}{4\omega}\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
\bar{q}_1 &= 2p\cos\left(\frac{\Omega t}{2\omega}\right) - 2q\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}p^3\cos\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}p^2q\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}pq^2\cos\left(\frac{3\Omega t}{2\omega}\right)\\
&+ \frac{\hat{b}}{4\omega^2}q^3\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{4\hat{F}_{act}p}{4\Omega_2^2+8\Omega_2\omega}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}q}{4\Omega_2^2+8\Omega_2\omega}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\Omega_2\hat{c}jp}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{4\hat{F}_{act}\Omega_2\hat{c}jq}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}\hat{c}jp}{4\Omega_2^2\omega+8\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\hat{c}jq}{4\Omega_2^2\omega+8\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&+ \frac{4\hat{F}_{act}\Omega\Omega_2\hat{a}_5jr}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}\Omega\Omega_2\hat{a}_5js}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}\Omega\hat{a}_5jr}{4\Omega_2^2\omega+8\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&+ \frac{4\hat{F}_{act}\Omega\hat{a}_5js}{4\Omega_2^2\omega+8\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}Lp}{8\Omega_2^2\omega^2+16\Omega_2\omega^3}\cos\left(\frac{9\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}Lq}{8\Omega_2^2\omega^2+16\Omega_2\omega^3}\sin\left(\frac{9\Omega_2 t}{4\omega}\right)
\end{aligned} \tag{11}$$

$$\begin{aligned}
\bar{q}_2 &= 2r\cos\left(\frac{\Omega t}{2\omega}\right) - 2s\sin\left(\frac{\Omega t}{2\omega}\right) + \frac{\hat{b}}{4\omega^2}r^3\cos\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}r^2s\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{3\hat{b}}{4\omega^2}rs^2\cos\left(\frac{3\Omega t}{2\omega}\right)\\
&+ \frac{\hat{b}}{4\omega^2}s^3\sin\left(\frac{3\Omega t}{2\omega}\right) - \frac{4\hat{F}_{act}r}{4\Omega_2^2+8\Omega_2\omega}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}s}{4\Omega_2^2+8\Omega_2\omega}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\Omega_2\hat{c}jr}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{4\hat{F}_{act}\Omega_2\hat{c}js}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\hat{c}jr}{4\Omega_2^2\omega+8\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\hat{c}js}{4\Omega_2^2\omega+8\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{4\hat{F}_{act}\Omega\Omega_2\hat{a}_5jp}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\sin\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\Omega\Omega_2\hat{a}_5jq}{4\Omega_2^2\omega^2+8\Omega_2\omega^3}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}\Omega\hat{a}_5jp}{4\Omega_2^2\omega+8\Omega_2\omega^2}\sin\left(\frac{5\Omega_2 t}{4\omega}\right)\\
&- \frac{4\hat{F}_{act}\Omega\hat{a}_5jq}{4\Omega_2^2\omega+8\Omega_2\omega^2}\cos\left(\frac{5\Omega_2 t}{4\omega}\right) - \frac{4\hat{F}_{act}Lr}{8\Omega_2^2\omega^2+16\Omega_2\omega^3}\cos\left(\frac{9\Omega_2 t}{4\omega}\right) + \frac{4\hat{F}_{act}Ls}{8\Omega_2^2\omega^2+16\Omega_2\omega^3}\sin\left(\frac{9\Omega_2 t}{4\omega}\right)
\end{aligned} \tag{12}$$

where, $k = \left(\frac{1}{-\frac{\Omega_2^2}{\omega^2} - \frac{2\Omega_2}{\omega}}\right)$, $Q = \left(\frac{1}{-\frac{4\Omega_2^2}{\omega^2} - \frac{4\Omega_2}{\omega}}\right)$, $j = \left(\frac{1}{-\frac{2\Omega_2^2}{\omega^2} - \frac{4\Omega_2}{\omega}}\right)$ and $L = \left(\frac{1}{-\frac{8\Omega_2^2}{\omega^2} - \frac{8\Omega_2}{\omega}}\right)$.

Full time-domain solutions of Eqs. (1) and (2) which do not have parametric excitation terms are presented in Eqs. (7) and (8), while Eqs. (9) and (10) presents solutions to equations with single parametric excitation terms. Solutions to equations of motion with double parametric excitation terms are presented in Eqs. (11) and (12). Amplitudes are denoted with the variable letters p, q, r and s, Ω is the excitation frequency such that $\Omega_2 = 2\Omega$ where Ω_2 denotes the principal parametric resonance frequency.

3 Experimental Work

The experiment employs a rotor-kit (built in the Cape Coast Technical University workshop) and piezoelectric exciters developed specifically for the purposes of this study. The kit comes with an electrical drive that served as power for the rotor, a rotor supported by journal bearings and a separate control box from which to select the preferred rotational speed. A solid coupling transmits the torque to the rotor from the electrical motor. Displacement transducers are provided to measure the rotor's movements. A piezoelectric exciter is attached to the rotor kit to serve as an active controller of vibration. A critical component of the exciter unit is a piezoelectric actuator which is supported by a helical compression spring and all are contained in a linear sliding bearing and an aluminum casing. A function generator drives the piezoelectric actuator via a piezoelectric actuator amplifier. To prevent direct contact between the piezoexciter and the shaft and permit free rotation and movement of the shaft end, a small self-aligned ball bearing is fixed between the piezoexciter and the shaft. The rotor's vibration response is then measured with a Polytec Laser Vibrometer, which allows the displacements to be recognized and inspected by a multi-channel data acquisition analyzer. Figures 2, 3 and 4 show the experimental set-up used to activate the flexible rotor system. The foremost principle here is to axially control the rotor vibrations, by using the piezoelectric actuators mounted at the two ends of the shaft.

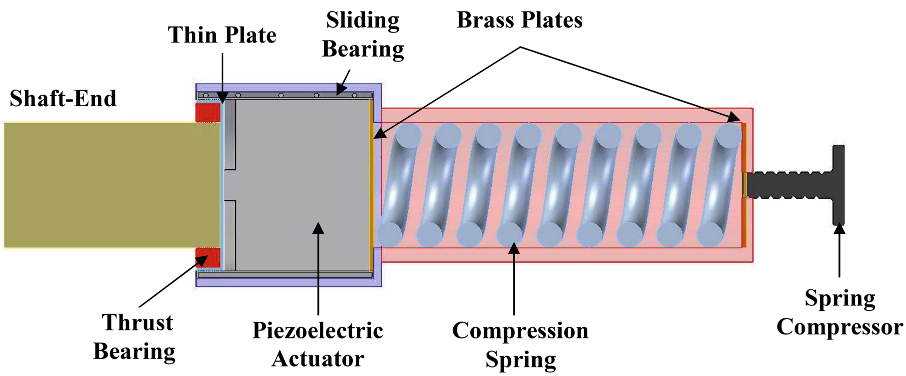

Fig. 2. Schematic of the Piezoelectric Exciter (Atepor 2008)

As stated earlier, the goal has been to design and build a test rig to demonstrate the practicability of active controller of vibration in rotor dynamics using double piezoelectric actuators. Special emphasis is placed on the likelihood of decreasing the amplitude of vibrations of a flexible dynamically unbalanced rotor within acceptable levels. This is accomplished by creating a Piezoexciter that is activated by a high frequency drive. The active Piezoexciter is made up of a sliding bearing that contains a piezoelectric stack actuator and is serially attached to a compression spring. The reaction spring is set up against the actuator, owing to the fact that, the actuator operates only in expansion, with small displacements. The spring compressor adjusts the spring to the requisite length,

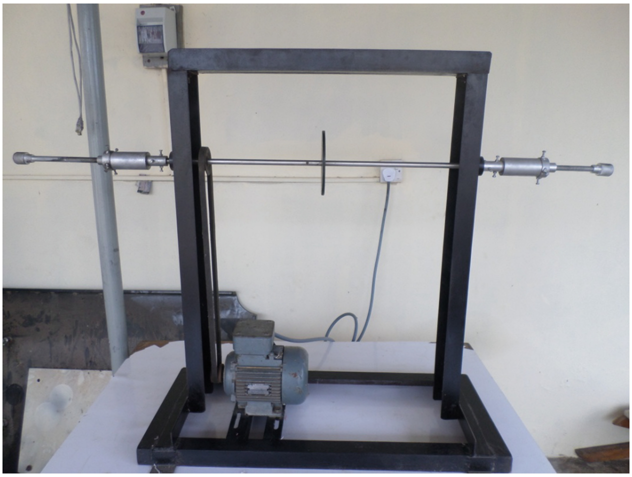

Fig. 3. The Piezoexciter Test Rig with the two exciters at both ends of the shaft.

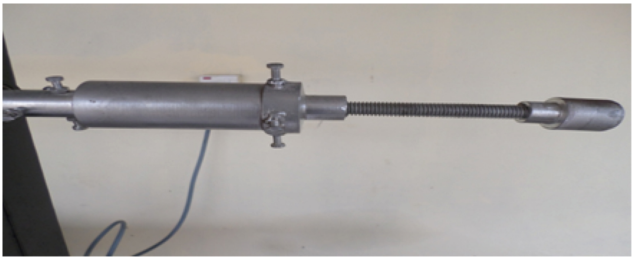

Fig. 4. Close up of the exciter assembly

and voltage is applied to the actuator via a piezoelectric voltage amplifier, which generates parametric excitation at twice the rotor system's first whirl frequency. The exciter is operated by a function generator with the help of a high voltage amplifier. The parametric excitation force to be axially introduced to the shaft, are generated by initiating the piezoelectric actuator at twice the excitation frequency of the rotor system.

A laser vibrometer is then used to measure the vibration response of the rotor-bearing system. The response is then analyzed using a multi-channel data acquisition analyzer. The rotor-bearing system and the exciter unit's compression spring are fixed to the first whirl resonance frequency and the requisite length respectively, then the rotor's response is measured. The piezoelectric actuator is then activated, first at a frequency twice the rotor system's first whirl frequency. A series of timed tests were administered, and mean readings were recorded. Sweep tests are then performed around the first whirl frequency, initially without activating the piezoexciter and later, with the exciter activated at the parametric excitation frequency.

4 Results and Discussion

4.1 Theoretical Results

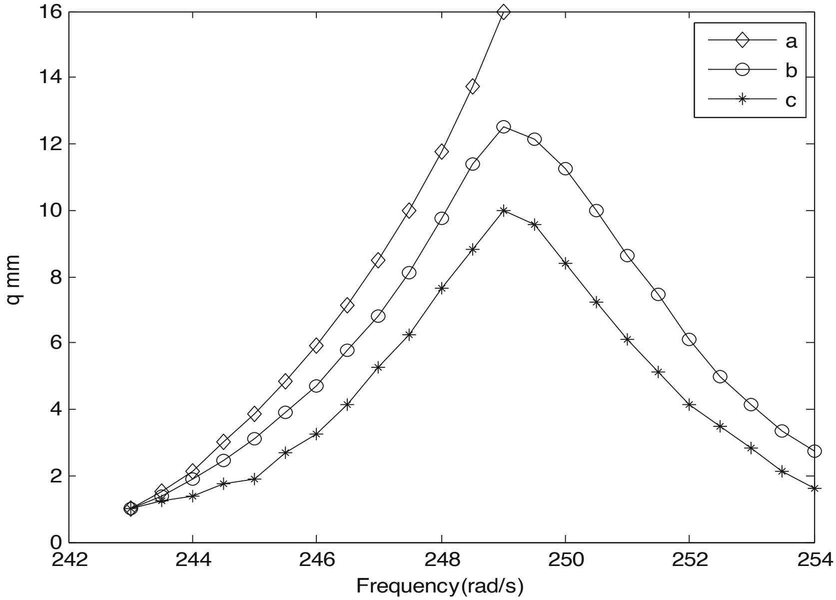

Fig. 5. Amplitudes (q) of the response as functions of the frequency (rad/s): a-without parametric force term, b-with single parametric force term, c-with double parametric force term.

The theoretical results as presented in Fig. 5. were obtained first obtaining solutions to the governing equations of motion using the perturbation method of multiple scales and the plot generated by using Mathematica ™ software. Concerning Fig. 5, and considering plot legend **a,** responses in the first mode of q show hardening characteristics, jump phenomena and both stable and unstable solutions when the equations of motion contain no parametric force terms. When no parametric force term is present in the equations of motion, responses in the first mode of q show hardening characteristics, jump phenomena, and both stable and unstable solutions (see Fig. 5a). A 23% decrease in the amplitude is observed when single parametric force terms are included in the equations of motion as can be seen in Fig. 5(b). Elimination of the jump phenomena and stable solutions are also observed. Referencing Fig. 5(c), upon the introduction of double parametric force terms, the same observation was made as seen in Fig. 5(b). However, the amplitude, in this case, was reduced by 60.2%. The further decrease implies stability of the system at a parametric frequency which was actioned by the theoretical introduction of parametric forces at both ends of the shaft.

4.2 Experimental Results

To ultimately evaluate the performance of the test rig shown in Fig. 3, the loading condition of the spring of the exciter is set at a length of 25.2 mm (Atepor 2008) where the author in considering the performance of a similar test rig investigated three different loading conditions and arrived at the conclusion that the 25.2 mm compressed length of the spring gives better maximum and minimum spring forces.

The effect of single and double piezoelectric exciter activation was thoroughly investigated. Figure 6(a) displays a 14.95 mm peak amplitude at a 250 rad/s resonance frequency in the absence of the piezoexciter, i.e. the shaft experiences no parametric excitation its speed is varied between 75 rad/s and 450 rad/s inclusive. In Fig. 6(b), activating only one piezoexciter at a parametric frequency of $\Omega_2 = 500$ rad/s, where $\Omega_2 = 2\Omega$, the disk vibration amplitude reduces to 12.8 mm. For the third case, when both piezoexciters are activated at both ends of the shaft at parametric frequencies of 500 rad/s, the disk vibration amplitude reduces to 11.45 mm and this is depicted in Fig. 6(c). In Fig. 6, the collective influence of the existing force vibration and the supplementary parametric excitation in principal parametric resonance occasioned the regulation of the responses of the already existing vibration of the mass unbalance as well as decrement in the critical whirl amplitude. As a recap, the existing force vibration is caused by the mass unbalance and the extra parametric excitation is sanctioned by the piezoexciter. When the parametric excitation is applied to a single end of the shaft, 14.4% decrease in the amplitude is observed, as shown in Fig. 6(b). In the case where both ends of the shaft are exposed to parametric excitations, a further reduction of 23.4% is experienced in the amplitude as shown in Fig. 6(c).

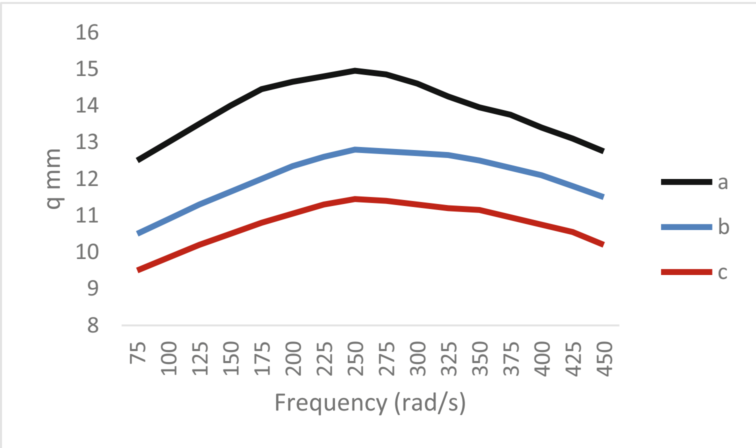

Fig. 6. Disc vibration amplitudes (q) as functions of frequency (rad/s) with a spring compression length of 25.2 mm: Experimental results, a-parametric force term absent, b-single parametric force present, c-double parametric force term present.

5 Conclusion

The comparison between the theoretical and experimental analyses summarized in this work provides proof of a consistent occurrence, but with a lower percentage reduction for the experimental benchmark than the theoretical consideration, which is primarily due to the assumptions made when the nonlinear equations of motion are analytically solved. The methods used to investigate and identify rotor systems response behavior have all revealed comparable trends in terms of the consequences of introducing double parametric forces. The innovative piezoelectric exciter concepts could be effectively attached to both ends of the shaft of industrial machines, predominantly those installations where axial loading on the rotor is also an inherent part of the control actuation for a very high reduction in vibration amplitude, according to prototypical experimental results from rotor systems.

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