

# The Square of Opposition: Past, Present, and Future



Jean-Yves Beziau and Ioannis Vandoulakis

**Abstract** We first explain the origin and development of the theory of opposition, its generalization to many concepts, and figures of opposition, particularly the hexagon of opposition. We also survey the organization of a series of events on the topic since 2007 in Montreux. We then talk in details about the sixth edition of the world congress on the square emphasizing the fact that it was organized at the Orthodox Academy of Crete. In the third part, we discuss the bright future of the theory.

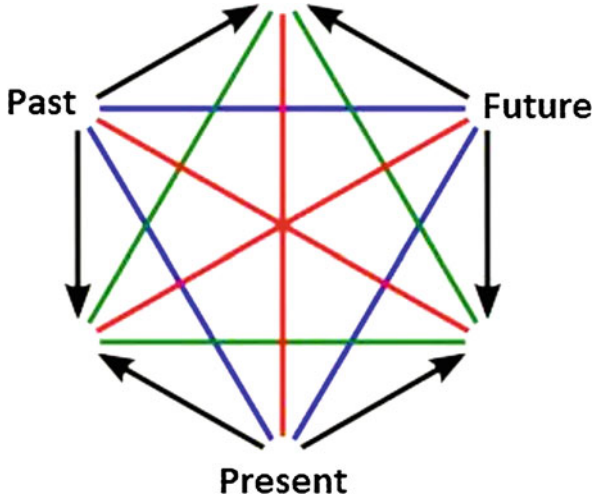
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## 1 The Square of Opposition: A Diagram and a Theory

The square of opposition is a diagram going back to Apuleius (*Peri Hermeneias*) and Boethius (*De Silogismo Categorico*) based on some ideas by Aristotle (*De Interpretatione* 6–7, 17 b 17–26 and *Prior Analytics* I.2, 25 a 1–25) (for details see [32]). The expression “square of opposition” is used today not only to talk about this diagram but also about the theory surrounding it, whose original diagram is just a primitive print.

There are two crucial aspects of the SQUARE theory: on the one hand, the visual aspect, and on the other hand, the logical aspect. The visual aspect is related to geometry, originally a square, and the logical aspect to the notion of opposition, hence the terminology “square of opposition.”

Opposition is a single notion (about the concept of notion, see [5]), the reason why the singular is used, although the theory includes three kinds of opposition: contradiction, contrariety, and subcontrariety. These three oppositions are presented in a unified logical framework in the theory of opposition. These are logical concepts involving truth and falsity, going beyond the truth/falsity dichotomy, not by introducing a third value but by combination and interaction of these two values.

These three kinds of opposition were originally defined on the basis of propositions. Two propositions are contradictory iff (if and only if) they cannot be true together and cannot be false together, *contrary* iff they can be false together but not true together, and *subcontrary* iff they can be true together but not false together. The square of opposition was indeed an essential part of the theory of categorical proposition, giving an architecture, a classification of Aristotelian propositions (Fig. 1).

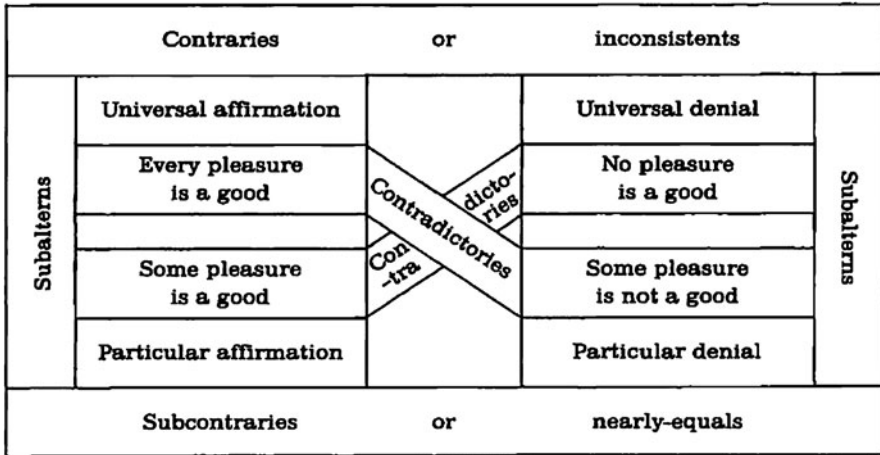
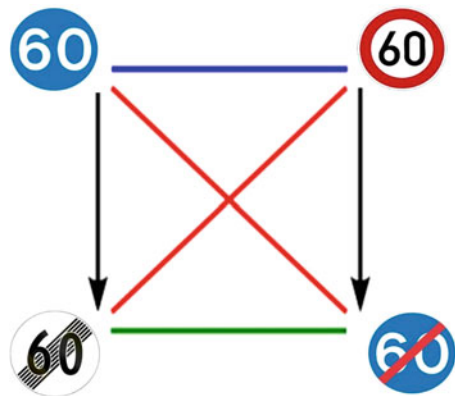


Fig. 1 Apuleius’s reconstructed square

But this structure can easily be extended to concepts conceived extensionally or intensionally. For example, extensionally contradiction corresponds to the set-theoretical notion of complementation. This variation to concepts is important and gives rise to numerous applications of the square to deontic notions (prohibition, obligation, and the like) (Fig. 2), color classification, music concepts, and many other topics. The *square of opposition theory* has evolved in particular through this conceptual appraisal, not by the development of new forms of opposition.

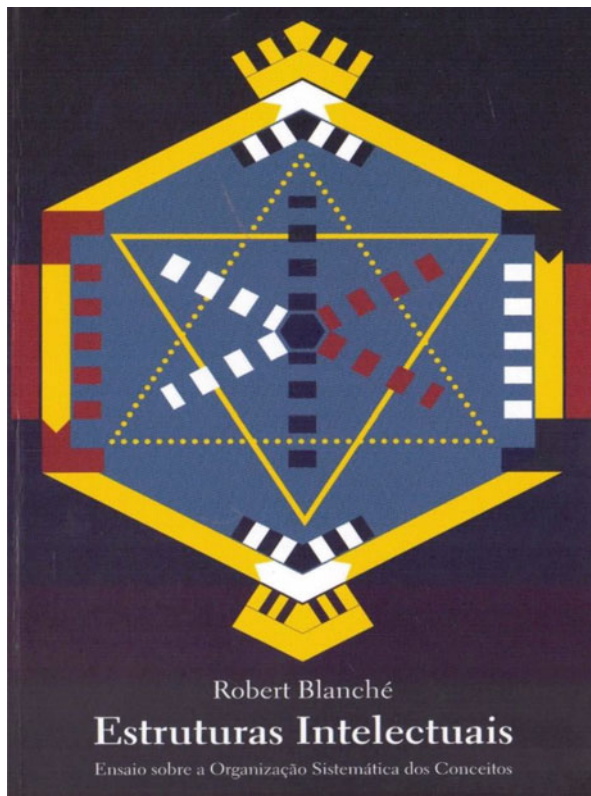
Fig. 2 Deontic traffic sign square of opposition



Another significant step in the development of the theory is the versatility of the relations between the three kinds of opposition and the notion of subalternation which escorts these relations. These variations are naturally produced by the geometrical aspect of the theory. From a square we can go to a hexagon, octagon, decagon, a cube, a dodecahedron, and so forth.

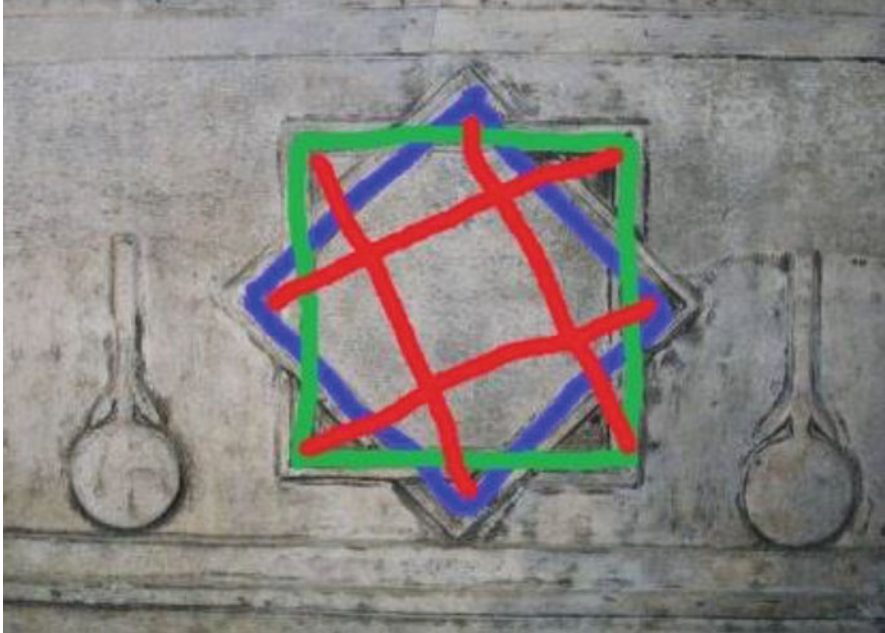
A central geometrical figure which emerged and turned out to be a central key to the theory is however the hexagon [3]. The hexagonal theory of opposition was mainly promoted by Robert Blanché (but see [21]). And the basis of Blanché's hexagonal theory is the triangle of contrariety, which is the heart of the hexagon. Blanché developed his theory at the end of the 1950s. He published a paper in the *Journal of Symbolic Logic* [15], but his main work on the topic, his book *Structures intellectuelles - Essai sur l'organisation systématique des concepts*, was published in 1966 [16]. His theory did not have much impact. His book has up to now not been translated into English or into another foreign language except Brazilian Portuguese (Fig. 3).

**Fig. 3** Brazilian version of Blanché's book. (Cover by Sergio Kon of the Book by Robert Blanché *Estruturas intelectuais: ensaio sobre a organização sistemática dos conceitos*. Reproduction courtesy of EDITORA PERSPECTIVA, <https://editoraperspectiva.com.br/produtos/estruturas-intelectuais/>)



The theory of the square was revived by the first author when he wrote in 2003 the paper “New light on the square of oppositions and its nameless corner,” published in the logic journal of the *Russian Academy of Sciences* in Moscow [2], introducing:

- Coloring.
- A three-dimensional object which is a combination of three hexagons.
- An octagon which like the hexagon is the combination of two dual figures of opposition, a square of contrariety and a square of subcontrariety (Fig. 4).



**Fig. 4** Octagon of opposition generalizing Blanché’s hexagon. (Photos courtesy of Jean-Yves Beziau)

He was then in touch with both Alessio Moretti, a French-Italian student who did a PhD with him on the topic in Neuchâtel, developing in particular the theory of  $n$ -opposition [23] corresponding to the third point and Hans Smessaert, a Belgian linguist much interested in polyhedra. Since then these two gentlemen have constantly been working on the topic (e.g., see [18, 24, 25, 29, 30]). They were present at the *first World Congress on the Square of Opposition (SQUARE)* organized by the first author in Montreux in 2007 [7, 8] and also at the sixth edition organized by both authors in Crete in 2018.

Between these two events, there were the second edition of the SQUARE in Corsica in 2010 [6], the third in Beirut in 2012 [10], the fourth at the Vatican in 2014 [11, 12], and the fifth in Easter Island in 2016 [13]. Many distinguished scholars from logic, mathematics, philosophy, computer science, semiotics, theology, and psychology have participated in these events, among them: Pierre Cartier, Pascal Engel, Jan Woleński, Larry Horn, Terence Parsons, Dale Jacquette (Fig. 5), Stephen Read, Peter Schroeder-Heister, Wolfgang Lenzen, and John Woods.



**Fig. 5** Dale Jacquette at the third SQUARE in Beirut. (Photos courtesy of Jean-Yves Beziau)

## **2 The Sixth World Congress on the Square of Opposition: Crete 2018**

Most of the papers included in this book are related to talks presented at the sixth edition of the *World Congress on the Square of Opposition* which took place at the *Orthodox Academy of Crete* (OAC) in the picturesque Kolymbari village, near Chania, in Crete, November 1–5, 2018. Other papers that were presented at this event were published in a special issue of *Logica Universalis* edited by the first author and Jens Lemanski [14].

The idea of organizing the event at the OAC in Crete is due to the second author. He was born in Crete and had already organized an event at this academy. The two authors of the present paper and editors of this book were the main organizers of the event (Fig. 6), supported, besides the OAC, on the one hand by Jens Lemanski, from the Institute of Philosophy of the *University of Hagen* in Germany, and on the other hand by Petros Stefanias and Ioannis Kriouvrekis, both from the Department of Applied Mathematics, of the *National Technical University of Athens*.



**Fig. 6** I.Vandoulakis and J.-Y.Beziau organizers of the sixth SQUARE. (Photos courtesy of the Orthodox Academy of Crete)

The Orthodox Academy of Crete was created in 1968 and functions under the auspices of the Ecumenical Patriarchate. It is a research, education, and conference center aiming at promoting the dialog between faith, science, and culture and inspired by the Platonic tradition of *συμφιλοσοφεῖν* (symphilosophēin – philosophizing together). The academy participates in EU (European Union) and national research projects and is a member of the *Ecumenical Association of Academies* and the *Laity Centres in Europe* (Oikosnet Europe).

The academy hosts a unique *Museum of Cretan Herbs* that includes about 6000 herbs of the collection of Cretan herbs, gathered by the French professor of botany Jacques Zaffran, who dedicated his life to the scientific study of the rich flora and especially the endemic species of Crete. Some of them, for instance, the dittany of Crete (*Origanum dictamnus*), is mentioned by Aristotle in his work *History of Animals* (612a4) and his pupil Theophrastus in his work *Enquiry into Plants* (9.16.1). Many important events have been organized by the OAC including international conferences related to philosophy, theology, environmental studies, physics, biology, medicine, computer science, bioethics, and social issues.

The event on the square was organized shortly after the commemoration of the 50th anniversary of the academy and its award of the silver medal by the Academy of Athens. The event lasted for 5 days, starting Wednesday, November 1, 2018, and ending Sunday, November 5, 2018, with invited talks, contributing talks, and tutorials.

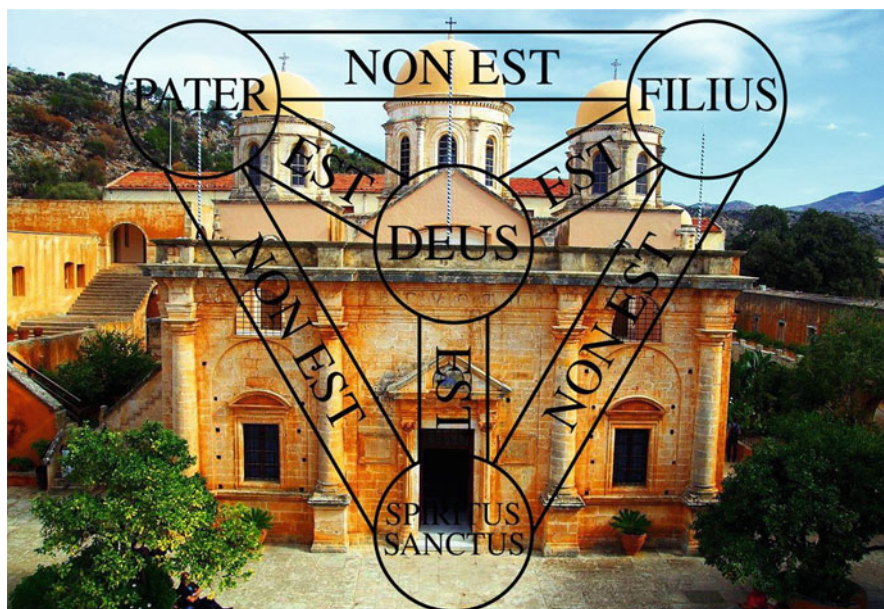
On Friday, November 3rd afternoon, an excursion was organized in the old Venetian town and harbor of Chania (*La Canea*). The Venetian architecture is quite visible from the first moment since, historically, after the Arabs and Byzantines,

Crete was conquered by Venetians in 1252. Chania is a place where different civilizations of the East and West have flourished throughout the centuries and left their impact visible today in every step. Besides the Venetian part of the city, there is the Jewish Quarter with the *Etz Hayyim Synagogue*, and the Turkish part called *Splantzia* with a maze of narrow streets leading to the Venetian port with the *Mosque of Kioutsouk Hassan*, the oldest Ottoman building in Crete, erected in 1645.

The excursion also included the seventeenth-century Monastery of *Agia Triada* (Holy Trinity) in the Akrotiri peninsula built by two brothers Jeremiah-Ioannis and Laurentius-Lucas of the Venetian noble Zancaroli family. The church is built in the Byzantine architectural cruciform style with three domes with two large Doric-style columns and one smaller, Corinthian-style column on either side of the main entrance. The facade bears an inscription in Greek, which is dated to 1631.

The Holy Trinity typically forms a *triangle of contrariety*, called “Shield of the Trinity” or *Scutum Fidei* in the Western Christian tradition (see [19]). This trinity was one of the reasons why the fourth edition of the SQUARE was organized at the *Pontifical Lateran University* in the Vatican in 2014 where the participants were welcomed by Bishop Enrico dal Covol, the rector of this university, nicknamed the “Pope University” (cf. [11, 12]).

Amazingly, this triangle can be recognized in the façade of the Monastery of *Agia Triada* (Fig. 7). This incidence can be possibly explained by the fact that the Zancaroli brothers were converted from the Catholic to the Greek Orthodox faith and were familiar with both religious cultures.



**Fig. 7** Monastery of *Agia Triada* Tsangarolon (of the Zancaroli family) in the Akrotiri peninsula represents fantastic blend of features of different architectural traditions. On its façade, the triangle of contrariety can be recognized. (Photos courtesy of the Orthodox Academy of Crete)



### 3 The Square of Opposition: An Ongoing Open Project

The adventures of the square of opposition will go on. The seventh edition of SQUARE is scheduled to be held in Leuven, in Belgium in 2022 (organized by KU Leuven, it was delayed due the COVID-19 pandemic) (Fig. 8) and the eighth edition on the Island of Madeira, Portugal.



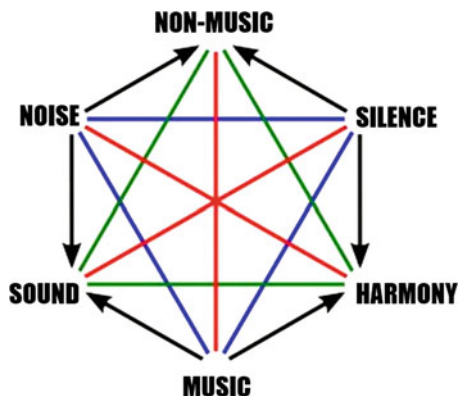
**Fig. 8** The seventh SQUARE will take place in KU Leuven, Belgium, Sept 2022. (Photo courtesy of Jacques Riche)

The future of the square of opposition looks bright, capturing more and more the interest of a great variety of scholars. The theory of opposition grows in depth and complexity and spreads over all fields of knowledge with extensive applications. Much work can be developed concerning the history of the square, its philosophical, logical, mathematical, and semiotic aspects.

Regarding the history of the square, the full story, within various cultures and traditions, still needs to be explored and told [9]. This can be done in different ways. We plan in particular to publish a “diagrammatic book” on the square, collecting the most significant diagrams of the history of the square with lengthy commentaries.

The square has flourished and will be flourishing indefinitely by its internal structure of opposition that grows and expands in many directions like any mathematical theory. There is a continuous interaction between geometrical objects, logical structures and all kinds of concepts. The theory has been applied in particular to color theory [20], music theory (see [26] and Fig. 9), quantum physics [28], painting theory [17], analogy [4, 27], and Kant's theory of antinomies [22].

Fig. 9 The hexagon of music

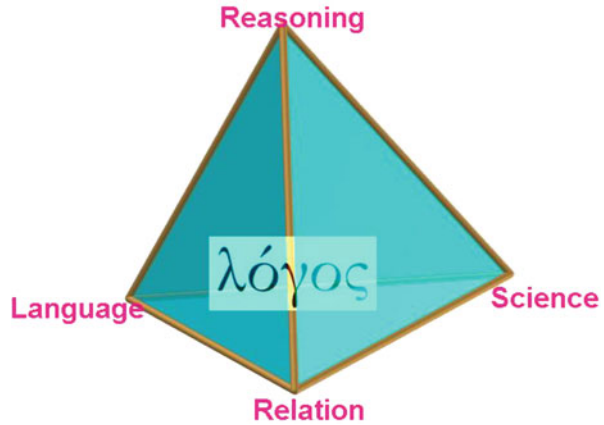


The reason to call the present book *The Exoteric Square of Opposition* is that in the future we expect to have more mysterious things going on, both at the theoretical level and at the level of applications. Then we will publish another book that will be entitled *The Esoteric Square of Opposition*.

The SQUARE project can be considered as a symbol of successful interdisciplinarity. The reason for its success is that it is based on a simple, yet rich, structure understandable by everybody. As with the chess game, there is a good balance/contrast between the simplicity of the basic rules and the complexity of what it is possible to do with these rules. The SQUARE can be seen as a kind of chess game of thought. However, it is a game with no losers, opening infinite possibilities.

This game, this theory, is based on logic, logic which is the foundation of rationality. The SQUARE theory is a good expression/reflection of the four aspects of the *Logos*: reasoning, relation, science, and language, that can themselves be represented by the four-theory of oppositions with a tetrahedron (of subcontrariety), the 3-simplex (Fig. 10).

**Fig. 10** The tetrahedron of the *Logos*



The organization of the sixth SQUARE in Greece was important for symbolic reasons. First, because the idea of the square was born in Greece in the work of Aristotle. Moreover to have that event in Crete was even more impressive, due to the fact that Crete is the origin of the Greek civilization. There were born the alphabet and Zeus. And in Crete there is Plato's cave and the Labyrinth, two fundamental philosophical symbols.

Crete is the cradle of the Minoan civilization, a Bronze Age Aegean civilization that represents the first advanced civilization in Europe. Several writing systems go back to the Minoan period, most of which remain still undeciphered, such as the Linear A script and the script on the Phaistos disc.

Crete is also associated with the semi-mythical philosopher and poet Epimenides of Knossos or Phaistos (seventh or sixth century BC), famous for his *Epimenides paradox* ("Cretans, always liars"), origin of the liar paradox (for a recent appraisal of this paradox, see [31]). According to Diogenes Laërtius, Epimenides met Pythagoras in Crete, and they went to the cave called *Dictaeon Antron* (the *Psychro* cave) of Mount Ida, where Zeus was raised by a goat named Amalthea. This cave is known to be the origin of one of the most famous texts of philosophy: the *Allegory of the Cave* by Plato (see [1]).

Crete is also famous for the labyrinth, as Arthur Evans called the Knossos palace, because of its architectural complexity. According to the myth, King Minos commanded the skillful craftsman Daedalus (the name *Δαίδαλος* connotes "labyrinth") to construct a monumental building of interconnected rooms – a *labyrinth* (*λαβύρινθος*) – to imprison *Minotaur* (a monster with the body of a man and the head of a bull). Minotaur's original name has obvious cosmological connotations: *Asterion*, which is the ruler of the stars; Minotaur also has a connotation to the constellation of *Taurus*.

The labyrinth is an ambiguous concept and construction that allows a double interpretation. From inside, it is a disorientating, chaotic construction; a man is imprisoned in it and unable to understand its structure and to find a way out of it; he is entrapped in a repetitious pattern of wrong choices. From outside or above, it is a sophisticated ordered construction of admirable complexity. Thus, the labyrinth is a metaphor that combines two opposite visions: overt chaotic complexity (internally) vs. underlying order (externally), imprisonment vs. freedom, confusion vs. clarity, multitude vs. unity, and limited perception vs. overall comprehension. The labyrinth pattern also appears in non-European cultures, for instance, in Indian manuscripts and esoteric Buddhist texts, such as the *Chakravayuha* that refers to a military formation narrated in the Hindu epic *Mahabharata*. A prehistoric petroglyph on a riverbank in Goa shows a labyrinthine pattern that has been dated to circa 2500 BC. All these shows that the pattern of the labyrinth is one of the oldest symbols of human civilization (see [33]).

In the European philosophical tradition, the labyrinth was also conceptualized in dynamic terms and used as a metaphor for mental processes. According to Gottfried Wilhelm Leibniz (1646–1716), there are “two famous labyrinths where our reason very often goes astray”:

- (i) The problem of human freedom.
- (ii) The structure of the continuum.

In the twentieth century, the computer scientist and Pulitzer Prize-winner Douglas R. Hofstadter represented the mind by the metaphor of an ant colony, i.e., a labyrinth of rooms, with endless rows of doors flinging open and slamming shut; a network of intricate domino chains, branching apart and rejoining, with little timed springs to stand the dominoes back up.

The SQUARE helps us circulate along the labyrinth of thought and to escape the cave’s darkness and illusion to reach understanding (Fig. 11).



**Fig. 11** Escaping the cave with the labyrinth of thought

**Acknowledgments** We thank all the participants of SQUARE 2018 and all people who helped to organize it. Special thanks to Dr. Antonis Kalogerakis, responsible for the local organization of the event at the OAC, and Dr. Konstantinos Zorbas, general director of the *Orthodox Academy of Crete*.

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