Static Indoor Pathfinding with Explicit Group Two-Parameter Over Relaxation Iterative Technique

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Abstract. A solution to Laplace's equation referred to as harmonic potential fields is commonly employed in robot pathfinding as an indication for robot navigation in an identifiable environment. The simulations computation of these harmonic functions frequently requires a high-performance computer. In the quest to address the pathfinding problem, the article presents a technique called Block Two-Parameter Over-Relaxation, otherwise known as Explicit Group Two-Parameter Over-Relaxation (EGTOR). The simulations of robot pathfinding were executed in a static known enclosed environment to validate the competency of EGTOR. Multiple tests are provided to assess the effectiveness of the suggested technique. Different departure and goal positions, in particular, are used to assess the paths generated by the simulations. The outcomes demonstrate the advantages of the proposed technique. In the context of iteration number, EGTOR improves by around 4.4% when compared to EGAOR and 17.1% in comparison with EGSOR. While as to computational timing, EGTOR outperforms EGAOR and EGSOR by 6.3% and 14.5%, respectively. The study concludes that the suggested method in computing harmonic functions is appealing and attainable for solving path planning problems.

Keywords: Self-directed · Harmonic potential · Collision free · Optimal path

1 Introduction

Developing intelligent autonomous motion planning is among of the most complex tasks in robotics practices. Presently, intelligent self-directed robots are in high demand in a variety of areas including space [1], industrial [2], manufacturing [3], transportation [4], military and security [5]. A successful autonomous mobile robot should be efficient and dependable in designing a path from any launch point to a finish point without colliding with obstructions. Path-planning with artificial potential fields [6] proposes an excellent approach for a navigational path selection through the formation of an artificial potential function which "draws" the robot towards the destination and "repulse" the robot from

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the obstacles. However, as discussed in [7], a potential field without residual local minima is extremely difficult to establish, which might mistakenly lead the robot passing over the field's negative gradient, eventually become stranded in the inaccurate position. Koren [8] has addressed additional well-known challenges, such as oscillations over the obstacle borders, escaping the entrance of extremely small tunnels, and oscillations while moving across tapering channels.

The notion of analytically addressing the Laplace equation for path planning purposes was led by Connolly [9]. He illustrated how numerical solutions may be used to search pathways in two and three-dimensional configuration spaces for certain simple and motionless environments. His approach has been executed utilizing the technology available at the time, which resulted in unacceptable time frames nowadays, with the computation of simple environments taking within 23 to 188 s. He projected that with adequately powerful computers, path formation consuming this technique would be an attainable choice.

Later, Sasaki [10] proposed by exploiting the elliptic PDE to tackle heat conduction issues in order to generate a potential field with no local minima. He successfully designed a potential field which ensures a route from start to the goal (assuming it exists) that free of local minima by viewing the start as a hot point, the goal as a cold, and the obstacles as an unknown adiabatic object. Sasaki's approach is, however, solely appropriate for motionless and thoroughly recognized environments. The starting point was given a high temperature and was designated as a local maximum. Since the study was completed in 1998, computer restrictions/limitations at the time prevent or at least discourage its usage for higher-dimensional problems or in difficult environments.

This article introduces the problem of mobile robot pathfinding expressed as a heat transfer analogy. The heat transmission is illustrated by Laplace's equation. One of the most essential features of heat transfer is its ability to exceed the difficulty of local minima, which makes it particularly promising for robot navigation control. The solutions to Laplace's equation a.k.a harmonic function, symbolize the temperature values in the environment of the path formation model. Several methods were applied for the achievement of harmonic functions, but the most general approach is through numerical techniques owing to the obtainability of rapid processing machines and their elegance and competence in addressing the problem [11–13]. Three experiments i.e. Explicit Group Successive Over-Relaxation (EGSOR), Explicit Group Accelerated Over-Relaxation (EGAOR), and Explicit Group Two-Parameter Over-Relaxation (EGTOR), were executed in this article to assess the proficiency of the accelerated iterative approach employed in constructing routes of a portable robot for various size of environments. This article is divided into four different sections. The next section mainly will address the materials and techniques that will be used in this study, including the formulation and algorithms that describe the entire pathfinding process. Whereas Sect. 3 describes the results and discussion, along with some figures and tables reflecting the study's outcome. Finally, in Sect. 4, the conclusion will be explained.

2 Materials and Methods

Rather than employing a genuine robot mobile, we recreate the idea of an autonomous mobile robot depicted by a roaming nodal point in a known motionless confined environment. The predicament of identifying the robot's course is classified as a problem of steady-state heat transfer. The goal should be regarded, in the resemblance of heat transmission, as a sink drawing heat in. The environment's obstacles/barriers and boundary walls are referred to as heat sources, and they are preserved at consistent temperatures. The temperature dispersion expands along the heat diffusion course, and the contour of heat flux spreading toward the sink that floods in the environment. Such condition may be considered as a means of communication among the goal, barriers, and the points functioning as robots. The temperature distribution inside the environment is then utilized as a controller to lead the robot to travel from the departure location to the target spot by directing the heat flow beginning at the highest to the lowest temperature point in the given environment. The temperature distribution in the environment is calculated by practicing the harmonic function to describe the setup space.

The domain Ω (denoted by $\partial\Omega$) in subject to generate the path planning consists of several components such as the walls, borders, obstacles/hindrances/barriers, multiple start locations, and goal point. A harmonic function in a domain $\Omega \subset R^n$ is a function that satisfies Laplace's equation, as follow

$$\nabla^2 U = \sum_{i=1}^{n} \frac{\partial^2 U}{\partial x_i^2} = 0 \tag{1}$$

with x_i indicates the i-th coordinates of Cartesian and the dimension be represented as n. Owing to the benefits of harmonic functions that satisfy the minimum-maximum condition, the formation of local minima in the domain can be prevented. As a result, harmonic functions are highly useful in robot pathfinding because they give exact path algorithms that allow smooth and efficient autonomous robot navigation [14]. It is widely known in the literature that Laplace's equation can be efficiently solved numerically through conventional techniques [15] for instance Jacobi**,** Gauss-Seidel (GS), and Successive Over-Relaxation (SOR). In pursuit to speed up the computation, this article aims to define Eq. (1) using an accelerated iterative approach called EGTOR.

In the pathfinding problem, the potential field is computed globally. Solving the Laplace expression, as indicated in Eq. (1), yielded the harmonic function. It is used to determine a route that progressively advances the point robot upon the starting location to the destination position while never colliding with any obstacles. The hindrances are always quantified as new sources, while the target is defined as the sink with the lowermost potential value. This all adds up to the application of Dirichlet boundary conditions, $U|\partial\Omega = c,\ c =$ constant. Later, by executing a Gradient Descent Search-Distance Transform (GDS-DT) on the potential field, a sequence of potential points with lower values will lead to the lowermost value, which is the goal point, to be discovered.

This study applied the above-mentioned framework for the path planning problem to aid in expressing the results of Laplace expression Eq. (1). The objective is to use the notion of how temperature and heat stream function in generating potential value and path lines for robot navigation. The experiment is carried out in a 2D domain with several types of obstacles. The proposed iterative scheme, EGTOR is employed to compute Laplace's equation as well as to obtain the values of temperature at each node. Comparisons with the EGSOR and EGAOR iterative techniques were used to evaluate its performance.

As mentioned before, a nodal point in the grid-form structure (see Fig. 1) portrayed the robot in this simulation. Meanwhile, Fig. 2 illustrates a part of a computational molecule for a five-point approximation from the configuration space, at which h indicates the length among nodal points at each direction. The numerical approach then calculates the function values for every node iteratively so as to meet Eq. (1). The departure location is assigned with the uppermost temperature, while the target spot is appointed as the lowest, and varying departure temperature values are allocated to the boundaries wall and barriers/obstacles. After obtaining the potential values in the configuration area, a smooth trajectory can be established by tracing the temperature dissemination through the steepest descent approach, in which the algorithm tracks the negative gradient at the lowermost temperature goal point from the start to lower temperature consecutive points.

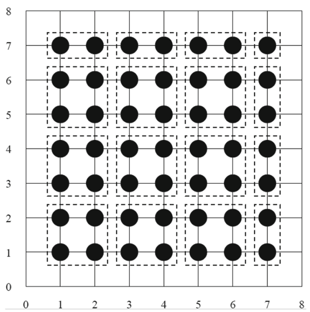

Fig. 1. Grid-form structure of nodes.

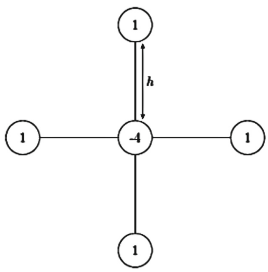

Fig. 2. Five-point approximation computational molecule.

2.1 Explicit Group Two-Parameter Over-Relaxation Iterative Technique

The conventional GS [9] and SOR [16] from robotics writings were employed as remedies for the problem (1). In this investigation, the solution to Laplace's equation is found by employing a quicker numerical approach, the Explicit Group Two-parameter Over-Relaxation (EGTOR) iterative technique. In reality, the TOR method is an extension of the AOR method (which has two parameters, r and ω). The AOR method, on the other hand, is an extension of the SOR method (that has one parameter, ω). SOR, AOR and TOR techniques are all within over-relaxation family scheme. Former work on block iterative techniques [17–20] uses various points of Explicit Group (EG) techniques to demonstrate that block iterative approaches outperform the conventional point techniques.

Let the two-dimensional Laplace's equation given in (1), be viewed as

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0. \tag{2}$$

Equation (2) approximation, as frequently represented in the following equation, allows to reduced using 5-point second-order finite difference formula,

$$u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = 0. \tag{3}$$

The TOR method comprising three distinct optimal relaxation parameters denoted by r, r', and ω. From Eq. (3), the iterative scheme for conventional TOR iterative method is given as

$$\begin{array}{c} U_{i,j}^{(k+1)} = \frac{r}{4}U_{i,j-1}^{(k+1)} + \frac{r'}{4}U_{i-1,j}^{(k+1)} + \frac{\omega}{4}\left(U_{i,j+1}^{(k)} + U_{i+1,j}^{(k)}\right) \\ +\left(\frac{\omega-r}{4}\right)U_{i,j-1}^{(k)} + \left(\frac{\omega-r'}{4}\right)U_{i-1,j}^{(k)} + (1-\omega)U_{i,j}^{(k)} \end{array}. \tag{4}$$

By considering the approximation in Eqs. (3) and (4), the general iterative scheme for EGTOR may be expressed as

$$\begin{bmatrix} U_{i,j} \\ U_{i+1,j} \\ U_{i,j+1} \\ U_{i+1,j+1} \end{bmatrix}^{(k+1)} = \frac{1}{24}\begin{bmatrix} 6S_1 + S_a \\ 6S_2 + S_b \\ 6S_3 + S_b \\ 6S_4 + S_a \end{bmatrix} + (1-\omega)\begin{bmatrix} U_{i,j} \\ U_{i+1,j} \\ U_{i,j+1} \\ U_{i+1,j+1} \end{bmatrix}^{(k)} \tag{5}$$

where

$$\begin{aligned} S_1 &= r\left(U_{i-1,j}^{(k+1)} - U_{i-1,j}^{(k)}\right) + r'\left(U_{i,j-1}^{(k+1)} - U_{i,j-1}^{(k)}\right) + \omega\left(U_{i-1,j}^{(k)} + U_{i,j-1}^{(k)}\right), \\ S_2 &= r'\left(U_{i+1,j-1}^{(k+1)} - U_{i+1,j-1}^{(k)}\right) + \omega\left(U_{i+1,j-1}^{(k)} + U_{i+2,j}^{(k)}\right), \\ S_3 &= r\left(U_{i-1,j+1}^{(k+1)} - U_{i-1,j+1}^{(k)}\right) + \omega\left(U_{i-1,j+1}^{(k)} + U_{i,j+2}^{(k)}\right), \\ S_4 &= \omega\left(U_{i+2,j+1}^{(k)} + U_{i+1,j+2}^{(k)}\right), \\ S_a &= 2(S_2 + S_3) + S_1 + S_4, \\ S_b &= 2(S_1 + S_4) + S_2 + S_3. \end{aligned}$$

The ambiguous optimal values of all parameters imposed no constraints on obtaining the smallest iterations number. According to Hadjidimos [21], the amounts of r and r' are generally chosen to lie as close as the value of related SOR ω, with $\omega = [1, 2)$. As a result, sensitivity analysis was performed in this study to determine the best values of optimal relaxation parameters using $\omega = [1, 2)$ as a benchmark and following Hadjidimos's [21] motion. The implementation of EGTOR to solve problem (2) is described in Algorithm 1.

Algorithm 1: EGTOR technique	
i	Setup the configuration space with specified start and goal position
ii	Initializing starting point U, $\varepsilon \leftarrow 10^{-15}$, *iteration* $\leftarrow 0$
iii	Set the variables $S_1 \leftarrow r\left(U_{i-1,j}^{(k+1)} - U_{i-1,j}^{(k)}\right) + r'\left(U_{i,j-1}^{(k+1)} - U_{i,j-1}^{(k)}\right) + \omega\left(U_{i-1,j}^{(k)} + U_{i,j-1}^{(k)}\right),$ $S_2 \leftarrow r'\left(U_{i+1,j-1}^{(k+1)} - U_{i+1,j-1}^{(k)}\right) + \omega\left(U_{i+1,j-1}^{(k)} + U_{i+2,j}^{(k)}\right),$ $S_3 \leftarrow r\left(U_{i-1,j+1}^{(k+1)} - U_{i-1,j+1}^{(k)}\right) + \omega\left(U_{i-1,j+1}^{(k)} + U_{i,j+2}^{(k)}\right),$ $S_4 \leftarrow \omega\left(U_{i+2,j+1}^{(k)} + U_{i+1,j+2}^{(k)}\right).$ $S_a \leftarrow 2(S_2 + S_3) + S_1 + S_4$ $S_b \leftarrow 2(S_1 + S_4) + S_2 + S_3$
iv	For all non-occupied node points of type • using Eq. (5), calculate $U_{i,j}^{(k+1)} \leftarrow \frac{1}{24}[6S_1 + S_a] + (1-\omega)U_{i,j}^{(k)},$ $U_{i+1,j}^{(k+1)} \leftarrow \frac{1}{24}\left[6S_2 + S_b\right] + (1-\omega)U_{i+1,j}^{(k)},$ $U_{i,j+1}^{(k+1)} \leftarrow \frac{1}{24}\left[6S_3 + S_b\right] + (1-\omega)U_{i,j+1}^{(k)},$ $U_{i+1,j+1}^{(k+1)} \leftarrow \frac{1}{24}[6S_4 + S_a] + (1-\omega)U_{i+1,j+1}^{(k)}.$
v	Compute the remaining group of points (with one or two points) near to the boundary via direct method by using equation $U_{i,j}^{(k+1)} \leftarrow \frac{1}{4}\left[U_{i-1,j}^{(k+1)} + U_{i+1,j}^{(k)} + U_{i,j-1}^{(k+1)} + U_{i,j+1}^{(k)}\right]$
vi	Check the convergence test for $\varepsilon \leftarrow 10^{-15}$. If yes, proceed to step (vii). Else, back to step (iii)
vii	Execute GDS-DT to create path from departure to target location

3 Results and Discussion

The simulation environments employed were 300 × 300, 600 × 600, 900 × 900, 1200 × 1200, 1500 × 1500, and 1800 × 1800. In the configuration space, various numbers of hindrances of various forms have been established. All the obstacles and the walls were initially set to high temperatures. Meanwhile, the target point has been set to a very low temperature, but the departure point has no initial value. All other points were set to a zero temperature. The experiments were done out on a personal computer executing at 2.50 GHz speed with 8GB of RAM using Robot 2D Simulator [22]. The looping progression proceeded to calculate the temperature values at each point while waiting for the stopping criterion to be fulfilled. Specifically, when there has no change in the temperature value from one iteration process to the next or when it has converged at a predetermined value 1.0^{-10}. Such a configuration is required to avoid flat areas, which result in failure to generate a path to the goal location. Tables 1 and 2 respectively showed the iteration counts and time taken (in seconds) needed by each iterative algorithm based on these simulations. These result tables are separated into three different methods executed in four different environments, namely Event 1 to Event 4. In terms of iteration count, it is shown that the EGTOR produced the best results compared to the EGAOR (approximately by 4.4%) and EGSOR (approximately by 17.1%). Whereas in terms of CPU time, the EGTOR reduces roughly 6.3% over EGAOR and 14.5% over EGSOR.

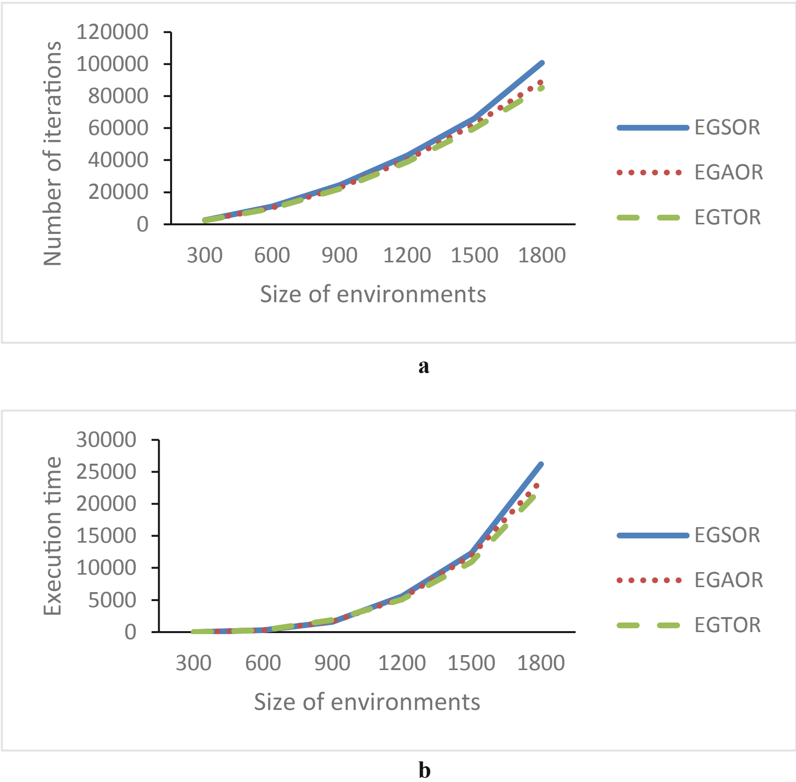

Fig. 3. (a) Overall performances concerning the iteration counts. (b) Overall performances concerning the time taken

The performance graph of the suggested methods relating to iteration counts (see Fig. 3a) and time taken (see Fig. 3b) are also illustrated. In reference to Fig. 3, EGTOR exhibited the least computing time with the fewest number of iterations needed in comparison with other existing methods. Clearly, it proved to be the fastest of all. This is because three distinct optimal parameters for this approach have been added. The EGAOR and EGSOR, on the other hand, required two and one parameters, respectively. These optimal parameters have a positive effect on the acceleration of computation.

Table 1. The implementation of the projected methods based on iteration counts.

	Methods	N					
		300	600	900	1200	1500	1800
Event 1	EGSOR	1258	5899	12844	22227	34055	48446
	EGAOR	1042	4994	10928	19107	29306	41775
	EGTOR	997	4812	10581	18549	28445	40524
Event 2	EGSOR	1729	6782	14874	26007	39968	56858
	EGAOR	1610	6368	13953	24429	32926	46923
	EGTOR	1489	5957	13062	22905	31552	45197
Event 3	EGSOR	2666	11076	24519	42897	65977	100842
	EGAOR	2480	10389	22995	40322	62423	89182
	EGTOR	2371	9977	22111	38917	59912	85272
Event 4	EGSOR	1629	6487	14194	24913	38195	54508
	EGAOR	1392	5648	12367	21724	33518	48120
	EGTOR	1328	5428	11907	20963	34842	49772

Table 2. The implementation of the projected methods based on time taken (in seconds).

	Methods	N					
		300	600	900	1200	1500	1800
Event 1	EGSOR	6.88	163.72	871.66	2694.80	6286.69	12675.73
	EGAOR	6.05	137.87	751.78	2442.66	5551.02	10459.68
	EGTOR	5.05	133.00	720.29	2394.62	5404.33	10316.42
Event 2	EGSOR	7.67	199.59	1009.48	2827.46	6925.80	14336.95
	EGAOR	8.25	185.36	926.49	3003.98	5909.85	12802.89
	EGTOR	7.64	169.39	867.20	2787.69	5700.19	12312.08
Event 3	EGSOR	13.24	315.87	1602.81	5591.93	12331.17	26171.60
	EGAOR	13.83	301.27	1633.35	5261.60	11975.63	23616.21
	EGTOR	11.66	296.46	1883.36	5094.93	10921.11	22338.11
Event 4	EGSOR	7.80	187.33	990.20	2979.14	6919.25	13843.15
	EGAOR	7.56	167.65	891.51	2609.11	6139.29	12833.82
	EGTOR	7.10	163.21	850.26	2573.12	6494.66	13381.51

When the configuration space potential values are obtained, the trail is generated by employing the steepest descent search from any departure location to the given target spot. The algorithm monitors the negative gradient and repeatedly selects the lowest temperature around the neighborhood points up until the constructed path reaches its goal point. Figure 4 illustrates the successful development of a path from numerical computation in a known stationary environment. All of the starting points (green/square) manage to end at the specific target point (red/circle) while escaping multiple obstacles in the given environment.

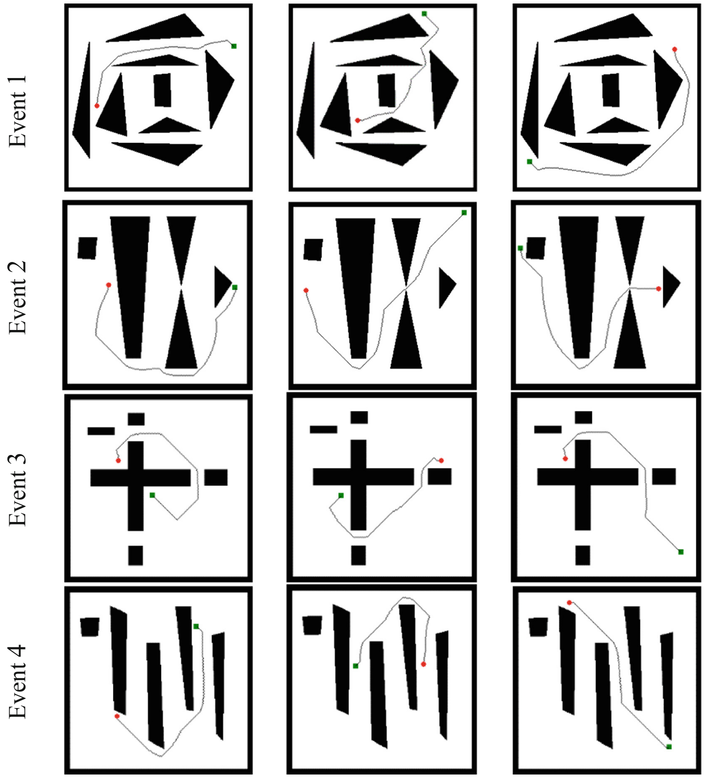

Fig. 4. The pathways created of four environments from various initial and target positions.

4 Conclusion

To summarize, the investigations in this article highlights that harmonic functions give a promising and feasible way of generating routes in a point robot environment due to advanced finding numerical techniques, together with the new sophisticated computing technologies. The simulation results confirmed that the EGTOR iterative scheme is faster than the conventional method (families of SOR and AOR). While the number of obstacles has risen, the effectiveness of the suggested technique is not affected. In reality, the computation becomes quicker as the obstacle zones are omitted along the calculation process. It is worth emphasizing once more that EGTOR results surpass the EGAOR (by approximately 4.4%) and EGSOR (by approximately 17.1%) as to iteration count. While in terms of computational time, the EGTOR saves around 6.3% over EGAOR and 14.5% over EGSOR. Furthermore, the originality/novelty of this study is the use of the TOR scheme families in robot pathfinding and on Algorithm 1.

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