Integrating Individual Preferences into Collective Argumentation

Chonghui Li(✉) and Beishui Liao

Institute of Logic and Cognition, Zhejiang University, Hangzhou, China
{lisabell,baiseliao}@zju.edu.cn

Abstract. In the field of collective argumentation, multiple agents may have distinct knowledge representations and individual preferences. In order to obtain reasonable collective outcome for the group, either individual frameworks should be merged or individual preference should be aggregated. However, framework merging and preference aggregation are different procedures, leading to disagreements on collective outcome. In this paper, we figure out a solution to combine framework merging, argumentative reasoning and incomplete preference aggregation together. Furthermore, a couple of rational postulates are proposed to be the criteria for the reasonability of collective outcome obtained based on our approach.

Keywords: Collective argumentation · Framework merging · Incomplete preference aggregation · Concordance

1 Introduction

Based on abstract argumentation, collective argumentation deals with the scenarios in which multiple agents have distinct individual frameworks representing their observed information and reasoning knowledge, aiming to obtain a reasonable reasoning outcome for the group [1]. For this purpose, an operation called *framework merging* is adopted to form representative collective frameworks first and then jointly accepted arguments can be obtained by argumentative reasoning with the collective frameworks. The criteria for the reasonability lie in the representativeness of collective frameworks and the acceptability of arguments at the group level. Existing literatures [2–6] are along with this line. However, if we extend individual frameworks to include individual preferences, what influences do they have on collective outcome? And what are the renewed criteria for the reasonability of collective outcome? Let us illustrate the questions with an example: Three Detectives.

Example 1. There are four suspects A, B, C and D in a stolen jewellery case. Each of them has an argument as follows.

– A: B is the criminal, because I saw B sold the jewellery to D two days ago. (argument a)

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- B: It's none of my business. The truth is A, C and D conspired the stealing. (argument b)
- C: I saw B wore a jewellery very similar to the stolen one yesterday. (argument c)
- D: I know nothing about this incident. (argument d)

Assume there is a committee of three detectives (subscript indexed as 1, 2, 3) in charge of the case. They need to identify the conflicting arguments with their own knowledge and reason with the case independently. Three individual frameworks representing detectives' distinct observed information are shown in Fig. 1. Here each detective represents the arguments (the dots) which have attack relation (the directed edges) in a di-graph, excluding the arguments which he/she personally supposes to be irrelevant. Apart from this, three detectives have their own preferences over the conflicting arguments, based on their personal credences on the suspects' arguments. Assume that each detective is a rational agent, whose preference is always *coherent* with his/her cognition. That is to say, if a detective supposes that argument a attacks argument b, then it is impossible for him/her to suppose b is more credible than a.

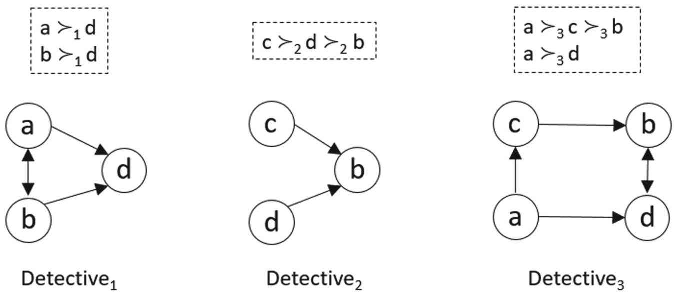

Fig. 1. The profile for three detectives

Note that the modelling of knowledge representations for three detectives may not be unique. Due to the vagueness of natural language and the subjectivity of personal cognition, multiple agents may have a variety of options for their knowledge representations. Now based on the example, our research question becomes more explicit: among four arguments, what are those arguments accepted by the committee as a reasonable collective choice? And what are the reasons for the choice?

Before the response, we need to make a further analysis on the nature of these questions. On one hand, if individual preferences were not considered, through the operations of framework merging and argumentative reasoning, three detectives may reach a reasonable collective outcome. On the other hand, if we were only informed with individual preferences over arguments, then through a procedure of preference aggregation, three detectives would agree on a social preference which leads to a reasonable collective choice too. It is obvious that framework merging and preference aggregation are different operations, they deviate from each other in at least three points: different inputs, different measurements on social agreement, and as a result, different outputs. However, in the scenario of Three Detectives, both individual frameworks and individual preferences are provided as given information, we need to figure out an approach to combine framework merging, argumentative reasoning and preference aggregation together and find an updated reasonable choice for the committee. Since individual preferences indicate the credence on arguments and are always coherent with the structure of individual frameworks, if they are aggregated to a reasonable social preference, it is supposed to have dominant influences on collective reasoning outcome. That is to say, an argument with greater credence according to social preference should be more acceptable than the ones with less credences for the group. Therefore, a solution for the combination could be: the collective outcome obtained from framework merging and argumentative reasoning is in *concordance* with social preference.

In this paper, we propose a novel method for framework merging which can form representative collective frameworks and has less complexity in computation and better explainability, compared to Coste-Marquis' method [2]. As individual preferences might be incomplete with respect to the profile, we adopt a pairwise majority based procedure for incomplete preference aggregation, proposed by Koncazk in [7]. Considering that the winner(s) of social preference is possibly discarded in the stage of argumentative reasoning, we apply social preference as modification of collective frameworks before argumentative reasoning. Then the criteria for the reasonability of collective outcome are renewed: it is the result obtained from reasoning with representative collective frameworks and in concordance with majority-based social preference.

The layout of the paper is as follows. Section 2 recalls some preliminaries of abstract argumentation, preference-based abstract argumentation, framework merging, preference aggregation. In Sect. 3, we propose a novel method for framework merging and evaluate the advantages of our method. In Sect. 4, we introduce a procedure of incomplete preference aggregation, define a method to obtain social preference over arguments and verify the reasonability of it. We establish the concordance between collective framework and social preference and have it evaluated in Sect. 5. Finally we conclude the paper in Sect. 6.

2 Preliminaries

First, let's recall some key elements of abstract argumentation frameworks as proposed by Dung in [8].

Definition 1. *An abstract argumentation framework (AF) is a pair* $\mathcal{F} = (\mathcal{A}, \mathcal{D})$ *where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{D} \subseteq \mathcal{A} \times \mathcal{A}$ *is a defeat relation.*

The key problem is to determine the sets of arguments that can be accepted together. According to some criteria, a set of accepted arguments is called an *extension*. Let us first introduce two basic criteria: conflict-freeness and acceptability.

Definition 2. *Given an AF $\mathcal{F} = (\mathcal{A}, \mathcal{D})$ and a set of arguments $S \subseteq \mathcal{A}$, we say that S is conflict-free iff $\nexists A, B \in S$ such that $(A, B) \in \mathcal{D}$. We say that an argument $A \in \mathcal{A}$ is acceptable w.r.t. S iff $\forall B \in \mathcal{A}$, if $(B, A) \in \mathcal{D}$ then $\exists C \in S$ such that $(C, B) \in \mathcal{D}$.*

A set of arguments S is *admissible* when it is conflict-free and each argument in the set is acceptable w.r.t. S. Several semantics have been proposed based on admissible sets. In this paper, we only focus on the standard semantics defined in [8]. We say S is a *complete* extension of $\mathcal{F}$ iff it is admissible and each argument acceptable w.r.t. S belongs to S. S is a *preferred* extension of $\mathcal{F}$ iff it is a maximal(w.r.t. set inclusion) complete extension of $\mathcal{F}$. S is a *grounded* extension of $\mathcal{F}$ iff it is the minimal (w.r.t. set inclusion) complete extension of $\mathcal{F}$. S is a *stable* extension of $\mathcal{F}$ iff it is conflict-free and it attacks all the arguments that do not belong to S. We denote $\mathcal{E}_\sigma(\mathcal{F})$ the set of extensions of $\mathcal{F}$ for the semantics $\sigma \in \{\mathbf{co}(mplete), \mathbf{pr}(eferred), \mathbf{gr}(ounded), \mathbf{st}(able)\}$.

Preference-based argumentation framework is first proposed by [9] as a extended framework of abstract argumentation framework.

Definition 3. *A preference-based argumentation framework (PAF) is a triple $\mathcal{F}_p = (\mathcal{A}, \mathcal{R}, \succ)$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation and $\succ$ is a strict partial order(irreflexive and transitive) over $\mathcal{A}$, called preference relation.*

Definition 4. *Let $(\mathcal{A}, \mathcal{R}, \succ)$ be a PAF and the reduction of PAF is an AF $\mathcal{F} = (\mathcal{A}, \mathcal{D})$ s.t. $\forall a, b \in \mathcal{A}$:*

- *Reduction 1* [9]*: $(a, b) \in \mathcal{D}$ iff $(a, b) \in \mathcal{R}$ and $b \not\succ a$;*
- *Reduction 2* [10]*: $(a, b) \in \mathcal{D}$ iff $((a, b) \in \mathcal{R}, b \not\succ a)$ or $((b, a) \in \mathcal{R}, (a, b) \notin \mathcal{R}, a \succ b)$.*
- *Reduction 3* [11]*: $(a, b) \in \mathcal{D}$ iff $((a, b) \in \mathcal{R}, b \not\succ a)$ or $((a, b) \in \mathcal{R}, (b, a) \notin \mathcal{R}))$*
- *Reduction 4* [11]*: $(a, b) \in \mathcal{D}$ iff $((a, b) \in \mathcal{R}, b \not\succ a)$ or $((a, b) \in \mathcal{R}, (b, a) \notin \mathcal{R}))$ or $((b, a) \in \mathcal{R}, (a, b) \notin \mathcal{R}, a \succ b)$.*

Note that if preference is not included, a PAF is exactly an AF since each attack in PAF is successfully converted to a defeat in AF. When preference is given, there exists a relationship between PAF and AF, called *reduction*. Definition 4 introduces four kinds of reduction in the existing literature. It is intuitive that an attack is successful (i.e. converted to a defeat) if and only if the attacked argument is not stronger than the attacker. It is exactly what Reduction 1 states. However, if a class of attacks which is called *critical attack* exists, namely $(a, b) \in \mathcal{R}$ and $b \succ a$, they won't be kept as defeats in AF. As a result, conflicting arguments may be all accepted which violates conflict-freeness of extensions. Critical attack is reversed in Reduction 2, deleted by Reduction 3 only if the opposite attack $(b, a) \in \mathcal{R}$ exists and made to be a symmetric attack by Reduction 4.

Next, we introduce basic definitions of framework merging and preference aggregation within the scope of collective argumentation.

Definition 5. *Given* $\{1, \ldots, n\}$ *a set of agents and a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$*. Framework merging is an operation* $Mer : \hat{\mathcal{F}} \to \mathcal{F}_{coll}$*, where* $\mathcal{F}_{coll} = (\mathcal{A}_{coll}, \mathcal{R}_{coll})$*.*

Note that different operations may give rise to different outputs and as a result, collective framework may not be unique.

Definition 6. *Given* $\{1, \ldots, n\}$ *is a set of agents,* $\mathcal{A}_1, \ldots, \mathcal{A}_n$ *are sets of arguments which belong correspondingly to agents* $\{1, \ldots, n\}$ *and a profile of individual preferences is* $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$*, where* $\succ_i$ *is agent i's preference over* $\mathcal{A}_i$*. Then preference aggregation is a procedure* $Agg : \hat{\mathcal{P}} \to \succsim_s$*. When* $\mathcal{A}_1 = \cdots = \mathcal{A}_n$*, it is called complete preference aggregation, otherwise we say it is incomplete preference aggregation.*

Previous work [12–14] focuses on complete preference aggregation in the area of collective argumentation. However, based on the settings of this paper, we consider the more complicated situation, namely incomplete preference aggregation.

3 A Novel Method for Framework Merging

Given distinct individual frameworks and individual preferences, in this section we temporarily put individual preferences aside and focus on obtaining collective frameworks from individual frameworks through the operation of framework merging. In the vein of framework merging, quantitative approach and qualitative approach tackle the problem differently. While the former treats the appearances of an attack in individual frameworks as *weight* [4–6], the latter treats it in a qualitative way. In extant literatures, Coste-Marquis proposes a qualitative approach [2]. There are three steps: consensual expansion, distance-based framework merging and argumentative reasoning. The main idea is to form representative collective frameworks first and then obtain collective reasoning outcome. However, it has high complexity in computation, limited capacity in explanation and difficulty in including individual preference.

As preference is regarded as a qualitative force influencing argument strength, we adopt qualitative approach to merging individual frameworks. We propose a novel method for framework merging. First of all, we define a class of relation in collective framework.

Definition 7. *Given a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$*. We say a relation* (a, b) *is exclusive w.r.t.* $\hat{\mathcal{F}}$ *iff* $a, b \in \mathcal{A}_i$ *and* $\nexists \mathcal{F}_k = (\mathcal{A}_k, \mathcal{R}_k)$ *where* $i \neq k$ *s.t.* $a \in \mathcal{A}_k$ *and* $b \in \mathcal{A}_k$*.*

Definition 7 identifies a special class of binary relation, which appears only once in the profile of individual frameworks. Note that exclusive relations includes either attack or non-attack. Based on it, we define our method of framework merging.

Definition 8. *Given a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$*. Our method of framework merging is the operation giving rise to a set of collective framework, denoted as* $\Gamma = \{\mathcal{F}_{coll_1}, \ldots, \mathcal{F}_{coll_k}\}$*, where* $\mathcal{F}_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{R}_{coll_j})$*.* Γ *is defined as:*

– $\mathcal{A}_{coll_1} = \cdots = \mathcal{A}_{coll_k} = \bigcup_i \mathcal{A}_i$;
– $\mathcal{R}_{coll_1}, \ldots, \mathcal{R}_{coll_k}$ *are exactly the members in* $R_1 \bigcup R_2 \bigcup R_3^*$*, where*[1]*:*
  - $R_1 = \{(a,b) | (a,b)$ *is an exclusive attack w.r.t.* $\hat{\mathcal{F}}\}$;
  - $R_2 = \{(a,b) | \#(\{i | (a,b) \in \mathcal{R}_i\}) > \#(\{j | (a,b) \notin \mathcal{R}_j\})$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i), \mathcal{F}_j = (\mathcal{A}_j, \mathcal{R}_j)$ *and* $a, b \in \mathcal{A}_i \cap \mathcal{A}_j\}$;
  - $R_3^* \in 2^{R_3}$*, where* $R_3 = \{(a,b) | \#(\{i | (a,b) \in \mathcal{R}_i\}) = \#(\{j | (a,b) \notin \mathcal{R}_j\})$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i), \mathcal{F}_j = (\mathcal{A}_j, \mathcal{R}_j)$ *and* $a, b \in \mathcal{A}_i \cap \mathcal{A}_j\}$.

Let's proceed with some elaborations on Definition 8. In each collective framework, the set of attacks is corresponding to each member in the union of: R_1, R_2 and R_3^*. R_1 is the set of attacks which are exclusive w.r.t the profile of individual frameworks. R_2 and R_3 are sets of attacks involving pairs of arguments which are in common in the profile of individual frameworks. For any attack, only if the votes of its appearance in the profile of individual frameworks are strictly greater than the ones of its absence can it be preserved in R_2. Therefore, R_2 is based on strict majority. R_3 deals with the attacks which have equal votes for their appearances and absences in the profile of individual frameworks. Each member of 2^{R_3} has equal possibility to appear in the set of attack in collective frameworks. For instance, if $R_3 = \{(a,b)\}$, then we have two collective frameworks: one includes the pair in its set of attack and the other denies the pair as its attack.

In the following, we illustrate two basic properties of our approach of framework merging.

Proposition 1. $\#(\Gamma) = 2^{\#(R_3)}$.

Proof. The cardinality of set Γ is the number of collective frameworks we obtained from framework merging operation Mer. According to Definition 8, the number of collective frameworks is determined by $\#(\{\mathcal{A}_{coll}\})$ and $\#(\{\mathcal{R}_{coll}\})$. Due to $\#(\{\mathcal{A}_{coll}\}) = 1$, the number of collective frameworks is determined by $\#(\{\mathcal{R}_{coll}\})$, i.e. $\#(\{R_1 \bigcup R_2 \bigcup R_3^*\})$. As $R_3^* \in 2^{R_3}$, $\#(\Gamma) = \#(2^{R_3})$, i.e. equals to $2^{\#(R_3)}$.

Corollary 1. *If* $R_3 = \emptyset$*, then the collective framework is unique.*

Let us illustrate the method with the running example.

Example 2. Given the profile of three detectives' individual frameworks as Fig. 1 shows, if we exclude individual preferences in this stage, we obtain four collective frameworks $\mathcal{F}_{coll_1}, \mathcal{F}_{coll_2}, \mathcal{F}_{coll_3}, \mathcal{F}_{coll_4}$ according to Definition 8, shown in Fig. 2. Here, $R_1 = \{(a,c)\}, R_2 = \{(c,b),(b,d),(d,b)(a,d)\}, R_3 = \{(a,b),(b,a)\}$. Hence according to Proposition 1, $\#(\Gamma) = 2^2 = 4$.

Our approach forms collective frameworks on the basis of classifying the attacks in the profile of individual frameworks into three categories. It is one-step operation and has better explainability on how we merge individual frameworks. Another benefit is that we are informed the number of collective frameworks as soon as R_3 is calculated. For the evaluation, we define five rational postulates, referring some of them to [15] and [16].

[1] For any set $S, \#(S)$ denotes the cardinality of S.

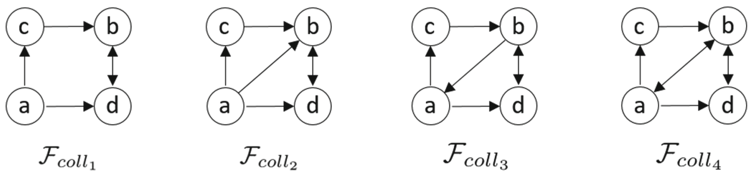

Fig. 2. Four collective frameworks for Three Detectives example

Definition 9. *Given a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$*. Through a framework merging operation* Mer*, we obtain the set of collective frameworks* $\Gamma = \{\mathcal{F}_{coll_1}, \ldots, \mathcal{F}_{coll_k}\}$*, where* $\mathcal{F}_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{R}_{coll_j})$*. Five rational postulates on attack are defined as:*

- ***Nomination*** [15] *(P1). If* $\#(\{i \in n | (a,b) \in \mathcal{R}_i\}) = 1$*, then* $(a,b) \in \bigcap_j \mathcal{R}_{coll_j}$*.*
- ***Unanimity*** [16] *(P2). If* $\#(\{i \in n | (a,b) \in \mathcal{R}_i\}) = n$*, then* $(a,b) \in \bigcap_j \mathcal{R}_{coll_j}$*.*
- ***Strict majority*** *(P3). Let* $\#(\{i \in n | a \in \mathcal{A}_i, b \in \mathcal{A}_i\}) = m$*. If* $\#(\{i \in m | (a,b) \in \mathcal{R}_i\}) > \frac{m}{2}$*, then* $(a,b) \in \bigcap_j \mathcal{R}_{coll_j}$*.*
- ***Weak majority*** *(P4). Let* $\#(\{i \in n | a \in \mathcal{A}_i, b \in \mathcal{A}_i\}) = m$*. If* $\#(\{i \in m | (a,b) \in \mathcal{R}_i\}) = \frac{m}{2}$*, then* $\exists \mathcal{F}_{coll_j} \in \Gamma$ *s.t.* $(a,b) \in \mathcal{R}_{coll_j}$*.*
- ***Closure*** [16] *(P5).* $\bigcup_j \mathcal{R}_{coll_j} \subseteq \bigcup_i \mathcal{R}_i$*.*

Nomination (P1) means once the attack appears in individual frameworks it will appear in collective frameworks. According to Definition 8, it is obvious that: P1 is satisfied by R_1; unanimity (P2) and strict majority (P3) are satisfied by R_2; weak majority (P4) is satisfied by R_3. Note that strict minority of attacks in the profile of individual frameworks will not preserved in the set of attack of collective frameworks but closure (P5) on attack is held. In short, our method satisfies above five rational postulates and the collective frameworks obtained from the method is representative for the profile of individual frameworks.

Proposition 2. *The collective frameworks obtained according to Definition 3 satisfies P1, P2, P3, P4 and P5.*

Now we introduce argumentative reasoning. To obtain a collective outcome for the group, we need to find the acceptability for arguments in collective frameworks. As we introduced in Sect. 2, acceptability of arguments is determined by abstract argumentation semantics (refer as Definition 2). If collective framework is unique, we can figure out the extensions instantly. If there are multiple collective frameworks, how to find out the joint acceptability of arguments? Here, we adopt Coste-Marquis' proposal in [2].

Definition 10. *Given a set of collective frameworks* $\Gamma = \{\mathcal{F}_{coll_1}, ..., \mathcal{F}_{coll_k}\}$*, obtained from* $\hat{\mathcal{F}}$ *according to Definitions 8, where* $\mathcal{F}_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{R}_{coll_j})$*. For any subset* $S \subseteq \mathcal{A}_{coll_j}$*:*
S is sceptically jointly accepted for Γ *iff* $\forall \mathcal{F}_{coll_i} \in \Gamma, \exists E \in \mathcal{E}_\sigma(\mathcal{F}_{coll_i})$ *and* $S \subseteq E$*.*
S is credulously jointly accepted for Γ *iff* $\exists \mathcal{F}_{coll_i} \in \Gamma, \exists E \in \mathcal{E}_\sigma(\mathcal{F}_{coll_i})$ *and* $S \subseteq E$*.*
The sets of arguments which are sceptically and credulously jointly accepted under a certain semantics σ *are denoted respectively as* $Sa_\sigma(\Gamma), Ca_\sigma(\Gamma)$*.*

4 Incomplete Preference Aggregation

In this section, we deal with individual preferences. In order to find a reasonable social preference for the group, a procedure for preference aggregation is needed. Since each individual framework is distinct, individual preferences are incomplete with respect to the profile. Thus a procedure of incomplete preference aggregation should be considered. In this section, we adopt *pairwise majority* based procedure to obtain Condorcet winners for the profile of incomplete individual preferences, proposed by Konczak in [7] and define a social preference over arguments of collective frameworks based on Condorcet winners. We evaluate the social preference obtained based on our method with three rational postulates.

In the following, we provide the basic notion of Condorcet winner in traditional preference aggregation, introduce the extended notions of necessary Condorcet winner and possible Condorcet winner, and the algorithms to compute two kinds of Condorcet winner. These are already introduced in [7]. We adapt the definitions in the context of collective argumentation.

Definition 11. *Given $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of complete individual preferences, where $\succ_i$ is a strict total order over a set of alternatives: $\{a, b, \ldots\}$. An alternative x is defined as a Condorcet winner iff $\forall y \neq x$, $\#(\{i|x \succ_i y\}) > \frac{n}{2}$.*

Definition 12. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of incomplete individual preferences w.r.t. $\bigcup_i \mathcal{A}_i$, where $\succ_i$ is a strict total order over $\mathcal{A}_i$, we say $\succ_i'$ is a completion of $\succ_i$ w.r.t. $\hat{\mathcal{P}}$ iff $\succ_i'$ is a strict total order over $\bigcup_i \mathcal{A}_i$ and $\succ_i'$ extends $\succ_i$. The set of all completions of $\succ_i$ is denoted as $Com(\succ_i)$.*

Definition 13. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of incomplete individual preferences w.r.t. $\bigcup_i \mathcal{A}_i$, let $Com(\hat{\mathcal{P}}) = Com(\succ_1) \times \ldots \times Com(\succ_n)$, for any $a \in \bigcup_i \mathcal{A}_i$ we define:*

- *a is a necessary Condorcet winner iff $\forall \hat{\mathcal{P}}' \in Com(\hat{\mathcal{P}})$, a is a Condorcet winner for $\hat{\mathcal{P}}'$;*
- *a is a possible Condorcet winner iff $\exists \hat{\mathcal{P}}' \in Com(\hat{\mathcal{P}})$, a is a Condorcet winner for $\hat{\mathcal{P}}'$.*

Definition 14. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of incomplete individual preferences w.r.t. $\bigcup_i \mathcal{A}_i$, for $x, y \in \bigcup_i \mathcal{A}_i$, we denote $N_{\hat{\mathcal{P}}}(x, y) = \#(\{i|x \succ_i y\}) - \#(\{i|y \succ_i x\})$, then we define:*

$$N^{max}_{\succ_i}(x, y) = \begin{cases} +1 & if \;\; not \;\; (y \succ_i x) \\ -1 & if \;\; y \succ_i x \end{cases} \; and \;\; N^{min}_{\succ_i}(x, y) = \begin{cases} +1 & if \;\; x \succ_i y \\ -1 & if \;\; not \;\; (x \succ_i y) \end{cases}$$

$$N^{max}_{\hat{\mathcal{P}}}(x, y) = \sum_{i=1}^{n} N^{max}_{\succ_i}(x, y) \;\; and \;\; N^{min}_{\hat{\mathcal{P}}}(x, y) = \sum_{i=1}^{n} N^{min}_{\succ_i}(x, y)$$

Argument a is a necessary Condorcet winner iff $\forall y \neq a$, $N^{min}_{\hat{\mathcal{P}}}(a, y) > 0$. The set of necessary winners for $\hat{\mathcal{P}}$ is denoted as $NW(\hat{\mathcal{P}})$.

Argument a is a possible Condorcet winner iff $\forall y \neq a$, $N^{max}_{\hat{\mathcal{P}}}(a, y) > 0$. The set of possible winners for $\hat{\mathcal{P}}$ is denoted as $PW(\hat{\mathcal{P}})$.

Note that due to $x, y \in \bigcup_i \mathcal{A}_i$, $y \succ_i x$ implies that $x, y \in \mathcal{A}_i$ and hence "not $y \succ_i x$" indicates the situations as follows: (1)$x, y \in \mathcal{A}_i$ but $x \succ_i y$; (2)either x or y is not in $\mathcal{A}_i$; (3)neither x nor y is in $\mathcal{A}_i$. The intuition of the algorithms is that for any pair of arguments (x, y), $N^{max}_{\hat{\mathcal{P}}}(x, y)$ covers the "best" case and $N^{min}_{\hat{\mathcal{P}}}(x, y)$ covers the "worst" case among all completions of individual preferences. If an argument is superior to any other arguments in the "worst" case, it is a necessary Condorcet winner for $\hat{\mathcal{P}}$.

In [7], Konczak states that possible Condorcet winners surely exist while necessary Condorcet winners do not. However, our question is: based on necessary and possible Condorcet winners, how to form a social preference over arguments of collective frameworks? Next, we propose a method.

Definition 15. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of incomplete individual preferences over $\bigcup_i \mathcal{A}_i$. Let $NW(\hat{\mathcal{P}})$ and $PW(\hat{\mathcal{P}})$ be the set of necessary and possible winners for $\hat{\mathcal{P}}$, then an aggregated social preference over $\bigcup_i \mathcal{A}_i$ (i.e. the set of arguments of collective frameworks), denoted as $\succsim_s$, is defined based on a strict partition $\gg$ on the sets of arguments:*

- *If $NW(\hat{\mathcal{P}}) \neq \emptyset$, then $NW(\hat{\mathcal{P}}) \gg PW(\hat{\mathcal{P}}) \setminus NW(\hat{\mathcal{P}}) \gg \bigcup_i \mathcal{A}_i \setminus (NW(\hat{\mathcal{P}}) \cup PW(\hat{\mathcal{P}}))$;*
- *If $NW(\hat{\mathcal{P}}) = \emptyset$, then $PW(\hat{\mathcal{P}}) \gg \bigcup_i \mathcal{A}_i \setminus PW(\hat{\mathcal{P}})$.*

Then for any two arguments $a, b \in \bigcup_i \mathcal{A}_i$:

- *If a, b belong to the same partition, then: $a \sim_s b$;*
- *If a, b belong to different partitions, then $a \succ_s b$ iff a is in the former partition and b is in the latter.*

We illustrate the operation for incomplete preference aggregation defined above with Three Detectives example.

Example 3. Proceed with Example 1. The AFs profile is $\hat{\mathcal{F}^3} = (\mathcal{F}'_1, \mathcal{F}'_2, \mathcal{F}'_3)$ as shown in Fig. 1. The profile of incomplete individual preferences w.r.t $\{a, b, c, d\}$ is $\hat{\mathcal{P}}^3 = (\succ_1, \succ_2, \succ_3)$, where:

- $\succ_1$: $a \succ_1 d$ and $b \succ_1 d$,
- $\succ_2$: $c \succ_2 b \succ_2 d$,
- $\succ_3$: $a \succ_3 c \succ_3 b$ and $a \succ_3 d$.

According to Definition 14, $N^{min}_{\hat{\mathcal{P}}}(x, y)$ and $N^{max}_{\hat{\mathcal{P}}}(x, y)$ are shown in Table 1, where $x, y \in \{a, b, c, d\}$. As $\nexists y \neq x, N^{min}_{\hat{\mathcal{P}}}(x, y) > 0$, $NW(\hat{\mathcal{P}}^3) = \emptyset$; As $\forall y \neq x$, when $x = a$ and c, $N^{max}_{\hat{\mathcal{P}}}(x, y) > 0$, $PW(\hat{\mathcal{P}}^3) = \{a, c\}$. Thus there is no necessary Condorcet winner and the possible Condorcet winners for $\hat{\mathcal{P}}^3$ are arguments a, c. According to Definition 15, we obtain a social preference: $a \sim_s c \succ_s b \sim_s d$.

Table 1. $N^{min}_{\hat{\mathcal{P}}}(x, y)$ and $N^{max}_{\hat{\mathcal{P}}}(x, y)$ for $\hat{\mathcal{P}}^3$

$N^{min}_{\hat{\mathcal{P}}}(x, y)$	a	b	c	d
a	–	-1	-1	1
b	-3	–	-3	-1
c	-3	1	–	-1
d	-3	-1	-3	–

$N^{max}_{\hat{\mathcal{P}}}(x, y)$	a	b	c	d
a	–	**3**	**3**	**3**
b	1	–	-1	1
c	**1**	**3**	–	**3**
d	-1	1	1	–

To evaluate social preference obtained according to Definition 14 and 15, we propose three postulates as follows.

Definition 16. *Given a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \dots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ *and* $\hat{\mathcal{P}} = (\succ_1, \dots, \succ_n)$ *is a profile of incomplete individual preferences over* $\bigcup_i \mathcal{A}_i$*. Let* Γ *be the set of collective frameworks, the social preference for* $\hat{\mathcal{P}}$ *be* $\succsim_s$ *and* σ *be a certain semantics. Three rational postulates on social preference are defined as:*

- ***Completeness*** *(P6).* $\succsim_s$ *is complete and transitive.*
- ***Pairwise strict majority*** *(P7). If* $a \succ_s b$*, then:* $\#(\{i \in n | a \succ_i b\}) > \#(\{i \in n | b \succ_i a\})$ *or* $\#(\{i \in n | \text{ not } (b \succ_i a)\}) > \#(\{i \in n | \text{ not } (a \succ_i b)\})$*.*
- ***Decisiveness in joint acceptance*** *(P8). If* $NW(\hat{\mathcal{P}}) = \{a\}$*, then* $a \in Sa_\sigma(\Gamma)$*.*

First of all, let us explain the implications of these rational postulates. *Completeness* means each argument in collective framework can be compared w.r.t. social preference. As individual preferences are partial orders which means some of arguments are incomparable w.r.t. individual preferences. Our method contributes the total comparability of arguments at the collective level. *Pairwise strict majority* indicates social preference has two characteristics of pairwise-majority-based consensus among agents, which actually maximises the agreement for the group. *Decisiveness in joint acceptance* states the necessary Condorcet winner always sceptically jointly accepted under a certain semantics as a collective outcome.

The satisfaction of P6 can be obtained instantly according to Definition 15 and the satisfaction of P7 are given in the following proposition.

Proposition 3. *Given a profile of AFs* $\hat{\mathcal{F}} = (\mathcal{F}_1, \dots, \mathcal{F}_n)$*, where* $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ *and* $\hat{\mathcal{P}} = (\succ_1, \dots, \succ_n)$ *is a profile of incomplete individual preferences over* $\bigcup_i \mathcal{A}_i$*. Let the social preference for* $\hat{\mathcal{P}}$ *be* $\succsim_s$*, if* $a \succ_s b$*, then:* $\#(\{i \in n | a \succ_i b\}) > \#(\{i \in n | b \succ_i a\})$ *or* $\#(\{i \in n | \text{ not } (b \succ_i a)\}) > \#(\{i \in n | \text{ not } (a \succ_i b)\})$*.*

Proof. According to [7], if necessary Condorcet winner exists, it is also a possible Condorcet winner and $PW(\hat{\mathcal{P}}) \setminus NW(\hat{\mathcal{P}}) = \emptyset$. Then according to Definition 15, if $a \succ_s b$, we have two possible situations:(1) a is a necessary Condorcet winner and b is neither a necessary Condorcet winner nor a possible Condorcet winner. If we want to prove $\#(\{i \in n | a \succ_i b\}) > \#(\{i \in n | b \succ_i a\})$ holds for this situation, we need to prove $N^{min}_{\hat{\mathcal{P}}}(a,b) > 0$. It is always held, since a is a necessary Condorcet winner and $\forall x \neq a$, $N^{min}_{\hat{\mathcal{P}}}(a,x) > 0$. (2) a is a possible Condorcet winner and b is neither a necessary Condorcet winner nor a possible Condorcet winner. If we want to prove $\#(\{i \in n| \text{ not } (b \succ_i a)\}) > \#(\{i \in n| \text{ not } (a \succ_i b)\})$, we need to prove $N^{max}_{\hat{\mathcal{P}}}(a,b) > 0$. It is always held, since a is a possible Condorcet winner and $\forall x \neq a$, $N^{max}_{\hat{\mathcal{P}}}(a,x) > 0$.

The verification for our method on P8 is based on the definition *preference-coherent*, which is a reasonable assumption to require agents to be rational. That is to say, an argument with less credence should not attack the argument more credible than it. Note that symmetric attack possibly exists when two arguments are incomparable w.r.t a partial order.

Definition 17. *Given a PAF $\mathcal{F}_P = (\mathcal{A}, \mathcal{R}, \succ)$, where $\succ$ is a strict partial order over $\mathcal{A}$. We say $\mathcal{F}_p$ is preference-coherent iff for any $a, b \in \mathcal{A} : (a,b) \in \mathcal{R}, (b,a) \notin \mathcal{R}$ iff $a \succ b$; $(a,b) \in \mathcal{R}, (b,a) \in \mathcal{R}$ only if a, b is incomparable w.r.t $\succ$.*

Theorem 1. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of individual preferences, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\succ_i$ is an individual preference over $\mathcal{A}_i$. Let each AF in $\hat{\mathcal{F}}$ be preference-coherent w.r.t. its individual preference, Γ be the set of collective frameworks, $\succsim_s$ be social preference for $\hat{\mathcal{P}}$ and $\sigma = gr$. If $NW(\hat{\mathcal{P}}) = \{b\}$, then $b \in Sa_\sigma(\Gamma)$.*

Proof. Given $NW(\hat{\mathcal{P}}) = \{b\}$, we know that b is on the top rank of $\succsim_s$ and it is the unique winner. $b \in Sa_{gr}(\Gamma)$ means b is not attacked in all collective frameworks. Assume the contrary, that is to say b is attacked in some of collective frameworks and we need to prove a contradiction with $NW(\hat{\mathcal{P}}) = \{b\}$. Let the attacker of b in some of collective frameworks be argument c, we have $b \succ_s c$. According to P7, the characteristic of pairwise majority for necessary winner b is held as: $\#(\{i \in n|b \succ_i c\}) > \#(\{i \in n|c \succ_i b\})$ (equation (i)). As each AF in $\hat{\mathcal{F}}$ is preference-coherent w.r.t. its individual preference, the set of individual frameworks which has individual preference $b \succ_i c$ has two situations: let the cardinalities of two situations be $k_1 = \#(\{i|(b,c) \in \mathcal{R}_i \text{ and } (c,b) \notin \mathcal{R}_i\}), k_2 = \#(\{i|(b,c),(c,b) \notin \mathcal{R}_i\})$; the set of individual frameworks which has individual preference $c \succ_i b$ also has two situations: let the cardinalities of two situations be $k_3 = \#(\{i|(c,b) \in \mathcal{R}_i$ and $(b,c) \notin \mathcal{R}_i, k_4 = \#(\{i|(b,c),(c,b) \notin \mathcal{R}_i\})$; There are two situations left for individual preference in which b and c are incomparable, let the cardinalities of two situations be $k_5 = \#(\{i|(b,c),(c,b) \in \mathcal{R}_i\}), k_6 = \#(\{i|(b,c),(c,b) \notin \mathcal{R}_i\})$. Then according to equation (i), we have: $k_1 + k_2 > k_3 + k_4 + k_5 + k_6$ (equation (ii)). Since c is the attacker of b in some of collective frameworks and (c,b) can't be an exclusive attack in R_1 since b is a necessary Condorcet winner, according to Definition 8, we have $(c,b) \in R_3$ or $(c,b) \in R_2$, which means $k_3 + k_5 \geq k_1 + k_2 + k_4 + k_6$ (equation (iii)), contradicting to equation (ii), which means b is not a necessary Condorcet winner. Contradiction. Hence the conclusion is held.

Three postulates consist of criteria for the reasonability of social preference. As discussed above, we reach the conclusion that our method satisfies all of them and the social preference is reasonable.

Proposition 4. *The Social preference obtained from the method according to Definition 14 and Definition 15 satisfies P6, P7 and P8.*

5 The Concordance Between Collective Framework and Social Preference

In Sect. 3 we define a method for framework merging and in Sect. 4 we provide a method for obtaining social preference based on Konczak's definitions of necessary and possible Condorcet winners. Furthermore, we evaluate the reasonabilities of two methods respectively with a couple of postulates. The results are positive, showing that they are both reasonable according to the criteria. However, in our scenario, individual preferences are provided as given information as well as individual frameworks. A unified reasonable collective outcome is expected for the group of agents. Since framework merging and preference aggregation are different operations with different inputs, it is supposed that collective outcomes obtained respectively may not agree with each other. Although in Theorem 1, we have already proved that the necessary Condorcet winner is always sceptically jointly accepted under grounded semantics as a collective outcome, the disagreement on the joint acceptability of other arguments could still exist. Let us check it with the running example Three Detectives.

Example 4. First we perform argumentative reasoning according to Definition 10 with four collective frameworks obtained in Example 2. Let $\sigma = pr$, we have: $\mathcal{E}_{pr}(\mathcal{F}_{coll_1}) = \{\{a, b\}\}, \mathcal{E}_{pr}(\mathcal{F}_{coll_2}) = \{\{a\}\}, \mathcal{E}_{pr}(\mathcal{F}_{coll_3}) = \{\emptyset\}, \mathcal{E}_{pr}(\mathcal{F}_{coll_4}) = \{\{a\}, \emptyset\}$. Therefore, we don't have sceptically jointly accepted arguments under preferred semantics and credulously jointly accepted arguments under preferred semantics are a, b. While the result of social preference obtained in Example 3 shows that a, c should be mostly acceptable. The results of two operations agree on one argument but disagree with each other on two arguments!

Now the question comes: how to obtain a renewed reasonable collective outcome with more agreements? Since social preference stands for majority based consensus on the credence over arguments in collective frameworks, it should have dominance on the acceptability of arguments. For this reason, shall we only take the result of social preference into account and disregard the reasoning result from collective frameworks? It is unreasonable. Take the running example for illustration, there is an attack between the winners of social preference according to each collective framework, which means argument a and c can not be both accepted. Therefore, we need to find a solution which integrates social preference into collective frameworks or collective extensions. As proposed in [10,11,17], preference may have two roles in single-framework argumentation: one role as modification of framework and the other role as refinement of extensions. In our scenario, after framework merging, we obtain a set of representative collective frameworks. To some extent, each of them can be seen as a single framework representing for the group as a whole. Thus we may consider the application of two roles for social preference. If we apply it as refinement on collective extensions, a possible situation could be that the winners of social preference are discarded in the stage of argumentative reasoning. Hence the option left for us is to apply social preference as modification of collective frameworks, which is also called *reduction* in [9–11]. In the background of collective argumentation, we call it the *concordance* between collective framework and social preference. We distinguish two forms of concordance, namely strong concordance and weak concordance. Their definitions are based on Reduction 2 and Reduction 4 in Definition 4. Formally, we define them as follows.

Definition 18. *Given a profile of AFs $\hat{\mathcal{F}} = (\mathcal{F}_1, \ldots, \mathcal{F}_n)$, where $\mathcal{F}_i = (\mathcal{A}_i, \mathcal{R}_i)$ and $\hat{\mathcal{P}} = (\succ_1, \ldots, \succ_n)$ is a profile of incomplete individual preferences over $\bigcup_i \mathcal{A}_i$. Let $\Gamma = \{\mathcal{F}_{coll_1} \ldots \mathcal{F}_{coll_k}\}$ be the set of collective frameworks, where $\mathcal{F}_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{R}_{coll_j})$ and the social preference for $\hat{\mathcal{P}}$ be $\succsim_s$.*

- *Strong concordance is a mapping $Con_{st} : \Gamma \rightarrow \Gamma'_{st}$, where $\Gamma'_{st} = \{\mathcal{F}'_{coll_1} \ldots \mathcal{F}'_{coll_k}\}$ and $\mathcal{F}'_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{D}_{coll_j})$, defined as: $\forall \mathcal{F}_{coll_j} \in \Gamma$ s.t. $(b,a) \in \mathcal{R}_{coll_j}, (a,b) \notin \mathcal{R}_{coll_j}$, if $a \succ_s b$, then: $(a,b) \in \mathcal{D}_{coll_j}$.*
- *Weak concordance is a mapping $Con_{we} : \Gamma \rightarrow \Gamma'_{we}$, where $\Gamma'_{we} = \{\mathcal{F}'_{coll_1} \ldots \mathcal{F}'_{coll_k}\}$ and $\mathcal{F}'_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{D}_{coll_j})$, defined as: $\forall \mathcal{F}_{coll_j} \in \Gamma$ s.t. $(b,a) \in \mathcal{R}_{coll_j}, (a,b) \notin \mathcal{R}_{coll_j}$, if $a \succ_s b$, then: $(b,a) \in \mathcal{D}_{coll_j}, (a,b) \in \mathcal{D}_{coll_j}$.*

We apply two concordances with Three Detectives example and check whether the disagreement is diminished.

Example 5. Proceed with Example 4. Let $\sigma = gr$, according to Definition 18:

- If we require a strong concordance between collective framework and social preference, $\Gamma'_{st} = \{\mathcal{F}'_{coll_1}, \mathcal{F}'_{coll_2}\}$, where $\mathcal{F}'_{coll_1}$ and $\mathcal{F}'_{coll_2}$ are shown in Fig. 3. $\mathcal{E}_{gr}(\mathcal{F}'_{coll_1}) = \{\{a,b\}\}$ and $\mathcal{E}_{gr}(\mathcal{F}'_{coll_2}) = \{\{a\}\}$. Therefore, argument a is sceptically jointly accepted under grounded semantics as collective outcome.
- If we require a weak concordance between collective framework and social preference, $\Gamma'_{we} = \{\mathcal{F}'_{coll_1}, \mathcal{F}'_{coll_2}, \mathcal{F}'_{coll_3}\}$, where $\mathcal{F}'_{coll_1}, \mathcal{F}'_{coll_2}$ and $\mathcal{F}'_{coll_3}$ are shown in Fig. 3. $\mathcal{E}_{gr}(\mathcal{F}'_{coll_1}) = \{\{a,b\}\}, \mathcal{E}_{gr}(\mathcal{F}'_{coll_2}) = \{\{a\}\}$ and $\mathcal{E}_{gr}(\mathcal{F}'_{coll_3}) = \{\emptyset\}$. Therefore, argument a is credulously jointly accepted under grounded semantics as collective outcome.

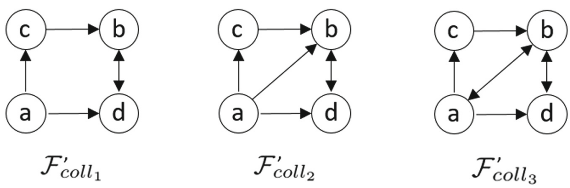

Fig. 3. Collective frameworks after concordance in Three Detectives

From Example 5 we can see that after the operation of concordance, under grounded semantics (which is the most sceptical of all), argument a can be sceptically (with strong concordance) or credulously (with weak concordance) jointly accepted as a collective outcome. Actually, if we choose preferred semantics (which is the most credulous of all), argument a will be sceptically jointly accepted with both strong and weak concordance. After the operation of concordance, an agreement (i.e. the sceptical joint acceptability of arguments under a certain semantics) has been reached between the reasoning result from framework merging and one of social winners!

To evaluate the operation of concordance and provide new criteria for the reasonability of collective outcome, we propose three postulates as follows.

Definition 19. *Let $\Gamma = \{\mathcal{F}_{coll_1} \dots \mathcal{F}_{coll_k}\}$ be the set of collective frameworks, where $\mathcal{F}_{coll_j} = (\mathcal{A}_{coll_j}, \mathcal{R}_{coll_j})$ and the social preference for $\hat{\mathcal{P}}$ be $\succsim_s$. Let the operation of concordance be Con_x where $x \in \{st, we\}$. Let the sets of collective frameworks obtained after concordance be Γ'_x. Let $\sigma \in \{co, pr, gr, st\}$ and the sets of sceptically and credulously jointly accepted arguments after concordance be $Sa_\sigma(\Gamma'_x)$ and $Ca_\sigma(\Gamma'_x)$. Let $win(\succsim_s) = \{a | \forall b \in \mathcal{A}_{coll_j}$ s.t. $a \succ_s b$ or $a \sim_s b$ but not the case $b \succ_s a\}$ denote the set of social winners.*

- ***Collective cardinality decline*** *(P9). $\#(\Gamma'_x) \subseteq \#(\Gamma)$.*
- ***Joint acceptance growth*** *(P10). $Sa_\sigma(\Gamma) \subseteq Sa_\sigma(\Gamma'_x)$ and $Ca_\sigma(\Gamma) \subseteq Ca_\sigma(\Gamma'_x)$.*
- ***Social winner(s) dominance*** *(P11). $\exists a \in win(\succsim_s)$ s.t. $a \in Sa_\sigma(\Gamma'_x)$.*

Let us elaborate the implications of these postulates. P9 states that after concordance, the cardinality of collective frameworks declines which means representative frameworks for the group become more concentrated and complexity of computation is reduced. P10 indicates that another benefit of concordance is more arguments are possible to be jointly accepted. P11 says although it seems that the group are less prudent on the acceptance of arguments than before according P10, we make sure that at least one of social winners is sceptically accepted under a certain semantics as collective outcome, which implies non-emptiness of collective outcome. Next, we verify our methods with three postulates.

Proposition 5. *The operations of strong concordance and weak concordance satisfy collective cardinality decline (P9).*

Proof. Only if $\exists a, b \in \mathcal{A}_{coll}$ s.t. $(a, b), (b, a) \in R_3$ and a, b is not indifferent according to $\succsim_s$, we have $\#(\Gamma'_x) \subset \#(\Gamma)$ and in other situations we have $\#(\Gamma'_x) = \#(\Gamma)$. Thus the conclusion is held.

Corollary 2. $\#(\Gamma'_{st}) \subseteq \#(\Gamma'_{we})$.

Proof. When $\exists a, b \in \mathcal{A}_{coll}$ s.t. $(a, b), (b, a) \in R_3$ and a, b is not indifferent according to $\succsim_s$, we have at least four collective frameworks. According to strict concordance, $\emptyset, (a, b)$ or $\emptyset, (b, a)$ will preserved as a result while symmetric attack $(a, b)(b, a)$ will be excluded. According to weak concordance, $(a, b)(b, a)$ will also preserved, which means $\#(\Gamma'_{st}) \subset \#(\Gamma'_{we})$. In other situations we have $\#(\Gamma'_x) = \#(\Gamma)$. Thus the conclusion is held.

Proposition 6. *The operation of strong concordance doesn't satisfy joint acceptance growth (P10) while the operation of weak concordance satisfies it only if under complete, preferred and stable semantics.*

Proof. First we prove the violation of strong concordance. According to the definition, $\forall \mathcal{F}_{coll_j} \in \Gamma$, if $a \succ_s b, (b,a) \in \mathcal{R}_{coll_j}, (a,b) \notin \mathcal{R}_{coll_j}$, then $(a,b) \in \mathcal{D}_{coll_j}$. It means argument a will take the place of argument b appearing in the set of sceptically jointly accepted arguments if a is not defeated by other arguments. $\exists b \in Sa_\sigma(\Gamma)$ and $b \notin Sa_\sigma(\Gamma'_{st})$, thus $Sa_\sigma(\Gamma) \nsubseteq Sa_\sigma(\Gamma'_{st})$. The proof of $Ca_\sigma(\Gamma) \nsubseteq Ca_\sigma(\Gamma'_{st})$ is the same. Next we prove the operation of weak concordance satisfies P10 only if under complete, preferred and stable semantics. According to the definition, $\forall \mathcal{F}_{coll_j} \in \Gamma$, if $a \succ_s b, (b,a) \in \mathcal{R}_{coll_j}, (a,b) \notin \mathcal{R}_{coll_j}$, weak concordance make the attack (b,a) to be symmetric. It means if argument a and b are not defeated by other arguments, under grounded semantics neither of them can be sceptically jointly accepted. $\exists b \in Sa_{gr}(\Gamma)$ and $b \notin Sa_{gr}(\Gamma'_{we})$. Thus $Sa_{gr}(\Gamma) \nsubseteq Sa_{gr}(\Gamma'_{we})$. The proof of $Ca_{gr}(\Gamma) \nsubseteq Ca_{gr}(\Gamma'_x)$ is the same. Let $\sigma \in \{co, pr, st\}$, $\forall \mathcal{F}_{coll_j} \in \Gamma$, if argument a and b are not defeated by other arguments and after weak concordance, $(a,b)(b,a) \in \mathcal{D}_{coll_j}$, we have $b \in Sa_\sigma(\Gamma)$ and $a, b \in Sa_\sigma(\Gamma'_{we})$. Thus $Sa_\sigma(\Gamma) \subseteq Sa_\sigma(\Gamma'_{we})$. The proof of $Ca_\sigma(\Gamma) \subseteq Ca_\sigma(\Gamma'_{we})$ is the same.

Proposition 7. *The operation of strong concordance satisfies social winner(s) dominance (P11) and the operation of weak concordance satisfies it only if under complete, preferred and stable semantics.*

Proof. As proved in Theorem 1, if social winner is the necessary Condorcet winner, without the operation of concordance, it will be sceptically jointly accepted. Here we only need to prove that if $NW(\hat{\mathcal{P}}) = \emptyset$ and $\exists a \in PW(\hat{\mathcal{P}})$ then $a \in Sa_\sigma(\Gamma'_x)$. (1) If a is attacked by a possible Condorcet winner b, since $a \sim_s b$ there is no concordance, i.e. $\Gamma = \Gamma'_x$. b will be sceptically jointly accepted. Thus $b \in win(\succsim_s)$ s.t. $b \in Sa_\sigma(\Gamma)$. (2)$\forall \mathcal{F}_{coll_j} \in \Gamma$, if a is attacked by an argument $b \in \bigcup_i \mathcal{A}_i \setminus PW(\hat{\mathcal{P}})$, since $a \succ_s b$ concordance is needed. The operation of strong concordance will reverse the attack and then $\forall \mathcal{F}'_{coll_j} \in \Gamma'_{st}, a \in \mathcal{E}_\sigma(\mathcal{F}'_{coll_j})$, i.e. $a \in Sa_\sigma(\Gamma'_{st})$. Let $\sigma \in \{co, pr, st\}$, the operation of weak concordance will make the attack be symmetric so that $\forall \mathcal{F}'_{coll_j} \in \Gamma'_{we}, a \in \mathcal{E}_\sigma(\mathcal{F}'_{coll_j})$, i.e. $a \in Sa_\sigma(\Gamma'_{we})$. Since when $\sigma = gr$, $\forall \mathcal{F}'_{coll_j} \in \Gamma'_{we}, a \notin \mathcal{E}_{gr}(\mathcal{F}_{coll_j})$, i.e. $a \notin Sa_{gr}(\Gamma'_{we})$. Therefore under grounded semantics, the conclusion is not held.

Table 2. The evaluation on two concordances

Concordance	P1-P5	P9	P10	P11
Con_{st}	✓	✓	×	✓
Con_{we}	✓	✓	$\checkmark^{\sigma \in \{pr,st,co\}}$	$\checkmark^{\sigma \in \{pr,st,co\}}$

For clarification, we list all above results in Table 2. From the table, we can see that both strong and weak concordances satisfy P1-P5 since the operation of framework merging is same for them and it satisfies all of five postulates.

Further, weak concordance satisfies all of three postulates under complete, preferred and stable semantics, including grounded semantics in P9. Although strong concordance doesn't satisfy P10, it satisfies both P9 and P11 under all standard semantics. Note that the collective outcome obtained after the operation of concordance, not only preserves majority based consensus among individual frameworks, but also modifies collective frameworks to ensure at least one of social winners will be accepted by the group. Therefore, it is more reasonable than the results obtained from framework merging or incomplete preference aggregation, since it reflects the interplayed consensus on reasoning result and social credence.

6 Conclusion

Individual preferences give rise to argument strength in collective argumentation and may have influences on collective outcome correspondingly. In order to tackle individual preferences as well as distinct individual frameworks in a multi-agent scenario, in this paper we first propose a method for the operation of framework merging and then define a method to obtain social preference through the operation of incomplete preference aggregation. However, two different operations may not agree with each other on the joint acceptability of arguments. Aiming to find a solution to reach more agreements between the results from framework merging and incomplete preference aggregation, we define an operation called concordance, which is actually the modification of collective frameworks according to social preference. As a result, the collective outcome obtained from argumentative reasoning with modified collective frameworks is renewed to reflect the dominance of social preference. We propose a couple of rational postulates and verify that the methods we propose are equipped with reasonability.

6.1 Related Work

Preference has been studied in single-framework argumentation [9–11,17]. [10,17] proposes two roles for preference and the following paper [11] agrees with them but proposes four methods for reduction. We adopt two reductions proposed in [10] and [11] for the reason that they can preserve conflicting relation between pairs of arguments and strengthen the argument strength of social winners. Although these previous work hasn't studied preference in the background of collective argumentation, they genuinely construct basis for the research in this paper.

In the perspective of framework merging, Coste-Marquis' approach [2] is close to us. Since distance-based framework merging always gives rise to majority-based results, we define two subsets of attack for collective framework based on the majority and adopt Coste-Marquis' proposal in argumentative reasoning. However, our approach is more concise on the definition and more explainable on the results. Another qualitative approach for framework merging is Delobelle's work [3]. He propose an approach for framework merging based on belief revision [18] and represent the expected extensions as formula consisting of arguments. His approach is deviated from ours and each individual framework in his settings shares the same set of arguments. Apart from these, Delobelle [6], Gabbay [5] and Cayrol [4] adopt a quantitative way for framework merging, treating the appearances of an argument or an attack in individual frameworks as weights. Delobelle selects collective extensions based on weights. In Gabbay's approach, weights can be propogated in the collective framework and the acceptability of arguments is determined by a threshold. Cayrol's approach only defines a quasi-semantics named vs-defend to justify a successful defense between pairs of arguments.

Based on Bench-Capon's innovative VAF [12–14, 19] study value preference in the field of multi-agent systems. Airiau [12] discusses the criteria for rationalisation of the profile of individual frameworks. The rationalisation actually comes from transitivity and acyclicity of strict total ordering. Lisowski [13] concerns about the correspondence between the reasoning results obtained from value preference aggregation and framework merging. They figure out a method to construct the correspondence. Liao [14] studies value preference's influences on the reasoning results based on different aggregation rules and ordering-lifting principles. However, preference aggregation in both [13] and [14] is dealing with complete preference according to their settings and the approach for framework merging adopted in [13] is a quota rule which is unable to tackle framework merging of distinct individual frameworks.

6.2 Future Work

First, as studied in [14], different aggregation rules may give rise to different results with certain properties. We'd like to adopt other incomplete preference aggregation procedures, such as Borda procedure proposed in [7] and minmax regret approximation [20], and evaluate the reasonability of them respectively. Second, as proposed by Baumeister in [21], there is a class of argumentation framework with uncertainty called incomplete argumentation framework. In future, we can extend our research to integrate incomplete individual preferences into the profile of incomplete individual frameworks. Third, since individual preference can be quantitatively represented as personal degree of belief, in other words, we can treat it as a probability. Then we can connect this area with probabilistic argumentation framework.

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