Guaranteed Cost Formation Control for Linear Multi-agent Systems with Switching Topologies

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Abstract. Guaranteed-cost formation problem for linear multi-agent systems with switching topologies is investigated. The guaranteed cost formation problem is transformed into a guaranteed cost control problem of an reduced-order switched systems equivalently by a linear transformation. Then, a necessary and sufficient condition for guaranteed cost formation is proposed. Moreover, based on an average dwell time scheme, a sufficient condition for the guaranteed cost formation of linear multi-agent system with switching topologies are presented in terms of linear matrix inequality techniques, and an upper bound of the guaranteed cost function is given. Finally, a numerical example is given to demonstrate the effectiveness of the theoretical results.

Keywords: Guaranteed-cost formation · Multi-agent systems · Switching topologies · Average dwell-time

1 Introduction

During the past decades, coordinations of multi-agent systems have received more attention due to its wide applications in military and civilian areas. In the multi-agent systems, formation control has already been a hotspot of research, many researchers focused on formation control problems, such as leader-follower [1], virtual structure [2], behavior-based [3], and consensus-based strategy [4], which have been well developed and applied.

However, in many practical cases, the communication topologies of multi-agent systems may be switching due to that the communication channel may

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fail or new channels may be created during movement. Time-varying formation control problems for multi-agent systems with switching topologies are investigate in [5] by using the common Lyapunov functional approach and algebraic Riccati equation technique. As we know, the dynamic structures of agents and the communication topology of multi-agent systems are key factors for formation. However, switching signals of communication topological are also one of the important factors of formation feasible for multi-agent systems with switching topologies. Average dwell time and dynamic dwell time scheme were presented to investigate the formation problem of linear multi-agent systems with switching communication topologies in [6,7].

Moreover, in practical applications, each agent of multi-agent systems may have limited energy supply to perform certain tasks, such as sensing, communication, and movement as well as be required to achieve some formation performance. Therefore, it is very important to realize a balance between formation performance and energy consumption, which can usually be modeled as optimal or suboptimal formation problems. To the best of our knowledge, there are few papers addressing guaranteed cost formation problems for multi-agent systems with switching topologies. Guaranteed cost formation control under fixed topology for multi-agent systems were investigated in [8,9]. Energy-constraint output formation problems for high-order linear multi-agent systems with switching topologies and the random communication silence were investigated in [10]. Guaranteed-cost consensus for multi-agent systems with switching topologies were investigated, where the topology was described by an undirected graph and the dwell time of each topology was assumed to be the same in [11–13].

Motivated by this, guaranteed cost formation control problem of high-order continuous-time linear multi-agent systems with switching communication topology is investigated in this paper. Compared with the literature mentioned above, the main contribution of the current paper is that: i) the guaranteed cost formation problem with switching topologies under directed graphs is considered; ii) the average dwell time scheme is introduced into guaranteed cost formation problem with switching topologies.

The rest of the paper is organized as follows. Section 2 shows the problem description based on graph theory. A linear transformation approach is presented in Sects. 3, sufficient condition for guaranteed-cost formation for linear multi-agent systems under switching topologies is proposed, and the upper bound of guaranteed cost are presented. Numerical results are presented in Sect. 5.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the n-dimension real column vector and the set of $n \times m$ dimensional real matrices, respectively. Let 0 be zero number, zero vectors, or zero matrices in appropriate dimension, respectively. Let $\mathbf{1}_N$ denote an N-dimensional column vector with $\mathbf{1}_N = [1, 1, \cdots, 1]^T$. Let P^T and P^{-1} denote the transpose and the inverse matrix of P, respectively. $P^T = P > 0$ stands for matrix P is symmetric and positive definite. The notation $*$ denotes the symmetric terms of a symmetric matrix. I_N represents the identity matrix of order N, $\otimes$ is applied to denote the Kronecker product of matrices. $\lambda(\cdot)$ denotes the eigenvalue of a matrix.

2 Problem Description

Consider a linear multi-agent system (LMAS) consisting of N agents, where each agent takes the following dynamics:

$$\dot{x_i}(t) = Ax_i(t) + Bu_i(t),\ i \in \{1, \cdots, N\}, \tag{1}$$

with

$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, B = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \in \mathbb{R}^{2n \times m},$$

where (A, B) is stabilizable. $x_i(t) \in \mathbb{R}^{2n}$ is the state variable of agent i, and $x_i(t) = [s_i^T(t), v_i^T(t)]^T$, $s_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ are the position state and the velocity state of agent i respectively, $u_i(t) \in \mathbb{R}^m$ is formation control protocol of agent i, which depends on agents x_i and x_j. Agent j is called a neighbor of agent i if there exists a communication channel from j to i, and $\mathcal{I} = \{1, \cdots, N\}$ is the index set of agents.

Let $N_i(t)$ denote the set of the neighbors of the agent i at time t, and $\mathcal{N}(t) = \{N_i(t), i \in \mathcal{I}\}$ is a communication configuration of the system (1) at time t. $\mathcal{N}(t)$ can be expressed by a dynamic digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}(t),\ W(t))$. Vertex set $\mathcal{V} = \{1, 2, \cdots, N\}$ represents the group of agents. Time-varying edge set $\mathcal{E}(t) \subseteq V \times V$ denotes the communication topology $\mathcal{N}(t)$, i.e., $(j, i) \in \mathcal{E}(t) \Leftrightarrow j \in N_i(t)$, and $W(t) = [w_{ij}] \in \mathbb{R}^{N \times N}$ is a weighted adjacency matrix.

A formation, which described by a vector $H = [h_1^T, h_2^T, \cdots, h_N^T]^T \in \mathbb{R}^{2nN}$, is a geometric pattern, it satisfies some predefined geometric constraints which is required to achieve and maintain for the LMAS (1). H represents the desired formation. In the current paper, the formation vector $h_i = [h_{is}^T, h_{iv}^T]^T$ is used to express the relative position h_{is} and the relative velocity h_{iv} of agent i respectively. It is generally known that the velocity state $v_i(t)$ are synchronous when multi-agent system (1) achieves the formation H, therefore, there is $H = [h_1^T, h_2^T, \ldots, h_N^T]^T$ with $h_i = [h_{si}^T, 0]^T \in \mathbb{R}^{2n}$,$i = 1, \cdots, N$.

For a desired formation H, a formation protocol is considered as follows:

$$u_i(t) = K \sum_{j \in N_i(t)} w_{ij}(t)[(x_j(t) - h_j) - (x_i(t) - h_i)], t \geq 0, \tag{2}$$

where $w_{ij}(t)$ represents the coupling strength with respect to a communication channel from j to i at time t.

Definition 1. Denote $H = [h_1^T, h_2^T, \cdots, h_N^T]^T \in \mathbb{R}^{2nN}$ be a specified formation. Linear multi-agent system(LMAS) (1) is said to achieve formation H, if there exist vector-valued functions $\xi(t) \in \mathbb{R}^{2n}$ and a control protocol (2), such that $\lim_{t\to\infty} \|x_i(t) - h_i\| = \xi(t), i \in \mathcal{I}$, and the vector-valued function $\xi(t)$ is called a formation center function.

Assume the communication topology of system (1) is time-varying. Without loss of generality, assume the communication topology of system (1) switches in a topology set, i.e.$\mathcal{N}(t) \in \{\mathcal{N}^k, k \in \mathfrak{M}\}$, $\mathcal{N}^k = \{N_i^k, i = 1, \cdots, N\}$, where $\mathfrak{M} = \{1, \cdots, M\}$ is an index set. Thus, the communication topology $\mathcal{N}(t)$ is piecewise time-invariant as the system evolves. Define a switching signal $\sigma : [0, +\infty) \to \mathfrak{M}$, which is a piecewise constant and right-continuous function of time, to describe the switching rules among the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M}\}$, i.e.,$\mathcal{N}(t) = \mathcal{N}^k \Leftrightarrow \sigma(t) = k$. The switching signal specifies the index of the actived topology $\{\mathcal{N}^k, k \in \mathfrak{M}\}$ at time t.

Assume that the switching is finite in any finite time interval, and there are no jumps in the state at the switching instants. Corresponding to the switching signal $\sigma(t)$, we have the switching sequence $\{(t_0, k_0), (t_1, k_1), \cdots, (t_r, k_r), \cdots, | \, k_r \in \mathfrak{M}, \; r = 0, 1, \cdots\}$, which means that the communication topology of system (1) is $\mathcal{N}^{k_r}$ when $t \in [t_r, t_{r+1})$.

Consider the following linear quadratic cost function

$$\begin{aligned} J_C = \sum_{i=1}^{N} \int_0^{\infty} \{ & \sum_{j=1}^{N} w_{ij}(t)[(x_j(t) - h_j) \\ & - (x_i(t) - h_i)]^T Q[(x_j(t) - h_j) - (x_i(t) - h_i)] + u_i^T(t) R u_i(t)\} dt, \end{aligned} \tag{3}$$

where Q and R are given symmetric positive matrices.

Definition 2. LMAS (1) is said to achieve guaranteed cost formation H via protocol (2) under the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M}\}$ with the switching signal $\sigma(t)$, if for any initial condition sequence $x(0)$, there is $\lim_{t\to\infty} \|(x_i(t) - h_i) - (x_j(t) - h_j)\| = 0, i, j \in \mathcal{I}$, and there exists a $J_C^* > 0$, such that $J_C \le J_C^*$, J_C^* is said to be a guaranteed cost.

Definition 3. LMAS (1) with respect to the formation H is said to be guaranteed cost feasible via formation protocol (2) under the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M}\}$ with the switching signal $\sigma(t)$, if there exist control gain matrix K such that multi-agent system (1) achieves guaranteed cost formation H.

Let $x = [x_1^T \dots x_N^T]^T \in \mathbb{R}^{2nN}$, and the dynamics of the LMAS (1) with formation protocol (2) can be described by a compact form as follows:

$$\dot{x}(t) = (I_N \otimes A)x(t) - ((L_{\sigma(t)} \otimes BK_{\sigma(t)})(x(t) - H), \tag{4}$$

where outer-coupling matrix $L_{\sigma(t)} = [l_{ij}]_{\sigma(t)} \in \mathbb{R}^{N\times N}$ is Laplacian matrix induced by the communication topology $\mathcal{N}(t) = \{N_i^{\sigma(t)}, \; i \in \mathcal{I}\}$, and its entries are defined by

$$l_{ij}^{\sigma(t)} = \begin{cases} \sum\limits_{k\in N_i} w_{ik}^{\sigma(t)}, j = i \\ -w_{ij}^{\sigma(t)}, & j \ne i, \; j \in N_i \\ 0, & j \notin N_i \end{cases}$$

Note that for a given protocol (2), switching signals of communication topological are one of the important factors for formation control problem of LMAS

(1) with switching topologies. Based on this, this paper mainly studies the influence of the change of communication topology on guaranteed cost formation control for continuous-time linear multi-agent systems. Based on the actual situation of the communication topology, the following case is analyzed:

Assumption 1. Communication topology $\mathcal{N}(t)$ switches among set $\{\mathcal{N}^k, k \in \mathfrak{M}\}$, and there exist a spanning tree for each communication topology in set $\{\mathcal{N}^k, k \in \mathfrak{M}^- = \{1, 2, \cdots, r\}\}$, $1 \leq r < M$ meanwhile there does not exist a spanning tree for each communication topology in set $\{\mathcal{N}^k, k \in \mathfrak{M}^+\}$, as well as $\mathfrak{M} = \mathfrak{M}^- \cup \mathfrak{M}^+$.

3 Problem Transformation

In this section, guaranteed cost formation control problem for LMAS (1) with switching topologies are converted into guaranteed cost control problem of a corresponding auxiliary switched systems.

Transform system (4) by the following linear transformation [7]:

$$\bar{x}(t) = S(x(t) - H), \tag{5}$$

where

$$S = \begin{bmatrix} \tilde{S}_0 \\ \mathbf{1}_N^T \end{bmatrix} \otimes I_{2n}, \tilde{S}_0 = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & -1 \end{bmatrix}.$$

The inverse matrix of S can be worked out as follows:

$$S^{-1} = \frac{1}{N} \begin{bmatrix} N-1 & N-2 & \dots & 1 & 1 \\ -1 & N-2 & \dots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -2 & \dots & 1 & 1 \\ -1 & -2 & \dots & -(N-1) & 1 \end{bmatrix} \otimes I_{2n} = \begin{bmatrix} \hat{S}_0 & N^{-1}\mathbf{1}_N \end{bmatrix} \otimes I_{2n}.$$

By the linear transformation (5), system (4) is transformed into the following system:

$$\dot{\bar{x}}(t) = S[(I_N \otimes A) - L_{\sigma(t)} \otimes BK]S^{-1}\bar{x}(t) + S(I_N \otimes A)H. \tag{6}$$

Let $\bar{x} = [y^T \ z^T]^T$, where $y = [\bar{x}_1^T \dots \bar{x}_{N-1}^T]^T$, unfold system (6) by $\bar{x} = [y^T \ z^T]^T$:

$$\begin{aligned}
\dot{\bar{x}} &= \begin{bmatrix} \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} \\
&= (\begin{bmatrix} \tilde{S}_0 \\ \mathbf{1}_N^T \end{bmatrix} \otimes I_{2n})[(I_N \otimes A) - (L_{\sigma(t)} \otimes BK)]([\hat{S}_0 \quad N^{-1}\mathbf{1}_N] \otimes I_{2n}) \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} \\
&+ \begin{bmatrix} (\tilde{S}_0 \otimes A)H \\ (\mathbf{1}_N^T \otimes A)H \end{bmatrix} = \begin{bmatrix} \mathfrak{A}_{11} & \mathbf{0} \\ \mathfrak{A}_{21} & A \end{bmatrix} \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} (\tilde{S}_0 \otimes A)H \\ (\mathbf{1}_N^T \otimes A)H \end{bmatrix}.
\end{aligned}$$

where

$$\mathfrak{A}_{11} = I_{N-1} \otimes A - (\tilde{S}_0 L_{\sigma(t)} \hat{S}_0) \otimes BK,$$
$$\mathfrak{A}_{21} = -(\mathbf{1}_N^T L_{\sigma(t)} \hat{S}_0) \otimes BK.$$

System (6) is equivalent to the following system:

$$\begin{cases} \dot{y}(t) = [I_{N-1} \otimes A - (\tilde{S}_0 L_{\sigma(t)} \hat{S}_0) \otimes BK] y(t) + (\tilde{S}_0 \otimes A)H, \\ \dot{z}(t) = Az(t) - (\mathbf{1}_N^T L_{\sigma(t)} \hat{S}_0) \otimes BKy(t) + (\mathbf{1}_N^T \otimes A)H. \end{cases}$$

Let $\bar{A} = I_{N-1} \otimes A$, $\bar{B}_{\sigma(t)} = -(\tilde{S}_0 L_{\sigma(t)} \hat{S}_0) \otimes B$, $\bar{K} = I_{N-1} \otimes K$.

According to the structure of A and H, it derived that $(\tilde{S}_0 \otimes A)H = 0$. Hence, the above equation can be expressed by

$$\begin{cases} \dot{y}(t) = (\bar{A} + \bar{B}_{\sigma(t)} \bar{K}) y(t), \\ \dot{z}(t) = Az(t) - (\mathbf{1}_N^T L_{\sigma(t)} \otimes BKy(t). \end{cases} \tag{7}$$

Obviously, there is no relationship between $y(t)$ and $z(t)$ in the first equation in system (7). Therefore, the following Lemma is obtained which transforms the formation problem with switching topologies into a asymptotic stability problem of reduced-order switched systems equivalently.

Lemma 1. LMAS (1) achieves formation H via protocol (2) under the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M}\}$ with the switching signal $\sigma(t)$ for any bounded initial states $x(0)$ if and only if switched systems

$$\dot{y}(t) = (\bar{A} + \bar{B}_{\sigma(t)} \bar{K}) y(t) \tag{8}$$

is asymptotically stable.

Cost function J_C in (3) can be rewritten as follows:

$$J_C = \int_0^{\infty} y^T(t) \{ 2L_{\sigma(t)} \otimes Q + [L_{\sigma(t)}^T L_{\sigma(t)} \otimes (K^T RK) \} y(t) dt. \tag{9}$$

According Lemma 1 and Definition 2, the following theorem can be obtained:

Theorem 1. LMAS (1) achieves guaranteed cost formation H via protocol (2) under the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M}\}$ with the switching signal $\sigma(t)$ for any bounded initial states $x(0)$, if and only if switched systems (8) is asymptotically stable and there exists a $J_C^* > 0$, such that $J_C \leq J_C^*$.

4 Main Results

In this section, guaranteed-cost formation criteria is presented based on the average dwell time method. The communication topologies $\mathcal{N}(t)$ of LMAS (1) satisfies Assumption 2. Communication topology $\mathcal{N}(t)$ switches among set $\{\mathcal{N}^k, k \in \mathfrak{M}\}$, and there exist a spanning tree for each communication topology in set $\{\mathcal{N}^k, k \in \mathfrak{M}^-\}$, meanwhile there does not exist a spanning tree for each communication topology in set $\{\mathcal{N}^k, k \in \mathfrak{M}^+\}$, as well as there is $\mathfrak{M} = \mathfrak{M}^- \cup \mathfrak{M}^+$.

It follows that matrices $(\bar{A} + \bar{B}_k\bar{K}), k \in \mathfrak{M}^-$ are Hurwitz, matrices $(\bar{A} + \bar{B}_k\bar{K}), k \in \mathfrak{M}^+$ are not Hurwitz. There exist normal number $\theta_1, \ldots, \theta_r, \theta_{r+1}, \cdots, \theta_M$, such that $(\bar{A} + \bar{B}_k\bar{K}) + \theta_k I$, $k \in \mathfrak{M}^-$ and $(\bar{A} + \bar{B}_k\bar{K}) - \theta_k I, k \in \mathfrak{M}^+$ are Hurwitz matrices. Hence, there exist positive definite symmetric matrices $T_k, k \in \mathfrak{M}$ satisfy

$$\begin{cases} (\bar{A} + \bar{B}_k\bar{K} + \theta_k I)^T M_k + M_k(\bar{A} + \bar{B}_k\bar{K} + \theta_k I) < 0, \ k \in \mathfrak{M}^-, \\ (\bar{A} + \bar{B}_k\bar{K} - \theta_k I)^T M_k + M_k(\bar{A} + \bar{B}_k\bar{K} - \theta_k I) < 0, \ k \in \mathfrak{M}^+. \end{cases} \tag{10}$$

Homogeneously, denote $\alpha_1 = \min_{k\in\mathfrak{M}} \lambda(M_k)$, $\alpha_2 = \max_{k\in\mathfrak{M}} \lambda(M_k)$, $\mu = \alpha_2\alpha_1^{-1}$. Let $T^-(t)$ $(T^+(t))$ signify the total activation time of the LMAS (1) under topologies $\mathcal{N}^k, k \in \mathfrak{M}^-(\mathcal{N}^k, k \in \mathfrak{M}^+)$. Denote $\theta^- = \min_{k\in\mathfrak{M}^-} \theta_k$ and $\theta^+ = \max_{k\in\mathfrak{M}^+} \theta_k$ then for any given $\theta \in (0, \theta^-)$, choose $\theta^* \in (\theta, \theta^-)$, proposing the switching condition:

$$\inf_{t\geq 0} \frac{T^-(t)}{T^+(t)} \geq \frac{\theta^+ + \theta^*}{\theta^- - \theta^*}. \tag{11}$$

Next, we try to characterize the switching signals within the communication topologies such that the LMAS (1) achieves formation via the protocol (2). We focus on the switched linear system (8) and introduce the following definition and Lemma.

Definition 4. [14] For any $t > t_0 \geq 0$, let $N_\sigma(t_0, t)$ denote the number of switchings of the signal $\sigma(t)$ over the time interval (t_0, t). If $N_\sigma(t_0, t) \leq N_0 + (t - t_0)\tau_a^{-1}$ holds for $\tau_a > 0$ and $N_0 \geq 0$, then τ_a is called the average dwell time, and N_0 is the chatter bound of the switching signal $\sigma(t)$. Denote $S_{ave}[\tau_a, N_0]$ be the set of all switching signals $\sigma(t)$ with the average dwell time τ_a and the chatter bound N_0.

Lemma 2. [15] (Schur Complement)

For given symmetric matrix $P \in \mathbb{R}^{(m+n)\times(m+n)}$:

$$P = P^T = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$$

where $P_{11} \in \mathbb{R}^{m\times m}$, $P_{22} \in \mathbb{R}^{n\times n}$. Then the following three conditions are equivalent:

(1) $P < 0$;
(2) $P_{11} < 0, P_{22} - P_{12}^T P_{11}^{-1} P_{12} < 0$;
(3) $P_{22} < 0, P_{11} - P_{12} P_{11}^{-1} P_{12}^T < 0$.

Theorem 2. Suppose Assumption 1 hold, LMAS (1) with respect to the formation H is guaranteed cost feasible via protocol (2) under the communication topologies $\{\mathcal{N}^k, k \in \mathfrak{M} = \mathfrak{M}^- \cup \mathfrak{M}^+\}$, if the following two conditions hold simultaneously:

i) There is a finite constant $\tau_a^* = \frac{\ln \mu}{2(\theta^* - \theta)}$, such that the switching signal $\sigma(t)$ satisfies switching condition (11) as well as $\sigma(t) \in S_{ave}[\tau_a, N_0]$.

ii) There exist $2n(N-1)$-dimensions matrices $X_k = X_k^T > 0$ and matrix W_k such that

$$\Phi_k = \begin{bmatrix} \phi_{11} & I & L_k \otimes K \\ * & -\frac{1}{2}(L_k \otimes Q)^{-1} & 0 \\ * & * & -R^{-1} \end{bmatrix} < 0, \tag{12}$$

where

$$\phi_{11_k} = \bar{A}X_k + \bar{B}_k W_k + (\bar{A}X_k + \bar{B}_k W_k)^T.$$

In this case, the control gain matrix in formation protocol (2) satisfies $K_k = W_k X_k^{-1}$, and guaranteed cost $J_{Ck}^* = y_0^T X_k^{-1} y_0$, $J_C^* = \max_{k \in \mathfrak{M}} J_{Ck}^*$.

5 Simulation Example

In this section, a numerical example is given to illustrate the effectiveness of the obtained theoretical results. As Theorem 2 is a special case of Theorem 3, we only present a example that satisfies Assumption 2. Consider a multi-agent systems with 4 agents and the dynamics of each agent is described by LMAS (1) with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{13}$$

where $x_i(t) = [x_{i1}(t)\ x_{i2}(t)]^T$, $x_{i1}(t)$ and $x_{i2}(t)$ denote position and velocity of agent i respectively. The initial state is chosen randomly:

$$x(0) = [0\ \ 50\ \ 150\ \ 40\ \ 300\ \ 30\ \ 450\ \ 20]^T.$$

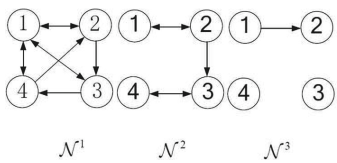

Fig. 1. Three communication topologies of LMAS (1)

Figure 1 shows the communication topologies $\mathcal{N}^1$, $\mathcal{N}^2$, $\mathcal{N}^3$, which satisfies Assumption 2, without loss of generality, let the communication topology weight is 1. One can figure out $\tau_a^* = 0.4808$ and $\frac{T^-(t)}{T^+(t)} = 3$. We choose a switching signal $\sigma(t)$ which satisfies Theorem 3 is shown in Figure Fig. 2. In the guaranteed cost function (3) we choose $Q = 0.02I_2$ and $Q = 0.04I_2$. The target formation $H = [50\ 100\ 150\ 200]^T \otimes [1\ 0]^T$.

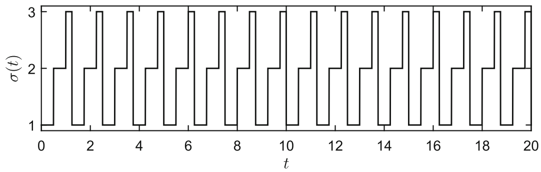

Fig. 2. A switching signal $\sigma(t)$

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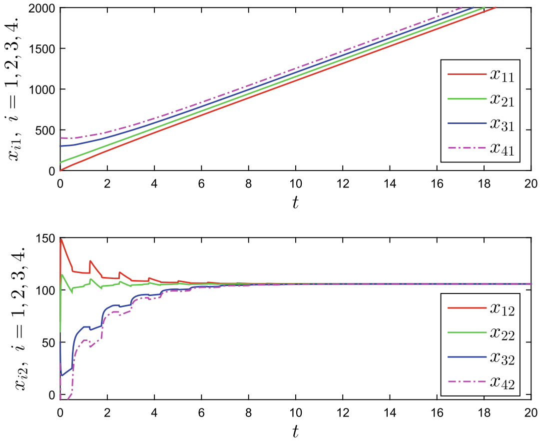

Fig. 3. Position and velocity state trajectories of LMAS (1) under topologies $\mathcal{N}^1$, $\mathcal{N}^2$, $\mathcal{N}^3$ with switching signal $\sigma(t)$

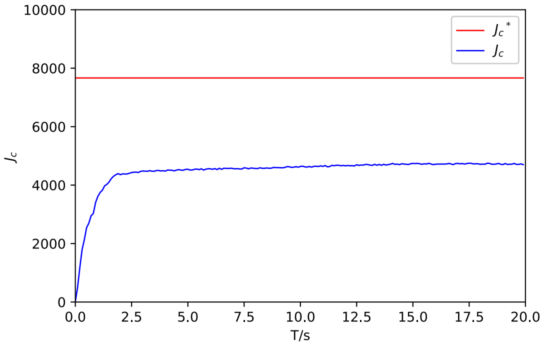

Fig. 4. Trajectory of the guaranteed cost function J_C^* and J_C^*

In Fig. 3, the state(position and velocity) trajectories of LMAS (1) are shown, One can see from Fig. 3 that the position trajectories of all agents converge with a fixed difference and the position trajectories of all agents converge to the ones. Figure 4 depicts the trajectory of the cost function J_C and J_C^*. The cost function J_C converges to a finite value less than J_C^*. The simulation results illustrate that LMAS (1) achieves guaranteed-cost formation with protocol (2) under topologies $\mathcal{N}^1$, $\mathcal{N}^2$, $\mathcal{N}^3$ with switching signal $\sigma(t)$.

6 Conclusion

The guaranteed cost formation control for multi-agent systems with switching topologies under directed graphs was investigated in this paper. By a linear transformation, the guaranteed cost formation problem for multi-agent systems were equivalently converted into guaranteed cost control problem of a reduced-order auxiliary switched systems. A sufficient condition for the guaranteed cost formation control was given based on two types of switching topologies, and an upper bound of the guaranteed cost function was determined. Finally, the effectiveness of the proposed theory has been illustrated by a 4 agents systems experiments. Further research will be conducted to design the formation protocol and optimize guaranteed cost function for formation problem of multi-agent systems with switching topologies and time-delays.

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