Chapter 14
Attitude Dynamics

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Abstract The theory of attitude control for satellites is presented. The definition of "attitude" is followed by a description of the several disturbances and of the methods to determine the current status of rotational motion. An attitude prediction into the near future allows for active control, either in one or in three axes, either done autonomously on board or by commanding. The principles of attitude propagation and control are described, as well as the possible types of control mechanism. Comparisons between theory and practice are made and several examples are given from real missions.

14.1 Introduction

The attitude of a satellite is always defined in relation to an external reference system. A few of the more obvious examples of such reference systems are a coordinate system based upon the trajectory, one with the Earth, the Sun, or another celestial body at the center, or an inertial coordinate system defined by the "fixed" stars (see also Sect. 13.2). The three angles between the satellite's body-fixed axes (for example denoted by a unit vector x_{sat}, y_{sat}, z_{sat}) and the chosen reference coordinate system (e.g., the unit vector x_{ref}, y_{ref}, z_{ref}) uniquely define the attitude. The transformation matrix from this reference frame into spacecraft body coordinates is often referred to as the attitude or direction cosine matrix:

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{x}_{sat} \cdot \boldsymbol{x}_{ref} & \boldsymbol{x}_{sat} \cdot \boldsymbol{y}_{ref} & \boldsymbol{x}_{sat} \cdot \boldsymbol{z}_{ref} \\ \boldsymbol{y}_{sat} \cdot \boldsymbol{x}_{ref} & \boldsymbol{y}_{sat} \cdot \boldsymbol{y}_{ref} & \boldsymbol{y}_{sat} \cdot \boldsymbol{z}_{ref} \\ \boldsymbol{z}_{sat} \cdot \boldsymbol{x}_{ref} & \boldsymbol{z}_{sat} \cdot \boldsymbol{y}_{ref} & \boldsymbol{z}_{sat} \cdot \boldsymbol{z}_{ref} \end{pmatrix} \tag{14.1}$$

whereby each dot product is the cosine of the angle between the two axes referred to (see e.g., Wertz 1978).

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Easiest to grasp intuitively are the roll, pitch, and yaw angles which are also widely used in the navigation of ships and airplanes (see Fig. 14.1 and also Arbinger and Luebke-Ossenbeck 2006). Roll is the movement around the direction of motion (the length axis in case of ships and planes). Pitch denotes the angle around the axis perpendicular to the orbital plane—the stamping motion of a ship—and yaw the angle around the local vertical axis.

It is always possible to choose an orthogonal reference system denoted by r_x, r_y, r_z, for example, such that the instantaneous attitude of the satellite is defined by

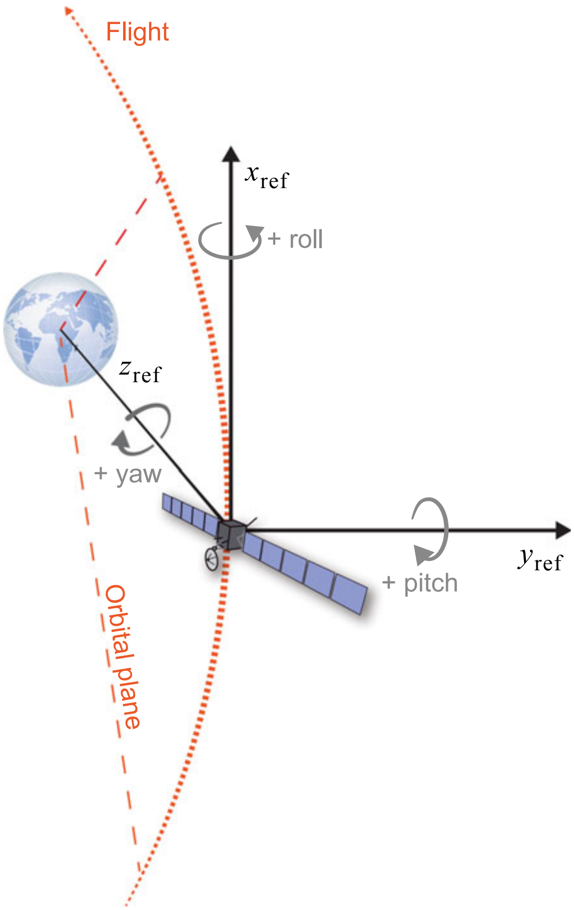

Fig. 14.1 Example of the roll, pitch and yaw angles. Reference is normally the orbit frame. Roll is the movement around the direction of flight, pitch around the direction perpendicular to the orbital plane and yaw the movement around the local vertical axis and pitch around the direction perpendicular to the orbital plane

the rotation around a single axis and thus by a single angle θ.[1] This leads to the definition of quaternions, or so-called Euler symmetric parameters. More than one representation is possible, but a possible choice is (see e.g., Wertz 1978):

$$\begin{aligned}
\boldsymbol{q}_1 &= \boldsymbol{r}_x \sin\frac{1}{2}\theta \\
\boldsymbol{q}_2 &= \boldsymbol{r}_y \sin\frac{1}{2}\theta \\
\boldsymbol{q}_3 &= \boldsymbol{r}_z \sin\frac{1}{2}\theta \\
\boldsymbol{q}_4 &= \cos\frac{1}{2}\theta
\end{aligned} \tag{14.2}$$

whereby $q_1{}^2 + q_2{}^2 + q_3{}^2 + q_4{}^2 = 1$ and q_4 is the scalar component of the quaternion. Quaternions were especially designed for three-dimensional space and thus offer, from a computational point of view, the most efficient way to handle all necessary attitude operations, such as propagation, vector and matrix multiplications, rotations, transformations, etc. Note that an attitude quaternion will normally be time dependent and its interpretation not straightforward anymore. Therefore, results are normally re-translated into more easily understood quantities such as the deviations from a default attitude or into roll, pitch and yaw angles.

In general, the attitude will not be constant in time. Changes are introduced by internal disturbances, e.g., caused by the firing of a thruster or a change in the speed of a reaction wheel (see Larson and Wertz 1992 and references cited therein for descriptions of all sensors and actuators that are commonly used), but also by external influences such as radiation pressure or gravity gradients. The deviations from a prescribed attitude which are still acceptable depend upon design and environment but most of all, on the task at hand. Clearly, providing sunlight on the panels or pointing the nadir-antenna towards the Earth has less stringent requirements than imaging a star or mapping part of the Earth's surface with a resolution of one meter. Most missions therefore have three distinct regimes of attitude control:

1. Rate damping
2. Coarse control in a basic safe mode (e.g., designed to survive a situation with low power or with thermal problems)
3. Fine control used for payload operations and orbit control.

The exact definitions vary from mission to mission, but a rough indication is provided by the following numbers:

1. Rates > 0.2–0.3°/s must be damped before operations can start; high rates can, for example, occur shortly after launch when the satellite is separated from the upper rocket stage.

[1] The single rotation axis is an eigenvector of the direction cosine matrix A; for example, $A \cdot r_x = r_x$, when r_x is the rotation axis.

2. Coarse attitude control points the satellite with a typical accuracy of a few degrees, which is good enough to guarantee power on the solar panels or to establish a link to a ground station.
3. Fine pointing generally is in the order of arcminutes or better.

Determination of the spacecraft's attitude and its variability depends upon the desired accuracy and the available environment. It might be feasible to use the magnetic field or radiation for attitude determination when flying close to a central body, but at some distance the signal strength might become too weak to be usable. Widely implemented is a configuration like the one discussed below, but each mission may and will have individual solutions.

1. Large rates are often measured with an optical gyroscope (IMU = inertial measurement unit which can handle up to ~ 15°/s; Larson and Wertz 1992). Smaller rates can also be obtained by determining the time derivative of subsequent attitude measurements.
2. A simple (and hence robust) bolometer can yield the directions of the Earth (or another central body) and the Sun with an accuracy of a couple of degrees. Close to the Earth a magnetometer (Larson and Wertz 1992) may deliver similar results. Reaction times of such devices may be of the order of several seconds (CESS = coarse Earth Sun sensor. Bolometric Sun sensors with higher precision have also been developed but are less frequently used).
3. A star tracker (Larson and Wertz 1992) typically provides information with an accuracy < 1 arcmin in all three axes. Computations can be done at least once per second, thus also enabling rates to be determined accurately.

Several general principles must be taken into account before using such measurements for attitude control. First of all, the on-board computer normally calculates the desired corrections at a fixed frequency, so that the measurements have to be propagated to the next grid point. Secondly, all measurements exhibit noise which is normally suppressed by applying filters. Spurious or missing points must be flagged as unusable and some kind of damping mechanism must be included in order to avoid "bang-bang" control.

Actuators are then used to actively control the attitude. Once again, the exact choice will depend upon environment and mission goals. Commonly used are thrusters, reaction wheels, and, when a sufficiently strong magnetic field is present, magnetic torque rods (Larson and Wertz 1992). Other, more exotic solutions have also been tried, but will not be covered here. A more exhaustive enumeration of equipment used for attitude control can be found in Chap. 22 on AOCS (attitude and orbit control system) operations.

14.2 Disturbances

The attitude of each satellite is influenced by several intrinsic and external factors, the relative importance of which depends once more upon design and environment. Some of these influencing factors may be actively used to control the attitude.

14.2.1 Satellite Intrinsic

Reaction wheels provide a powerful and accurate way to control the attitude in a fine pointing mode. Disturbances which are several orders of magnitude larger than the allowed tolerances may occur in case of malfunctioning, or when one of the wheels is switched off completely. Turning off all wheels simultaneously will also lead to severe attitude deviations due to the unavoidable asymmetrical run-down.

Thrusters used for orbit correction maneuvers are designed to act through the center of gravity. However, the center of gravity (CoG) may shift during the mission due to the emptying of the fuel tanks, for example. Originally equal thrusters may develop dissimilarities with time, which also leads to attitude disturbances if maneuvers are made with several thrusters at the time.

Movable or flexible parts of the satellite may influence its attitude. A camera, antenna, or other instrument might be movable and lock into a prescribed slot at the end of a slew. A boom or another flexible appendix leads to disturbances as does the sloshing of fuel in an emptying tank.

14.2.2 External Influences

In the vicinity of the Earth, or another massive central body, the major disturbance of the attitude stems from the differential gravitational force acting on the satellite.

The gravity gradient tends to point the length axis, or more precisely the axis with the smallest moment of inertia, towards the central body.

The influence of solar radiation pressure and to a far lesser extent the pressure of the reradiated infrared (IR) radiation of a central body depends upon distance, the area-to-mass ratio, and properties of the satellite such as reflectivity. The geometry of the orbit also plays a role. The disturbance of radiation pressure will be more or less constant when flying in Sun-synchronous orbit but will be variable in other orbits.

Ram pressure or air drag can become important for an orbit close to a central body. It is a differential effect depending on asymmetric properties of the spacecraft. In general, this effect is several orders of magnitude smaller than the effects mentioned above. Satellites flying around the Earth at altitudes $\leq$ 400 km will be affected, whereby the height of the remnant atmosphere depends upon solar activity.

The smallest effect of all is caused by the interaction with the magnetic field. Satellites which use magnetic torque rods for attitude control are confined to low Earth orbits $\leq$ 500 km where the field strength is of the order of 40,000 nT. Disturbances are typically an order of magnitude smaller than the effects of air drag. However, if the satellite possesses a significant magnetic dipole of its own the torques can be as large as the disturbance caused by the gravity gradient. That occurs, for example, if the satellite has a relatively large residual magnetic moment due to currents on the solar panels (see the example below), or in "SAR" missions that use a synthetic aperture radar which sends out a strong electromagnetic signal.

A quantitative example is included here for a satellite with a mass of 600 kg and a cross section in flight direction of 1 m^2, flying in a polar orbit at 495 km altitude.[2] The residual magnetic moment was measured to reach up to 2 Am^2 in the roll- and pitch-axes leading to disturbance torques $\leq 10^{-4}$ Nm. The gravity gradient is in the order of a few 10^{-5} Nm, and the disturbances by the solar radiation ~ 5×10^{-6} Nm (see also d'Amico 2002). The aerodynamic drag at this altitude is only ~ 10^{-7} Nm during years with low solar activity. This can be up to two orders of magnitude larger at lower altitudes during solar maximum, though (Fig. 14.2).

14.3 Attitude Determination

In principle, two external reference points suffice to determine the attitude. The practical implementation is considerably more complicated, though. This will be illustrated by looking at three different methods. The first delivers a coarse attitude by finding the direction to the Sun and/or Earth with temperature sensors. The second, much more precise method is based upon the measurement of stars. These two computations are carried out on board. It is also possible to do the attitude determination on ground, either as a desired profile to be sent beforehand to the satellite, or as a refinement of the on-board calculation. This constitutes the third method.

14.3.1 Coarse Attitude

A rough idea of the satellite's attitude was all that was possible in the early days of space flight, mostly due to limitations in on-board computing power (see Wertz 1978). Star cameras were available to the military only and it was not before the last decade of the twentieth century that these and arrays of photocells became generally available. Sensors for a coarse determination (see Chap. 22 for descriptions and accuracies)

[2] Properties of the two GRACE (Gravity Recovery And Climate Experiment) follow-on satellites launched in 2018. Peculiarities of this mission are the microwave and laser links between their frontends over 220 $\pm$ 50 km. This implies that both fly with a ~ $-1°$ pitch bias with respect to the flight direction.

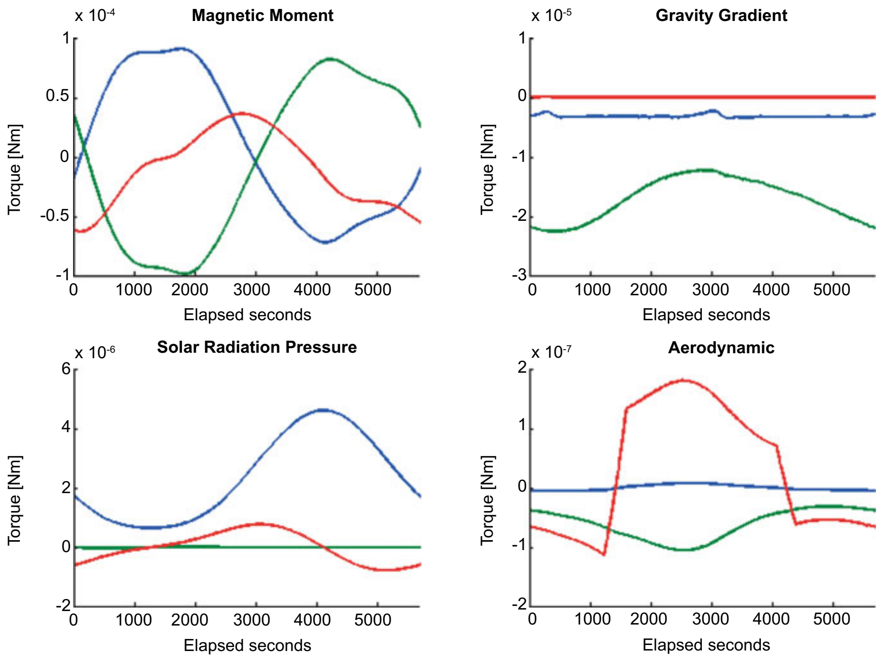

Fig. 14.2 The torques that are caused by the four relevant disturbances for low-flying satellites, are shown in all three axes over one orbit. The blue, green and red colors refer to the roll-, pitch- and yaw-axes, respectively. The properties are those of the GRACE follow-on satellites (see text and footnote[2]) and the environmental conditions represent those of January 2019. The theory is compared with the situation on-board in the next panels

continue to play an important role and are still used to guarantee a reasonable attitude for thermal and power control in case the more precise method on board fails.

One Axis, or Spin Stabilization

A stable attitude is achieved easiest for a spinning satellite with one axis pointing towards an external reference (e.g., Earth or Sun). Denote the reference vector in body coordinates as

$$r_{ref} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{14.3}$$

in this case implying that the z-axis should be pointed towards the reference. The actual direction with respect to the satellite can be computed from the temperature measurements.

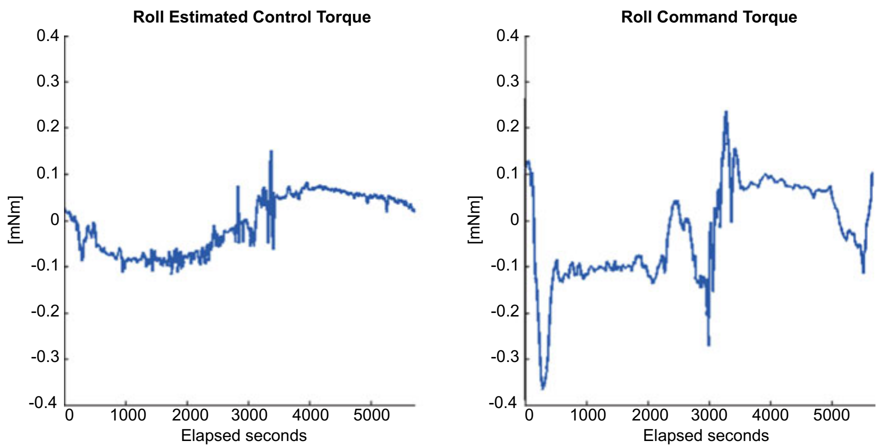

Fig. 14.3 The total disturbance in roll is the sum of the four components as shown in Fig. 14.2. The autonomous attitude control system has to counteract the disturbance torques and must also correct attitude and rate errors to stay within prescribed limits. The estimated control torque (left side panel) includes the reaction to the theoretical disturbances as well as the contribution to correct the observed attitude deviations. It is compared with the actually commanded torque which is shown in the right-hand panel. The one orbit shown uses an average of the measurements over one full day

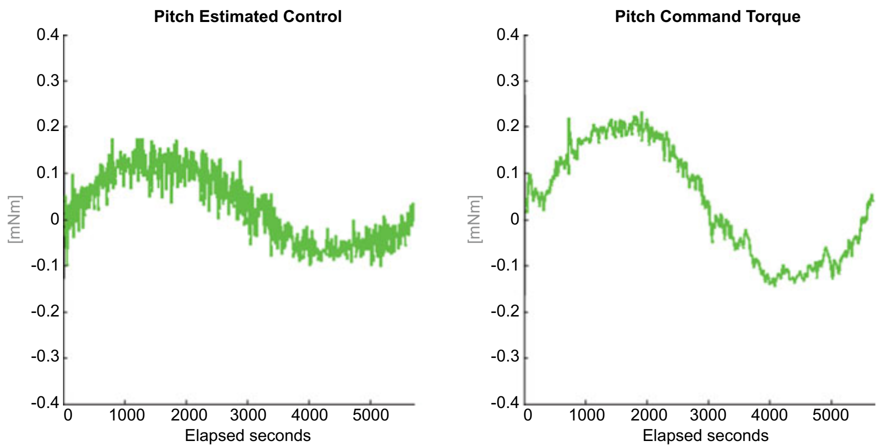

Fig. 14.4 The same comparison as in Fig. 14.3 is made for pitch. Note that the contribution of the gravity gradient to the disturbance torque is more pronounced due to the permanent $-1°$ pitch bias[2]

$$r_{meas} = \begin{pmatrix} x_{meas} \\ y_{meas} \\ z_{meas} \end{pmatrix} \tag{14.4}$$

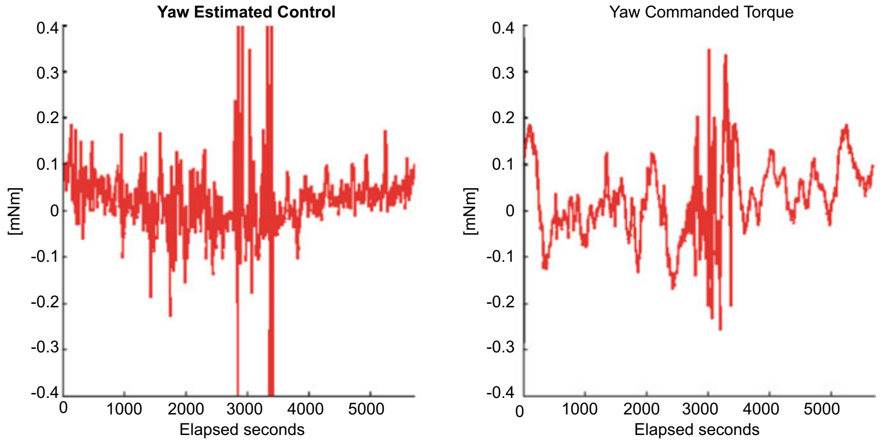

Fig. 14.5 Comparison for the yaw axis (see descriptions above). The disturbances are relatively small (see Fig. 14.2) and most of the corrective torque is applied near the nodes where magnetic authority for this axis is largest

and the angle between the measured and desired attitude follows from the cross-product.

$$\sin\varphi = \frac{|r_{ref} \times r_{meas}|}{|r_{ref}||r_{meas}|} \tag{14.5}$$

The angle φ can then be minimized by on-board control (see Sect. 14.5). The instantaneous rotation axis is given by

$$e = r_{ref} \times r_{meas} \tag{14.6}$$

No model is required for this but the expected temperature of the reference. The measurements must be converted using an a priori calibration.

Three Axes Stabilization

Slightly more difficult is stabilization in three axes. The references are taken to be the Earth and the Sun and the angles that are to be minimized are computed as in Eq. (14.5). The reference vectors require modelling because Sun and Earth directions are seldom perpendicular. A possible way is to let one satellite axis point towards the center of the Earth and then optimize the angle between the Sun direction and the plane defined by this axis and a second one. That requires a simple model of the Sun's direction over an orbit and during a year.

Note that the attitude thus determined is ambiguous, which is normally irrelevant or resolved by taking other factors into account. No three axes solution is possible if the reference directions are too close together, i.e. when Sun, satellite and Earth are aligned.

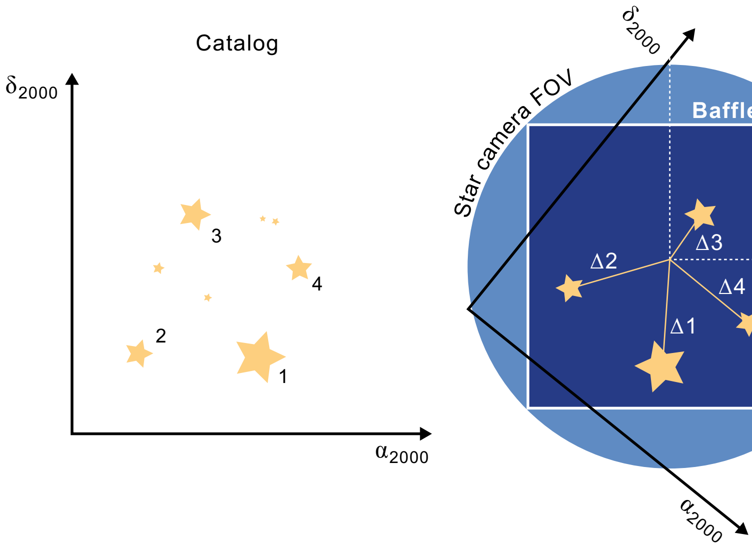

Fig. 14.6 The bore sight and orientation of the star tracker is determined in Y2000 coordinates by comparison with a catalog. Magnitude information is also used to guard against spurious detections. The high redundancy allows an estimate of the quality of the solution to be made

14.3.2 Precise Attitude

A detailed description is given in this section of the steps to get a precise determination of the attitude in all three axes from star camera measurements.

1. A CCD (charge-coupled device) image of a portion of the sky is made each duty cycle of the attitude control system. The field-of-view is typically ~ 100 deg^2 comprising some 20–100 bright stars (Fig. 14.6). The limiting magnitude and minimum signal-to-noise ratio might be autonomously adjusted or be set per ground-command. A baffle provides protection against stray light from, e.g., the Sun or the Moon.
2. Comparison with a star catalogue, which is stored on board, yields the pointing direction of the star tracker's bore sight and the orientation of the field of view (FOV). The apparent stellar positions must be corrected for the effect of aberration.[3] Very precise measurements require the inclusion of a correction for the proper motion of the stars. In principle it is enough to have two identified stars (position on the sky and magnitude) to fix the camera's looking direction

[3] Aberration is the apparent displacement of a star due to the motion of the observer; for a satellite moving at a velocity v with respect to the fixed stars the displacement $\approx v/c \sin \varphi$, where c is the velocity of light and φ denotes the angle between motion and star direction.

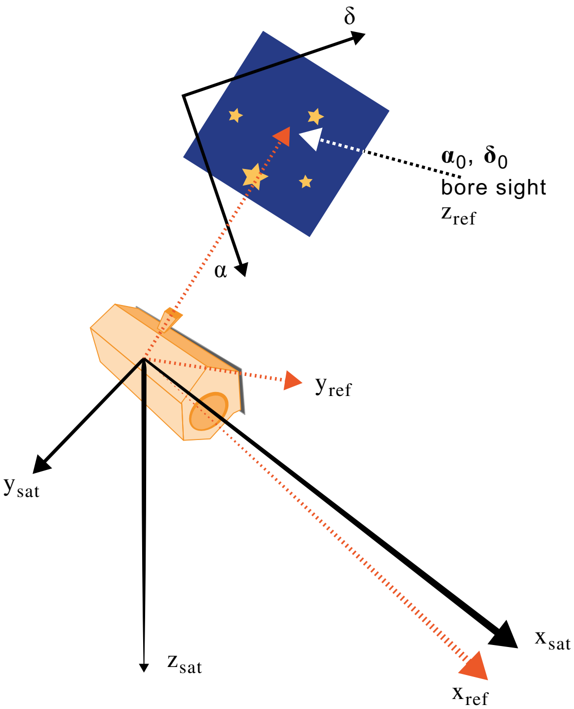

Fig. 14.7 Transformation of the measured star tracker's attitude (here indicated by the system with suffix "ref" which has for illustration purposes one of its axes aligned with the bore sight) into the body frame yields the attitude of the satellite in Y2000 coordinates (suffix "sat"). The relation with the measured star field is indicated

and orientation. The surplus information from 20 to 100 is used to get an estimate of the accuracy of the determination and will also guard against spurious identifications.[4]

3. The attitude of the star tracker is now known in the coordinate system of the catalogue, usually right ascension and declination (α, δ) at epoch 2000.0.[5]
4. The location and built-in angles of the star trackers on the satellite are known from prelaunch measurements, so that it is easy to transform the result into the attitude for the three body axes (see Fig. 14.7). This might already be sufficient if an instrument has to be pointed towards a galaxy or a star. Note that the built-in angle of the star tracker and the motion of the satellite along its orbit imply that the accuracy of the attitude determination will in general not be identical in the three body axes.

[4] Another satellite, a comet, an asteroid or meteoroid could transitorily be identified as a star. Blinding could lead to afterglow on one or more pixels with the same result.

[5] Right ascension is measured along the Earth's equator towards the east; zero point is the direction of the vernal equinox at the specified epoch. Declination is measured perpendicular to the equator and is positive towards the north.

5. If, however, the purpose is to point an instrument at a certain location on the Earth's surface, further transformations are necessary. The first step is to insert the precession and nutation from the epoch of the catalogue until the exact moment of the determination. Use of the instantaneous direction of the vernal equinox and the current value for the obliquity of the ecliptic (the angle between the equator and the plane of the ecliptic) leads to the denomination "true-of-date" (Fig. 14.8).
6. Additional information on the current position of the satellite in its orbit allows the transformation from inertial to Earth-fixed coordinates. Position information can be sent to the spacecraft in the form of an ephemeris, or as a single state vector at a given epoch which is then propagated with an on-board model. The current generation of satellites often use GPS receivers to determine this information autonomously on board (only feasible in orbits around the Earth

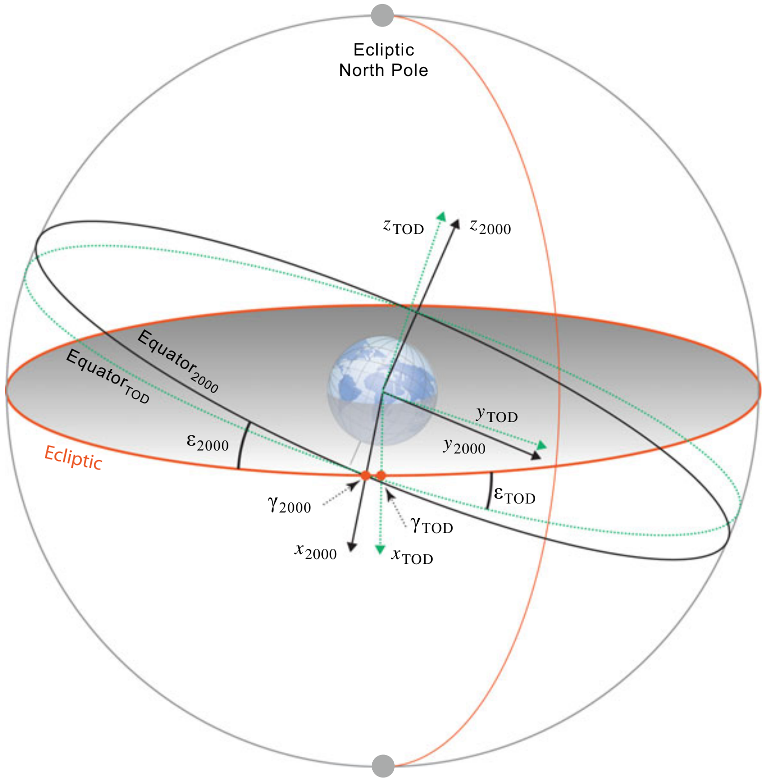

Fig. 14.8 The relation is shown between the inertial Earth-centered coordinate systems Y2000 and "true-of-date" (TOD; see text for further details). Targeting of a point on the surface requires time information and knowledge of the current position in orbit also. Thereafter the attitude can be expressed in the familiar roll, pitch and yaw angles. High precision applications might require additional corrections such as for the UT1-UTC time difference or for the oblateness of the Earth

with an altitude < 10,000 km). After this step the attitude can be expressed in the intuitively understood roll, pitch and yaw angles.

7. Further corrections in order to relate the attitude to a target on the surface might need to take the oblateness of the Earth or the random polar motion into account. The latter can be implemented by correcting the time from UTC to UT1 (Seidelmann 1992).
8. Payload operations require a final transformation into an instrument frame.

The above is by no means unique, but different missions, different payloads, different sensors, or a different central body might all lead to adaptations of this scheme. Several operational aspects always have to be considered though.

- The sensor used for attitude determination in fine pointing mode will normally have at least one backup. The mutual alignments of prime and backup(s) have to be matched to such a degree that payload operations are not disturbed if a temporary switch-over is necessary. The same holds true for redundant sensors in other modes.
- Such mounting matrices are of course measured when building the satellite but might change during the mission, especially at launch. Also, characteristics in orbit might differ from those in the laboratory. Note that in case of misalignments detected in flight one sensor has to be chosen as reference and the orientation of all others will be related to this one. A good choice would be a star tracker which serves as cold backup, i.e., it is not operationally used at the time.
- Similarly, results from different types of sensors have to match within their respective accuracies.
- Computations might use measurements from different types of sensors which requires their relative weights to be set. The direction to the center of the Earth can, for example, be derived from CESS and magnetometer data with a relative weight of 2:1 (numbers are arbitrarily chosen in this example).
- Additional information is usually required before the sensor information can be evaluated. The below would be exemplary for a satellite in low Earth orbit.

  (a) Star tracker:

  Catalogue (e.g., in Y2000 coordinates) containing all stars brighter than a given magnitude
  Current epoch
  Precession and nutation matrices as a function of time
  Mounting matrix with respect to the satellite
  Orbit position.

  (b) GPS receiver:

  Mounting matrix
  GPS SV (or corresponding PRN) usability
  GPS SV ephemerides
  GPS clock correction
  Model for orbit propagation.

(c) CESS:

Position of heads and thermistors
Calibration of temperatures
Model of terrestrial (or central body) albedo
Position of the Sun as a function of time (a simple sinus function normally suffices).

(d) Magnetometer:

Mounting matrix
Model of the terrestrial magnetic field Orbit position.

(e) IMU:

Mounting matrix

- Propagation to next grid point (model and or rates)
- Rates from time differentiation or from separate measurement
- Noise may require use of filters; obviously wrong measurements should be discarded. This requires quality control, e.g., by monitoring the point-to-point variation or signal-to-noise ratio and flagging of "invalid" points
- Gaps require special handling.

(f) Expected outages:

E.g., the star tracker used will be blinded by the Sun (provided the orbit continues as is)

(g) Expected outages:
(h) Unexpected outages:

Star tracker blinded by another satellite
Long stretch of invalid measurements e.g., due to sensor malfunctioning or wrong parameter adjustment
Software problem or any other problem causing sensor outage.

Mitigation measures are designed to minimize the effect on payload operations. A switch to the redundant sensor, either autonomously done on board, or by a priori commanding, is the easiest solution.

It might be possible to wait until the sensor delivers valid data again and, in the meantime, to bridge the gap by propagating the last valid attitude measurement. This usually requires a model of the disturbances acting on the satellite and/or the availability of independent rate measurements. However, even with sophisticated means a propagation of attitude information usually does not remain within the limits necessary for payload operations for a long time.

Cruising through the outage, i.e., disable active attitude control, is another possibility, but obviously deviations will grow even quicker than in case of propagation.

Finally, there could be an option to switch to another mode that uses different sensors which are not affected.

14.3.3 Ground-Based Attitude

Attitude control runs in real time and is only feasible as an autonomous process on board. There are a few exceptions though that will be discussed briefly here.

Reconstruction
The orbit can be determined with much higher precision on ground than on board. Long intervals of GPS measurements can be used as well as tracking data from ground stations (e.g., range, range-rate, or angle measurements). The reconstruction of the attitude is used to improve the coordinates of single images, or to facilitate the combination of images taken at different times.

Initialization
Lasers are used more and more frequently on satellites. They may serve as communication link between them, transfer data to a ground station or to a relay satellite, or they may be employed as a ranging instrument. The requirements for pointing are μrad or better, whereas the best that can be done by on-board attitude control is in the order of 100 mrad.

Precise orbit prediction allows accurate target information to be sent to the satellite in order to initialize the link. Movable mounting or mirrors can maintain the link once it is established and optimize the pointing of the laser.

Predictions become inaccurate in relatively short time so that fresh information has to be sent to the satellite several times per day.

Guidance
Sometimes specific operations, such as orbit maneuvers or image acquisitions, require computation on ground and an attitude profile is subsequently sent to the satellite as reference. This can take the form of a single target point or a guidance list over a certain interval. Time corrections may be included to account for small along-track deviations detected shortly before execution.

Such scenario might be applicable for a satellite with defunct components or with limited on-board autonomy or computing capacity.

14.4 Attitude Propagation

A prediction over a certain time interval in the future can be made as soon as the current attitude and angular rates have been determined. The simplest propagation would be a linear extension of the measurements, a so-called kinematic model. The combination of an attitude matrix A [e.g., the one from Eq. (14.1)] with the skew matrix for rotation Ω leads to the former's time derivative:

$$\frac{d\boldsymbol{A}}{dt} = \boldsymbol{\Omega} \cdot \boldsymbol{A} \tag{14.7}$$

Ω is defined in terms of the angular rates around the three principal axes of the reference coordinate system ω_x, ω_y, and ω_z (denoted by ω in the following):

$$\boldsymbol{\Omega} = \begin{pmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{pmatrix} \tag{14.8}$$

The equivalent when Euler symmetric parameters are used, see Eq. (14.2), becomes $dq/dt = ½\Omega \cdot q$ with the skew matrix defined in this instance as

$$\boldsymbol{\Omega} = \begin{pmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{pmatrix} \tag{14.9}$$

A kinematic model can, in practice, only be used in the absence of disturbance torques (e.g., interplanetary flight without active control) or in order to bridge short time intervals (normally less than one second for a propagation to the next grid point in the on-board computer).

A more elaborate and better prediction can be attained by adding a dynamic model, which takes all internal and external disturbances into account (see Sect. 14.2 for an overview). The complexity increases with the accuracy to be obtained. Each of the disturbance torques has to be modeled and projected onto the chosen reference system, for example, body-fixed coordinates. All terms then have to be included in the right-hand side of Eq. (14.7). The number of disturbance torques taken into account will depend upon the actuators used, the design of the satellite, and the accuracy that has to be reached. Euler's equation of dynamic motion is mostly written in the form of

$$\boldsymbol{I}\frac{d\boldsymbol{\omega}}{d\boldsymbol{t}} = \boldsymbol{T}_d + \boldsymbol{T}_c - \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \omega) \tag{14.10}$$

where T_d and T_c denote the overall disturbance and control torques, respectively, I is the tensor for the satellite's moments of inertia, and the vector $\omega = (\omega_x, \omega_y, \omega_z)$. T_c will normally include the torques by the wheels, and/or the thrusters and maybe also the magnetic torque rods when present, whereas T_d will comprise the torques caused by the gravity gradient, by the air drag and possibly by the solar radiation pressure and by the magnetic field also.

The vector product $I \cdot \omega$ is the total angular momentum of the satellite, often denoted as L. Note that the last term in Eq. (14.16) is a vector cross-product. It can be seen from Eq. (14.16) that the relative influence of disturbance torques diminishes if either ω or I become large. The attitude of fast rotating satellites or of large structures such as the international space station is not that sensitive to disturbances.

Finally note that Eq. (14.16) is valid for a rigid body only. Inclusion of the dynamics of flexible or irregularly moving structures is normally too complicated to be considered. A more elaborate and detailed treatment falls outside the scope of this chapter, but examples can be found in Wertz (1978), Sidi (1997) and Wiesel (1997).

14.5 Attitude Control

Attitude control can either be active or passive. Passive control can be achieved by stabilization due to gravity gradients, by the alignment of an on-board magnet with an external field, or by a high spin rate sustained over a long time which, for example, could be applicable during the cruise phase in interplanetary flight. Active control is far more common though.

Once the satellite's attitude and rates are determined and a model for all disturbing influences implemented, future trends can be predicted and thus counteracted by applying one or more of its actuators. Commonly used are

- **Thrusters**: A pressurized propellant in a central reservoir is led to several thrusters that singly, or in combination, can control one of the axes. The torque is achieved by expelling a small amount of mass and is described well by the Tsiolkovsky rocket equation in the low-mass approximation (see, e.g., Wiesel 1997). Thrusters can be used in any space environment but have the obvious disadvantage that the amount of propellant is limited.
- **Reaction wheels**: A change in the rotation velocity of one (in case of a three-wheel configuration) or several (in case of the more often used four-wheel configuration) wheels leads due to conservation of angular momentum to an opposite rate change in one of the satellite's axes. The transfer of momentum will in practice be less than 100% due to friction. Reaction wheels can also be used universally, but require a significant amount of power which must either be provided by an internal source (e.g., a battery) or by the Sun. Also, the amount of lubricant carried is limited meaning that at some time the friction will become too large for further operation.
- **Magnetic torque rods**: These are mounted parallel to each of the satellite's axes. A current sent through the rod will produce a Lorentz force in the presence of a magnetic field. This requires a power source and a field that is strong enough. The torque is perpendicular to the current and to the field, which implies that an axis aligned with the field can't be controlled at that particular position in orbit.

Attitude control is normally implemented as a fully autonomous process on the satellite (see Chap. 22, but also Sect. 14.3.3). Manual commanding is impracticable due to the short duty cycle (≤ 1 s in general), but the possibility to intervene in the control loop in case of problems is still present on most satellites. The way to correct the measured deviations from a desired attitude by autonomous on-board control is described at the end of this section. Such algorithms can be used for any deviation, but are normally limited to a certain range, that might be $\leq 30°$ for a coarse mode and $\leq 0.5°$ for a fine pointing mode (the numbers are exemplary only). A slew, i.e., a planned change of the default attitude, can in principle also be carried out by just specifying a target and let the controller do its work. This method is used on several satellites, but it has some drawbacks. The acceleration is governed by the moments of inertia and thus different for each axis. The speed of the slew depends upon the maximum torque delivered by the actuators, which might be dependent upon the position in orbit (e.g., there is no magnetic authority in yaw near the poles). Thus, the maximum angular rate, the duration and even the direction of the slew (180° could be made in either direction depending upon the accidental attitude error at the start) are not well predictable. The maximum torque must be throttled in order to prevent that slews become too fast, whereby different limits need to be applied for the three axes and perhaps also for several ranges (e.g., angles $< 0.5°$, $< 3°$, or $\leq 180°$; examples and number of ranges are again exemplary only).

Therefore, a better way is to handle a change of the default attitude manually. The commanding of a 90° yaw slew will be treated in detail below as an example, but it might also apply to the pointing of the instrument towards a target or preventing irradiation of certain parts of the satellite by turning it away from the Sun.

Consider a satellite flying around the Earth with its orbit control thrusters all located on the backside.[6] Then, in order to change the inclination of the orbit an "out-of-plane" maneuver is necessary, preferably near one of the nodes. This requires a $\pm 90°$ yaw slew with the offset reached at a very precise moment, namely just before the start of the burn. It is implemented by defining a profile connecting start and target attitude (ψ_0 and ψ_t respectively) with a user-defined rate $\dot{\psi}_0$ and acceleration $\ddot{\psi}$.

The acceleration and deceleration phases are equally long and determined by the time it takes to reach the cruising rate.

$$t_1 - t_0 = t_3 - t_2 = \frac{\dot{\psi}_0}{\ddot{\psi}} \tag{14.11}$$

The angle traveled in the acceleration phase is

$$\Delta\psi = \frac{1}{2}\ddot{\psi}(t_1 - t_0)^2 \tag{14.12}$$

so that $(\psi_t - \psi_0) - 2\Delta\psi$ is the angle remaining to be traversed in the cruise phase (Fig. 14.9). This of course takes

[6] TerraSAR-X, TanDEM-X, and PAZ are all build like that.

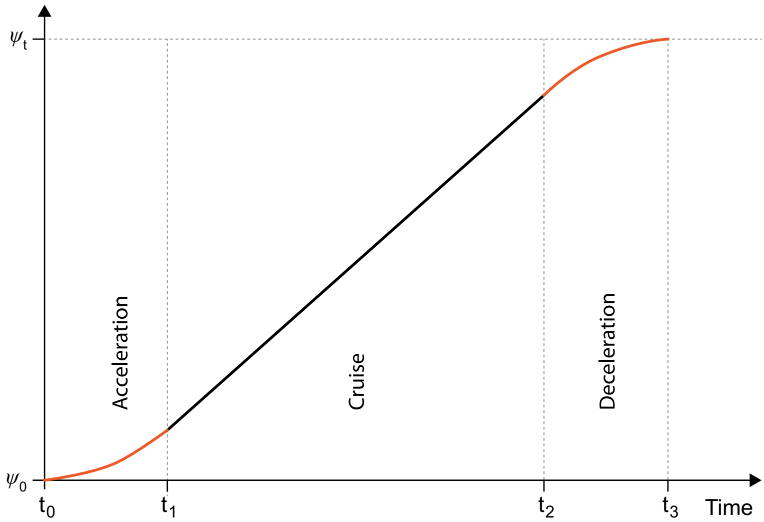

Fig. 14.9 Profile for a yaw slew from ψ_0 (default) to ψ_t (target, e.g., 90°). Acceleration starts at t_0 and ends at t_1, where the cruise phase at constant rate starts. The deceleration phase starting at t_2 lasts just as long as the acceleration and ends at t_3

$$t_2 - t_1 = \frac{(\psi_t - \psi_0) - 2\Delta\psi}{\dot{\psi}_0} \tag{14.13}$$

The maximum cruise rate and acceleration that can be chosen of course depend upon the power of the actuators, but more critically on the limits, set by the sensors, that are required to deliver attitude and rate information during the slew. Most star trackers for example will not function above a rate of ~ 0.5°/s. Finally, note that the dwelling time at a non-nominal attitude might be limited e.g. by power constraints.

A practical example can be found in Fig. 14.10, where the measured yaw angles and rates are shown for a − 90° yaw slew. The prescribed profile is followed closely, but some slight deviations occur. For $t < 700$ s and for $t > 1800$ s the yaw angle is not 0°; this is due to the fact that the TerraSAR-X satellite performs so-called yaw-steering, a roughly sinusoidal motion superimposed on the orbit with an amplitude of ~ 3° to compensate for the Earth's rotation during data takes. Yaw-steering is switched off in preparation of the slew ($\psi = 0$ at $t = 750$ s, rates > 0 from $t = 700$ to 750 s). Acceleration starts at $t = 750$ s and ends at ~ 920 s, see Eq. (14.12). The choice of parameters in this case is such that the cruise phase [Eq. (14.13)] that now begins is very short and ends at $t = 925$ s already with the onset of the deceleration phase. The yaw angle shows a small overshoot at − 90° and some rate disturbances are visible around $t = 1200$ s due to the maneuver. The return slew to $\psi = 0°$ starts at $t = \sim 1250$ s. The operation is almost symmetrical, the only difference being that the re-enabling of the yaw-steering has a delay of 1–2 min (allows different maneuver sizes to be made with the same procedure).

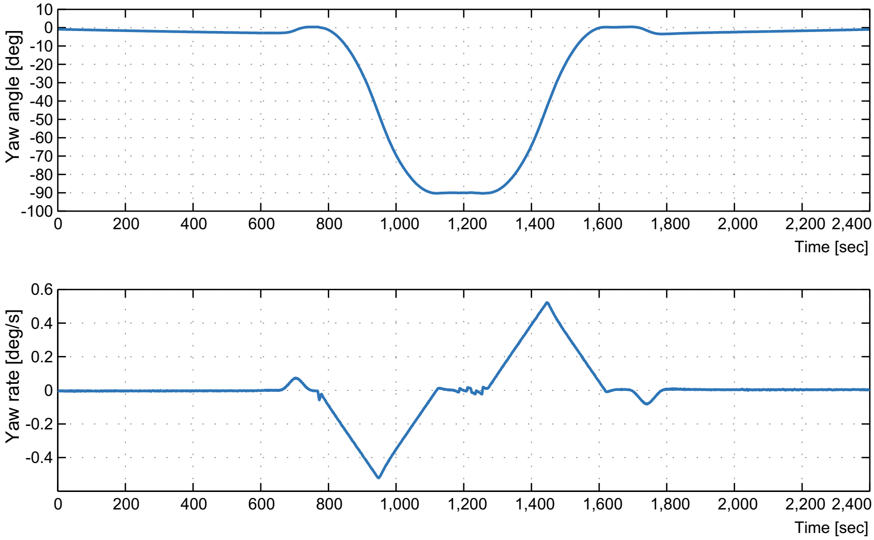

Fig. 14.10 The top panel shows the actual achieved profile for a yaw slew from ψ_0 (default, here 0°) to ψ_t (target, here $-$ 90°) preparatory to an inclination maneuver on TerraSAR-X. The corresponding rate measurements are shown in the bottom panel

The elaborate method described above is not feasible for the real time correction of small attitude errors which is rather done by autonomous control on board. This will be based upon the equations for attitude determination [Eq. (14.7)] and propagation [Eq. (14.8)]. The latter will normally include several disturbances (see Sect. 14.2). The deviation from the desired attitude, which can either be a default attitude or a profile as defined above, and the rate of change in this deviation yield the direction and magnitude of the torque to be applied to the spacecraft in order to correct these errors.

The desired torque might then be distributed over several types of actuators if that is wanted. One could, in order to save fuel for example, send as much to the magnetic torque rods as possible and only if that is not sufficient, invoke the thrusters for the remainder. The computed torques must finally be transformed into the coordinate system of the actuators; in case of a four-wheel configuration this will be a 3×4 matrix.

The calculation of the desired torques from the measurements can take several layers of complexity. The principle will be illustrated here for a one-dimensional controller. Denote the predicted deviation in roll by δ_{roll}. The time of the computation could, for example, be the next grid point of the on-board processor. The prediction itself might be complex too (see Sect. 14.2). The easiest controller to build is a proportional one that reacts directly on the predicted deviation. The desired torque T_{des} can be written as:

$$T_{des} \sim -G\delta_{roll} \tag{14.14}$$

The gain parameter G determines the strength of the reaction and is normally configurable. The result will be a rather crude bouncing between the upper- and lower- dead bands. Therefore, a better way is to include the rate of change as well as the predicted deviation. The result is a so-called PD (proportional differential) controller which takes the form

$$T_{des} \sim -G_1 \delta_{roll} - G_2 \frac{d\delta_{roll}}{dt} \tag{14.15}$$

The gain parameters G_1 and G_2 are once more configurable and can themselves be complex functions of actuator properties. A final refinement not only takes the predicted values, but also includes the time interval before. The length of the interval $(t_1 - t_0)$ can be chosen and it is also possible to, of course, include a function that gives less weight to older values. This yields a so-called PID (proportional integral differential) controller which is widely used nowadays.

$$T_{des} \sim -G_1 \delta_{roll} - G_2 \frac{d\delta_{roll}}{dt} - G_3 \int_{t_0}^{t_1} \delta_{roll} \mathrm{dt} \tag{14.16}$$

The performance of the attitude control system can and normally will be evaluated by high precision attitude determination a posteriori. The on-board restriction that requires a full new computational cycle for each grid point (≤ 1 s) vanishes and long stretches of data can be used. Such a high precision attitude reconstruction might also be required by the users of the payload, for example, for very accurate image processing or in order to combine images taken at different times.

14.6 Tasks of AOCS

Each mission normally has one or more engineers dedicated to each particular subsystem. It normally comprises such areas as instrument or payload operations, power/thermal, data and on-board computer, as well as AOCS. Tasks of AOCS are manifold and generally it is one of the busiest subsystems during LEOP (launch and early orbit phase) and also during the further mission.

- In LEOP the performance and alignments of all sensors and actuators used for attitude determination and control must be verified (including redundant ones). A reconfiguration might be required in case of defunct components (more likely to occur later in the mission).
- AOCS has a close interaction with the users of the instrument. Control parameters must be adjusted to optimize payload output; sometimes this can continue throughout the mission [prime examples are the GRACE and GRACE follow-on missions where the satellites themselves are the instrument (d'Amico 2002).
- There is also a close interaction with FD (flight dynamics). The performance of maneuvers must be evaluated and taken into account in future planning. The

direction of thrust might be different from the a priori calibration and might shift during the mission. Single thrusters can under- or over-perform and might even become defunct.

- Non-standard attitude maneuvers can be required, especially during LEOP, which need manual computation and commanding (also with FD).
- Precise attitude determination might be required for payload checkout and instrument calibration (again in co-operation with FD).
- Close interaction with P/T (power/thermal) subsystem; the temperature and power range of each sensor and actuator must be monitored, especially when switched on or off. The performance in terms of current (cost function) can become important when the batteries start degrading in the later stages (Herman and Steinhoff 2012).
- The performance of all actuators must be monitored during the entire mission (e.g., to detect increasing friction or leakage).
- Resources must be administered (fuel expenditure, book-keeping of the number of thruster cycles).
- Sensor gaps (e.g., intrusions) must be avoided by switching to a redundant sensor or by a configuration change.
- Special wishes (orbit correction maneuvers, imaging at special attitude, new targets) must be accommodated.
- A complete re-design might be required in case of serious changes in the mission's design. Examples of these are the new magnetorquer safe mode on the TerraSAR-X and TanDEM-X satellites that is not using the thrusters anymore (became necessary to safely fly in a formation with mutual distances < 150 m; Schulze et al. 2012), or the magnetic yaw-steering for GRACE, which was designed to align the satellite with the magnetic field (necessitated by the failure of several CESS thermistors).

The above also requires offline support from the flight dynamics group and for most missions also from the users of payload instruments. An in-depth overview of the AOCS subsystem is given in Chap. 22. Here only two examples will be given of the interaction of AOCS and FD with the on-board attitude control system.

14.6.1 Example 1

Interaction with the satellite is normally not per single command but with prepared procedures which not only contain the several commands required for a certain action, but also the timing, pre- and post-conditions that must be met, possibly the setting of the correct attitude (which for example in case of a 90° yaw offset will take a certain amount of time to achieve and might need to be carried out at a certain orbit position), possible changes in fault detection, isolation and recovery (FDIR) settings, timely interruption of payload operations, mode changes, switch on of additional telemetry packets, and—in case the action is done in real time—telemetry checks to be performed at the console.

The procedure shown in Fig. 14.11 can only be used time-tagged, i.e., the complete sequence is prepared beforehand, uploaded to the satellite and then stored in the on-board buffer. It will be carried out autonomously at the specified time.

In case a flight procedure can be carried out in real time, i.e., during a contact, every single command will be accompanied by several telemetry checks. Parameters are checked before sending the command and after on-board processing the effects become visible. Expected values, parameter names, and pages are embedded in the procedure. An example is shown in Fig. 14.12.

The flight procedures must be developed well in advance and be tested either on the satellite itself, on a real-time test bed, or on a software simulator.

14.6.2 Example 2

The second example deals with part of the evaluation of star tracker data as collected during the LEOP of GRACE (d'Amico 2002, but see also footnote[2] in Sect. 14.2). The measured attitude quaternions showed erratic behavior soon after the star tracker had been switched on with a one-orbit periodicity and each time lasting about 5 min (see Fig. 14.13).

The activity of the attitude thrusters and hence fuel expenditure increased considerably during and immediately after these disturbance periods. The cause could soon be identified. The star tracker started to deliver invalid quaternions as soon as the Moon was in, or close to its FOV (see Fig. 14.14).

The conclusion was twofold: In the first place it was seen that it is necessary to switch to the redundant star tracker as soon as the Moon enters the FOV of the prime and in the second place it was clear that the sensitivity for stray light extended over an area ~ 30% larger than specified (see Fig. 14.15).

This strategy was implemented and then used for the entire mission. In the late phases of the mission the FOV had to be enlarged even further, because the sensitivity of the star trackers to stray light worsened over the years. Finally, it should be noted that a switch to the redundant star tracker is only possible if that is not blinded by the Sun at that moment.

Procedure for an inclination maneuver with +90° yaw slew on TerraSAR-X (Note that the only parameter input is the start time of the maneuver and its duration)		
1	Pre-conditions	Satellite must be in the correct AOCS mode. The thruster branch must be available and active. Time-tagged commanding only Payload and other AOCS operations are not possible from 9 m 40 s before until 14 m 50 s after the start of the maneuver. The maximum burn time is 180 s.
2		The reference time, t_0, is the start time of the maneuver.
3	**t_0-00:59:52**	Additional heaters near the thrusters are switched on.
4	**t_0-00:11:00**	Additional telemetry packets containing parameters pertaining to the maneuver are switched on.
5	**t_0-00:09:50**	The autonomous reaction which triggers if deviations larger than a specified limit occur on the Sun-vector are disabled.
6	**t_0-00:09:50**	The table showing the settings of the on-board surveillance is dumped.
7	**t_0-00:09:45**	Payload operations on TerraSAR-X require continuous correction of the yaw angle in order to image the desired swash on the Earth's surface. This so-called "yaw-steering" is disabled.
8	**t_0-00:09:44**	The autonomous recovery from "attitude hold mode" into the mode for payload operations is disabled.
9	**t_0-00:09:40**	The satellite is commanded to attitude hold mode; this mode uses the same sensors and actuators as the mode for payload operations, but has larger tolerances on attitude and rate deviations. It also allows one to command any orientation with respect to the orbit frame. A margin of three minutes is included to let the spacecraft stabilize.
10	**t_0-00:09:35**	The commands in step #10, …, #14 set and activate an on-board
11	**t_0-00:09:34**	safety mechanism that would stop the thrusters in case they
12	**t_0-00:09:31**	would still be active after the specified end of the burn. Terra-
13	**t_0-00:09:28**	SAR-X uses off-modulation to control the attitude during the
14	**t_0-00:09:26**	burn, so a 10 % margin of the maneuver duration is added.
15	**t_0-00:06:40**	A yaw slew to +1.574 mrad is commanded. Note that the 90° maneuver is not made with exactly 1.571 mrad, but with 1.574 to account for the difference in performance of the thrusters. Slew and stabilization last exactly 400 s.
16	**t_0**	Start of the maneuver with the input burn time. As soon as the accumulated thruster on-time reaches the specified duration the thrusters close automatically.
17	**t_0+1.1*(burn time)**	Additional, normally superfluous, maneuver stop command; safe- guards e.g., against a zero to many in the input duration
18	**t_0+00:00:01**	The yaw-steering (see #7) is re-enabled. Commanded here, but takes effect only once the satellite is back in attitude hold mode
19		Autonomous start of the slew back to the nominal yaw attitude. Slew and stabilization last 400 s
20		Autonomous transition into attitude hold mode at the end of the slew
21	**t_0+00:13:00**	Well after the end of maneuver and slew the additional heaters switched on in step #3 are turned off again.
22	**t_0+00:13:51**	The on-board surveillance of deviations of the Sun-vector is re-enabled.
23	**t_0+00:13:52**	The table showing the settings of the on-board surveillance is dumped.
24	**t_0+00:14:00**	The autonomous transition from attitude hold mode into the mode where payload operations are performed is re-enabled. The transition will occur as soon as attitude deviations are small enough; statistics over a large number of maneuvers shows this normally occurs between four to eight minutes.
25	**t_0+00:14:54**	The additional telemetry switched on in step #4 is switched off again.
26	Post-condition	Next maneuver can′t be made within 60 minutes (unless step #21 is deleted and the timing adapted).

Fig. 14.11 Example of a flight procedure for an "out-of-plane" orbit maneuver, i.e., a burn with a 90° yaw offset

…	Previous part	see Fig. 14.11
		Verify telemetry ASD00367 PSI_FROM_QUAT = 0.0 rad Page = AOC5505A
15	**If TLM OK then**	Slew to -90° is commanded. Slew and stabilization last exactly 400 s
	Wait 400 s then	Verify telemetry ASD00367 PSI_FROM_QUAT = -1.574 rad Page = AOC5505A
…	Following part	see Fig. 14.11

Fig. 14.12 Example of telemetry verification during a station contact. Such checks may be part of a procedure as shown in Fig. 14.11, when it is completely or partly carried out in real time

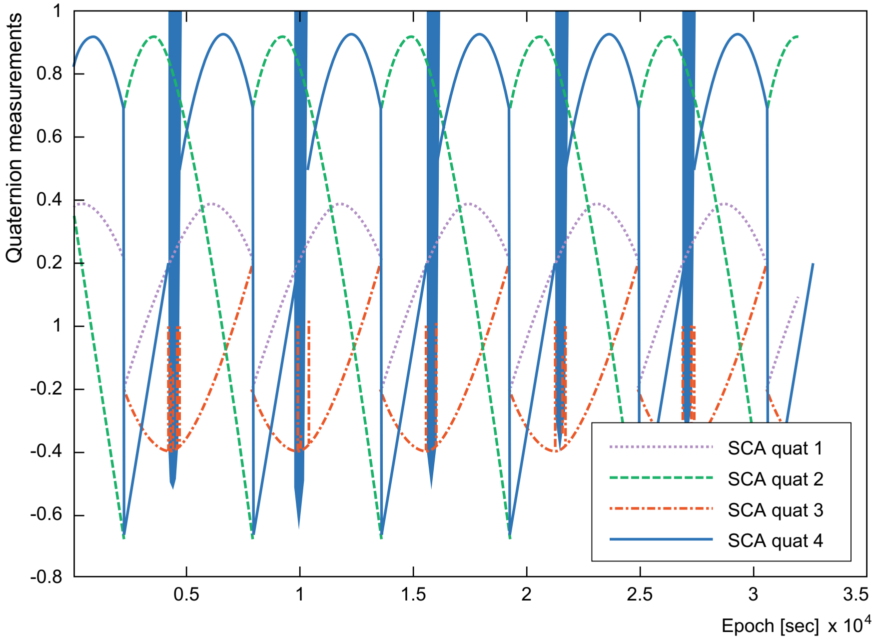

Fig. 14.13 [© (d'Amico 2002)] The measured star tracker quaternions on GRACE displayed strong disturbances during the first days of LEOP. These recurred each orbit and lasted about 5 min each. Note that the discontinuities (e.g., around epoch 0.25, 0.75 etc.) are no error but are caused by a parity switch; changing the sign of all four components does not change a quaternion

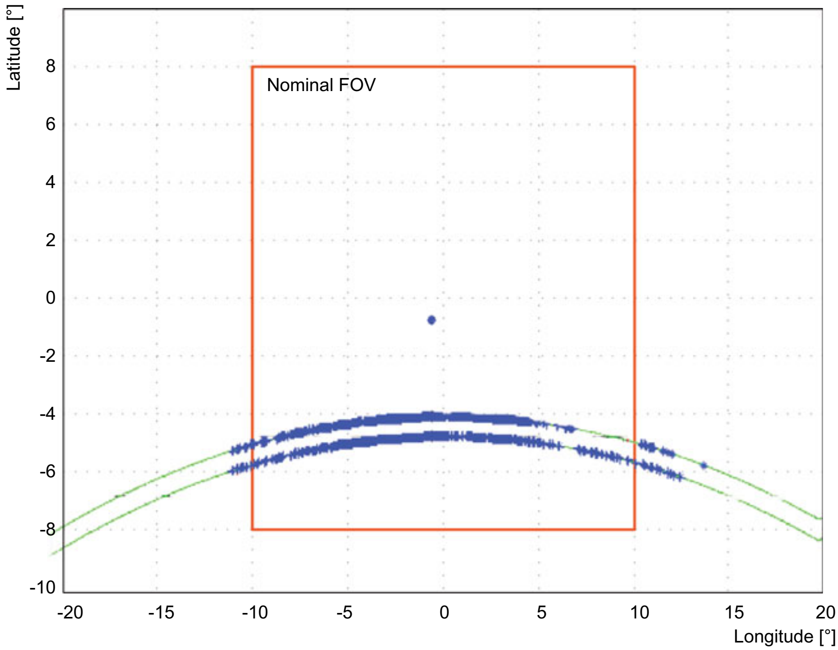

Fig. 14.14 [© (d'Amico 2002)] The path of the Moon through the star tracker's FOV is shown for two consecutive orbits (green line). The blue marks denote the points where the solution became invalid. The nominal FOV is shown by the red lines; it is rectangular due to the baffle in front of the camera

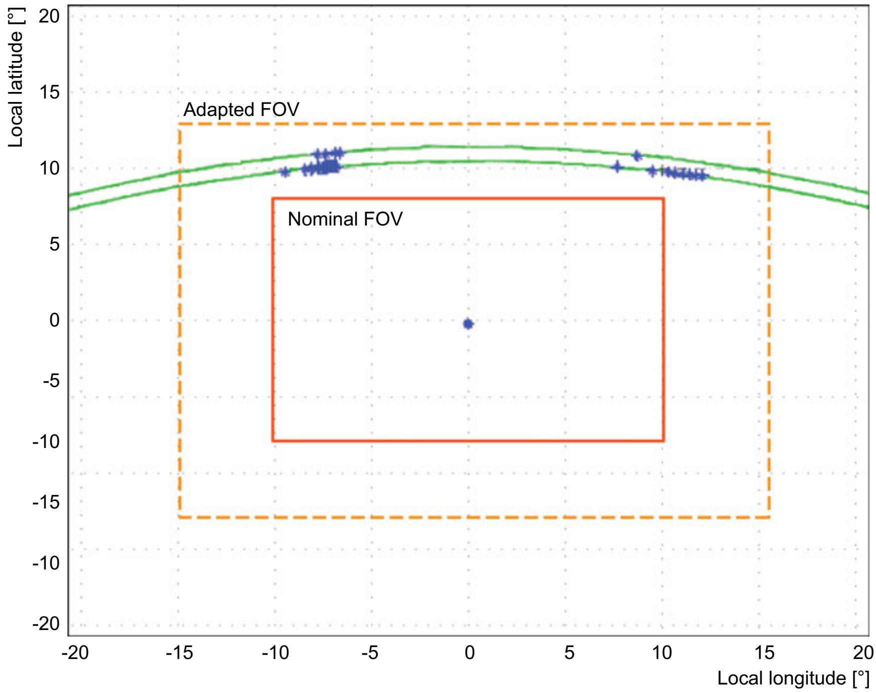

Fig. 14.15 [© (d'Amico, 2002)] Two orbits are plotted a day later than the ones shown in Fig. 14.14 Symbols are the same as in Fig. 14.14. A switch to the redundant star tracker is commanded from this moment on as soon as the Moon enters the enlarged FOV of 30° × 26°. Note that on the right-hand side, where the Moon leaves the FOV, an additional safety margin of ~ 1° is implemented

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