Rapid SIF Calculation of Inclined Cracked Steel Plates Bonded with CFRP Materials, Prestressed and Non-prestressed

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Abstract. Stress intensity factor (SIF) is a fundamental parameter in evaluating the fatigue crack propagation of steel structures. This paper focuses on the theoretical SIF calculation of steel plates with an inclined crack, subjected to tensile fatigue loading. Erdogan and Sih's equations for calculating the mixed-mode I/II SIF are extended to the bonded strengthening of steel plates using non-prestressed and prestressed CFRP. The prestress loss caused by the compression of the steel plate is considered. The theoretically calculated SIFs are proven to be in very good accordance with the numerical simulation. Due to the fact that all mixed-mode fatigue cracks develop soon to the tensile predominant mode under tensile fatigue loading, a simplified method is proposed for fatigue life estimation, in which the cracks propagate under mixed-mode in the very beginning and then perpendicular to the loading direction. The simplified method shows a fairly good capability of the fatigue life estimation.

Keywords: Stress intensity factor (SIF) · Carbon fiber reinforced polymer (CFRP) · Fatigue life · Inclined crack · Prestress · Unbonded and bonded fatigue strengthening

1 Introduction

Carbon fiber reinforced polymers (CFRP) have been used in improving the fatigue performance of cracked metallic structures. It covers the crack and bears a part of the load. Therefore, the stress intensity around the crack tip is reduced, leading to an increased fatigue life (Li et al. 2019a, 2019b). Stress intensity factor (SIF) is a fundamental parameter in fatigue. It can be used to predict fatigue crack growth rate and direction. Numerical methods, e.g. finite element method (FEM) (Wang et al. 2018) and boundary element method (BEM) (Chen et al. 2018a), have been widely employed to calculate SIF. However, the modeling and calculation process is cumbersome and time-consuming. Things would be easier if there are analytical approaches to calculate the SIF.

A. Ilki et al. (Eds.): CICE 2021, LNCE 198, pp. 2247–2257, 2022.
https://doi.org/10.1007/978-3-030-88166-5_194

1.1 Theoretical Solutions for SIF Calculation

A large number of theoretical equations exist in Handbook (Tada et al. 2000) for estimating SIF of cracked elements with different configurations and loading forms. Yu et al. (2013) and Liu et al. (2009) proposed analytical equations for SIF calculation of central cracked steel plates strengthened by CFRP. Their calculation fitted well with the experiment. Wang et al. (2018) proposed an analytical model to calculate SIF in double-edge cracked steel plates strengthened by CFRP. The analytical SIF were in good accordance with their FE simulation.

1.2 Bonded and Unbonded Strengthening

CFRP can either be bonded on steel elements using adhesive (bonded strengthening) or gripped by mechanical clamps to cover the crack (unbonded strengthening). Ghafoori et al. (2012) investigated the fatigue performance of cracked steel beams strengthened by unbonded and bonded CFRP. The study revealed that the bonded strengthened beam had higher stiffness than the unbonded beam, and SIF of the bonded strengthened beam was lower than that of the unbonded strengthened one.

1.3 Fatigue of Specimens with Mixed-Mode I/II Cracks

The type I fatigue cracks were mainly focused in the above studies. Nevertheless, there exist a large number of mixed-mode I/II cracks. Aljabar et al. (2018), Chen et al. (2018b) and Li et al. (2019a, 2019b) investigated the fatigue strengthening of central inclined cracked steel plate with CFRP bonded on one side or two sides. Different crack angles with respect to the loading direction and different initial crack lengths were considered. They found that the smaller the crack angle with respect to the loading direction, the higher the fatigue life; the shorter the initial crack length, the higher the fatigue life. Li et al. (2019a, 2019b) explained the phenomenon by means of finite element (FE) simulation: the different crack angles and initial crack lengths affected the SIF, therefore, to influence the fatigue life. Chen et al. (2018a) and El-Emam et al. (2017) focused on the prestressed strengthening of steel plates with mixed-mode I/II cracks. They found SIF can be more severely decreased by prestressed CFRP than non-prestressed CFRP.

In this paper, an analytical approach is proposed for SIF estimation of central inclined cracked steel plates bonded strengthened by CFRP, non-prestressed and prestressed. The SIFs calculated by the equations are compared to 72 FE models. A simplified theoretical approach is proposed for fatigue life evaluation. The estimated fatigue lives are compared with a total of 60 specimens from experiment and BEM simulation.

2 Mixed-Mode SIF Calculation After Strengthening

2.1 Mixed-Mode SIF Calculation of Unstrengthened Plate

Erdogan and Sih (1963) derived equations to calculate the mixed-mode I/II SIF in a plate with finite width (see Fig. 1), (Eqs. 1–2).

$$K_{\mathrm{I}} = F \cdot \sigma \cdot \sin^2 \beta \cdot \sqrt{\pi \cdot a} \tag{1}$$

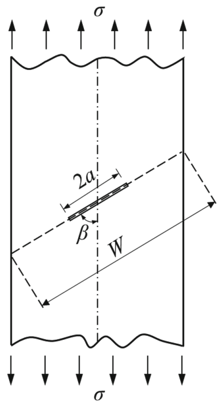

Fig. 1. Schematic view of center inclined cracked plate under far-field tensile loading.

$$K_{\mathrm{II}} = F \cdot \sigma \cdot \sin\beta \cdot \cos\beta \cdot \sqrt{\pi \cdot a} \tag{2}$$

K_{I} and K_{II} represent opening and sliding mode SIF, respectively; F is the correlation factor, as presented in (Eq. 3), employing Koiter equation (Tada et al. 2000); σ is the far-field stress; β is the angle between loading direction and inclined crack; a is the half crack length. F and a are constant under a certain geometry. It means SIF is the function of far-field stress and crack inclination angle.

$$F = \frac{1 - 0.5(2a/W) + 0.326(2a/W)^2}{\sqrt{1 - 2a/W}} \tag{3}$$

W is the steel plate width along the crack line, as illustrated in Fig. 1. For any ratio of $2a/W$, this equation has an accuracy better than 1%.

2.2 Unbonded Strengthened, Without Prestress

For steel plates strengthened by unbonded CFRP, the far-field stress can be calculated by (Eq. 4) (Hosseini et al. 2019):

$$\sigma = \rho \cdot \sigma_0 \tag{4}$$

where, σ and σ_0 are the far-field stress in the strengthened and tensile stress on unstrengthened cross-section, respectively; ρ is the stiffness ratio defined as (Eq. 5):

$$\rho = \frac{E_{\mathrm{s}} \cdot A_{\mathrm{s}}}{E_{\mathrm{s}} \cdot A_{\mathrm{s}} + E_{\mathrm{f}} \cdot A_{\mathrm{f}}} \tag{5}$$

E_{s} and E_{f} are elastic moduli of steel plate and CFRP, respectively; A_{s} is the area of uncracked section of steel plate; A_{f} is the total area of CFRP on both sides of steel plate. Here, the fatigue is sensitive to SIF range rather than SIF, and the two SIF range components can be calculated as (Eqs. 6–7):

$$\Delta K_{\mathrm{I}} = F \cdot \Delta\sigma \cdot \sin^2\beta \cdot \sqrt{\pi \cdot a} \tag{6}$$

$$\Delta K_{\mathrm{II}} = F \cdot \Delta\sigma \cdot \sin\beta \cdot \cos\beta \cdot \sqrt{\pi \cdot a} \tag{7}$$

2.3 Unbonded Strengthened, with Prestress

If the unbonded CFRP is prestressed, there are tensile stress in CFRP and compressive stress in steel plate. The tensile stress in CFRP and compressive stress in steel can be expressed as (Eqs. 8–9):

$$\sigma_{\mathrm{f,t}} = P \cdot \sigma_{\mathrm{f,\,u}} \tag{8}$$

$$\sigma_{\mathrm{s,\,c}} = \sigma_{\mathrm{f,t}} \cdot \frac{A_{\mathrm{f}}}{A_{\mathrm{s}}} \tag{9}$$

where, $\sigma_{\mathrm{f,t}}$ and $\sigma_{\mathrm{f,u}}$ are the tensile stress and tensile strength of CFRP, respectively; P is the design prestress level (after prestress loss); $\sigma_{\mathrm{s,c}}$ is the compressive stress in steel plate. Subtracting (Eq. 4) by (Eq. 9), the far-field stress, when loaded, in the strengthened cross-section of steel plate can be calculated by (Eq. 10):

$$\sigma = \rho \cdot \sigma_0 - P \cdot \sigma_{\mathrm{f,\,u}} \cdot \frac{A_{\mathrm{f}}}{A_{\mathrm{s}}} \tag{10}$$

Due to the fact that only tensile component of type-I SIF contributes to the fatigue, the two SIF ranges can be calculated by (Eqs. 11–12):

$$\Delta K_{\mathrm{I}} = \begin{cases} F \cdot \Delta\sigma \cdot \sin^2\beta \cdot \sqrt{\pi \cdot a}, R_{\mathrm{r}} > 0 \\ F \cdot \sigma_{\max} \cdot \sin^2\beta \cdot \sqrt{\pi \cdot a}, R_{\mathrm{r}} \le 0 \end{cases} \tag{11}$$

$$\Delta K_{\mathrm{II}} = F \cdot \Delta\sigma \cdot \sin\beta \cdot \cos\beta \cdot \sqrt{\pi \cdot a} \tag{12}$$

R_{r} is the stress ratio in steel plate after strengthening. Up to here, the derivation is almost the same as Hosseini et al. (2019), but expressed in a different form to be in accordance with the content hereinafter.

Due to the prestress loss, CFRP needs to be over-tensioned. The over-tension prestress level, P_{over}, is larger than the design prestress level, P, and can be determined as follows (only considering the prestress loss caused by compression of steel plate): (a) Before prestraining, CFRP and steel plate are shown in Fig. 2; (b) CFRP is then tensioned ($\Delta L_{\mathrm{f}} + \Delta L_{\mathrm{s}}$) to the length same as the steel plate ($L + \Delta L_{\mathrm{s}}$), and their two ends are gripped together. After removing the tensile force at the end of CFRP, CFRP contracts and steel plate is compressed to the length of L.

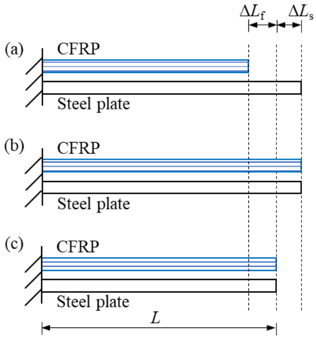

Fig. 2. Schematic view of prestressing: (a) non-prestressed state; (b) CFRP is tensioned as long as the steel plate; (c) removing the tensile force, CFRP is tensioned and steel plate is compressed.

Tensile strain in CFRP and compressive strain in steel plate can be written as (Eqs. 13–14):

$$\varepsilon_{f,\,t} = \frac{\Delta L_f}{L - \Delta L_f} \tag{13}$$

$$\varepsilon_{s,\,c} = \frac{\Delta L_s}{L + \Delta L_s} \tag{14}$$

ΔL_f and ΔL_s are the tensile deformation of CFRP and compressive deformation of steel plate, respectively. Because ΔL_f and ΔL_s are relatively small compared to L, they can be ignored in the denominator. Thus, the above two equations are simplified as (Eqs. 15–16):

$$\varepsilon_{f,\,t} = \frac{\Delta L_f}{L} \tag{15}$$

$$\varepsilon_{s,\,c} = \frac{\Delta L_s}{L} \tag{16}$$

From equilibrium, we obtain (Eq. 17):

$$\varepsilon_{f,\,t} \cdot E_f \cdot A_f = \varepsilon_{s,\,c} \cdot E_s \cdot A_s \tag{17}$$

Substituting (Eq. 15–16) into (Eq. 17), we obtain:

$$\Delta L_s = \frac{E_f \cdot A_f}{E_s \cdot A_s} \cdot \Delta L_f \tag{18}$$

When over-tensioned, the strain and stress of CFRP are:

$$\varepsilon_{f,\,over} = \frac{\Delta L_f + \Delta L_s}{L} \tag{19}$$

$$\sigma_{f,\,over} = \varepsilon_{f,\,over} \cdot E_f \tag{20}$$

and the over-tension prestress level is:

$$P_{\text{over}} = \frac{\sigma_{\text{f, over}}}{\sigma_{\text{f, u}}} \tag{21}$$

Substituting (Eq. 5, 8, 15, 18–20) into (Eq. 21), we obtain:

$$P_{\text{over}} = \frac{P}{\rho} \tag{22}$$

where, ρ is the stiffness ratio expressed in Eq. (5).

Likewise, CFRP can first be tensioned to the over-tension level (P_{over}), then the design prestress level 'P' after the prestress loss can be easily calculated by (Eq. 22). Substituting the prestress level 'P' in (Eqs. 10–12), we get the SIF ranges.

2.4 Bonded Strengthened, Without Prestress

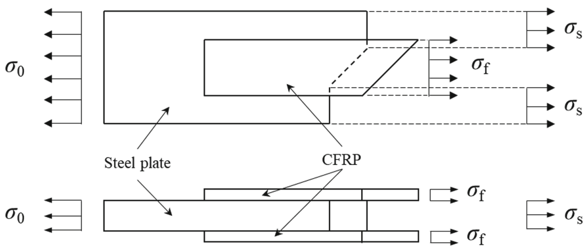

Fig. 3. Equilibrium of the free body.

For the steel plate strengthened by bonded CFRP, we need a far-field stress to get the mixed-mode SIFs. Similar to the methods of Yu et al. (2013) and Liu et al. (2009), the free body in Fig. 3 should be in equilibrium. Thus, we obtain:

$$F_0 = F_{\text{s}} + F_{\text{f}} \tag{23}$$

$$F_0 = \sigma_0 \cdot A_{\text{s}} \tag{24}$$

$$F_{\text{s}} = \sigma_{\text{s}} \cdot (A_{\text{s}} - A_{\text{crack}}) \tag{25}$$

$$A_{\text{crack}} = 2a_{\text{p}} \cdot t_{\text{s}} \tag{26}$$

$$F_{\text{f}} = \sigma_{\text{f}} \cdot A_{\text{f}} \tag{27}$$

$$\sigma_{\text{s}} = E_{\text{s}} \cdot \varepsilon_{\text{s}} \tag{28}$$

$$\sigma_f = E_f \cdot \varepsilon_f \tag{29}$$

F_0 is the tensile force; F_s and F_f are the forces steel and CFRP bear, respectively, on the cracked section. σ_0 is the tensile stress on the unstrengthened section of steel plate; σ_s is the average tensile stress on cracked section of steel plate; σ_f is the tensile stress on the cracked section of CFRP. A_s is the uncracked area of the steel plate while A_{crack} is the projection area of the cracked section of the steel plate. a_p is the projection length of half crack. A_f is the total area of CFRP on two sides of the steel plate. ε_s and ε_f are the strains of steel and CFRP, respectively, on the cracked section. To be noted here, even though adhesive bonds CFRP and steel plate together, the contribution of adhesive in this model can be ignored, due to its small thickness and low elastic modulus. Assuming that CFRP and steel share the same strain in the cracked section, we have:

$$\varepsilon_s = \varepsilon_f \tag{30}$$

Substituting (Eqs. 24–30) into (Eq. 23), we obtain:

$$\varepsilon_s = \varepsilon_f = \frac{\sigma_0 \cdot A_s}{E_s \cdot (A_s - A_{crack}) + E_f \cdot A_f} \tag{31}$$

Again, substituting (Eq. 31) into (Eq. 28), we have:

$$\sigma_s = \frac{E_s \cdot A_s}{E_s \cdot (A_s - A_{crack}) + E_f \cdot A_f} \cdot \sigma_0 \tag{32}$$

The far-field stress with respect to the cracked section can be expressed as:

$$\sigma = \rho_{crack} \cdot \sigma_0 \tag{33}$$

where ρ_{crack} is the cracked stiffness ratio, which is expressed as (Eq. 34):

$$\rho_{crack} = \frac{E_s \cdot (A_s - A_{crack})}{E_s \cdot (A_s - A_{crack}) + E_f \cdot A_f} \tag{34}$$

Thus, the conclusion could be drawn: when the unbonded strengthening is used, the tensile stress is distributed on steel plate and CFRP by the stiffness ratio, ρ; when the bonded strengthening is used, the cracked stiffness ratio, ρ_{crack}, should be used to assign the tensile stress.

2.5 Bonded Strengthened, with Prestress

If the bonded CFRP is prestressed, we need to determine the prestress level in CFRP. Due to the existence of adhesive, the strain of CFRP has a gradient along the length direction, and so does the steel plate. This is different from the unbonded case, in which the strain of CFRP and steel plate keeps constant.

The authors conducted several numerical simulations with different prestress levels (not loaded) using ABAQUS. Figure 4 presents a typical stress distribution (normal stresses of CFRP and steel, and shear stress of adhesive) in the system. At the end of

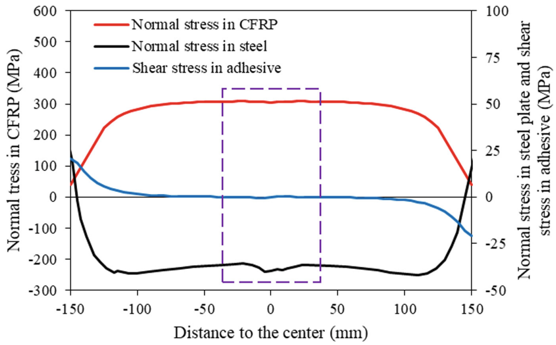

Fig. 4. Typical stress distribution in a bonded strengthening system with prestress (not loaded).

CFRP and the corresponding position in adhesive and steel plate, these three stresses vary obviously. Away from this end, the shear stress in adhesive is almost zero, and the tensile stress in CFRP and compressive stress in steel plate are almost constant, as depicted in dashed rectangular in Fig. 4.

Thus, away from the CFRP end, the bonded strengthening with prestress can be simplified as the equilibrium of steel plate and prestressed CFRP (CFRP is tensioned and steel plate is compressed). This is identical to the unbonded prestressed case. Therefore, (Eq. 22) can still be used to calculate the prestress level. When loaded, the far-field stress with respect to the cracked section is:

$$\sigma = \rho_{\mathrm{crack}} \cdot \sigma_0 - P \cdot \sigma_{\mathrm{f,\,u}} \cdot \frac{A_{\mathrm{f}}}{A_{\mathrm{s}}} \tag{35}$$

ρ_{crack} is the cracked stiffness ratio, as defined by (Eq. 34); the rest are the same as those in (Eq. 10). The two components of mixed-mode SIF range can be calculated by (Eqs. 11–12).

3 SIF Comparison with FE

A total of 72 strengthened specimens (with and without prestress) with inclined cracks were simulated. Two kinds of CFRP, three inclination angles (30°, 45° and 60°), four prestress levels (0, 10%, 20% and 30%), and three damage levels (ratio of crack projection length on plate width, 10%, 20% and 30%) were investigated ($2 \times 3 \times 4 \times 3 = 72$). The specimen geometry and material properties can be found in (Li et al. 2019a, 2019b). The mixed-mode SIFs were compared with the calculation based on the equations in Sect. 2.4 and Sect. 2.5.

The SIF comparison between simulation and analytical calculation is presented in Fig. 5, which shows a very good fitting. Thus, it is believed that the equations for mixed-mode SIF calculation are effective and precise enough.

4 Fatigue Life Estimation

The numerical study of Li et al. (2019a, 2019b) revealed that all mixed-mode fatigue cracks develop soon to the tensile predominant mode under tensile fatigue loading.

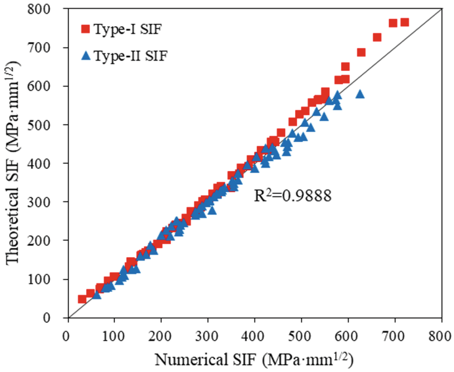

Fig. 5. Theoretical SIF vs. numerical SIF.

Therefore, a simplified theoretical method is proposed by manually letting the crack propagate perpendicular to the loading direction. The initial SIF is calculated by substituting the real crack inclination angle in (Eqs. 6–7) or (Eq. 11–12); later on, 90° is used as the crack inclination angle.

Aljabar et al. (2018), Chen et al. (2018b) and Li et al. (2019a, 2019b) conducted fatigue test on steel plates with central inclined cracks strengthened by CFRP. Chen et al. (2018a) used BEM to estimate the fatigue lives of steel plates with inclined cracks strengthened by non-prestressed and prestressed CFRP; no corresponding test was conducted. A total of 60 specimens are collected, among which 21 are unstrengthened, 20 are strengthened on two sides without prestress and 19 are strengthened on two sides with prestress. The authors of the current study used the simplified method to estimate the fatigue lives of these 60 specimens. Fatigue crack propagation models and the corresponding parameters are chosen based on the original studies, as listed in Table 1. ΔK_{th} is 148.6 MPa·mm$^{1/2}$, as described previously. Note here, C and m are not provided in (Aljabar et al. 2018), therefore, the authors of the current study fitted these two parameters using the a-N curves of their unstrengthened specimens. The two components of the initial SIF, i.e. type-I and -II SIF, can be easily calculated from the equations in Sect. 2.4 and Sect. 2.5. The effective SIF can be calculated using the equation proposed by Tanaka (1974):

$$\Delta K_{\mathrm{eff}} = \sqrt[4]{\Delta K_{\mathrm{I}}^4 + 8\Delta K_{\mathrm{II}}^4} \tag{36}$$

Figure 6 gives a comparison of theoretical and experimental fatigue lives and it shows a very good fitting. All points are near the solid line (point on this line means the theoretical value is exactly the same as the experiment). 82% of the errors between calculation and the test result are within ± 30%. The conclusion could be drawn: this simplified theoretical method is suitable for SIF evaluation and fatigue life estimation.

Table 1. Crack propagation model and corresponding parameters

	Fatigue crack propagation model	C	m
Chen et al. (2018b)	$da/dN = C(\Delta K_{eff} - \Delta K_{th})^m$	3.98×10^{-13}	2.88
Li et al. (2019a, 2019b)		3.98×10^{-13}	2.88
Aljabar et al. (2018)		1.07×10^{-12}	2.86
Chen et al. (2018a)	$da/dN = C(\Delta K_{eff})^m$	6.77×10^{-13}	2.88

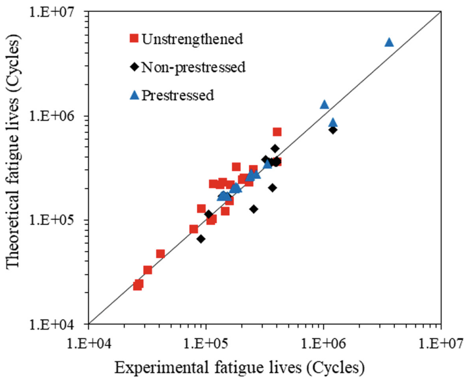

Fig. 6. Comparison between theoretical and experimental fatigue lives.

5 Conclusions

In this study, theoretical investigation was carried out to evaluate the stress intensity factor of steel plates with central inclined cracks strengthened by non-prestressed and prestressed CFRP. A group of equations were proposed to calculate the SIF, and a simplified theoretical method was used to evaluate the fatigue life of steel plate with a central inclined crack. The calculated SIFs were compared with the finite element simulation. The fatigue lives estimated by the simplified theoretical method were compared with the fatigue test and boundary element simulation. The following conclusions could be drawn:

1. The equations proposed are suitable for mixed-mode I/II SIF calculation. The calculated SIFs match very well with finite element simulation.
2. In the simplified theoretical method for fatigue life estimation, the fatigue cracks propagate perpendicular to the loading direction. The initial stress intensity factor is calculated using the real crack inclination angle; later on, the stress intensity factors are calculated using the angle of 90°. The fatigue lives estimated by this method fit well with the fatigue test and boundary element simulation.

Acknowledgement. This study is financially sponsored by National Natural Science Foundation of China (NSFC Grant No. 51978509).

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