Chapter 4
The Modern Era of Colour Theory

> *—Undulations are excited in this ether whenever a body becomes luminous.*
>
> Thomas Young, On the theory of light and colours

4.1 Leonardo da Vinci

Leonardo di ser Piero da Vinci, or simply Leonardo da Vinci (1452–1519) was an Italian polymath who is considered among the founders of the Renaissance. His extensive diversity of talents transformed his name into a brand name to characterise those extremely rare individuals in human history to poses an exquisite genius. He is considered both an artist and a thinker, with a pervasive interest in conclusions through observation and experiential cognition. As expected, his interest in light and vision was primarily due to his artistic nature that was also coupled with his fact-seeking impulse through accurate observation.

To Leonardo's mind, the human brain collects visual and other stimuli, processes them into sensory perception and subsequently transmits responses to the muscles through the nerves (Isaacson, 2017). Leonardo's attention to observation details in all his aspects of work seems to have been based on an attempt to highlight and illustrate the superiority of vision among the senses. As Isaacson (2017) quotes Leonardo saying, *the eye, which is said to be the window to the soul, is the primary means by which the sensory receptor of the brain can fully and magnificently contemplate the infinite works of nature*. He was in awe of the function of vision, as evident in his poetic text in the *Optics* of his notebooks (MacCurdy, 1955) (c.a.345 v.b),

G. Pavlidis, *A Brief History of Colour Theory*,
https://doi.org/10.1007/978-3-030-87771-2_4

> Who would believe that so small a space could contain the images of all the universe? O mighty process! What talent can avail to penetrate a nature such as these? What tongue will it be that can unfold so great a wonder? Verily, none! This it is that guides the human discourse to the considering of divine things. Here the figures, here the colours, here all the images of every part of the universe are contracted to a point. O what point is so marvellous! O wonderful, O stupendous Necessity thou by thy law constrainest all effects to issue from their causes in the briefest possible way! These are the miracles, ... forms already lost, mingled together in so small a space, it can recreate and reconstitute by its dilation.

His observations on the formation of shadows are defining aspects of his paintings (as are of his concepts). Due to these observations, his painting practices, mathematical aspects and his theory of optics, he admitted that there are no precise borders of the objects. Through experimentation[1] and anatomy, he maintained that the image of a scene is formed as a whole in the eye, and this is way no clear borders can be imaged.

By having a perfect understanding of the principle of the camera obscura and the inverted image, he was fascinated how the eyes do not suffer from this phenomenon. His way out of this impasse was to propose a double-crossing of the rays in the eye so that eventually the formed image turns upright before moving on to the brain (MacCurdy, 1955; Isaacson, 2017) as shown in the diagram in Fig. 4.1 based on Leonardo's original drawing. According to his theory, the eyes are spherical so that light from multiple angles can be captured and form images of a wide scene (MacCurdy, 1955) (D I r.).

> Nature has made the surface of the pupil situated in the eye convex in form so that the surrounding objects may imprint their images at greater angles than could happen if the eye were flat.

In his notes on the *Optics*, and his notebooks (MacCurdy, 1955), he seems well-aware of the classical laws of light propagation, such as refraction, as revealed in the following text.

> And this process of contraction proceeds from the fact that the rays of the images approach the perpendicular when they pass from the thin to the dense, and that the albugineous humour is

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[1] For example, based on Alhazen's work, he experimented among others with a needle, which when brought close to the eye starts to become blurry and translucent, and finally disappears just in front of the eye.

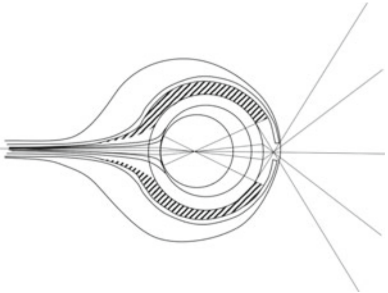

Fig. 4.1 Leonardo Da Vinci's concept of double ray crossing in the eye

> here much thinner and more subtle than the space enclosed by the surface of the vitreous sphere.

In addition, his observations led him to differentiate central to peripheral vision,

> The eye has one central line and all the things that come to the eye along this line are seen distinctly. Round about this line are an infinite number of other lines that adhere to this centre line and these have so much less strength in proportion as they are more remote from the central line.

In Leonardo's notebooks, there is one about the *Atmosphere*, in which he stated something quite remarkable for his time. Although to that date, a ray-based, mostly particle-like theory of light was widely accepted, he somehow imagined a *wave theory of light* (MacCurdy, 1955; Keele, 1955), in saying that

> Just as the stone thrown into the water becomes the centre and cause of various circles, and the sound made in the air spreads itself out in circles, so every body placed within the luminous air spreads itself out in circles and fills the surrounding parts with an infinite number of images of itself, and appears all in all and all in each smallest part.

At the same time, he made numerous drawings and anatomical designs (like the one in Fig. 4.1), in which the ray-based light propagation was also adopted, making it seem like he already imagined a *dual nature for light*, which was established some 5 centuries later.

Regarding human perception, Leonardo's concepts were clearly expressed in that there is a hierarchy in human physiology or a process by which the mind is the decisive component, which is fed with information by the senses (MacCurdy, 1955).

> The soul apparently resides in the seat of the judgment, and the judgment apparently resides in the place where all the senses meet, which is called the common sense; and it is not all of it in the whole body as many have believed, but it is all in this part; for if it were all in the whole, and all in every part, it would not have been necessary for the instruments of the senses to come together in concourse to one particular spot; rather would it have sufficed for the eye to register its function of perception on its surface, and not to transmit the images of the things seen to the sense by way of the optic nerves; because the soul—for the reason already given—would comprehend them upon the surface of the eye. So therefore the articulation of the bones obeys the nerve, and the nerve the muscle, and the muscle the tendon, and the tendon the common sense, and the common sense is the seat of the soul, and the memory is its monitor, and its faculty of receiving impressions serves as its standard of reference.

Nevertheless, he was somehow reluctant to accept the existence of colours (but only of light) and to propose a clear colour theory. He seems to have accepted a complicated view, in which there is only white light and shadows. In his notes about *Painting*, he stated that the medium between the eyes and the objects transforms the view of the objects to appear in a particular colour. Like all translucent or transparent media are to be treated as coloured media which impose the apparent colouring. But then, he also stated that the reason for the formation of a rainbow is the light rays travelling through the water drops in the air; yet in another passage, he definitely concludes that it is not the sunlight that is responsible for the rainbow. He also definitively concludes that the eyes have no share in the creation of colours. In another passage regarding light and colour, he suggested that "the quality of colours becomes known by means of light". In his notes about *Colour* (MacCurdy, 1955) (XXX.Colour), Leonardo focuses on colour differences, particularly colour opponency, apparently through his experience in colour adaptation (particular colour against a completely different background). His ideas about colour influenced Göethe and exhibit a slight resemblance to the opponent colour theory discovered later.

4.2 Johannes Kepler

Johannes Kepler (1571–1630) was a German astronomer and mathematician from the Free Imperial City of Weil der Stadt, Holy Roman Empire (later Germany). Kepler is a central figure in science and particularly for the scientific revolution of the 17th century that he helped set in motion. He is known for his work in astronomy and especially his laws of planetary motion, as laid out in his 1609 book *Astronomia Nova ΑΙΤΙΟΛΟΓΗΤΟΣ seu physica coelestis, tradita commentariis de motibus* *stellæ Martis ex observationibus G.V. Tychonis Brahe* (or simply *New Astronomy*), where he proposed the elliptical planetary orbits in the heliocentric model and laws regarding the speed and axis of rotation of planetary motion. Apart from his seminal work in astronomy, Kepler was also deeply interested in the study of optics and human vision. He is considered to be the first to accept the inverted-reversed image projection onto the retina of the eye. The correction of the image is done, according to him, in the brain, although he did not seem to be so concerned about how, since his primary interest was in the optics.

Of particular interest is his largely important work published in 1604 under the long title *Ad Vitellionem Paralipomena, quibus Astronomiæ Pars Optica Traditur* (*Paralipomena to Witelo whereby the Optical Part of Astronomy is Treated* or *Paralipomena to Witelo and the Optical Part of Astronomy*) (Kepler, 1604). This treatise is considered to be a foundation of optics in their modern form. He was particularly interested in projective geometry, which he tried to found upon conic sections, while he described how projective space changes would change shapes within this family of conic sections (ellipse becomes a parabola, merging of the two foci of ellipse results in the formation of a circle, merging of the foci of a hyperbola transforms it into a pair of straight lines, a straight line extended to infinity will meet itself at a single point at infinity). A translation of the original Latin text in English can be found in Kepler & Donahue (2000).

Kepler, analysed the nature of light and its propagation, along with its interaction with bodies, but was also concerned about the visual sense mechanism. He expressed his theory on the nature of light in the form of a series of not less than 38 Propositions. In the opening Propositions, Kepler theorises that *light has a specific origin and travels instantly on spherical surfaces that extend radially from the source to infinity, without losing strength; this motion can be envisioned in directions of straight lines, defined by dense rays*. In his own words,

> Luci effluxus vel e iaculatio competit à sua origine in locum distantem...Punctum quodlibet infinitis numero lineis effluit. Scilicet vt orbem omnem circumcirca illustret, quod sieri debere diximus Sphæricum autem infinitas habet lineas...Lux seipsa in infinitum progredi apta est...Lineæ harum eiaculationum rectæ sunt, dicantur radij...Lucis motus non est in tempore, sed in moment.

Kepler explained the instantaneous propagation of light in terms of the Aristotelean law of motion[2] and his hypothesis that *light has no mass*, by which he concluded that time is not involved in the motion of light. Interestingly, in Proposition VIII, he suggests that the light rays should be also be considered as indications of the motion, but what is moving through space is a surface.

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[2] Kepler expressed it as in terms of time, supporting that time is proportional to the ratio between the moving mass to the medium in which motion takes place, or the ratio of the moving power to the mass.

> *Lucis radius nihil est de luce ipsa egrediente.* Nam radius per IV. nihil aliud est nisi ipse motus lucis. Sané vt & in motu physico, motus ipsius est recta linea, physicum vero mobile, est corpus: ita in luce motus ipse est recta itidem linea, mobile veró, est superficies quædam. Et vt illic recta motus non pertinet ad corpus, sic hic recta motus non pertinet ad superficiem.

In Proposition XV, Kepler mentioned colour for the first time, where he expressed an interesting hypothesis that *colour is light in potentiality, enclosed in a transparent material*. In a way, colour is a quality of matter and the colour on objects includes the potential to transmit light if excited by the light from the Sun. *Differences in light intensity and in material density and transparency are the qualities that define the various colours*.

> Color est lux in potentia, lux sepulta in pellucidi materia: si iam extra visionem consideretur; & diversi gradus in dispositione materiæ, causa raritatis & densitatis, seu pellucidi & tenebrarum; diuersi item gradus luculæ, quæ materiæ est concreta, efficiunt discrimina colorum.

Kepler in the subsequent Propositions reaffirmed the laws of reflection and refraction and summarised all the interactions of light with matter in Proposition XXVII. Light in the same medium is partly reflected (Prop. XVIII), partly refracted (Prop. XX), partly adheres to the colour of the medium (Prop. XVI) and attenuated by the colour of the medium (Prop. X).

> Lux in codem medio partim repercutitur, partim infringitur, partim & in colore medii adhaerescit, seu à colore reuibratur, atque ita in tenuiores luces diuiditur.

To Kepler the law of light mixing is additive, and the resulting colour would depend on the proportion of the mixed lights' densities or strengths. Furthermore, he attributed *heat* to be *a property of light*, or a carrier of heat. And as heat in objects is generated in time, longer exposures to light ultimately destroys and burns the objects and bleaches the colours (as colours are properties of the matter). Kepler in the Appendix to his chapter on the nature of light included a detailed critical analysis of Aristotle's theory of light and colour.

Kepler's theory of image formation includes Definitions and Propositions of deep insight, following the tradition of Witelo. To Kepler, an Image is practically nothing in itself, as it is the vision of an object subject to the errors in the sense of vision. An image is similar to an imagination, whereas an object is actually an objective entity. Nevertheless, an image carries important information that includes colour, position, distance and quantity.

> Primum ex Catoptrice, in quam ingredimur, definitionem Imaginis desumptam in vestibulo colloco. Dicunt enim imaginem optici, cùm res ipsa quidem cum suis coloribus & figuræ partibus cernitur, sed situ alieno, alicubi & alienis indura quantitatibus & partium figuræ proportione inepta. Breuiter, imago est visio rei alicuius, cum errore facultatum ad visum concurrentium coniuncta. Imago igitur perse penè nihil est, imaginatio potiùs dicenda. Res est cõposita ex specie coloris vel lucis reali, & quantitatibus intentionalibus...Etenim in imagine sunt hæc quatuor potissimùm, Color, situs seu plaga, distantia, quantitas, subsidiis comprehendantur, explicandum: quamuis eadem Vitellio libro 3. & 4. explicauerit.

Kepler made clear that seeing is receiving, which requires contact (“visio sit passio & passio siat per contactum”) between the eye and the image of the light rays. The eye is capable of receiving light and colours because it consists of transparent humours (“ocullus constat humoribus pellucidis: hoc ita que respectu lucis & colorum capax est”). Kepler formalised the stereo vision theory by supporting that *two eyes are needed to provide distance estimates through triangulation*. In addition, the eye has a sense of the viewing angles in which it operates (“ocullum sensum habere angulorum apud se constitutorum”).

In a subsequent chapter (*Chapter V. De modo Visionis*), Kepler got involved in defining the function of the human vision, as there was no clear description of the subject at that period. Kepler never performed or attended any practice of dissection and thus he cites the work of eminent physicians and anatomists to base his theory of vision. Regarding the anatomy of the human eye, he cites Felix Platter (1536–1614), Johannes Jessenius a Jessen (1566–1621) and Hieronymus Fabricius ab Aquapendente (1537–1619). Kepler comments on the origins of the word *oculus* from the greek ὀπή → ὄπτεσϑαι → ὄψις → ὄμμα, ὄφϑαλμός, which conveys the meaning of an opening from the opaque organism to the air

> ...quòd hæ sint rimæ seu aperta foramina, è tenebroso capite in clarum aërem pertingentia;

In addition, he makes clear that there are two eyes in animals not as a means of protective redundancy, but as a distance measuring mechanism. The eyes are in a high location so that more distant objects may be observed since the experienced world is on a sphere. Kepler unfolds a line of reasoning on why the eyes needed to be spherical, or in their location close to the brain, why they are aligned with the horizon, why they are individually protected and why they developed rapid motion capabilities to cover large angles of view. He reminds that it was already known that the brain extends, by means of nerves, to the inner surface of the eyes forming a complex layer (“...ipsa verò cerebri substantia neruum opticum...hæcque ispa

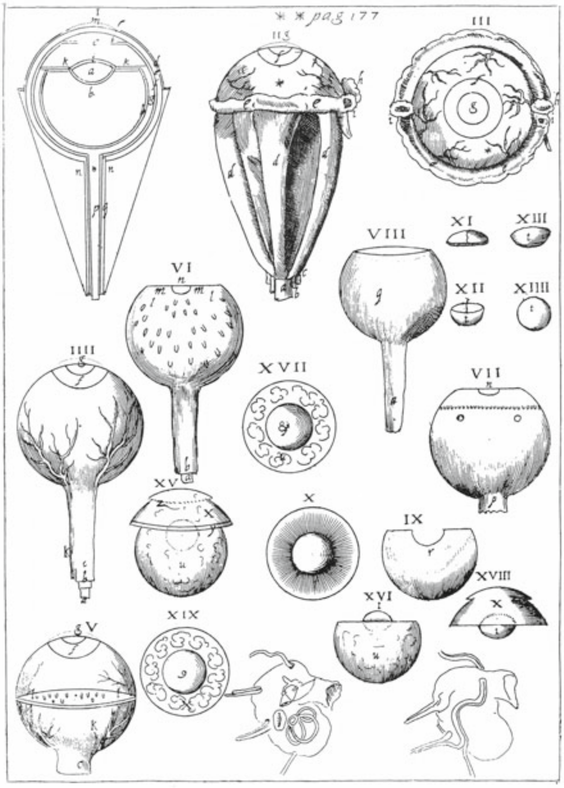

Fig. 4.2 Anatomical drawings of the human eye from Kepler's *Paralipomena to Witelo and the Optical Part of Astronomy*, attributed to Felix Platter

non simplex."). Kepler provided a detailed description of the parts of the eye and included a figure of multiple anatomical drawings of the human eye, borrowed from Felix Platter (Fig. 4.2).

Subsequently, in Sect. 2 of Chapter V, Kepler focuses on the means of vision ("modus visionis"), in which he unfolds the complete theory of vision, as he emphatically states, for the first time to his knowledge. Vision occurs when the image of the whole hemisphere hits the retina of the eye.

> Visionem fieri dico, cum totius hemisphærii mundani, quod est ante oculum, & amplius paulò, idolum statuitur ad album sub rufum retinæ cauæ superficiei parietem.

He proceeded to confront many of Witelo's propositions regarding the function of refraction of light in the eyes. Through logical reasoning, he supported that the optical information retrieved by the eyes is altered on its course to the brain, and he theorised about the usage of the *optic chiasm*, the joining of the left and right eye

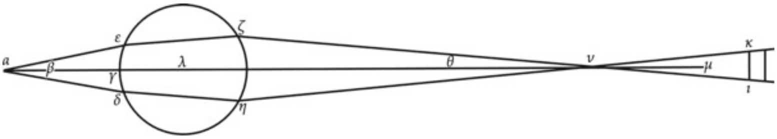

Fig. 4.3 Kepler's diagram that demonstrates the optics of image inversion

nerves halfway to the brain. He specified that *images are inverter in the eyes* and all the colours are being imprinted on the retina.

> Visio igitur sit per picturam rei visibilis ad albam retinæ & cauum parietem; & quæ foris dextra sunt, ad sinistrum parietis lacus sinistra ad dextrum, supera ad inferum, infera ad superum depinguntur: viridia etiã colore viridi, & in vniuersum res quæcunque suo colore intra pingitur...

Kepler envisions the focusing of light by the lens as a simplified graph of two (light) cones with a common base coinciding with the lens of the eye, with the vertex of the one cone on the point of the object being viewed and the vertex of the other on the retina, where the point is being detected. This mechanism works for all points on the objects of the scene in front of the viewer.

Furthermore, Kepler used analogies with crystal balls and vessels filled with clear water to unfold his theory of geometric optics in vision, in the form of 28 Propositions, accompanied with graphs and explanations, particularly to highlight the role of refraction. He also explained the inversion of the images due to rays converging on and diverging from the visual axis in purely geometric terms and the assumption of straight-line ray propagation. Figure 4.3 shows a reconstruction of the original Kepler's diagram that demonstrates the optics of image inversion (Proposition XVII). In the Corollaries of Proposition XVII (and in reference to Fig. 4.3), Kepler detailed how and when images appear inverted, in relation to the location of the observed object or the eyes relative to the points of ray convergence.

> 1. Patet hinc, oculo α longius etiam distante quàm γβ si tamen ικ fuerit inter θμ partim euerso situ (in extremis nempe) visum iri, partim (& in intermediis) situ recto: Et candem etiam circularem, iuxta p 43 decimi Vitellionis: itaque confusè.
>
> 2. Quod si sic manente oculo ικ sit etiam intra ϑ citimum intersectionis terminum, tota erecta videbitur.
>
> 3. Si verò oculus intra βγ sit, intersectionibus in infinitum excurrentibus, & aliquibus ex oculo radiationibus parallel iter refrac-

> tis: Si tunc res in axe fuerit sita, & minor parallelorum distantia, videbitur erecta & euersa simul, siquidem remotior fuerit citima intersectione: Sin propior, erecta tantùm apparebit.
>
> 4. At si excesserit complexum parallelorum, ultra terminum tota euersa, medium erectum, & partim circulare apparebit.
>
> 5. Deniquibus oculo & re cis terminos intersectionŭ existentib. ille parallelorum, hæc radiationum oculi, res erecta & maximæ quartitatis videbitur.

Kepler made an interesting definition and distinction between an *image* and a *picture*. In the Definition just above Proposition XIX, he assigns the term image to the result of reason, whereas he connects the term picture with the figures of objects on surfaces. Last but not least, he explained the usage of correcting lenses for common eye defects.

Kepler also left a remarkable treatise on dioptrics in his 1611 *Dioptrice*, where one may find his theory arranged in axions, theorems and proposition, along with excellent illustrations (Kepler, 1611).

4.3 René Descartes

René Descartes (1596–1650), born in Touraine France, was a mathematician and philosopher who seemed to share similar philosophical ideas with Aristotle and the Stoics,[3] and, besides his important contributions to mathematics, he is widely recognised as a founding figure of modern philosophy. Apart from the Cartesian geometry, he is best remembered by his quote "cogito, ergo sum", "I think, therefore I am", included in his most famous work *Discourse on the Method* written in 1637 Descartes (1637a, 1667). Descartes supported a *particle theory of light*, by stating that light is made up of discrete *corpuscles* (small particles) that travel in straight lines with a finite speed, thus sparking the modern *corpuscular theory of light*. Apparently, he was influenced by the growing trend towards *atomism* in that period, which was a trend in favour of the ancient atomic theories, as expressed by Leucippus and Democritus.

Descartes was particularly interested in explaining how human vision works as he tried to explain how the human body works in general, in his quest to understand its nature. In his work *Tractacus De Homine* (Descartes, 1677) he draws a particularly

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[3] The school of the Stoics was a Hellenistic philosophical movement, founded by Zeno of Citium (Ζήνων ὁ Κιτιεύς, c.334–262 BCE) in Athens, during the 3rd century BCE, which supported a philosophy of personal ethics that draws on logic and interpretations of the natural world.

Fig. 4.4 Descartes' graph of human visual perception from *Tractacus de homine* (Descartes, 1677)

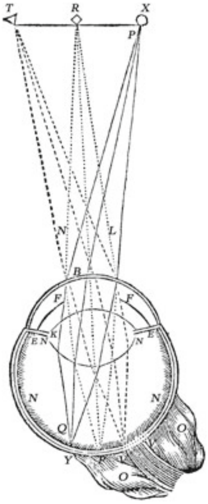

Fig. 4.5 Descartes' graph of the human eye from *De homine figuris* (Descartes, 1664)

interesting diagram of the human visual perception mechanism, a representation of which is shown in Fig. 4.4. In addition, in *De homine figuris* (Descartes, 1664), among others, he displays a graph of the human eye which is shown in Fig. 4.5, on which Descartes explains the formation of images in the eyes.

In his *Principles of Philosophy* (Descartes, 1644, 1637c) (1644) Descartes explained that[4]

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[4] Passage in Latin taken from https://www.loc.gov/resource/rbc0001.2013rosen1431/?sp=1, Pars quarta. De Terra. CXCV. De visu.

> Denique nervorum opticorum extremitates, tunicam, retinam dictam, in oculis componentes, non ab aere nec a terrenis ullis corporibus ibi moventur, sed a solis globulis secundi elementi, unde habetur sensus luminis & colorum: ut jam satis in Dioptrica & Meteoris explicui.

which can be rendered in English[5] as stating that

> The optic nerves are the organs of the subtlest of all the senses, that of sight. The extremities of these nerves, which make up the coating inside the eye called the retina, are moved not by air or any terrestrial bodies entering the eye but simply by globules of the second element which pass through the pores and all the fluids and transparent membranes of the eye. This is the origin of the sensations of light and colours, as I have already explained adequately in my Optics and Meteorology.

In *La Dioptrique* (Descartes, 1637b),[6] Descartes begins by stating that

> Toute la conduite de notre vie dépend de nos sens, entre lesquels celui de la vue étant le plus universel et le plus noble, il n'y a point de doute que les inventions qui servent à augmenter sa puissance ne soient des plus utiles qui puissent être.

which translates to that all the conduct of our life depends on our senses, among which that of sight is the most universal and the noblest; there is no doubt that the inventions which serve to increase its power are of the most usefulness that can be. Descartes derives the law of reflection in agreement with what was already known using a simple example of a ball and the ground. He uses the same logic to derive the law of refraction by substituting the ground with a thin canvas (a linen sheet), which the ball can easily penetrate losing only a part of its speed. By assuming that the speed is only determined by the resistance due to each medium through which the ball travels, Descartes formulated a proposition that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is equal to a constant determined by the resistances of the media in the experiment. This, actually, is *a mechanistically derived law of refraction*, which raised a lot of controversies and even accusations of plagiarism (of Snell's law, which was derived around 1621). Overall, Descartes'

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[5] According to the *Selections from the Principles of Philosophy* found at http://www.earlymoderntexts.com/authors/descartes, *PART IV. OF THE EARTH. CXCV. Of sight*, found at http://www.earlymoderntexts.com/assets/pdfs/descartes1644part4.pdf.

[6] A 1657 edition of *Discours De la Methode* that also includes the original texts of *La Dioptrique* and *Les Meteores* ca be found online @ https://ia600503.us.archive.org/26/items/discoursdelamet00desc/discoursdelamet00desc.pdf.

view is that rays of light mechanically stimulate the eyes and those mechanical stimulations are then passed to the brain and give rise to perceptual experiences.

4.4 Isaac Newton

Isaac Newton (1642–1726/27) from Woolsthorpe-by-Colsterworth, Lincolnshire, England, was a mathematician, physicist, astronomer and theologian, who was particularly interested in systematically defining light and colour. His 1687 volume titled *PhilosophiæNaturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) was more or less the foundation of classical mechanics (Newton, 1687).[7]

In 1675 N proposed that light is a continuous flow of particles (the photons) that travel in straight lines, laying foundations for the *corpuscular theory of light* initially suggested by Descartes. The intensity of light is measured by the number of those photons reaching a surface.

Of his first published works on light, colour and visual perception, a letter to the Philosophical Transactions of the Royal Society in 1671 stands out (Newton, 1671), in which Newton laid out his new theory and presented insights from his work on optics and light, in the form of thirteen propositions that connected refraction and colour, the nature of compound colours, the nature of white and the origin of the colour of objects.[8]

> As the Rays of light differ in degrees of Refrangibility, so they also differ in their disposition to exhibit this or that particular colour. Colours are not Qualifications of Light, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed,) but Original and connate properties, which in divers Rays are divers.
>
> To the same degree of Refrangibility ever belongs the same colour, and to the same colour ever belongs the same degree of Refrangibility. The least Refrangible Rays are all disposed to exhibit a Red colour, and contrarily those Rays, which are disposed to exhibit a Red colour, are all the least refrangible: So the most refrangible Rays are all disposed to exhibit a deep Violet Colour, and contrarily those which are apt to exhibit such a violet colour, are all the most Refrangible. And so to all the intermediate colours in a continued series belong intermediate degrees of refrangibility. And this Analogy 'twixt colours, and refrangibility, is very precise and strict; the Rays always either exactly agreeing in both, or proportionally disagreeing in both.

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[7] One may find copies of the *Principia* in English online. A historical 1729 edition can be accessed online @ https://archive.org/details/bub_gb_Tm0FAAAAQAAJ/mode/2up.

[8] The quoted text from Newton's *New Theory about Light and Colours* is from the Newton Project page @ http://www.newtonproject.ox.ac.uk/view/texts/normalized/NATP00006.

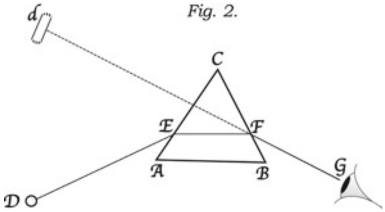

Fig. 4.6 Reproduction of Newton's figure on the refraction through a prism (AX. VIII. in the *Opticks* states that the object *D* seen through a prism appears to be in position *d* due to refraction)

> There are therefore two sorts of Colours. The one original and simple, the other compounded of these. The Original or primary colours are, Red, Yellow, Green, Blew, and a Violet-purple, together with Orange, Indigo, and an indefinite variety of Intermediate gradations.
>
> But the most surprising, and wonderful composition was that of Whiteness. There is no one sort of Rays which alone can exhibit this. 'Tis ever compounded, and to its composition are requisite all the aforesaid primary Colours, mixed in a due proportion...Whiteness is the usual colour of Light; for, Light is a confused aggregate of Rays indued with all sorts of Colours, as they are promiscuously darted from the various parts of luminous bodies. And of such a confused aggregate, as I said, is generated Whiteness, if there be a due proportion of the Ingredients; but if any one predominates, the Light must incline to that colour; as it happens in the Blew flame of Brimstone; the yellow flame of a Candle; and the various colours of the Fixed stars.

Newton presented a more concrete version of his light and colour theory in *Opticks* (Newton, 1704), where he connected refraction with colour through experiments with prisms (Fig. 4.6). In his view, the degree of refrangibility corresponds with a one-to-one relation to colour, creating a necessary and sufficient condition. Thus, there is only one colour that corresponds to the same degree of refrangibility.[9]

> **DEFIN. VII.** The Light whose Rays are all alike Refrangible, I call Simple, Homogeneal and Similar; and that whose Rays are some more Refrangible than others, I call Compound, Heterogeneal and Dissimilar. The former Light I call Homogeneal, not because I would affirm it so in all respects; but because the Rays which agree in Refrangibility, agree at least in all their other Properties.

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[9] The quoted text from Newton's *Opticks* is from the Newton Project page @ http://www.newtonproject.ox.ac.uk/view/texts/normalized/NATP00033.

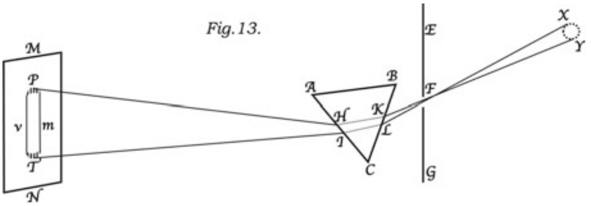

Fig. 4.7 Reproduction of Newton's drawing for his third experiment on support of Proposition II, Theorem II (that the sunlight consists of differently refrangible rays)

Newton defined all light rays that are alike refrangible as *homogeneal*, whereas light rays differently refrangible as *compound*. In addition, he defined the *primary* colours, like those which correspond to homogeneal light.

> **DEFIN. VIII.** The Colours of Homogeneal Lights, I call Primary, Homogeneal and Simple; and those of Heterogeneal Lights, Heterogeneal and Compound. For these are always compounded of the colours of Homogeneal Lights.

Newton further experimented on the relation between colour and refraction and presented conclusive experiments to support various theorems and propositions.

> **PROP. I. Theor. I.** Lights which differ in Colour, differ also in Degrees of Refrangibility.

Based on those experiments he also performed a number of experiments to prove that the light of the sun consists of rays differently refrangible, thus being compound. Referring to PROP. II. Theor. II., which states that (*the light of the sun consists of rays differently refrangible*), Newton in his third experiment, (graphically depicted in Fig. 4.7) defined the colours in terms of the refrangibility.

> This Image or Spectrum PT was coloured, being red at its least refracted end T, and violet at its most refracted end P, and yellow green and blue in the intermediate spaces. Which agrees with the first Proposition, that Lights which differ in Colour do also differ in Refrangibility.

In addition, in PROP. VI, PROB. II, which tackled the issue of the identification of a compound colour based on a given mixture of known primary colours,[10] presented his colour circle and how its geometrical structure can be used to find out a compound colour (Fig. 4.8). In the example illustrated in this figure, the small circles labelled with small letters p, q, r, s, t, u, v, x represent the gravity centres of

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[10] In his own words: "In a mixture of primary Colours, the quantity and quality of each being given, to know the Colour of the compound.".

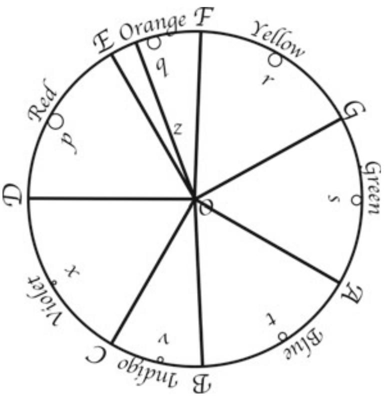

Fig. 4.8 Reproduction of Newton's colour circle; drawing adapted from the *Opticks*

the respective arcs that are defined by the capital letters and correspond to the various colour regions (i.e. the arc DE corresponds to the region of red colours). In addition, the size of the small circles corresponds to the proportion of each colour to the mixture in the example that is being considered. The location of the centre of gravity of these circles is found in the location marked as z. Apparently, z is in the region of the orange colours and a bit to the side of reds (rather than the side of yellows). In addition, as z falls closer to the centre of the circle, which corresponds to the region of least saturated colours, the mixing is thus inferred to correspond to a *relatively faint, slightly reddish-orange compound colour*.

4.5 Christiaan Huygens

Christiaan Huygens (1629–1695) was a Dutch physicist, mathematician, astronomer and inventor. Huygens is appreciated as one of the greatest scientists of all time. He was involved in mechanics, optics, and astronomy, with his astronomical observations of the rings of Saturn and the discovery of Saturn's moon Titan being among the most important contributions. He was among the founders of mathematical physics and a pioneer in the theory of optics. Huygens also improved the design of the telescope by introducing what was subsequently known as the Huygenian eyepiece. One of his most pervasive inventions was the pendulum clock. He left a significant work on mechanics, including the geometrical derivation of the centripetal force and the laws of elastic collision.

What is relevant to this treatise is his significant work in optics. Although he was earlier to Newton, his theory of light and optics was published after Newton's theory. In his *Traité de la lumière* (*Treatise on Light*), published in 1690,[11] Huygens described in detail a new *wave theory of light* (Huygens, 1690, 1912; Huygens et al., 1900). In this treatise, Huygens begins by adopting *a finite speed for light*, as suggested by *Ole Christensen Rømer* (1644–1710, Danish astronomer) after he executed several astronomical observations in the1670 s. In particular, Rømer observed

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[11] The treatise begins by stating that the theory was presented to l' Academie Royale des Sciences (the Royal Academy of Sciences) already in 1678.

about 140 eclipses of Jupiter's moon Io for several months while in Uraniborg near Copenhagen, whereas *Giovanni Domenico Cassini* (1625–1712, Italian mathematician, astronomer and engineer) observed the same eclipses in Paris. Rømer worked with Cassini to further investigate the phenomena, which led to a hypothesis for a finite speed of light. Rømer continued to pursue this hypothesis after joining *Jean Picard* (1620–1682, French astronomer) in a collaboration that resulted in a presentation in the French Academy of Sciences in 1676. As in any other case, many scientific breakthroughs had to coincide for a new theory to emerge. It was in 1671 that Jean Picard published his treatise *Mesure de la terre*, where he laid out the foundations of geodesy by using triangulation to measure the diameter of the Earth (Picard, 1671). This led others to describe astronomical distances in terms of Earth diameters. This is most evident in Rømer's and also Huygens work. Through Rømer's estimates, an expression of the ratio of the speed of light to the speed of one Earth orbit around the Sun was $(365 \cdot 24 \cdot 60)/(\pi \cdot 22) \approx 7605$.[12]

Huygens' *Traité de la lumière* consists of six chapters, including the general theory of propagation in a homogeneous medium, the phenomena in media interfaces like reflection, refraction, atmospheric refraction, or special cases like birefringence ("strange refraction of Iceland crystals") and transparent bodies. The basis of the wave theory of light proposed by Huygens is that light propagates as a wave, which corresponds to the activations and collisions of the particles of the ether (the all-pervading medium). Luminous objects are responsible for the propagation of multiple waves as shown in Huygens depiction of a candle flame in Fig. 4.9, in which three distinct points are shown to create three distinct lightwaves. Every point of the candle flame is supposed to create its own wave. And what happens subsequently is that every particle of the ether that is excited becomes a source of further excitations, which are all spherical waves. In a way, every excited point becomes a light source and this is the way light propagates, as shown in Fig. 4.10. Overall, the progressing wavefront still exhibits a spherical nature, not due to the initial excitation but due to the interference between the numerous waves by each of the excited etherial particles, thus the apparent propagation in straight lines is also explained.

> Il y a encore à considerer dans l' emanation de ces ondes, que chaque parcicude de la matiere, dans laquelle une onde s'etend, ne doit pas communiquer son mouvement seule-ment à la particule prochaine, qui est dans la ligne droite tireé du point lumineux; mais qu'elle en donne aussi necess-airement à toutes les autres qui la touchent, & qui s'opposent à son mouvement.

After laying out the general theory of light propagation in a homogeneous medium, Huygens considered the cases of media interfaces, or what happens when light traverses different media. He analysed how his wave theory of light explains the phenomenon of reflection by following the reasoning of a sequential generation of spherical waves. In this case, the geometry of the setup for a reflection preserves equality of angles (of incidence and reflection). The case of reflection is shown in

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[12] Compare this to the modern estimate of $299792458ms^{-1}/29780ms^{-1} \approx 10066.91$.

Fig. 4.9 Huygens' depiction of the light waves from a candle flame

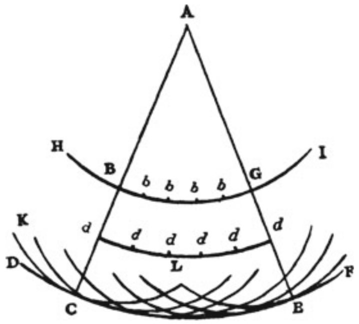

Fig. 4.10 Huygens' depiction of the propagation of light waves

Huygens' diagram in Fig. 4.11; as parallel rays (from infinity) hit the reflective surface AB they form spherical waves that at the time the ray at point C reaches point B on the surface, all waves will have line BN as their a common tangent, thus BN represents the propagation of the wave AC at that particular moment. Of course, triangles ACB and BNA are necessarily equal—they are both rectangular, have AB as common side and CB = NA—thus the angles CBA and NAB are also equal. Since CB marks the direction of incidence and AN the direction of reflection, this means that the angle of incidence is equal to the angle of reflection. In a similar way, Huygens proceeds to explain refraction, as depicted in Fig. 4.12. For this case, he first makes clear, that in presence of a medium with a dense substance the particles of the medium act exactly as the particles of the ether. The slowing of light in the denser medium results in the phenomenon of the bending of the light rays following the same reasoning used for reflection. The tangent BN represents the wavefront in the dense medium just like AC represented the wavefront in the initial rarer medium. The direction AN, perpendicular to BN is the direction of the refraction, satisfying the law of refraction, by which the new change in direction depends on the ratio of the density (refractive indices) of the media.

Huygens' wave theory of light provided elegant explanations to light phenomena, quite differently than how the particle (or corpuscular) theory of light by Newton

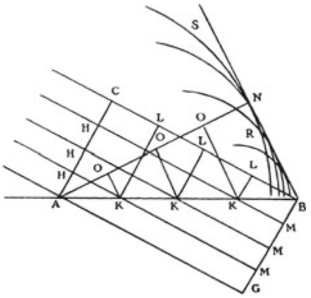

Fig. 4.11 Huygens' depiction of reflection

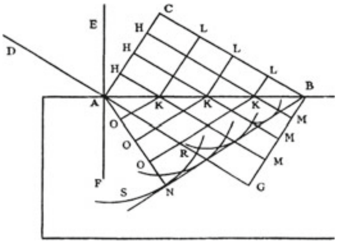

Fig. 4.12 Huygens' depiction of refraction

could. In some cases, like in refraction, Newton's theory supposed forces acting on the particles of light to make it bend, but this would result if the speed of light in denser media were to be increased, which is the exact opposite to what the wave theory supports. In addition, there is no explanation for diffraction by the particle theory, whereas a wave theory readily explains the phenomenon. Of course, at the time those theories were proposed, there was no technology to disprove one or the other.

4.6 Immanuel Kant

Immanuel Kant (1724–1804), from Königsberg, Eastern Prussia, (Kaliningrad, Russia) was a German philosopher and one of the founders of the *Enlightenment*. His most important work is the *Critique of Pure Reason* Kant (1787), in which, in section *Of Space* (in Section I-of Transcendental Aesthetics states), Kant states that (from the English translation of J. M. D. Meiklejohn, 1900)

> Colours are not qualities of a body, though inherent in its intuition, but they are likewise modifications only of the sense of sight, as it is affected in different ways by light.

On the core body-world problem (the basic question of the separation of the inner and the outer world), Kant argues that space is a priori, therefore man's familiarity with space is also a priori. However, this is something that he is not accepting for the objects in space, the experience of which is a property of our aesthetics. In his transcendental idealism, he argued that there are two worlds, that of experiences (thoughts, feelings, and sensory experiences of material things) and that of things not experienced through some known sense, while in this separation the body plays a dual role, as it is a material 'thing' of the world and at the same time is part of the self and a means of perceiving other things.

4.7 George Palmer

During the late1770 s and1780 s, *George Palmer* (1746–1826, or 1740–1795), among various short publications, issued two books on light, colour and colour vision with a long-lasting contribution to the development of colour science. Palmer based his research on physics and chemistry in an attempt to resolve the differences between the colour of light and the colour of objects. In his *Theory of Colours and Vision* (Palmer, 1777), he introduced a theory of colour and vision in a rather interesting form of a dialogue. He also included new experiments with prisms and new explanations of the results. This treatise opens with a list of his *seven principles of colour*[13]

1. La lumière ne comporte aucune couleur.
2. Chaque rayon de lumière est compose seulement de trois autres: dont un est analogue au jaune, l'autre au rouge, & l'autre au bleu.
3. Ces rayons sont dans des proportions differentes; & les conservent exactemet, malgre l'accroissement, ou l'affoiblissement de leur rayon principal.
4. Les corps colorées absobent les rayons analogues aux couleurs qu' ils nous presentent, & ne sont appercus que par les autres rayons qu'ils reflechissent.
5. Une surface blanche, réfléchissante toute la lumiere, offre une negation absolue de couleurs.
6. Une surface composee de trois principes colorans, dans une proportion & une intensite convenables, absorbant ces trois rayons conformement au quatrieme principe, offre une negation absolue de lumiere, & un noir parfait.
7. Un feut de ces trois principes colorans peut separement approcher du noir, sans changer de nature, & absorder ses rayons qui ne lui sont pas analogues, losque son intensite encede la proportion de son propre rayon.

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[13] The quoted principles are from the french version of this treatise that is available as a digitised book in Google Play Books Store @ https://bit.ly/3wY7zuU.

The principles of colour translated in English from the French text, with an influence by MacAdam (1970), can be rendered as follows:

1. Light has no colour.
2. Each ray of light is composed of only three others, one of which is analogous to yellow, the other to red, and the other to blue.
3. These rays are in different proportions; which are kept exactly, despite the increase, or the decrease of their main ray [luminance].
4. The coloured objects absorb the rays analogous to the colours which they present to us, and are only perceived by the other rays which they reflect.
5. A white surface, reflecting all the light, offers an absolute negation of colours.
6. A surface painted with the three primary colourants, in a suitable density, by absorbing the three rays in accordance with the fourth principle, offer an absolute negation of light, and a perfect black.
7. One of these three colouring principles can separately approach black without changing its nature, by absorbing rays not analogous to it, when its density exceeds the proportion of its own ray.

Palmer's theory suggests a basis of three primary colours, which in this case are *red, yellow and blue*. As in similar theories, all possible rays of light contain different proportions of the primary colours. The colours of objects appear so due to the absorption of the rays relating to their colour and the reflection of the other rays. According to the seven principles, rejection of all light indicates a white surface. On the other side, black is created when the three colouring primaries absorb the rays of other colours, creating an intensity that exceeds the proportion of the colour. Most importantly, *light has no colour*; it is that coloured surfaces absorb rays and white surfaces reflect them.

In *Théorie de la lumière applicable aux arts et principalement à la peinture* (Palmer, 1786), Palmer extended his reasoning presented in *Theory of Colours and Vision* with a theory of the prism, in an attempt to clarify and support his idea of the *destructive nature of light*, while enhancing, even more, the role of chemistry in the creation of colour. Palmer clarified that his attempt was towards a unified description for colour, enough to explain colour and light for the arts and science (chemistry and physics). One of his major concerns was to improve the arts by connecting them to science.

Palmer's most interesting contribution was the speculation that there are three different mechanisms in the human eye that account for colour vision. As he states in the *Theory of Colours and Vision*

> The superficies of the retina is compounded of particles of three different kinds, analogous to the three rays of light; and each of these particles is moved by its own ray.

This is a preliminary statement of a trichromatic colour vision, posed about twenty-five years before any other such statement was eventually made by Thomas Young.

4.8 Goethe

Johann Wolfgang von Goethe (1749–1832) was born in the Free Imperial City of Frankfurt, Holy Roman Empire (Germany). Among many things, Goethe (or Göthe) was a poet, playwright, and scientist. He is considered to be the greatest German literary figure of the modern era. Apart from the novels, poems, dramas and other types of literature he left and is most known for, Goethe was also concerned with natural science and particularly of the nature of colours, for which he wrote a treatise. He published in 1810 his most important work titled *Zur Farbenlehre* (von Goethe, 1810), which was in 1840 translated to English and published as *Theory of Colours* (Goethe, 1840), which can be found summarised in (Zajonc, 1976).

Goethe was an opponent of Newton's theory of colour, and was particularly interested in an aesthetic approach towards the understanding of the colour phenomena, with little, though, relation to analytic and mathematical analysis. Most important are his observations on the effect of opponent colours, which led him to redefine and arrange the colours of the typical colour wheel so that three pairs are in diametrically opposite positions, yellow opposite to violet, orange opposite to blue, green opposite to red (as described and depicted in Part I —Physiological Colours, Section V. Coloured Objects, par. 50 of his Theory of Colours). His main concern was to establish a theory that would be consistent with the *qualitative* evaluation of the perception of colour phenomena, one in which perception and explanation would be at the centre. What triggered his divergence from Newton's approach and the pursuit of an alternative theory was his (supposed) discovery that Newton's prismatic experiment was erroneous in that there is no green colour directly exiting a prism, but green is rather a composition of yellow and blue only after some distance from the prism, where the two colours overlap, as shown in Fig. 4.13.[14] Apparently, this observation is false and depends on the topology of the experiment and typical light effects at boundaries.

Ultimately, Goethe considered *darkness* not the absence of light, but rather, a second elusive entity that interacts with light to produce colours. In his experiments with black-white edges he pointed out that yellow appears in the white region of the edge, whereas blue appears in the dark region of the edge, and their mixture produces the green. The darkening of the two basic colours yellow and blue gives rise to reddish hues. Overall he proposed two alternative colour models, either a three-colour model based on yellow, red and blue or a six-colour model based on yellow, red, blue, green, white and black.

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[14] The reproduction was created based on the original drawing in Plate IV., Fig. 1, of the 1840 English edition found @ https://archive.org/details/Goethe_theory_of_colours_prism.

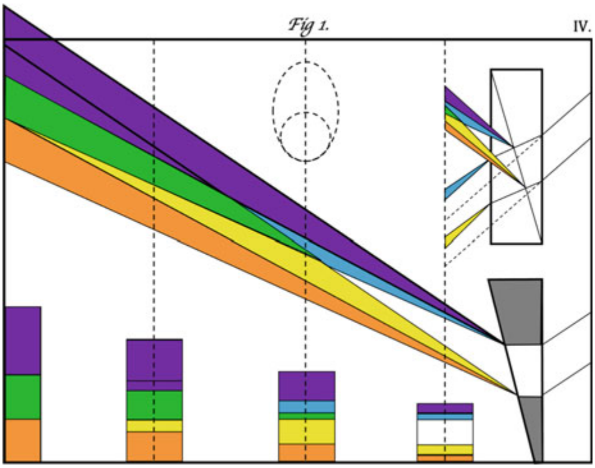

Fig. 4.13 Reproduction of Goethe's fundamental observation on the light dispersion in prisms

Essentially, Goethe rejected the 'sterilised' approach of a theory of colour, in which colour is deprived of its sensation, and is treated as an objective phenomenon even without the need for an observer to experience it. He was deeply certain that talking about colour has no meaning outside of the context of its perception, through the active sensation of vision.

4.9 Thomas Young

The period followed Newton's theories, *Thomas Young* (1773–1829) from Milverton, Somerset, England, was among the first great minds to have also tried to define light, colour and vision. His major contribution in the domain is considered to be in two treatises, *On the theory of light and colours* Young (1802) and *A course of lectures on natural philosophy and the mechanical arts* (Young, 1807a, b), some of his contributions also found in Huygens et al. (1900). In these treatises, he presented his main conclusions regarding colours, following the usual line of reasoning that accepts some of them being primary and the other being combinations of the primaries. Young was among those scientists to support a *wave theory of light*, in favour of Christian Huygens' view and against Newton's particle theory, which was the pervasive theory of that time. Young, although influenced by the Newtonian theory, formulated a theory, which he believed to be opposite to that of Newton and rather close to Christian Huygens' view. His theory was based on a set of hypotheses, largely influenced by Newton, beginning with the existence of the *luminiferous ether* that is supposed to be a rare and elastic medium pervading the universe. These four fundamental hypotheses are,

> **HYPOTHESIS I.** A luminiferous ether pervades the uni-verse, rare and elastic in a high degree.
>
> **HYPOTHESIS II.** Undulations are excited in this ether whenever a body becomes luminous.
>
> **HYPOTHESIS III.** The sensation of different colours depends on the different frequency of vibrations excited by light in the retina.
>
> **HYPOTHESIS IV.** All material Bodies are to be considered, with respect to the Phenomena of Light, as consisting of Particles so remote from each other, as to allow the ethereal Medium to pervade them with perfect freedom, and either to retain it in a stale of greater density and of equal elasticity, or to constitute, together with the Medium, an Aggregate, which may be considered as denser, but not more elastic.

Young preferred the term *undulations* instead of *vibrations* because he wanted to differentiate from the classical view of what vibration is—a permanent forward and backward motion. Instead, an undulation is something that traverses space. This is apparent when he talks about vibrations in the retina of the eye in Hypothesis III. In his scholium (comment) on this Hypothesis he laid out a theory by which there is a limited number of particles (substances) in the retina capable of capturing the undulations of light; he referenced, although as an example, the existence of three such particles, capable of capturing the three principal colours, which he named to be red, yellow, blue. The sensation of all colours comes as the appropriate combination of matching light undulations to retina vibrations. Thus Young's 1802 Bakerian Lecture Young (1802) references *three primary colours, red, yellow and blue* and *transforms the problem of colour perception from an objective natural phenomenon to a subjective human function, by which three receptors are responsible for the tridimensionality of colour vision.*

Young called Hypotheses I–III as essential hypotheses and only provided comments and thoughts on them. It is Hypotheses IV that is totally unique to Young, which differentiated his view from that of Newton's and aligned with that of Huygens. He presented this latter hypothesis extensively with several Propositions, Scholia and Corollaries. In summary, his theory stated that

> **PROPOSITION I.** All are in an elastic Medium Impulses propagated homogeneous with an equable Velocity.
>
> **PROPOSITION II.** An Undulation conceived to originate from the Vibration of a Particle, must expand through a homogeneous

> Medium single in a spherical Form, but with different quantities of Motion in different Parts.
>
> **PROPOSITION III.** A Portion of a spherical Undulation, admitted through an Aperture into a quiescent Medium, will proceed to be further propagated rectilinearly in concentric Superficies, terminated laterally by weak and irregular Portions of newly diverging Undulations.
>
> **PROPOSITION IV.** When an Undulation arrives at a Surface which is the Limit of Mediums of different Densities, a partial Reflection takes place, proportionate in Force to the Difference of the Densities.
>
> **PROPOSITION V.** When an Undulation is transmitted through a Surface terminating different Mediums, it pro-ceeds in such a Direction, that the Sines of the Angles of Incidence and Refraction are in the constant Ratio of the Velocity of Propagation in the two Mediums.
>
> **PROPOSITION VI.** When an Undulation falls on the Surface of a rarer Medium, so obliquely that it cannot be regularly refracted, it is totally reflected, at an Angle equal to that of its Incidence.
>
> **PROPOSITION VII.** If equidistant Undulations be supposed to pass through a Medium, of which the Parts are susceptible to permanent Vibrations somewhat slower than the Undu-lations, their Velocity will be somewhat lessened by this vibratory Tendency; and, in the same Medium, the more, as the Undulations are more frequent.
>
> **PROPOSITION VIII.** When two Undulations, from different Origins, coincide either perfectly or very nearly in Direction, their joint effect is a Combination of the Motions belonging to each.
>
> **PROPOSITION IX.** Radiant Light consists in Undulations of the luminiferous Ether.

Proposition II makes clear enough that light is to be considered as a spherically expanding wave in space. Furthermore, Proposition III describes how apertures influence the propagation of light, which he presented both verbally and graphically,

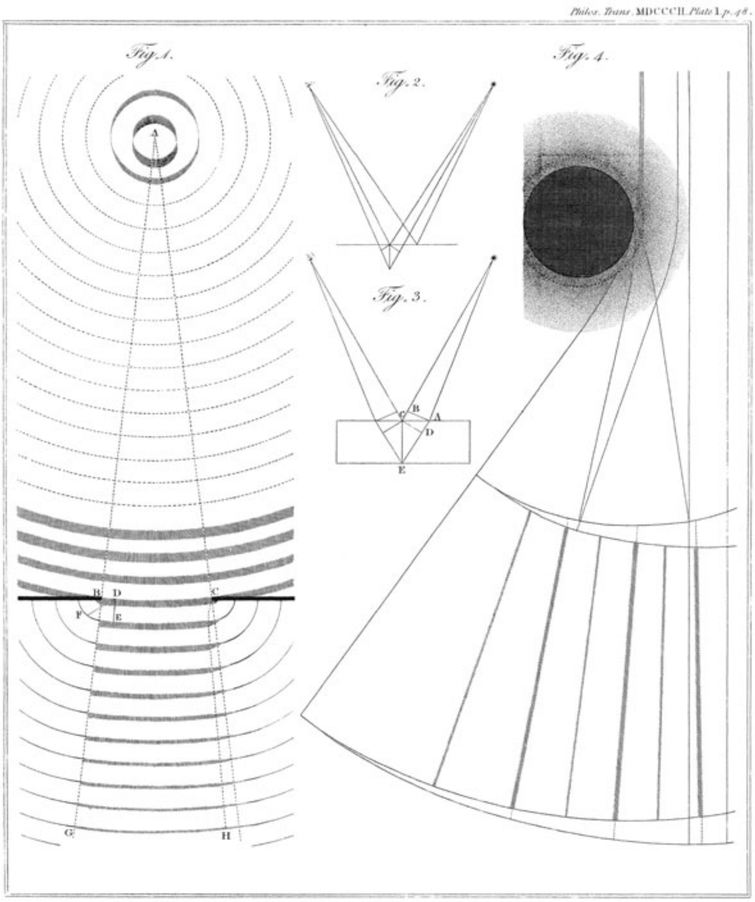

Fig. 1. The progress of a series of undulations admitted through an aperture.
Fig. 2. The difference of the paths of the light reflected from two points situated near each other.
Fig. 3. The difference of the paths of the light reflected from the opposite surfaces of a thin plate.
Fig. 4. The paths of two portions of light supposed to pass through an inflecting atmoshpere.

Fig. 4.14 Thomas Young's illustration of light propagation

using Plate I/Fig. 1 of the treatise (shown in Fig. 4.14).[15] Similarly, Propositions IV and V, although not analysed, they are graphically depicted in Plate I/Figs. 2 and 3 (Fig. 4.14), in which Fig. 3 shows refraction in a medium rarer than the surrounding mediums. Proposition VI is an expression of the law of internal reflection. Proposition VII is an expression of the phenomenon of the dispersion of colours, by which light of short wavelength is more refrangible than long-wavelength light. Furthermore, Proposition VIII is a statement of the appearance of interference, which Young analyses in detail using examples for striated surfaces (like gratings), thin

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[15] This figure was taken from *The Bakerian Lecture: On the Theory of Light and Colours* that is available online @ https://archive.org/details/jstor-107113/page/n1/mode/2up.

Table 4.1 Young's table of colours and frequencies, augmented with modern equivalent representations

Colours	Length of an Undulation in parts of an Inch, in Air.	Number of Undulations in an Inch.	Number of Undulations in a Second. Mil. of Millions.	Wavelength in nm	Wavenumber in cm^{-1}
Extreme	0.0000266	37640	463	675.64	14800.78
Red	0.0000256	39180	482	650.24	15378.94
Intermediate	0.0000246	40720	501	624.84	16004.10
Orange	0.0000240	41610	512	609.60	16404.20
Intermediate	0.0000235	42510	523	596.90	16753.22
Yellow	0.0000227	44000	542	576.58	17343.65
Intermediate	0.0000219	45600	561	556.26	17977.20
Green	0.0000211	47460	584	535.94	18658.81
Intermediate	0.0000203	49320	607	515.62	19394.13
Blue	0.0000196	51110	629	497.84	20086.77
Intermediate	0.0000189	52910	652	480.06	20830.73
Indigo	0.0000185	54070	665	469.90	21281.12
Intermediate	0.0000181	55240	680	459.74	21751.42
Violet	0.0000174	57490	707	441.96	22626.48
Extreme	0.0000167	59750	735	424.18	23574.90

plates of rarer media, as well as thick plates, and provides more insight on the notion of blackness and the colours by inflection, of which he provided Plate I / Fig. 4 (Fig. 4.14) to demonstrate how a varying density medium, like an atmosphere, could inflect the path of light. Young in his discussion on thin plates provides a table of colours and corresponding wavelengths, wavenumber and frequencies, using the estimate for the speed of light of that period, which he states to be expressed as a distance of $500,000,000,000$ feet traversed in $8\frac{1}{8}$ minutes. This is an equivalent $312,615,384.62$ m/s.[16] Table 4.1 shows Young's table of colours, augmented with two additional columns that represent the equivalent 'modern' quantities used to describe light and colour (wavelength and wavenumber). His last Proposition IX is a general conclusion of an underlying wave theory of light, which he further supported with additional evidence.

Thomas Young, in his *A course of lectures on natural philosophy and the mechanical arts* (Young, 1807a, b) changed his set of *primary colours to red, green and violet* (now referenced as 'primitive colours'), as he soon observed that his previous colour basis was not of independent colours. In Volume I (Young, 1807a), in *Lecture XXXVII.On Physical Optics*, Young stated that

> It is certain that the perfect sensations of yellow and of blue are produced respectively, by mixtures of red and green, and of green and violet light, and there is reason to suspect that those sensations are always compounded of the separate sensations combined: at least this supposition simplifies the theory of colours;

---

[16] Compare this to the recent estimate of $299,792,458$ m/s.

> it may, therefore, be adopted with advantage, until it be found inconsistent with any of the phenomena; and we may consider white light as composed of a mixture of red, green, and violet, only, in the proportion of about two parts red, four green, and one violet, with respect to the quantity or intensity of the sensations produced.
>
> If we mix together, in proper proportions, any substances exhibiting these colours in their greatest purity, and place the mixture in a light sufficiently strong, we obtain the appearance of perfect whiteness; but in a fainter light the mixture is grey, or of that hue which arises from a combination of white and black; black bodies being such as reflect white light but in a very scanty proportion.

In addition, in Volume II (Young, 1807b), in *Section VII.Of Dioptrics and Catoptrics, 403.Definition*, Young stated that

> Light is distinguished by its effect on the sense of vision, into white and coloured light; and coloured light into a great number of various hues: but they may all be referred to the three primitive colours, red, green, and violet.

4.10 Augustin-Jean Fresnel

Augustin-Jean Fresnel (1788–1827) was born in Normandy, France. He was a civil engineer and physicist. His research in optics had a significant impact on pushing towards a wave theory of light for almost 100 years, against Newton's corpuscular theory. He is most known for his work with lenses, and particularly the invention of the catadioptric (combination of reflective and refractive) lens. His work on lenses led to a significant improvement of the visibility of lighthouses, with an important impact on safer maritime travels. A simpler version of that composite lens, the dioptric stepped lens (refractive), has been widely used in overhead projection devices and screen magnifiers. The ingenious invention of the catadioptric lenses revolutionised the technology of lighthouses by limiting the propagation of light into a region that is most important for maritime applications. Typically, light propagates spherically, which means much of the light travels towards the ground and the sky; this part is of no practical use. Fresnel designed his composite lens system so that it exploits refraction and reflection in a way that all light rays are forced to propagate horizontally (parallel to the ground). Figure 4.15 shows a graph of the principle of operation of the composite Fresnel lens system, adapted from the 1881 E. Atkinson's English translation of Adolf Ganot's *Cours Élémentaire de Physique* (*Natural Philosophy*) (Ganot, 1881). Based on the same edition, Fig. 4.16 shows a sketch of a lighthouse employing Fresnel's catadioptric system.

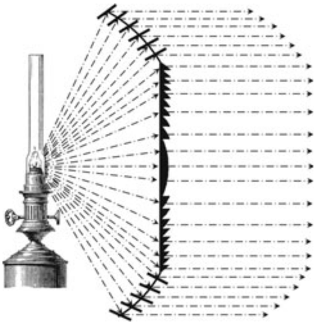

Fig. 4.15 Fresnel's catadioptric system for parallel light rays

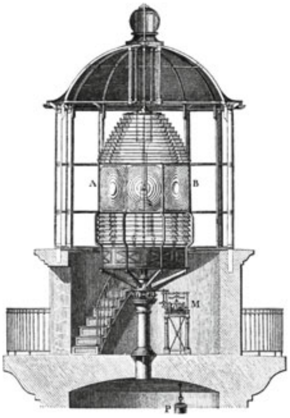

Fig. 4.16 Adolf Ganot's depiction of a lighthouse using the Fresnel's composite catadioptric lens

Fresnel's work is concentrated in the three volumes of the *Œuvres Complètes* (Fresnel, 1866, 1868, 1870) and in the 1900 collection of wave-light-theorists edited by Huygens et al. (1900). In his address to the French Academy of Sciences in 1815, title *La Diffraction de la Lumière*, he examined the phenomenon of the coloured fringes by the shadows of bodies illuminated by a luminous point source (Fresnel, 1866), Fresnel explained the phenomenon in purely wave-theoretic manner, through the interference of spherical waves from point sources. This way, he explained the phenomenon of diffraction and from that starting point, he argued that also reflection and refraction could be explained by adopting the wave theory. He proved that, although the light is reflected and refracted in infinite directions, only straight-line directions are actually seen, as interference cancels all other directions. "C'est que leurs vibrations se contrarient, comme il est facile de le prouver", he stated, meaning, the vibrations are opposed and it is easy to show. Fresnel studied in detail the phenomena of coloured fringes in the shadow of illuminated bodies and concluded that even under purely white light the diffraction of the light waves is the cause of the appearance of colours.

> Les franges, dans la lumière blanche, sont la réunion des bandes obscures et brillantes produites par toutes les espèces d'ondes lumineuses dont elle se compose. La largeur de ces bandes étant proportionnelle à la longueur d'ondulation varie avec elle; en sorte que les bandes obscures et brillantes de diverses couleurs, au lieu de se su-perposer parfaitement, empiètent les unes sur les autres; d'où résultent des mélanges dans d'autres proportions que celles qui constituent la lumière blanche, et par conséquent un phénomène de coloration.

According to Fresnel, light is definitely propagating as a transverse wave and this is fundamental to his theory. As expected, he was particularly involved in phenomena of diffraction, interference and polarisation because those phenomena could not be explained by Newton's particle theory of light. In some of the *memoirs* in the *Œuvres Complètes*, Fresnel extended Huygens' theory and provided analytical solutions to the phenomena of diffraction and reflection to support *Huygens' principle* of secondary waves and Young's theory of interference.

Further, Fresnel was deeply concerned with *polarisation of light* (see for example the second section of Volume I of the *Œuvres Complètes*) and designed a number of interesting experiments with double refraction in crystals, on which he found out that there is no other reason than the occurrence of polarisation in perpendicular planes for the theory and the observation to be in agreement. Apparently, he imposed polarisation by reflection. Furthermore, he provided an explanation (which Thomas Young failed to do) on the chromatic polarisation, again experimenting with crystals.[17]

> L'explication que je viens de donner des phénomènes de simple dépolarisation est fondée sur la supposition que la lumière polarisée est divisée par une réflexion complète en deux systèmes d'ondes polarisés l'un parallèlement, l'autre perpendiculairement au plan d'incidence, et séparés par un intervalle d'un huitième d'ondulation.

In the third section of the *Théorie de la Lumière*, which is in the second Volume of the *Œuvres Complètes*, published in 1868 (Fresnel, 1868), Fresnel exposed his complete wave theory of light, including explanations of most important phenomena of light. He provided his own estimates of the wavelengths of light that correspond

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[17] It is generally accepted that Fresnel was the person who coined the terms for linear, circular and elliptical polarisation.

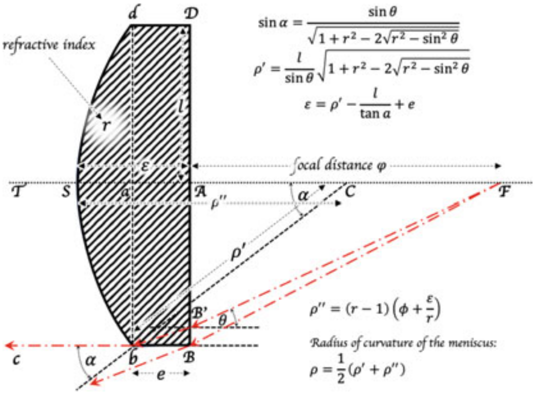

Fig. 4.17 The optics and math of the meniscus element in a Fresnel lens

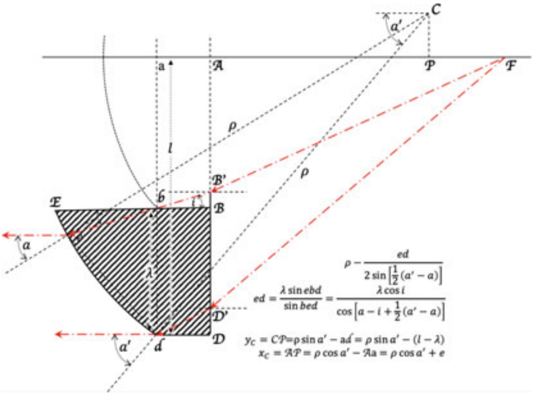

Fig. 4.18 The optics and math of the annular element in a Fresnel lens

to the principle colours, adopting Newton's set, and extended the estimates to intermediate colours. Although he was involved in every aspect of research targeting the nature and propagation of light, he (like Huygens and contrary to Newton) was not deeply concerned with how colours are perceived by humans, and what colours actually are.

The third Volume of the *Œuvres Complètes*, published in 1870 (Fresnel, 1870), is mostly related to Fresnel's work on lighthouses (*Phares et Appareils d'Éclairage*). This is where one may find all the details of Fresnel's ingenious *catadioptric lenses*. Figure 4.17 is an adaptation to Fresnel's original drawing of the meniscus element of the composite lens. The principle of operation and the required mathematics for the implementation of this element are also included in the figure. Similarly, Fig. 4.18 shows the principle and design data for the annular elements, Fig. 4.19 shows the principle and design data for the catadioptric elements and Fig. 4.20 shows the principle and design data for the catoptric elements of the system.

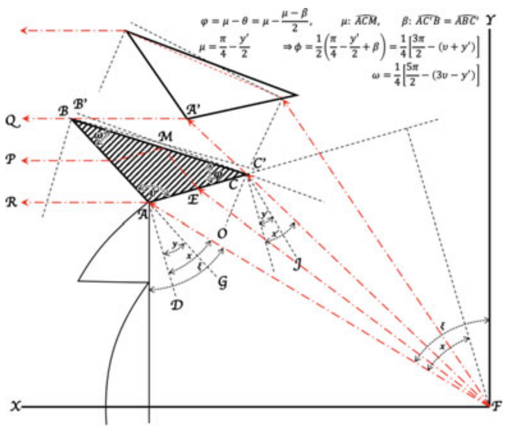

Fig. 4.19 The optics and math of the catadioptric element in a Fresnel lens

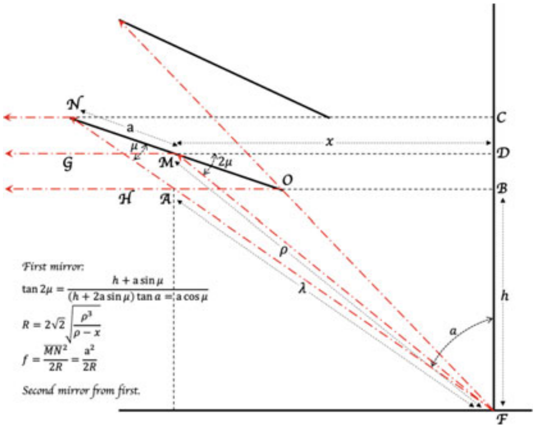

Fig. 4.20 The optics and math of the catoptric element in a Fresnel lens

4.11 Arthur Schopenhauer

Arthur Schopenhauer (1788–1860) was a German philosopher, born in Danzig (Gdańsk), Polish–Lithuanian Commonwealth (Baltic coast of Northern Poland). He built on Kant's transcendental idealism and was concerned with the phenomenal world, which he show through a philosophical pessimism, and he has been largely influential for subsequent thinkers. Schopenhauer's most famous work of 1818 *Die Welt als Wille und Vorstellung* (The World as Will and Representation[18]) Schopenhauer (1969a, b) begins with

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[18] The digitised original Gernal text can be found @ https://www.lernhelfer.de/sites/default/files/lexicon/pdf/BWS-DEU2-0958-03.pdf.

> "Die Welt ist meine Vorstellung:" - dies ist die Wahrheit, welche in Beziehung auf jedes lebende und erkennende Wesen gilt...Die Welt ist Vorstellung.

The world is my conception, says Schopenhauer and this is a truth valid with reference to every living and knowing being. The world is representation. The text continues with

> ...sondern zu welcher nur tiefere Forschung, schwierigere Abstraktion, Trennung des Verschiedenen und Vereinigung des Identischen führen kann, - durch eine Wahrheit, welche sehr ernst und Jedem, wo nicht furchtbar, doch bedenklich seyn muß, nämlich diese, daß eben auch er sagen kann und sagen muß: "Die Welt ist mein Wille".

To Schopenhauer only deeper investigation, more difficult abstraction, the separation of what is different, and the combination of what is identical can lead us to this truth; this truth, which must be very serious and grave if not terrible to everyone, is that a man also can say and must say that "the world is my will". One may probably detect some philosophical similarities to Protagoras' theory. Apparently, Schopenhauer describes as *will* that which characterises the inner nature of all things, and the world seems to consist of two sides, the *world is will* and *the world is representation*. As will, the world is in itself, a unity. As representation, the world is that of appearances, of ideas or of objects, a diversity. What he talks about, is the world as *Reality* and the world as *Appearance*.

As expected, Schopenhauer was deeply concerned with vision and colours and wrote a treatise on this subject titled *Über das Sehn und die Farben* (*On vision and colours*)[19] (Schopenhauer, 1816). In Schopenhauer & Runge (2010) there is a 2010 English translation that also includes the treatise *Farben-Kugel* (*Color sphere*) by Philipp Otto Runge (1777–1810), a Romantic German painter, who derived a trichromatic colour model in the form of a sphere around 1807 (Runge, 1810) and shared it with Goethe.[20] As can be easily deduced from Schopenhauer's treatise, he was among the very first to propose a model for visual perception that separates sensation and representation. In this model, *the role of perception is to transform the subjective sensations of the objects of the outside world into objective representations within, through the interference of the understanding*.

In the part of the treatise devoted to vision, Schopenhauer praises the value of vision among the senses, but, at the same time, clarifies that its value is confined only

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[19] The original 1816 text may be found online @ https://archive.org/details/bub_gb_qIw5AAAAcAAJ/, or @ https://upload.wikimedia.org/wikipedia/commons/f/f0/Ueber_das_Sehn_und_die_Farben.pdf.

[20] This English translation of the two treatises included in *On vision and colours* can be found online @ https://pdfget.com/pdf-epub-on-vision-and-colors-on-vision-and-colors-color-sphere-download/.

within acquiring a sensation, not perception. He argues that if the eye was the organ of perception, then the world would be perceived inverted as the image formation on the retina follows the laws of optics, thus the eye should only be thought of as the object of sensation. Schopenhauer states that

> ...sondern die Anschauung entsteht dadurch, daß der Ver-stand den auf der Retina empfundenen Eindruck augen-blicklich auf seine Ursache bezieht, welche nun eben dadu-rch sich im Raum , seiner ihn begleitenden Anschauungs form, als Objekt darstellt. Bei diesem Zurückgehn nun von der Wirfung auf die Ursache, verfolgt er die Richtung, welche die Empfindung der Lichtstrahlen mit sich bringt; wodurch wieder alles an seine richtige Stelle kommt, indem jetzt am Objekt sich als oben darstellt, was in der Empfin-dung unten war.

The perception arises from the fact that the understanding instantly relates the impression felt on the retina to its cause, which now presents itself as an object in space, its accompanying form of perception. Following the reverse path from the effect to the cause, the understanding follows the direction that the sensation of the rays of light brings with it, whereby everything comes back into its right place, and the sensation of bottom corresponds to the top of the object. Schopenhauer is also among the first to provide a definition of illusions in contrast to errors, stating that *an illusion is a deception of the understanding, thus opposed to reality, whereas an error is a deception of reason, thus opposed to truth.*

In the part of the treatise devoted to colour, Schopenhauer unfolds his theory of colour perception, by making it clear that most previous scholars were wrong in trying to infer theories about colour by only studying the effects of light when the research should have been focused on the physiological dimensions of the sensation itself (the eye). He emphatically disregarded Newton's theory on this basis, by stating that

> Daß er dabei die Siebenzahl einzig und allein der Tonleiter zuliebe gewählt hat, ist nicht dem mindesten Zweifel unterworfen: er durfte ja nur die Uugen aufmachen, un zu sehn, daß im prismatischen Spektrum durchaus nicht jieben Farben sind, sondern bloß vier, von denen, bei größerer Entfernung des Prismas, die zwei mittleren, Blau und Gelb, übereinander greifen und dadurch Grün bilden. Daß noch jetzt die Optiker sieben Farben im Speftrum aufzählen, ist der Gipfel der Lächerlichkeit. Wollte man es aber ernsthaft nehmen, so wäre man, 44 Jahre nach dem Auftreten der Goethejchen Farbenlehre, berechtigt, es eine unverschämte Lüge zu nennen: denn man hat nach gerade Geduld genug gehabt.

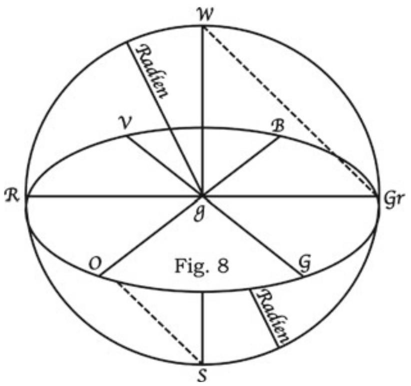

Fig. 4.21 Reproduction of Runge's colour space (sphere)

Apparently, he felt that believing Newton's theory in that there are seven primary colours is totally absurd.

In the development of his theory of colour vision, he stated that when a body under the influence of light does not react on the eye at all, it should be called black, whereas white is the light that reacts (the effect of light and white is the same). Thus the degree of the activity of the retinal reaction indicates the intensity of the activity, forming thus the grey hues. Furthermore, he defined yellow and violet as those produced by a (non-equally) divided retina half-activation, complementing each other. Schopenhauer accepted Runge's colour system, which, in essence, is a three-dimensional coordinate system with hue, saturation and brightness being the three coordinates, represented as a sphere, in which the poles are white and black, pure colours rest on the equator and grey colours run along the white-black axis. A reproduction of Runge's colour sphere is shown in Fig. 4.21[21] and Fig. 4.22[22] from Runge (1810).[23]

Regarding the perception of colour, Schopenhauer stated that colour should be considered as *the qualitatively divided activity of the retina*. In his colour system, the number of colours is infinite, but any two opposite colours (like yellow-violet) contain the full potential of all the others, like in an opponent colour theory. Thus, it is not correct to refer to individual colours, but only of colour pairs, in a sense that each pair represents the totality of the activity of the retina divided into two (qualitative) halves. In this colour system, there are *six primary colours, violet, blue, green, red, orange and yellow*,[24] whereas there are only *three basic chemical colours, blue, red and yellow* (most probably accepted from Runge's colour system). In addition,

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[21] Adapted from https://archive.org/details/farbenkugeloderc00rung.

[22] Adapted from https://archive.org/details/farbenkugeloderc00rung and https://commons.wikimedia.org/wiki/File:Runge_Farbenkugel.jpg.

[23] The original text can be found online @ https://archive.org/details/farbenkugeloderc00rung.

[24] Schopenhauer thinks of the primary colours as excitations of the retina and assigns zero (0) to black and one (1) to white, which he does not include in the set of primary colours, but he uses them only as the limits of the excitation range. In this representation, the colours in the set {violet, blue, green, red, orange, yellow} correspond to a set of activation intensities as {1/4, 1/3, 1/2, 1/2, 2/3, 3/4} respectively.

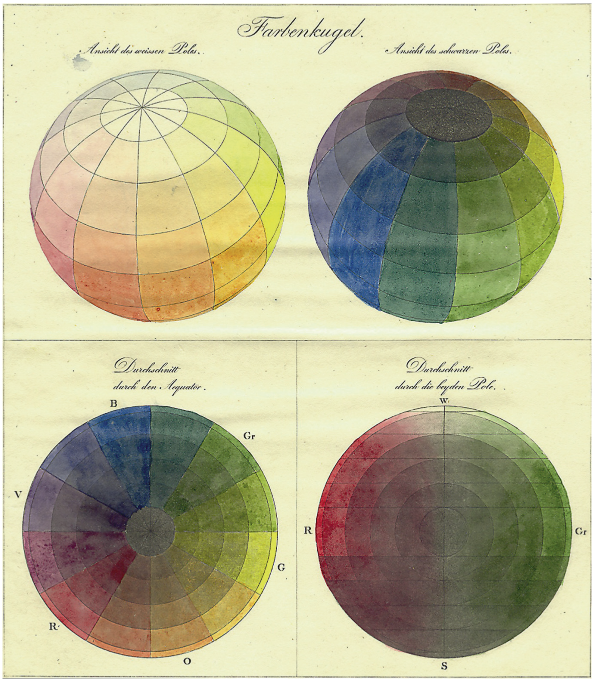

Fig. 4.22 Various representations of Runge's colour space

the system of primary colours is additive, meaning that combined excitations result in summing their individual contributions such that

$$\begin{aligned} \mathcal{RED} &= \text{the full activity of the retina minus } \mathcal{GREEN} \\ \mathcal{GREEN} &= \text{the full activity of the retina minus } \mathcal{RED} \\ \mathcal{RED} + \mathcal{GREEN} &= \text{the full activity of the retina} = \\ &= \text{the effect of light (or white)} \end{aligned} \tag{4.1}$$

4.12 Johannes Müller

Johannes Peter Müller (1801–1858) was born in Koblenz, Rhin-et-Moselle, First French Republic (later part of Germany). He was a German physiologist, anatomist, ichthyologist, and herpetologist. His inquisitive mind and influence of other scholars led him, on one hand, to dismiss the approaches to the physiology of his times and, on the other hand, to express his theories in his most important work, the *Handbuch der Physiologie des Menschen*, which was published between 1833–1840 (Müller, 1835; J. Müller, 1838; Müller, 1840b) and was later translated to English as *Elements of Physiology* by William Baly, between 1837–1843 (Müller, 1840a; J. P. Müller, 1842). This publication, which became the textbook in physiology, signified the beginning of a new era in physiology, bringing together anatomy, chemistry and microscopy.

Among the most important contributions is Müllers work on the mechanisms of the nervous system and the senses. He recognised how the sense organs are fit to a single sensation, so distinctively that any type of stimulus would always be perceived in the way the sense organ reacts to it (for example mechanical stimulation of the retina will result in the perception of images, exactly like when light excites it). He conducted a massive amount of experiments to prove this idea.

Müller, in Volume II of the Handbook, expressed some important laws that are of particular interest in relation to the topics of this treatise. He stated that *humans cannot be directly aware of objects in perception, but only of the qualities specific to particular nerves*. Sensations correspond in their features mainly to states in the nerves induced by stimulation and not to states of their distal causes. The characters of sensations are tied specifically to the nerves producing them and to the ‘energies’ (in quotes here, although this is the expression Müller used) of those nerves. Sensations need not have external causes at all but can be caused internally by direct stimulation of a nerve or even by an electrical impulse.

His general contribution to the physiology of the nerves can be found in Book III of Volume I of the Handbook, where he describes the structure and excitability of the nerves and the propagation of signals and actions.

Book V. (hosted in Volume II) Müller begins with his preliminary considerations *Of the Senses*, stating that

> Die Sinne unterrichten uns von den Zuständen unseres Körpers durch die eigenthümliche Empfindung der Sinnesnerven, sie inderrichten uns auch von den Eigenschafted und Veränderungen der Natur ausser uns, insofern diese Zustände unserer Sinnesnerven hervorrufen. Die Empfindung ist allen Sinnen gemein, aber der modus der Empfindung ist in den enzelnen verselieden, nämlich Lightempfindung, Tonempfindung, Geschhnnack, Geruch, Gedühl.

As far as qualities and changes in internal states and the external nature give rise to changes in the conditions of nerves, senses arise to inform of those changes. Sensation is common to all senses, but the type of sensation is different in each sense, thus differentiating the sensations of light, sound, taste, smell, and of feeling or touch. Furthermore, paragraph V begins by stating that

> Die Sinnesempfindung ist nicht die Leitung einer Qualität oder eines Zustandes der äusseren Körper zum Bewustsein, sondern die Leitung einer Qualität, eines Zustandes eines Sinnesnerven zum Bewustsein, verunlasst durch eine äussere Ursache, und diese Qualitäten sind in den verschiedenen Sinnesnerven verschieden, die Sinnesenergieen.

A bold statement indeed, conveying that *sensory perception is not the conduction of quality or a state of an external body to consciousness, but the conduction of a* *quality or a state of the corresponding sensory nerve, caused by an external cause*. Clearly, perception gives an interpretation of the natural world by coupling stimuli from it with knowledge of the mechanics of the corresponding sensor.

The entire first section of Book V is devoted to sight (*I.Abschnitt. Vom Gesichtssinn—Of Vision*). First, Müller analyses the physical conditions necessary for the formation of luminous images. He then provides an excellent description of the structure and operation of the eye (a) by means of a pinhole camera and (b) by means of a focusing system based on a refractive lens. Thus, he states the basic laws of reflection and refraction to provide a basis for the description of the mechanisms of the vision. He also mentions aberrations of lenses. For Müller the lens is the most perfect part of the eye, having the shape and transparency for the required refraction to focus images on the retina.

> Körper, welche das Licht in jenem Sinne zu sammeln vermögen, sind die durchsichtigen das Licht brechenden Mittel, deren vollkommenste für das Sehorgan zweckmässigste Gestalt die linsenförmige ist, wie sich specieller sogleich ergeben wird.

Müller accepted Newton's colour theory and cited the effects on prisms. He reiterated the conclusion that white light is composed of the different coloured rays, which combined give the sensation of the white, but which may be separated by refractive media, due to their different refrangibility.

> Diess führt zu dem Schluss, dass das Weisse dann gesehen werde, wenn dieselben Stellen eines Körpers ungleichartige Strahlen aller Art zugleich erhalten und ins Auge werfen, das hingegen die Farbe dann erscheine, wenn das gleichartige Licht einer Art den Eindruck hervorbringt, mit anderen Worten, dass das weisse Licht aus den verschiedenen Farben zusammengesetzt sei, welche zusammen weiss geben, durch brechende Mittel abet wegen ihrer verschiedenen Brechbarkeit zur Sonderung gebracht werden.

Nevertheless, In subsequent paragraphs, he dismissed Newton's seven dioptric colours theory and proposes that there should only be three primary colours, *yellow, blue, and red*. He defined complementary those colours that their combinations produce white. He was careful to denote in this definition that the one should be a homogeneous colour and the other a mixed prismatic colour, thus discussing combinations like green and red, or violet and yellow, blue and orange. In addition, he recognised that black colour is by definition the absolute darkness, the state of repose or freedom from excitement. Figure 4.23 is a reproduction of the colour circle Müller provided in this Handbook, in which he put the primary colours on the vertices of an equilateral triangle, circumscribed a circle on that triangle, and placed the composite colours between the primary colours that mix to produce them. Diameters in this circle denote the complementary colours, as he defined them.

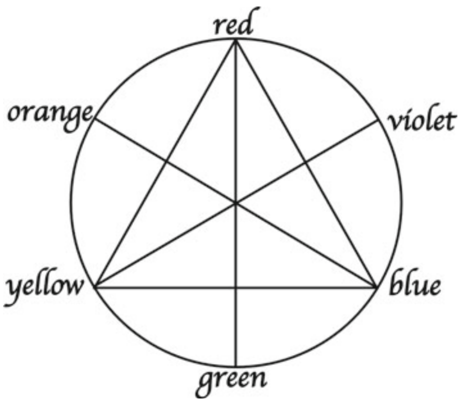

Fig. 4.23 Müller's colour circle

Table 4.2 Hershel's measurements of common colours

Colours of the Spectrum	Length of an undulation in millionths of an inch	Number of undulations in an inch	Number of undulations in a second (trillion)
Extreme red	26.6	37640	458
Red	25.6	39180	477
Intermediate	24.6	40720	495
Orange	24.0	41610	506
Intermediate	23.5	42510	517
Yellow	22.7	44000	535
Intermediate	21.9	45600	555
Green	21.1	47460	577
Intermediate	20.3	49320	600
Blue	19.6	51110	622
Intermediate	18.9	52910	644
Indigo	18.5	54070	658
Intermediate	18.1	55240	672
Violet	17.4	57490	699
Extreme violet	16.7	59750	727

Müller dismissed Goethe's objections to Newton's theory of colours as erroneous, by stating that there was already proof of how light interacts with translucent media causing the appearance of a particular colour, which was the main source of objection in Goethe's approach. In conclusion, he reiterated John Frederick William Herschel's (1792–1871) measurements on the wavelengths, wavenumbers and frequencies of selected colours (as found in the updated English translation), as shown in Table 4.2.

In *II.Capitel. Vom Auge als optisehem Werkzeuge*, Müller analysed the eye as the optical instrument of vision, where he distinguished three types of eyes based on the complexity of their structure and provided a description for these types. The

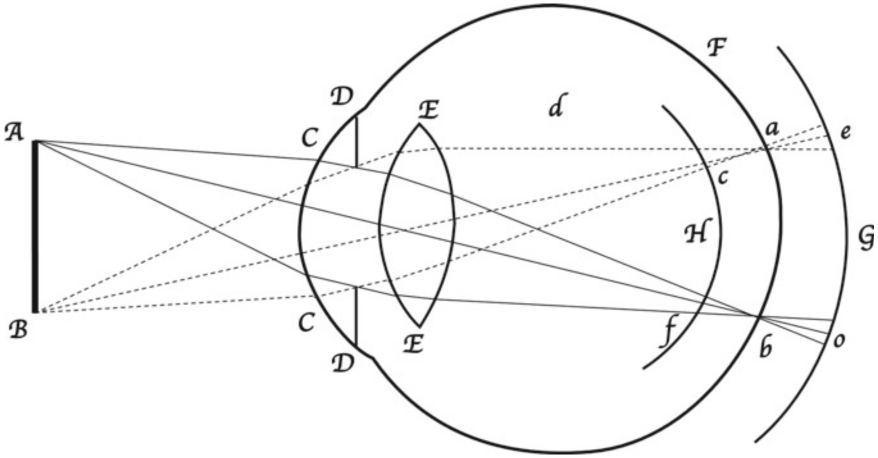

Fig. 4.24 Müller's diagram of the refractions in the human eye

human eye, which is in the category of those based on refractive lenses, is described in a manner relating to the function of vision. He provided a brief description of the basic parts of the eye and particularly focused on the retina, which he described as consisting of three main layers and focused on the layer of the cone or rod-shaped nerve terminations. A reproduction of Müller's diagram of the refractions taking place in the human eye is shown in Fig. 4.24. As he stated, the refraction of the rays of light in the eyes is threefold, one by the cornea and two by the lens, as shown by the bending of the rays in the figure. He recognised that the focal distance of the eye should be exactly on the retina, otherwise any image would not be perfectly focused and would appear blurry, as shown in the figure by the two spherical surfaces H and G, in which the retina is positioned closer and further to the lens.

Müller included in his *Handbuch* all the known knowledge of that time regarding the structure and function of the eye as a focusing apparatus and suggested that any image is a composition of the tiny sensations captured by the rod-like optic nerve terminations in the eye. He devoted a part of the text in the description of the adaptation at different distances, which he suggested (as Young did) could be attributed to a change in the convexity of the lens, and not the enlargement of the eye, although other experts also suggested the change in cornea's convexity. He further discussed the issues of *chromatic aberration*, a phenomenon that results from the dispersion property of light, by which different wavelengths undergo different refraction. In the case of the human eye, which has a lens of a fixed refractivity, this results in having different focal lengths for lights of different wavelengths, thus short wavelengths focus in front of long wavelengths as they are more refrangible. In colour perception, this translates to having violet light focus in front of yellow, which in turn, focuses in front of red. Müller analyses the achromatic property of the eye, the property by which the refractive media of the eye do not disperse the light entering the eye, and supports that the chemical composition and geometric structure makes it possible.

> Worin die Achromasie ihren Grund hat, lässt sich mit Bestimmtheit nicht angeben, wohl aber die Möglichkeit der Achromasie des Auges aus dem optischen Bau desselben einsehen. Seine brechenden Mittel sind von ungleicher Brechkraft, von ungleichen Convexitäten und ungleicher chemischer Constitution. Das eine ist die Linse mit ungleichen Convexitäten, das zweite die Cornea mit dem Humor aqueus. Letztere bilden zusammen eine convex-concave Linse, deren Brechkraft von der Linse verschieden ist. Vielleicht ist die Farbenzerstreuungskraft beider brechender Mittel ihrer Brechkraft nicht proportional und hierdurch die Achromasie bedingt.

In *Chapter III. Of the action of the retina, optic nerve, and sensorium in vision*, Müller reported the important findings of his time regarding the actual chromatic visual perception. He stated that "light and colour are actions of the retina, and of its nervous prolongations to the brain. The kind of colour and luminous image perceived depends on the kind of external impression." Simultaneous impression of undulations of different wavelengths on the same point of the retina results in the sensation of white. Colours are just manifestations of excitation with light of different wavelengths. Müller further outlined the limits of the current knowledge during his time by stating that *there was no clue on where perception takes place*.

> Wo wird der Zustand der Nervenhaut empfunden, in der Nervenhaut selbst oder im Gehirn?

He elaborated on this issue by analysing the facts known at that time; it was already known that the central vision is acute whereas the peripheral was not and that the optic nerve connecting with the brain consists of far fewer fibres than those on the retina. He examined various possibilities but ultimately concluded that at that time it was impossible to form a concrete theory. By analysing how the eye-brain interaction occurs, according to experience and experiments, he suggested that their cooperation is so tight and constant that it is difficult to distinguish what actually influences a visual sensation. For the relation of the sense of vision to the perception of the external world, Müller reminds that *it is by the operation of the judgment that the objects of vision are recognised as exterior to the body of the observer*. In his analysis, he presented several interesting facts regarding visual perception, the results of the simultaneous action of the two eyes (stereo vision) and even discussed the aesthetics of colour combinations.

An interesting essay by Scott Edgar published in 2015 took a more philosophical look at Müller's work. Edgar pointed out that (for the Neo-Kantians, second half of 19th century) the crucial insight from this work from an epistemological perspective is that if the nature of any representation (say of a physical world object) is not subject to the independent properties of the represented object but rather to the properties of the sensory and cognitive system, then the representation will not resemble the independent object (Edgar, 2015).

4.13 William Hamilton

William Rowan Hamilton (1805–1865) was born in Dublin, Ireland. He was a mathematician and astronomer and left important work in pure mathematics and mathematics for physics. He was the founder of a theory of dynamical systems in classical mechanics (later known as *Hamiltonian mechanics*), which became crucial to the study of electromagnetism and the development of quantum mechanics. He is also known for his invention of the *quaternions*. Quaternions form a generalisation of the theory of complex numbers in the case of four dimensions, to describe rotations in three dimensions. As complex numbers form a two-dimensional space to describe one-dimensional vibrations (as projections of 2D on 1D), quaternions form a four-dimensional space to describe three-dimensional rotation (as projections of 4D on 3D). Although vector analysis replaced quaternions the following centuries, the compact and quicker computations they involve in comparison to matrix representations, make quaternions extremely useful (and stable) in practical applications involving three-dimensional rotations (aviation, computer graphics, etc.). As extensions to complex numbers, quaternions are similarly represented, for real numbers a, b, c, d and unit quaternions $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as

$$\begin{aligned} &a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \\ &\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{i}\,\mathbf{j}\,\mathbf{k} = -1 \end{aligned} \tag{4.2}$$

Quaternions, although a powerful and straightforward tool to use, are very difficult to grasp, due to their four-dimensional nature, and a number of treatises and resources can be found that deal with their description, analysis and representation.[25]

At the end of the 1820s, Hamilton presented a theory of a single function, *Hamilton's principal function* (also known as the *Hamilton–Jacobi equation*), which bridges mechanics, optics, and mathematics.

$$S(q, t; q_0, t_0) = \int_{t_0}^{t} \mathcal{L}\left(\gamma(\tau), \dot{\gamma}(\tau), \tau\right)\, d\tau \tag{4.3}$$

where t is time, q is a position in generalised coordinates, (t_0, q_0) are initial conditions, $\dot{q}$ is the velocity, γ are the solutions to the Euler-Lagrange equations (extrema), $\gamma|_{\tau=t_0} = q_0$ and $\exists \hat{t} \in [t_0, t_1)\ \gamma|_{\tau=\hat{t}} = q$. *This is a milestone in mechan-*

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[25] An interesting explanation, demonstration and interactive simulation can be found online @ https://eater.net/quaternions.

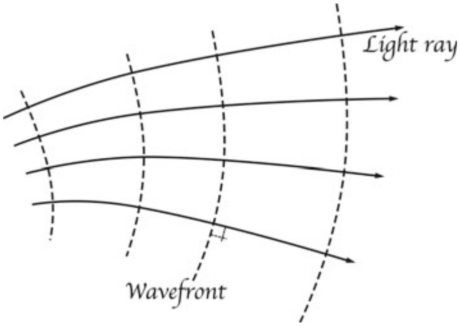

Fig. 4.25 Hamilton's wave propagation of light

ics, since it is the only formulation of mechanics, in which the motion of a particle can be represented as a wave. This gave rise to Hamilton's representations of light propagation as shown in the diagram of Fig. 4.25.

Hamilton viewed all processes, thus also light propagation, in this way, and based his theory of light, which he published as *Theory of Systems of Rays* in 1828, on such an approach (Hamilton, 1828a, b). There he focused on reflection and refraction, the most basic phenomena of light propagation studied by all scientists in optics. He presented the ellipsoid model to his reflection theory, by which any point on the ray of incidence and the corresponding point of reflection are foci of an ellipsoid that touches the surface of reflection.[26]

Hamilton formulated the principle of stationary action (or principle of least action) for dynamical systems. Of course, this principle appeared many times in history (of the science of the optics), even during Euclid's and Hero's era and the formulation for light is credited to Pierre de Fermat in the1600 s. Hamilton's principle generalises the case for the dynamics of physical systems. According to this principle, *the path taken by a system between two times and two configurations is the one for which the action is stationary (no change) to first order*. Given two states of a system expressed in N-generalised coordinates $\mathbf{q}_i(t), \ (i = 1, 2)$ that correspond to two times t_1, t_2, the true evolution of the system is a stationary point of the action-describing functional,

$$\mathcal{S}[\mathbf{q}] = \int_{t_1}^{t_2} \mathcal{L}\,(t, \mathbf{q}(t), \dot{\mathbf{q}}(t))\ \mathrm{d}t \tag{4.4}$$

where $\mathcal{L}(t, \mathbf{q}, \dot{\mathbf{q}})$ is the Lagrangian of the system. By this definition, $\mathcal{S}$ is clearly a functional, for which Hamilton's principle states that the true evolution of the system is a solution of the functional

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[26] This is due to the *reflective* nature of ellipses, by which when connecting any point p on an ellipse to its foci f_1, f_2 the tangent to that point forms a surface of reflection for one of the 'radii' (say $|p\ f_1|$) to the other (say $|p\ f_2|$). Due to this reflective property of ellipses, all possible rays passing through a focal point are expected to be reflected by the ellipse towards a direction that necessarily passes through the other focal point.

$$\frac{\delta \mathcal{S}}{\delta \mathbf{q}(t)} = 0 \tag{4.5}$$

which gives the path, in configuration space, for which the action is stationary.

Hamilton's work on optics is complemented with his three important supplements to the *Theory of Systems of Rays*, sequentially published during 1830 and 1831 (Hamilton, 1830b, c, a, 1831a, b, 1837). In these supplements, he extended his theory and made it clear that he supported a *wave theory of light*. One of the remarkable results of his work is the prediction of *conical refraction*, a phenomenon occurring in biaxial crystals, by which light rays exit those materials in the form of a hollow cone. Although Hamilton left a voluminous work on optics, he was not particularly concerned about the way humans perceive and interpret colours.

4.14 Hermann Günter Grassmann

Hermann Günter Grassmann (1809–1877) was a German polymath, linguist, mathematician and physicist, born in Stettin, Kingdom of Prussia (Western Poland). His legacy includes contributions to linear algebra, projective and differential geometry, but also linguistics. It should be noted that he is probably the first to apply vector methods to mechanics and was the first to formulate a theory of linear algebra, so ahead of his time, that it initially did not attract any attention as it was not understood. In 1853 he published *Zür Theorie der Farbenmischung* (*Theory of Compound Colours*), his theory on colour mixing and colour sensation, known today as *Grassmann's laws* in optics (Grassmann, 1853, 1854; MacAdam, 1970). Grassmann's theory plainly states that that chromatic sensation in human vision can be described as an effective stimulus consisting of linear combinations of different wavelength lights. His *first law* states that colour matches are trivariate; provided three colour primaries (say, R, G, B), any colour C can be matched by a weighted summation of the three primaries (measured in any form of quantity of light power r, g, b).

$$C \equiv r(R) + g(G) + b(B) \tag{4.6}$$

Grassmann's *second law* states that mixing any two colours is matched by a linear combining of the mixtures of any three other colours that individually match the two colours considered.

$$\begin{aligned} C &\equiv C_1 + C_2 \\ C_i &\equiv R_i + G_i + B_i, \quad i = 1, 2 \end{aligned} \tag{4.7}$$

Grassmann's *third law* states that the hue of a colour resulting from additive colour mixing depends only on the colour impression of the initial colours, but not on their physical (spectral) compositions. Thus mixing of even the metameric colours[27] can be described exactly on the basis of their colour impression, and conversely, no direct conclusions about the spectral composition of colour can be drawn from the mixing.

$$C' \equiv k \cdot C \equiv k \cdot R + k \cdot G + k \cdot B \tag{4.8}$$

Grassmann's *fourth law* states that the intensity of an (additively) mixed colour corresponds to the sum of the intensities of the mixed colours.

$$I(C_3) \equiv I(C_1) + I(C_2) \tag{4.9}$$

Of particular interest is that Grassmann opens his 1853 paper (Grassmann, 1853) by clearly confronting Helmholtz, stating that

> Im 87. Bande dieses Journals theilt Hr.Helmholtz eine Reihe zum Theil neuer und sinnreicher Beobachtungen mit, aus welchen er den Schlufs zieht, dass die seit Newton allgemein angenommene Theorie der Farbenmischung in den wesentlichsten Punkten irrig sey, und es namentlich nur zwei prismatische Farben gebe, nämich Gelb und Indigo, welche vermischt Weiss liefern.
> Hierbei wird es nöthig seyn, den Farbeneindruck, dessen das Auge fähig ist, in seine Momente zu zerlegeg. Zunächst unterscheidet das Auge farbloses und farbiges Licht. An dem farblosen Lichte (Weiss, Grau) unterscheidet es nur die größere oder geringere Intensitaet, und diese lässt sich mathematisch bestimmen. Ebenso unterscbeiden wir an einer homogenen Farbe nur ihre größere oder geringere Intensität. Aber auch fuer die Verschiedenheit der einzelnen homogenen Farben haben wir ein mathematisch bestimmbares Maaß, welches uns am vollkommensten in der jeder Farbe entsprechenden Schwingungsdauer geboten wird; schon die populäre Sprache hat diese Differenz auf eine sehr passende Weise durch den Ausdruck Farbenton bezeichnet. Wir werden also an einer homogenen Farbe zweierlei: ihren Farbenton und ihre Intensität unterscheiden können. Vermischt man nun eine homogene Farbe mit farblosem Lichte, so wird der Farbeneindruck durch diese Beimischung abgeschwächt.

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[27] Metameric colours are those with the same colour impression but with a different spectral composition.

which may be rendered in English as

> In the 87th volume of this journal, Mr.Helmholtz conveys a series of partly new and ingenious observations, from which he concludes that the theory of colour mixing, generally accepted since Newton, is erroneous in the essential points, and in particular only two prismatic colours give, as yellow and indigo, which deliver mixed white.
>
> Here it will be necessary to dissect the colour impression of which the eye is capable, into its elements. First, the eye distinguishes colourless and coloured light. At the colourless light (white, grey) it distinguishes only the greater or lesser intensity, and this can be determined mathematically. Likewise, we only consider their greater or lesser intensity in a homogeneous colour. But also for the difference of the individual homogeneous colours, we have a mathematically determinable measure, which is offered to us most completely in the oscillation period corresponding to each colour; even the popular language has designated this difference in a very fitting way by the term colour tone (hue). So we will be able to distinguish two things from a homogeneous colour: its colour tone (hue) and its intensity. If one then mixes a homogeneous colour with colourless light, the impression of colour is weakened by this admixture.

Grassmann claimed that in white light one is able to only distinguish the intensity, whereas in a homogeneous colour both its intensity and its hue can be distinguished. *Every impression of colour may be analysed into three mathematically determinable elements, the hue, the intensity of colour, and the intensity of the intermixed white.* Apparently, this is the basis of any hue-saturation-brightness (or lightness, or intensity) colour model. Quite interestingly, due to his linguist background, Grassmann recognised and acknowledged that it is rather difficult to prove this argument and invoked the usage of the language to his defence, in that there has never been an observer to name other elements (apart from these three) for the description of the impression of colour.

Grassmann supported that to any homogeneous colour one may find another, which, if mixed with the former, would result in colourless light. He carefully analysed and compared the work of Newton and Helmholtz and derived his set of complementary colours and his version of the *colour circle*, based on that of Newton's, as shown in Fig. 4.26. In this diagram, the letters *A* to *G* correspond to the *Fraunhofer lines*.[28] Grassmann's colour circle was derived from Newton's circle according

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[28] The Fraunhofer lines are a set of spectral lines–named after the German physicist Joseph von Fraunhofer (1787–1826)–which were originally observed as absorption lines in the optical spectrum of the sun, observed as a result of gas in the photosphere of the sun. Practically, around 1817 Fraunhofer examined further the observation of chemist and physicist *William Hyde Wollaston* (1766–1828), who noticed gaps in the spectrum of the sun through a prism. By looking even closer, Fraunhofer found a large number of missing slices (or dark lines) in the spectrum. These are

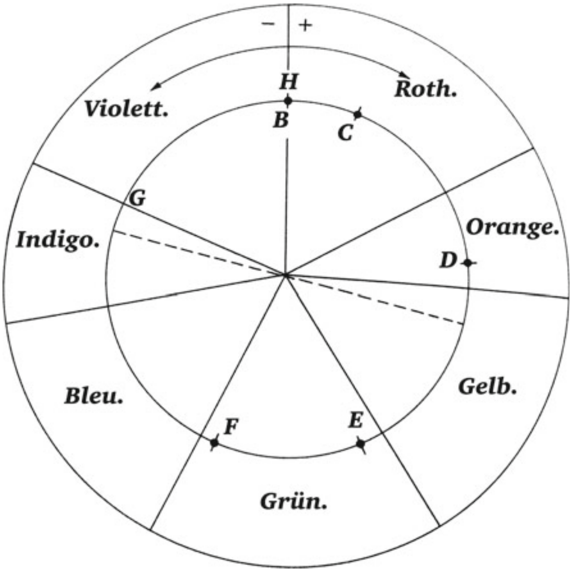

Fig. 4.26 Reproduction of Grassmann's colour circle adapted from Grassmann's original work and MacAdam (1970)

Table 4.3 Grassmann's complementary colours

Yellow	Yellowish Green	Green	Bluish Green	Azure	Indigo
Indigo	Violet	Purple	Red	Orange	Yellow

to the rules in the *Opticks* and the corresponding positions of the Fraunhofer lines, and Grassmann provided detailed relations and proportions among the various distances on the circle. Grassmann's table of complementary colours as outlined in Grassmann (1853) are listed in Table 4.3, where the complementary colours stand one above the other.

4.15 Hermann von Helmholtz

Hermann Ludwig Ferdinand von Helmholtz (1821–1894) was born in Potsdam, Kingdom of Prussia (Germany). He was a physicist, physician and philosopher with a significant scientific contribution in many fields of research, including theories of energy conservation, electrodynamics, thermodynamics, philosophy of science, aes-

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the lines named after him and are attributed to the absorption of particular wavelengths of light by the various materials. It was around 1860 that *Gustav Robert Kirchhoff* (1824–1887) and *Robert Wilhelm Eberhard Bunsen* (1811–1899) found out why these dark lines appear, by working in the opposite direction, studying the emission lines of various heated gasses. Thus, the dark lines in the solar spectrum seen on the surface of the Earth are due to the absorption of light by the gasses that make up Earth's atmosphere.

thetics and more. Helmholtz was particularly intrigued by the physics of perception and focused on optics and acoustics. He is particularly known for his theories of vision and visual perception. Helmholtz quickly became famous for his 1851 invention of the *ophthalmoscope*, a device with which it is possible to see the retina of the eye (*fundus* image of the eye[29]).

In the 1860s, with the appearance of Helmholtz's *Handbuch der physiologischen Optik* (*Handbook of Physiological Optics*) (von Helmholtz, 1867, 1909a, b, c),[30] old and new experimental observations have been unified and explained under a single theory. This theory has long been known as *the Young-Helmholtz theory of colour vision*, though some researchers argue that it should be known as *the Young-Helmholtz-Maxwell theory of colour vision* (Sherman, 1981; Kremer, 1993; Heesen, 2015). The famous "Fig. 95" from the *Handbook*, a drawing of Helmholtz's ophthalmoscope, is reconstructed in Fig. 4.27 from the 1867 edition (this figure became "Fig. 104" in the 1909 edition of Volume I of the *Handbook*). Figure 4.28 shows Helmholtz's diagrams for the ophthalmoscope apparatus (from the 1909 edition of Volume I of the *Handbook*). The description of the principle of operation by Helmholtz himself is rather simple and straightforward.

> Sehr viel bequemer wird die Beobachtung, wenn der Beobachter einen durchbohrten undurchsichtigen Spiegel anwendet, um das Auge $\mathcal{A}$ zu erleuchten. Es sei in Fig. 95 wieder $\mathcal{A}$ das beobachtete, $\mathcal{B}$ das beobachtende Auge, $\mathcal{C}$ die Convexlinse, und $\mathcal{SS}$ ein durchbohrter Spiegel. Von dem Netzhautpunkte a wird ein Bild bei d entworfen, welches der Beobachter durch die Oeffnung des Spiegels hin betrachtet. Von dem ganzen von a kommenden Strahlenkegel geht nur der schmale Theil für die Beleuchtung verloren, welcher durch die Oeffnung des Spiegels fällt, der ganze übrige Theil wird reflectirt und kann dem leuchtenden Körper zugelenkt werden. Zu dem letzteren Ende ist entweder der Spiegel $\mathcal{SS}$ ein Hohlspiegel (Ruete), oder aber ein Planspiegel (Coccius) oder Concavspiegel (Zehender), neben dem man eine Linse $\mathcal{L}$ angebracht hat, welche die Strahlen auf den leuchtenden Körper vereinigt.

Helmholtz describes how to set up an apparatus using a perforated opaque mirror $\mathcal{SS}$ before the observer's eye $\mathcal{B}$, in order to look into the eye of the subject $\mathcal{A}$, by illuminating through a source $\mathcal{D}$. Lens $\mathcal{L}$ is used to force the rays of light to hit the mirror in a near parallel arrangement, which then converge to mirror's focal point d, pass through lens $\mathcal{C}$ which guides light rays to the correct position into the eye.

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[29] A *fundus* is a part of a hollow object (particularly an organ) that is furthest from the opening. In the context of vision, a fundus image of the eye shows the retina, the part of the eye that is opposite to the opening, the pupil.

[30] Also in subsequent smaller treatises like *Ueber die Theorie der zusammengesetzten Farben* (*On the Theory of Compound Colours*) and *The recent progress of the theory of vision* (von Helmholtz, 1852b, a; Helmholtz, 1885).

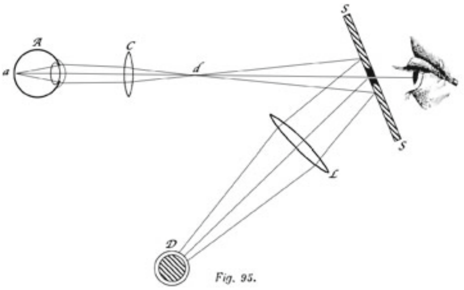


Fig. 4.27 Reconstruction of Helmholtz's drawing of the ophthalmoscope

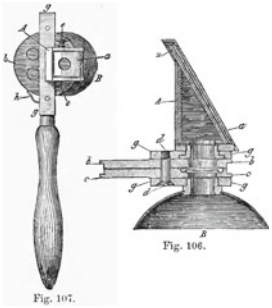


Fig. 4.28 Helmholtz's drawings of the ophthalmoscope apparatus

The reflected light will follow the prescribed path straight to the eye of the observer through the perforated mirror.

In his 1868 lecture on the *Recent progress of the theory of vision* (Helmholtz, 1885), Helmholtz presented an account of the known physiological and mental process of vision. This was a summarisation of the current knowledge, which was finalised, to the extent the technological and theoretical means of his time could support. This is a text worth reading, as it is a view of the structure and physiology of the human eye and its function, which has not changed since then. Figure 4.29 side by side with Fig. 4.30 show how Helmholtz presented the eye as a camera obscura, but with great admiration for the efficiency by which nature created the eye, unparalleled by whatever apparatus human was able to create at that time. The inverted image in the eye formed by refraction was clearly accepted at that time. Furthermore, Fig. 4.31 shows a section of the central part of the retina (part of the *macula*, near *fovea centralis*), the photosensitive part of the eye, which he attributed to the anatomical work of Friedrich Gustav Jakob Henle (1809–1885). According to his description, this is the part responsible for the most significant part of the overall function of vision. Helmholtz informed that it was already estimated that the viewing angle of a single human eye covers roughly 160° laterally and 120° vertically, whereas the two eyes combined cover a horizontal field of view of roughly 180°. He also provided information regarding the visual acuity in central vision, which was estimated to that of the resolution provided by a single cone, roughly in the order of a minute of a degree. Helmholtz recognised *three stages in vision*, beginning with

Fig. 4.29 Helmholtz's depiction of the camera obscura of his time

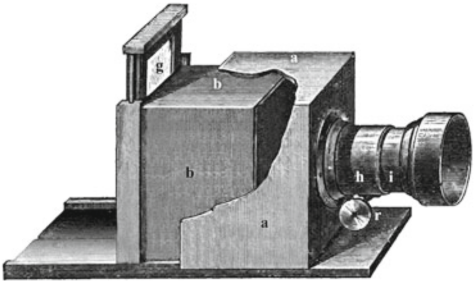
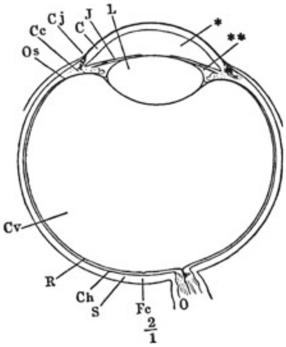


Fig. 4.30 Helmholtz's depiction of the anatomical structure of the human eye



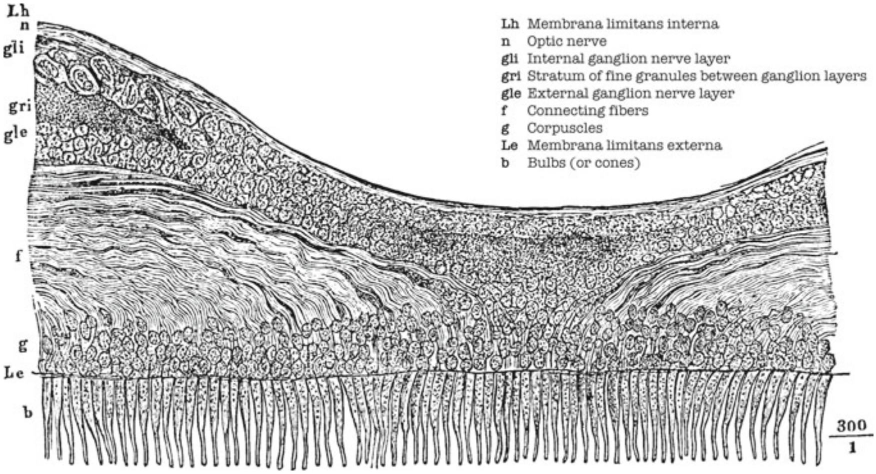


Fig. 4.31 Helmholtz's depiction of the retina borrowed by Friedrich Gustav Jakob Henle

the *physical* (the optics of the eye), then the *physiological* (the conversion of light to nervous impulses) and ending with the *psychological* (the perception).

In the *Handbook*, Helmholtz provided an excellent and critical review of the theories of light and colour vision previously developed, going back to the time of Aristotle. He unfolded this review using Newton as a reference point and explicitly mentioned the theory of Goethe and his ‘cloudy’ media hypothesis, along with Goethe’s strong opposition to Newton’s theory for the composition of white light. As he stated

> The complexity of white light, which Newton announced, was the first decisive empirical step in the direction of recognising the merely subjective significance of the sense-perceptions. Goethe’s presentiment was, therefore, correct when he violently opposed this first advance that threatened to ruin the “fair glory” of the sense-perceptions.

Helmholtz concluded the review with a presentation of the basic elements of the theories of Hegel, Descartes, Hooke, de la Hire, Huygens, Euler, Hartley, Young, Fresnel and Brewster. Then, he proceeded to unfold his theory of the sensation of compound colours by using an analogy and a profound distinction of the visual system with the auditory system for the perception of sound. He emphasised a difference in the ‘mechanics’ of mixing powdered or liquid pigments to the mixing of lights, denoted an analogy of the light passing through liquids to the light passing through prims, and ultimately defined what today is called the *subtractive colour representation* and a basis for the definition of the effects of optical filters. He emphatically suggested that

> Evidently, therefore, the result of mixing pigments cannot be used to deduce conclusions as to the effect of combining different kinds of light. The statement that yellow and blue make green is perfectly correct in speaking of the mixture of pigments; but it is not true at all as applied to the mixture of these lights.

In Helmholtz’s colour theory, white can be produced by combining different pairs of simple colours in definite ratios, which are complementary colours. *He defined the complementary colours of the spectrum to be red and greenish-blue, orange and cyan-blue, yellow and indigo-blue and greenish-yellow and violet.* He defined the complementary colour to green to be purple, which is not a single colour (but rather a compound colour). In his theory, the most saturated colours in order of degree of saturation are violet, indigo-blue, red and cyan-blue, orange and green, yellow. In this view, the number of different colours is exhausted by mixing pairs of two simple or homogeneous colours. Helmholtz’s colour representation system for light sources includes three variables, namely *luminosity, hue and saturation*, which may produce every colour impression. In introducing this colour model he rejects a colour model based on primary colours, like the one based on red, yellow and blue, as, he states, no combination of three colours can actually reproduce the effect that the prismatic colours have. Nevertheless, he proceeds and proposes a change in the set of primary colours into violet, green and red, as more suitable for better approximation of the prismatic colours. He defines a new colour circle, based on the intuition that many colours are not represented in triangular regions as

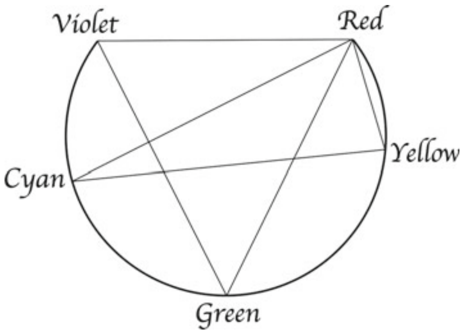


Fig. 4.32 The flattened colour circle that Helmholtz proposed

shown in Fig. 4.32. Through this graph, he states that it is clear how a red-yellow-blue (blue was actually cyan during that time) model is totally inappropriate for colour representation. Based on experiments (which he does not mention further at this point) he proposed that the colour circle should actually be flattened at the violet-red region. In a way, this diagram, if inverted, was very close to the most modern colour representation theories. Regarding the proper and objective naming of the colours, Helmholtz provided Table 4.4, in which colour names were matched to wavelength and corresponding Fraunhofer lines.

Table 4.4 Objective naming of colours by Helmholtz

Fraunhofer lines	Wavelength (nm)	Naming
A	760.40	Extreme red
B	686.853	Red
C	656.314	Border of red and orange
D	589.625 589.023	Golden yellow
E	526.990	Green
F	486.164	Cyan blue
G	430.825	Border of indigo and violet
H	396.879	Border violet
L	381.96	
M	372.62	Ultraviolet
N	358.18	
O	344.10	
P	336.00	
Q	328.63	
R	317.98	
U	294.77	

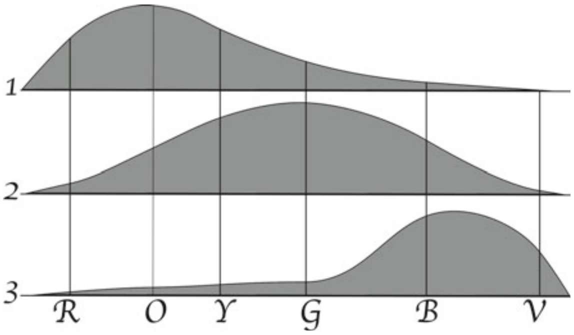


Fig. 4.33 The spectral sensitivity of the eye's nervous fibres according to Helmholtz

Although Helmholtz extended the space of the colour model to accommodate for more perceived colours, he emphatically pointed out that *no set of primary colours should be taken as having any objective significance* whatsoever, when the human eye is not taken into account. Here, he accepts *Thomas Young's theory of colour sensation*, in which, he agrees, there is the meaning of choosing and talking about sets of primary colours. In this view, the eye consists of *three distinct sets of nervous fibres*, which when excited correspondingly create the sensation of red, green and violet colours. Depending on its wavelength, light excites these fibres accordingly, although, the fibres should not be expected to have a strict wavelength selection mechanism; light should be expected to excite all fibres to a different degree depending on their nature. Although Helmholtz did not provide any evidence whatsoever for a quantitative analysis on this subject, he presented a graph of the response he envisioned these fibres should have in order to create the sensation of colour, as shown in Fig. 4.33. In this figure, the horizontal axis is in decreasing wavelengths from red (ℛ) to violet (𝒱).

Helmholtz, in Sect. 20 *Die zusammengesetzten Farben* of the 1867 edition of the *Handbook*, stated that

> Will man dagegen in der Farbentafel als gleich gross solche Quantitäten verschiedenfarbigen Lichts betrachten, welche dem Auge bei einer gewissen absoluten Lichtintensität als gleich hell erscheinen, so erhält die Curve der einfachen Farben eine ganz andere Gestalt ähnlich wie in Fig. 117.
> Die gesättigten Farben Violett und Roth müssen weiter vom Weiss entfernt sein, als ihre weniger gesättigten Complementärfarben, weil nach dem Urtheile des Auges bei der Mischung von Gelbgrün und Violett zu Weiss die Quantität violetten Lichtes viel kleiner ist, als die des gelbgrünen, und wenn das Weiss im Schwerpunkte beider liegen soll, die kleinere Quantität Violett an einem grösseren Hebelarme wirken muss, als die grössere Lichtmenge des Gelbgrün. Uebrigens würden auch hier wieder die Spectralfarben an der Peripherie der Curve, das Purpur auf einer Sehne stehen müssen, Complementärfarben an den entgegengesetzten

> Enden von Sehnen, welche durch den Ort des Weiss gelegt sind, wie bei der kreisförmigen Fig. 114.
>
> Die Zurueckfuehrung des Farbenmischungsgesetzes auf Schwerpunktconstructionen wurde zuerst von Newton nur als eine Art mathematischen Bildes vorgeschlagen, um die grosse Menge der Thatsachen dadurch auszudrücken, und er stützte sich nur darauf, dass die Folgerungen aus jener Darstellung qualitativ mit den Erfahrungsthatsachen übereinstimmten, ohne dass er quantitative Prüfungen ausgeführt hätte. Dergleichen quantitative Prüfungen sind dagegen in neuester Zeit von Maxwell ausgeführt worden.

In essence, he stated that, if one needs to create colour charts (or circles), in which the areas of the coloured regions correspond to the perceived colours, then the colour space should be like the one shown in Fig. 4.34 (Fig. 117 in the original text of the 1867 edition); in particular, the saturated violet and red colours must be further from white than their less saturated complementary colours, since when mixing yellow, green and violet to get white, the amount of violet light needed is much less. In this colour representation, the spectral colours would have to be on the periphery of the curve, the purple region should be on a chord connecting red and violet, complementary colours on the opposite ends of chords that pass through the location of the white, as in the Newtonian circular representation of Fig. 4.35 (Fig. 114 in the original text of the 1867 edition). In addition, Helmholtz commented that the reduction of the law of colour mixing to constructions of the centre of gravity was first proposed by Newton only as a kind of mathematical representation in order to express the great number of facts by it, and he relied only on the fact that the conclusions from that representation correspond qualitatively with the empirical facts, without being quantitative tested. In contrast, such quantitative tests have recently been carried out by Maxwell. This interesting section of the Handbook also includes a particular insightful paragraph regarding the construction of perceptually consisted colour charts (*Construction der Farbentafel*) in which Helmholtz presents the methodology and the simple mathematics appropriate to create colour charts that correspond to the human perception of colours.

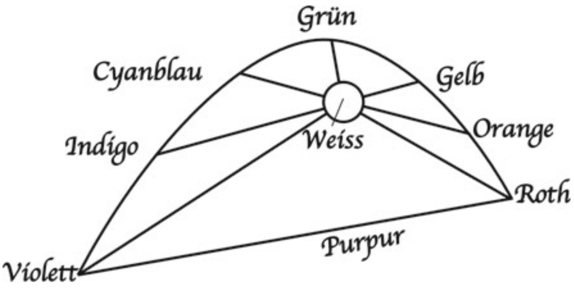


Fig. 4.34 Reproduction of Helmholtz's perceptual colour space representation

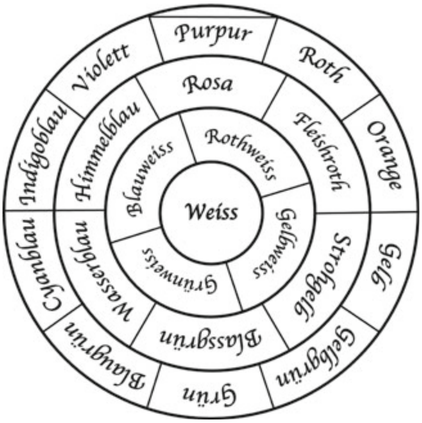


Fig. 4.35 Reproduction of a typical Newtonian colour space representation from Helmholtz's 1867 *Handbook*

Last but not least, Helmholtz in his theory emphatically differentiated (using through experiments) the mixing of coloured light to the mixing of pigments, for which he stated that they are two completely different things.

4.16 James Clerk Maxwell

James Clerk Maxwell (1831–1879) was a Scottish scientist that made a significant contribution to mathematical physics with his formulation of the classical theory of electromagnetic radiation that unified the theories of electricity and magnetism, and in essence, founded the field of electrical engineering. He was among those deeply involved in shaping the knowledge about colour, colour sensation and perception. He left a large volume of works regarding optics and colour perception, where he laid out his adoption of the trichromatic model of vision proposed by Young and his quantification and mathematical modelling of phenomena of the optics and colour perception (Maxwell, 1855, 1856a, b, c, 1857a, c, b, 1858, 1860, 1861, 1867, 1869, 1871a, b, 1872, 1874).

In his *Theory of the Perception of Colours* (Maxwell, 1856c), Maxwell defined colour as a function of three independent variables, which he identified as *luminance, hue and tint*. Maxwell declared *tint* as a synonym to *purity*, white being the purest colour, contrary to the definition of saturation by other scientists. Maxwell also recognised that composite colour light consists of infinite variables but adopted Young's theory of trichromacy in that there are three elementary sensations in human vision, by combinations of which all the sensations of colours are produced. He accepted *red, green and violet* as the primary sensations and envisioned all sensations of colours being linear combinations of the primary sensations. Maxwell specifically acknowledged Newton, Young, Helmholtz and Grassmann for their contribution,

> We are indebted to Newton for the original design, to Young for the suggestion of a means of working it out, to Helmholtz for a rigorous examination of the facts on which it rests, and to Professor Grassmann for an admirable theoretical exposition of the subject.

In the subsequent *On the Theory of Compound Colours, and the Relations of the Colours of the Spectrum* (Maxwell, 1860), Maxwell reiterated the suggestion of the curve within the colour triangle following his reasoning based upon Newton, Young and Helmholtz but yet did not provide a quantitative analysis. He unfolded his theory and provided a mathematical analysis and an extensive set of experimental results regarding the observations of compound colours and the accuracy of the observations.

> The investigation of the chromatic relations of the rays of the spectrum must therefore be founded upon observations of the apparent identity of compound colours, as seen by an eye either of the normal or of some abnormal type; and the results to which the investigation leads must be regarded as partaking of a physiological, as well as of a physical character, and as indicating certain laws of sensation, depending on the constitution of the organ of vision, which may be different in different individuals. We have to determine the laws of the composition of colours in general, to reduce the number of standard colours to the smallest possible, to discover, if we can, what they are, and to ascertain the relation which the homogeneous light of different parts of the spectrum bears to the standard colours.

Maxwell suggested that his experiments highlighted a resolution to the dispute about yellow being a primary element of colour, against this hypothesis, since his experiments showed that the composition of yellow from red and green is indistinguishable to pure yellow by the observers, and only a prism could expose the difference.

Almost a decade later, Maxwell in his treatise *On Colour Vision* Maxwell (1871a, 1872) summarised the current knowledge (basically his recognition of Newton's and Young's work) and his contribution to the domain. He expressed his alignment with the theory that humans are capable of three different colour sensations, which light of any kind excites in different proportions in order to produce all the varieties of sensed colours. To Maxwell *seemed almost a truism to say that colour is a sensation.* To him *colour is related to human physiology and by no means to the nature of light.* He opens this paper by stating that

> All vision is colour vision, for it is only by observing differences of colour that we distinguish the forms of objects. I include differences of brightness or shade among differences of colour.

He escalates his reasoning by emphatically stating that

> The science of colour must therefore be regarded as essentially a *mental science*. It differs from the greater part of what is called mental science in the large use which it makes of the physical sciences, and in particular of optics and anatomy. But it gives evidence that it is a mental science by the numerous illustrations which it furnishes of various operations of the mind.

Regarding the apparent colour of objects, Maxwell stated that when objects are illuminated by white light, they separate that light into its components absorbing some and scattering others. In addition, using the analogy of the sensation of sound, in which humans are able to recognise the components of a composite sound (analysed into elementary sensations), Maxwell explained that this is not the case with colour, the sensation of (composite) colour being a *single thing* and cannot be decomposed into its elementary sensations (or the sensation of its elementary components). Further, Maxwell affirmed that it is easy to experimentally prove that *the quality of colour can vary in three and only three independent ways*, restating the three independent variables being *hue, tint and shade* (in a slightly different way, as he initially proposed the terms luminance, hue and tint), and emphatically highlighting that if one adjusts one colour to another, so as to agree in hue, tint and shade, the two colours would be absolutely indistinguishable.

Maxwell was not fully satisfied with the experimental results derived from colour wheels so he devised a series of instruments, the ‘light boxes’ (Longair, 2008) or ‘colour boxes’ (MacAdam, 1970) to be able to make accurate measurements on colour perception. A schematic diagram and the principle of operation of Maxwell’s light box is shown in Fig. 4.36. Reference white light is shone onto the top slit as well as three adjustable lights (blue, green and red) at corresponding slits at $\mathcal{B}$. The reference white is guided through the box to the eyes of the observer, through a set of mirrors $\mathcal{C}$. The three adjustable lights are being mixed and also guided through mirrors and prisms to the observer. There, according to the width of the slits at $\mathcal{B}$ the tested composite light can be subjectively compared to the reference light. While the observer reports a difference, the test lights are being adjusted by means of the widths of the corresponding slits. When the observer reports a perfect match, the position of the slits at $\mathcal{A}$ and the width of the slits at $\mathcal{B}$ are used to register a *colour equation*. Maxwell carried out a number of such experiments, which resulted in accurate information about the composition of various colours and provided the first types of modern chromaticity diagrams, which define how different colours can be synthesised from chosen primary colours.

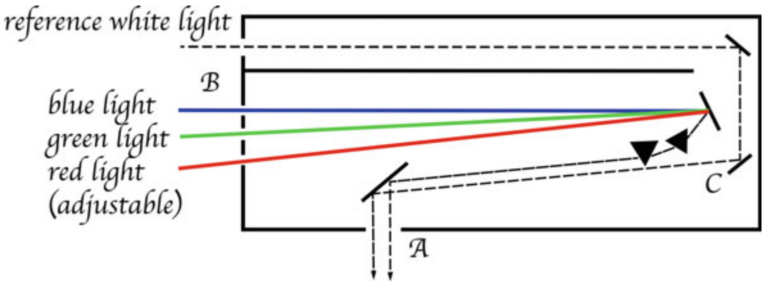
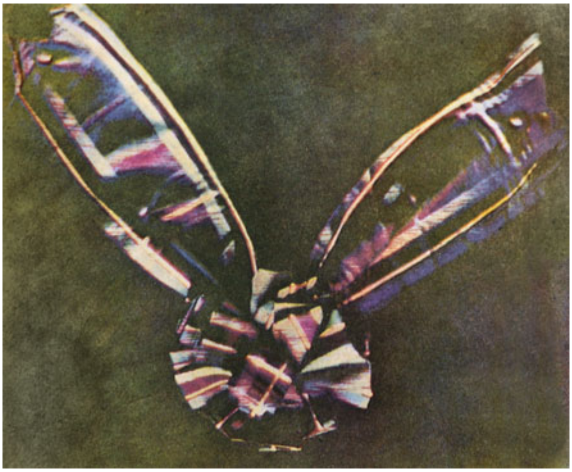


Fig. 4.36 A schematic diagram and the principle of operation of Maxwell's light or colour box

Fig. 4.37 The famous tartan ribbon, the first ever permanent coloured photograph captured by using Maxwell's technique in 1861



It should be noted that in 1861 Maxwell was able to capture *the first permanent coloured photograph* (Fig. 4.37). Maxwell's technique involved the use of three positive photographic plates and three filters (red, green and blue-violet). The three positives that were produced, were projected through the same filters onto a screen and thus combined into a reasonably fully coloured image.

4.17 Ewald Hering

Karl Ewald Konstantin Hering (1834–1918) was born in Alt-Gersdorf, Kingdom of Saxony. He was a German physiologist with a main interest in the physiology of the visual system and colour vision. He proposed what is known as the *opponent colour theory*, which was a significant paradigm shift in comparison to the standard trichromatism accepted by most scientists before him. His work, among other subjects, included studies of binocular vision, hyper-acuity and eye movement. In

1879 he made a significant proposition regarding the visual direction that was later named after him (*Hering's law* or *law of visual direction*), which described the perceived visual direction of a natural object in relation to the observer, having in mind that there are two visual sensors (binocular vision). According to this law, *everything that is in the line of sight of each of the eyes appears mixed in one and only one virtual egocentric direction*. This is like having a single eye, referred to as the *cyclopean eye*, positioned in the middle of the two eyes. Hering published important work on the spatial sensing and the movement of the eyes (E. Hering, 1879), on the binocular vision (E. Hering, 1868), on visual acuity (K. E. K. Hering, 1899; Strasburger et al., 2018) and colour theory (K. E. K. Hering, 1964, 1878, 1920).

In his 1892 *Grundzüge der Lehre vom Lichtsinn* (*Outlines of a Theory of the Light Sense*) (K. E. K. Hering, 1964, 1878, 1920), Hering dismissed the prevailing theory developed by Young-Maxwell-Helmholtz, by which colour vision is based on a model of three-dimensional colour sensing (three primary colours and three types of photoreceptors). He dismissed this theory, primarily based on experiments in colour adaptation and observations in the usage of colour-related linguistic terms; the latter is a very interesting argument, as one may easily accept that there cannot be a whitish-black, or a reddish-green or even a yellowish-blue colour, as it is natural to accept, for example, a yellowish-red, or a greenish-blue, or even a greenish-yellow. Since there are no linguistic terms to define some composite colours, while there is a multitude of such terms for all other colours, this is strong evidence that those particular colours cannot exist. Hence, Hering concluded that colour vision must be based on detection of colour opponency, and particularly the one of red-green, yellow-blue and white-black. This way, he indirectly defined a four-colour system, by proposing that since yellow is not perceived as a red-green mixture (although it can be produced by such a mixture), it should be regarded as one of the colours to define the new opponent model. He found this mechanism to be more efficient, also because it was already suggested by models and evidence that the various receptors in the eye exhibit an overlapping sensitivity (see for example Fig. 4.33).

Hering's *Grundzüge der Lehre vom Lichtsinn* begins with a statement that emphasises the role of colour perception, highlighting that our world of vision consists only of different colours, and the things we see are nothing other than colours of different types and shapes.

> Unsere Sehwelt besteht lediglich aus verschieden gestalteten Farben, und die Dinge, so wie wir sie sehen, d.h. die Sehdinge, sind nichts anderes als Farben verchiedener Art und Form.

He also emphasised the role of perception over sensing, by reminding that the whole world of vision and its content is a creature of the *inner eye*, a name for the complete nervous organ of vision (retina, optic nerve and the related parts of the brain), in contrast to the dioptric apparatus, the *outer eye*. The creative capacity of the inner eye creates these colour structures under the compulsion of the stimuli that receives from the radiation sent into the eye by real external objects.

> Die ganze Sehwelt mit ihrem Inhalt ist ein Geschöpf unseres inneren Auges, wie wir das nervöse Sehorgan (Netzhaut, Sehnerv und die bezüglichen Hirnteile) nennen können, im Gegensatze zu dem dioptrischen Apparat als dem äußeren Auge. Das schöpferische Vermögen unseres inneren Auges schafft jene Farbengebilde unter dem Zwange der Anregungen, welche es durch die von den wirklichen Außendingen in unser Auge geschickten Strahlungen erhält.

Based on this, he clarified that the real world and the world accessible by the senses should be regarded as totally independent. He also made clear that perception of colour is bound by the process of adaptation, thus an object may appear to have a different colour in different viewing conditions. In addition, he noted that colours appear different when perceived by different parts of the retina, and particularly in central and peripheral vision.

Reproductions of Hering's colour circles are shown (a) in Fig. 4.38, in which the four primary colours are shown, along with their interaction and contribution to the production of all other colours, and (b) in Fig. 4.39, in which some colours are drawn in a classic colour circle corresponding to Hering's model, where no colour is clearly reddish and greenish, nor yellowish and bluish at the same time, red and green are just as mutually exclusive as yellow and blue.

> Keine Farbe ist deutlicherweise rötlich und grünlich, keine gelblich und bläulich zugleich, Röte und Grüne schließen sich ebenso aus wie Gilbe und Bläue.

Regarding the white-black opponent pair, he suggested that although this pair has a natural gradation from one to the other through the scale of grey, there is no such a gradation for the other opponent pairs, yellow-blue and red-green, which need to fade completely to grey in order to go from one to the other. It is there that Hering defined the opponent colours (*die Gegenfarben*), as the mutually exclusive pairs that cannot, in any "normal" circumstances, appear at the same time.

> Da also Röte und Grüne, bezw. Gilbe und Bläue in keiner Farbe gleichzeitig deutlich sind, sich vielmehr gegenseitig auszuschließen scheinen, habe ich dieselben als Gegenfarben bezeichnet. Hiermit soll zunächst lediglich die Art ihres Vorkommens gekennzeichnet sein ohne jede Beziehung auf irgendwelche Erklärung.

In his colour circle (Fig. 4.39), any pair of diametrically opposite colours (yellowish-red and bluish-green) should be considered doubly opponent, in contrast to any

Fig. 4.38 Reproduction of Hering's colour circle with opponent colours

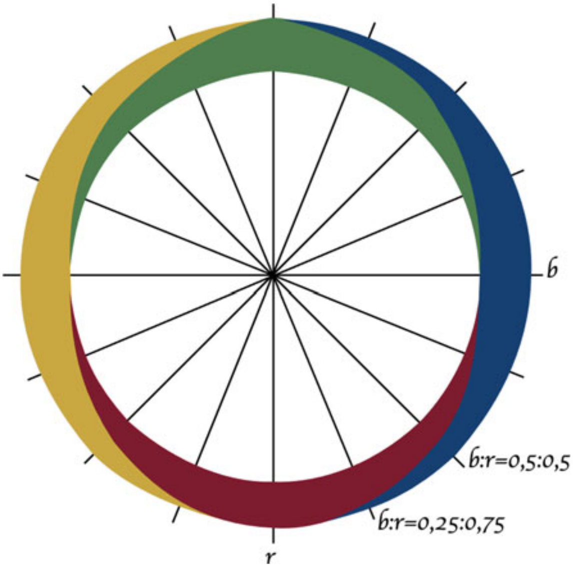


Fig. 4.39 Reproduction of Hering's colour circle

colour pairs symmetrical around a primary colour (reddish-yellow and greenish-yellow), which should be considered singly opponent.

Hering found it most interesting that there are those particular pairs of colours (the opponents), for which there cannot be any intermediate mixtures and concluded this should be hardwired in visual perception, leading to his opponent colour perception theory.

> Es erscheint von vornherein höchst auffällig, dass es z.B. zwischen Rot und Grün nicht ebenso eine Reihe bunter Zwischenfarben giebt, wie zwischen Rot und Gelb oder zwischen Rot und Blau, dass es also keine Farben giebt, welche uns in ähnlicher Weise zugleich rötlich und grünlich erscheinen, wie das Orange zugleich rötlich und gelblich oder das Grau zugleich weißlich und schwärzlich. Wir dürfen daraus schließen, dass im inneren Auge ein physiologischer Prozess, dessen psychisches Korrelat von gleichzeitig deutlicher Röte und Grüne bezw. Gilbe und Bläue wäre, entweder überhaupt nicht oder nur unter ganz besonderen, ungewöhnlichen Bedingungen möglich ist.

Hering was also interested in the non-linear response of vision and he tried to gather data to support a model for this phenomenon. Figure 4.40 shows a reconstruction of his plot of perceived brightness against luminance, in which the two curves correspond to a medium grey at 1/2 and 1/3 of the $W - S$ distance (black-to-white normalised perceived brightness scale). Clearly, the perceived brightness increases rapidly with little luminance increase at low luminance conditions, and then slowly and asymptotically converges to maximum brightness.

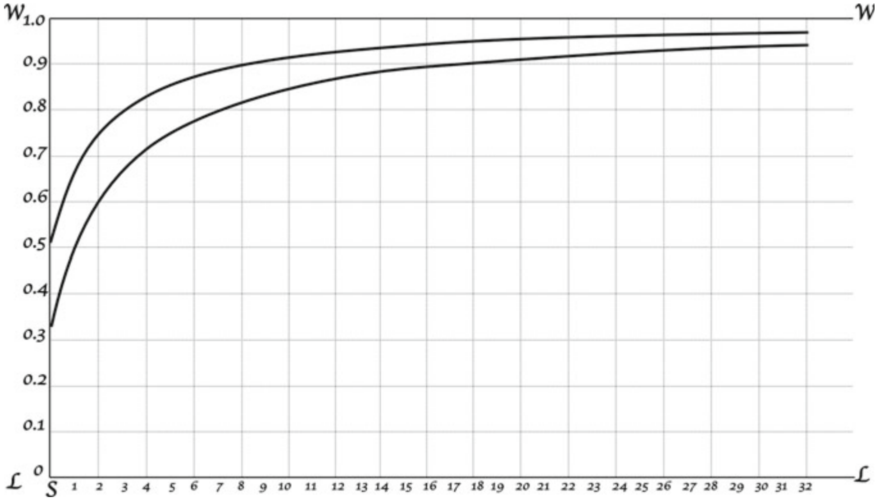


Fig. 4.40 Reproduction of Hering's plot of perceived brightness against luminance

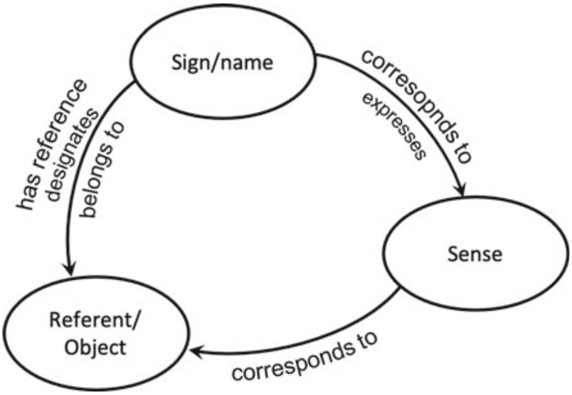
4.18 Gottlob Frege

Friedrich Ludwig Gottlob Frege (1848–1925) was born in Wismar, Grand Duchy of Mecklenburg-Schwerin, German Confederation. He was a philosopher, logician and mathematician. He is considered to be the father of analytic philosophy in language, logic and mathematics. Interestingly, he was not so famous in his lifetime and his work became known later through other eminent thinkers, like Bertrand Russell (1872–1970) and Ludwig Wittgenstein (1889–1951), and is considered to be among the greatest mathematical philosophers and logicians, on the scale of Aristotle. He left very important writings, and although not directly connected to the topic of this treatise, he still deserves to be included due to the indirect implications of his theories.

His most famous works include the 1879 *Begriffsschrift, Eine der Arithmetischen Nachgebildete Formelsprache des Reinen Denkens* (*Begriffsschrift, a Formula Language, Modeled upon that of Arithmetic, for Pure Thought*) (Frege, 1879), the 1884 *Die Grundlagen der Arithmetik* (*The basics of arithmetic*) (Frege, 1884), the 1892 *Über Sinn und Bedeutung* (*On Sense and Reference*) (Frege, 1892), the 1893 *Grundgesetze der Arithmetik* (*Basic laws of arithmetic*) (Frege, 1893) and the most influential 1918 *Der Gedanke: Eine logische Untersuchung* (*The thought: a logical investigation*) (Frege, 1918). Frege, in his attempt to turn mathematics into an application of logic, he invented a new symbolic language to describe mathematics based on pure logic, which he named *Begriffsschrift* (conceptual notation, or ideography). It was like inventing a programming language in more modern terms.

Frege's work that is most relevant to a theory of visual perception is the 1892 *Über Sinn und Bedeutung* (*On Sense and Reference*) (Frege, 1892; McCarty et al., 2000). In this treatise, Frege made extensive use of linguistics and philosophy to present the basic threefold nature of any object in human perception (Fig. 4.41). His theory distinguishes an object (reference) to its name (sign) and its cognitive content (sense), with extensions from simple names to sentences and the notion of the truth. Practically, in his theory, Frege accepts the objectivity of an entity by means of

Fig. 4.41 Frege's famous Sign-Sense-Reference



commonly shared subjective senses, like considering something to be objectively true if the senses of multiple observers agree. On the other hand, the sense is a purely subjective experience and only due to strife for the truth, sense reaches out to find a reference.

Frege clarified even further his philosophical views in his 1918 *Der Gedanke: Eine logische Untersuchung* (*The thought: a logical investigation*) (Frege, 1918). There he differentiated senses from objects and ideas, somehow reviving the Platonic world of ideas (ideals). He discussed the paradox of the meaning of a colour name when comparing the sensation of colour between a person with normal vision and a colour-blind. To the colour-blind person, the green strawberry plant leaves and the red strawberries will appear in the exact same colour. Then, Frege wonders what the name of that colour might be when the normal vision person already names the leaves green and the strawberries red.

> Mein Begleiter und ich sind überzeugt, daß wir beide dieselbe Wiese sehen; aber jeder von uns hat einen besonderen Sinneseindruck des Grünen. Ich erblicke eine Erdbeere zwischen den grünen Erdbeerblättern. Mein Begleiter findet sie nicht; er ist farbenblind. Der Farbeneindruck, den er von der Erdbeere erhält, unterscheidet sich nicht merklich von dem, den er von dem Blatt erhält. Sieht nun mein Begleiter das grüne Blatt rot, oder sieht er die rote Beere grün? oder sieht er beide in einer Farbe, die ich gar nicht kenne? Das sind unbeantwortbare, ja eigentlich unsinnige Fragen.

For Frege, thought is the glueing factor between the world and its perception. The reception of sense impressions is not sufficient for seeing things. He makes clear that in the perception of a tree, physical, chemical, and physiological processes slip between the tree and the imagination (or thought). However, only processes in the nervous system are directly related to consciousness; and every observer of the tree has an individually special process in a special (or unique) nervous system.

> Zwischen den Baum und meine Vorstellung schieben sich physikalische, chemische, physiologische Vorgänge ein. Mit meinem Bewußtsein unmittelbar zusammen hängen aber, wie es scheint, nur Vorgänge in meinem Nervensystem; und jeder Beschauer des Baumes hat seine besonderen Vorgänge in seinem besonderen Nervensystem.

Most interestingly, Frege concluded that any observer may only be conscious of the end process of perception and cannot make objective connections of perceived impressions with independent stimuli that produce the sensory impressions in the first place.

> Wir glauben, daß ein von uns unabhängiges Ding einen Nerv reize und dadurch einen Sinneseindruck bewirke; aber genau genommen, erleben wir nur das Ende dieses Vorganges, das in unser Bewußtsein hereinragt.

4.19 Johannes von Kries

Johannes Adolf von Kries (1853–1928) was born in Freiburg, Germany, He was a physiological psychologist, particularly interested in the neural mechanisms of perception. He was the founder of the *duplicity theory* of vision, in which two types of photoreceptors are responsible for vision; these are the rods (or rod cells) that are efficient in low light conditions and the cones, which are three types of cells efficient at brighter lighting conditions. Apart from his main interest in the mechanism of visual perception, he made notable contributions to the foundations of the theory of probability. Von Kries is considered to be Helmholtz's warmest advocate (Turner, 1994).

Von Kries' relevant contribution can be found in two publications, (a) the 1878 *Beitrag zur Physiologie der Gesichtsempfindungen* (*Contribution to the physiology of visual sensations*), included in Volume I & II of the *Archiv für Anatomie und Physiologie*, edited by Wilhelm His, Wilhelm Braune and Emil Du Bois-Reymond (Kries, 1878) and (b) the 1905 *Die Gesichtsempfindungen* (*The visual sensations*), included in Volume III (Physiology of the Senses) of the *Handbuch der Physiologie des Menschen*, edited by Wilibald Nagel (von Kries, 1905).

In Kries (1878), he stated that one of the most important tasks for a theory of visual sensations is to explain why the diversity of our sensations is much smaller than that of light stimuli. While the latter can be varied in infinite abundance, the variety of visual sensations is only threefold. This can be formulated as follows: if any visual sensation can be produced by the action of α, β, γ quantities of the three primary types of light A, B, C, then every possible continuous change of the sensation can be produced by the constant change in the quantities α, β, γ.

> Für die Theorie der Gesichtsempfindungen ist es eine der wichtigsten Aufgaben, zu erklären, warum die Mannichfaltigkeit unserer Empfindungen eine viel geringere ist als die der Lichtreize. Während die letzteren in unendlicher Fülle variirt werden können, ist die Mannichfaltigkeit der Gesichtsempfindungen, wie man zu sagen pflegt, eine nur dreifach ausgedehnte. Den Satz, auf welchen es hier ankommt, können wir so formuliren: Wenn eine beliebige Gesichtsempfindung hervorgebracht werden kann durch die Einwirkung einer Mischung der Quantitäten α, β, γ, der

> drei einfachen Lichtarten A,B,C, so kann jede überhaupt mögliche continuirliche Aenderung der Empfindung hervorgebracht werden durch die stetige Aenderung der Quantitäten α, β, γ.

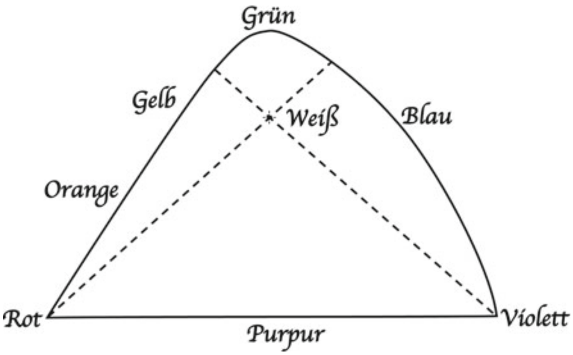
Von Kries affirmed that the two most important theories of visual sensation during that period, advanced by Thomas Young, renewed by Helmholtz and Maxwell, and those proposed by Hering, all agreed that sensation is determined by the values of three independent variables, which are supposed to consist in the various intensities of a limited number of simple nervous processes.

> Die beiden gegenwärtig bedeutendsten Theorien der Gesichtsempfindungen, die von Thomas Young aufgestellte, (von Helmholtz und Maxwell erneuerte), und die kürzlich von E. Hering gegebenemachen zur Erklärung dieser Thatsache übereinstimmend die Annahme, dass die Empfindung bestimmt sei durch die Werthe dreier unabhängig veränderlicher Functionen, welche bestehen sollen in den verschiedenen Intensitäten einer beschränkten Anzahl einfacher nervöser Vorgänge. Wenn auf jede dieser drei Functionen drei verschiedene Lichter einen verschiedenenEinfluss üben, so erklärt sich daraus unmittelbar der obige Satz.

In von Kries (1905), von Kries provided a concise theory of visual sensation. He began his treatise by providing background knowledge on the subject and by analysing the laws of colour mixing, with references to Newton and Grassmann. He also provided a representation of the colour chart of his time (Fig. 4.42), as the already established curved diagram of the spectral colours connected with the horizontal line of purples.

Given a mixture of, say greenish-blue (Gbl) and red (R) one may generate the same sensation with a mixture of green (Gr) and violet (V), which may result in an interesting mathematical formulation.

Fig. 4.42 Reproduction of Von Kries' colour chart



$$\begin{aligned} \alpha Gbl + \beta R &= \gamma Gr + \delta V \Rightarrow \\ \alpha Gbl &= \gamma Gr + \delta V - \beta R \end{aligned} \tag{4.10}$$

By using the second formulation, it is shown that, provided one accepts negative values, all sensations can be produced by mixtures of three lights.

Von Kries moved on to the study of complementary colours, beginning with mixtures of red and green. He adopted the definition by which complementary should be called the colours that when mixed in certain proportions result in colourless (white) sensation, and adopted and reiterated Helmholtz's table of complementary colours and graphical representations. Furthermore, von Kries provided his own table of complementary colours. On this, he briefly reviewed the Young-Helmholtz trichromacy theory, which he accepted. Nevertheless, he also confirmed the validity of Hering's opponent colour theory, which he described and commented, regarding it as an embodiment of a four-colour theory.

> Es ist, um Mißverständnisse zu vermeiden, wichtig, sie auseinander zu halten von der weit allgemeineren, oben als Vierfarbentheorie bezeichneten Anschauung, welche letztere, wenn sie mit dem Namen eines bestimmten Autors in Verbindung gebracht werden soll, wohl am ehesten an den Auberts zu knüpfen wäre. Die Theorie Herings ist eine auf gewisse allgemein biologische Vorstellungen gestützte Ausgestaltung der Vierfarbentheorie.

Von Kries was largely interested in the light adaptation characteristics of vision. He made a distinction between day-time vision (*Tagessehen*) and night-time vision (which he called the twilight vision–*Dämmerungssehens*) and studied the limits of those types of vision. He experimented with colour-blind people to identify the sensitivity of the eyes and produced light sensitivity curves for night-time vision, a reproduction of which is shown in Fig. 4.43; the graph shows the perceived brightness for various wavelengths of light for the case of night-time vision of a normal observer (solid line) and for the case of a colour-blind (dashed line), under the illumination of gaslight. This observation led him to support the dual sensor vision (cones and rods) and also the duplicity theory of vision. Since colour-blind people can perceive light of various wavelengths (which is the decisive feature for colour perception), the rods, which are active during night-time vision (he called them the twilight organs–*Dämmerungsorgane*), should not provide any information regarding the colour representation of light. This, he added, was also supported by other experiments regarding the visual acuity and response time to fast-changing stimuli. Von Kries broaden his study by examining the impact of the position in the field of view on the perception of colour (linked with the position of the cones and rods in the retina) and concluded that, as objects move away from the centre of the field (in any direction), colour is lost to a point that all objects appear in shades of grey in the extreme periphery. Another interesting graph reproduction of von Kries' original sensitivity curves is shown in Fig. 4.44, in which he showed the brightness percep-

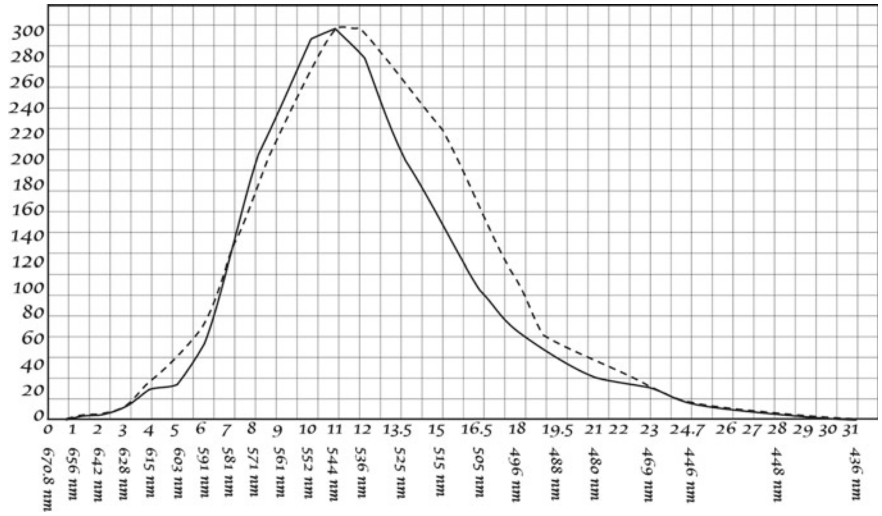
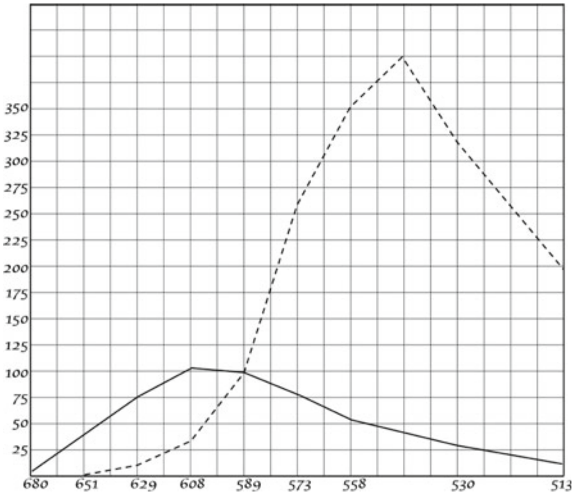


Fig. 4.43 Reproduction of Von Kries' light sensitivity curves

Fig. 4.44 Reproduction of Von Kries' light sensitivity curves



tion against the wavelength for either the light-adapted peripheral vision (solid line) or the normal night-time vision (dashed line).

One of von Kries' most important contributions is in the field of *adaptation*. As the term suggests, adaptation is the ability of the retina to adjust its sensitivity to be able to function efficiently in various lighting conditions (practically, by redefining the bases of what should be considered to be 'black'). In accordance with the duplicity theory that von Kries supported, adaptation takes place as a sensitivity transition from rods to cones (and vice versa) and within each of the two photoreceptors to around nine (9) orders of magnitude. What von Kries suggested in *Der Koeffizientensatz* (the set of coefficients) (von Kries, 1905) was that the process of adaptation is somewhat linear and depends on a coefficient per photoreceptor (with values in [0, 1]).

> Es liegt nämlich nahe, anzunehmen, daß, soweit die Wirkung äußerer Reize in Frage kommt, die Stimmung, sei es des Sehorgans in toto, sei es einzelner Bestandteile, sich als eine größere oder geringere Erregbarkeit gegenüber jenen Reizen geltend machen wird, und zwar so, daß der Erfolg sich immer etwa nach einem Produkt α R richtet, wo R den Reizwert, α aber die für diesen Erfolg bestehende Disposition oder die für diese Reizart vorhandene Erregbarkeit bezeichnen würde.

Practically, this means that for three photoreceptors, sensitive in red (R), green (G) and blue (B) light (R, G, B being the stimuli), there are three coefficients α, β, γ, which regulate the response of the retina. If R_r, G_r, and B_r are sensitivities under a reference illuminant then this relationship is expressed in (4.11).

$$\begin{aligned} R_r &= \alpha R \\ G_r &= \beta G \\ B_r &= \gamma B \end{aligned} \tag{4.11}$$

The coefficients α, β, and γ are the *von Kries coefficients* and correspond to the reduction in sensitivity of the three-cone mechanisms due to chromatic adaptation (relative to reference sensitivity) and are constant for all pairs of corresponding colours. This, in effect, means that any colour sensation will remain the same under various lighting conditions and is another expression of what is known as *colour constancy*. Von Kries expressed this linear relationship by stating another typical linearity criterion.

> Es müßte nämlich dann, wenn L_1 auf einer Netzhautstelle den gleichen Erfolg auslöst wie L_2, an einer anderen, und ebenso M_1, auf die erstere wirkend, den gleichen Effekt wie M_2 an der anderen, jedesmal auch $L_1 + M_1$ hier die gleiche Wirkung haben müssen wie $L_2 + M_2$ dort.

If L_1 triggers the same effect on one site of the retina as L_2 on another, and also M_1, acting on the former, it would have to have the same effect as M_2 on the latter, then each time $L_1 + M_1$ must also have the same effect on the first site as $L_2 + M_2$ on the second. This is a rather significant statement and this is what von Kries called the *Coefficient Theorem* (*Koeffizientensatz bezeichnen*). Recognising the approximating nature of this theory, von Kries mentioned that this is an approximate theory and cannot be verified in all experiments. Nevertheless, it has been extensively used in practical applications and particularly in digital camera technology for the process of estimating the white balance.

4.20 Arthur König

Arthur Peter König (1856–1901) was born in Krefeld, a city in North Rhine-Westphalia, Germany. His life work was on physiological optics. He studied under Hermann von Helmholtz, and eventually became his assistant. His work on optics was rather fruitful with published papers of significant importance, among which the works co-authored with Conrad Dieterici *Die Grundempfindungen und ihre Intensitäts-Vertheilung im Spectrum* (*The fundamental sensations and their sensitivity distribution in the Spectrum*) (König & Dieterici, 1886) and *Die Grundempfindungen in normalen und anomalen Farbensystemen und ihre Intensitätsverteilung im Spektrum* (*The fundamental sensations in normal and abnormal colour systems and their sensitivity in the spectrum*) (König & Dieterici, 1892). This work is of particular interest since it is a foundation work on the sensitivity of the human rod and cone visual system, which improved the earlier measurements by Maxwell, using more advanced equipment and procedures. Moreover, this work made it possible to propose a theory about defects in vision (dichromacy and colour blindness) and their connection with the absence of cone types in the retina. The data resulted by König's work were replaced some 30–40 years later by the more accurate data by John Guild and William David Wright, which eventually became the foundation of the modern CIE colour system. König, apart from his own work, he left a significant editorial work among which the *Beiträge zur Psychology und Physiologie der Sinnesorgane* after Helmholtz.

König and Dieterici (1892) made extensive measurements for the determination of the qualitative and quantitative nature of *white light*, for example with high-resolution relative estimates of the sunlight in relation to gaslight in a wide range of wavelengths. In addition, they performed estimates of the dispersion and interference spectrum of the gaslight and provided detailed illustrations of the visual sensation in monochromatic and dichromatic colour systems. They called their sensitivity curves the *elementary sensation curves* produced by several researchers. König and Dieterici made detailed measurements of the complementary colours under sunlight and gaslight and a number of experiments on the determination of the visual sensitivity by means of the complementary colours. They were among the very first to provide trustworthy estimates of the colour sensitivity curves, which are shown in Fig. 4.45; the graph presents their elementary RGV (V for violet) sensation curves, which were created by using their measurements (in Tables XVI and XVII) for the case of sunlight. The measurements were interpolated (typical spline interpolation) for a smoother presentation. The bold curves correspond to König's measurements, whereas the lighter curves correspond to Dieterici's measurements, for normal trichromatic vision.

According to their theory, basic sensations are a product of elementary sensations in a linear manner. Thus, the basic sensations $\mathfrak{R}$, $\mathfrak{G}$, $\mathfrak{B}$ for a normal trichromatic colour system can be defined (in their original notation) as

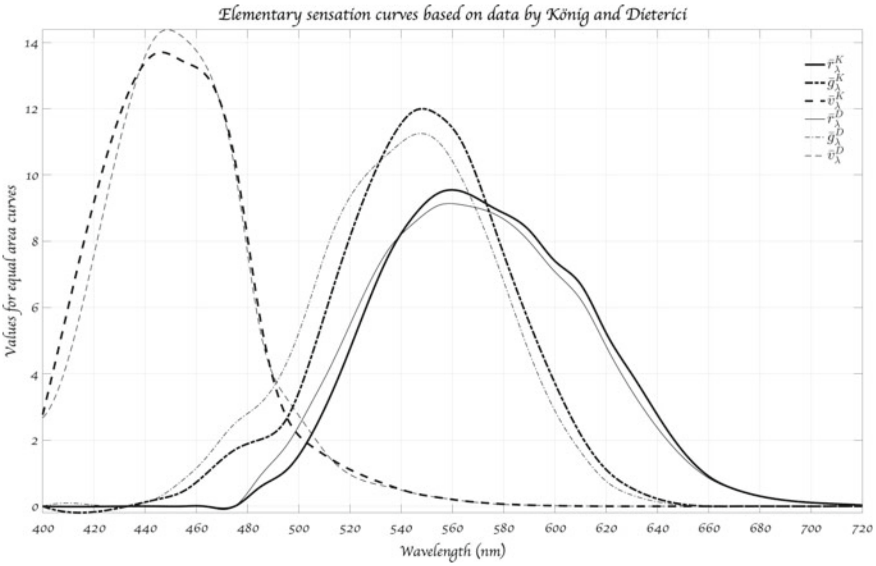


Fig. 4.45 Elementary sensation curves based on data by König and Dieterici

$$\begin{aligned}\mathfrak{R} &= a' \cdot R + b' \cdot G + c' \cdot V \\ \mathfrak{G} &= a'' \cdot R + b'' \cdot G + c'' \cdot V \\ \mathfrak{B} &= a''' \cdot R + b''' \cdot G + c''' \cdot V\end{aligned} \tag{4.12}$$

They also proposed the normalised form

$$\begin{aligned}\mathfrak{R} &= \frac{a' \cdot R + b' \cdot G + c' \cdot V}{a' + b' + c'} \\ \mathfrak{G} &= \frac{a'' \cdot R + b'' \cdot G + c'' \cdot V}{a'' + b'' + c''} \\ \mathfrak{B} &= \frac{a''' \cdot R + b''' \cdot G + c''' \cdot V}{a''' + b''' + c'''}\end{aligned} \tag{4.13}$$

again, for normal trichromacy, and proposed the corresponding coefficient values as follows

$$\begin{array}{lll} a' = 1 & b' = -0.15 & c' = 0.1 \\ a'' = 0.25 & b'' = 1 & c'' = 0 \\ a''' = 0 & b''' = 0 & c''' = 1 \end{array} \tag{4.14}$$

They went further and provided a definition of the standard observer (normal trichromat) from their data. Figure 4.46 shows the normal observer curves that can be created by interpolation of their sparse data. In addition, they were able to use

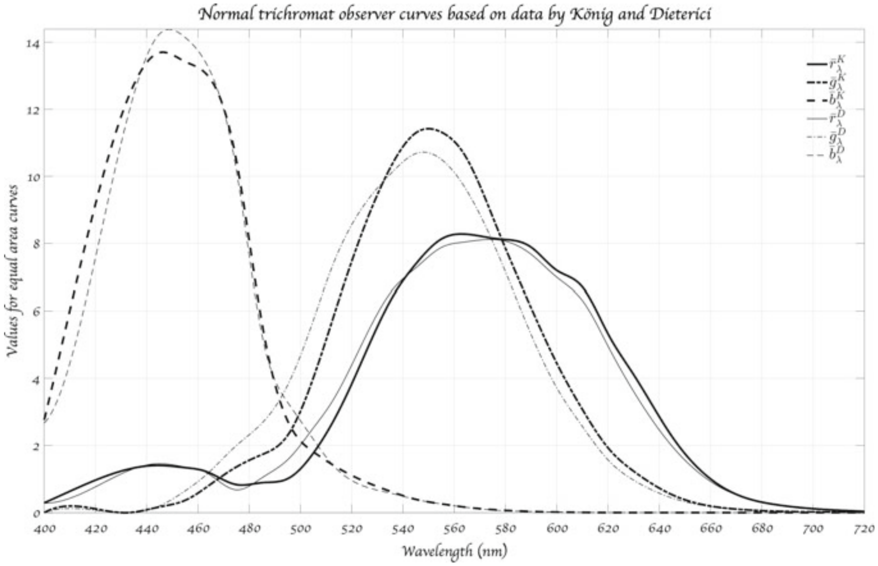
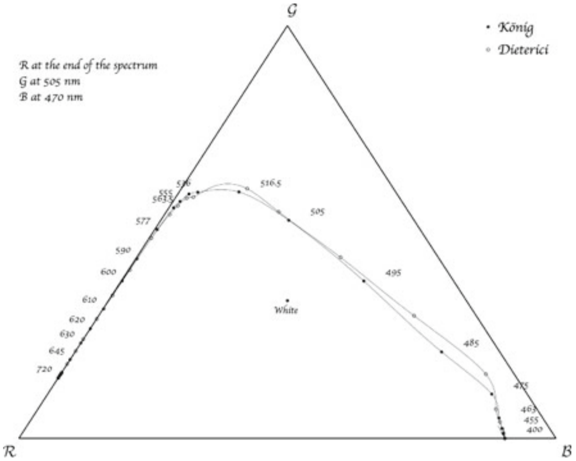


Fig. 4.46 Normal trichromat observer curves based on data by König and Dieterici

Fig. 4.47 König and Dieterici colour space in the classic colour triangle



those data and place the position of the various wavelengths of monochromatic light into the typical R-G-B colour triangle, as shown in Fig. 4.47.

Last but not least, *they confronted Hering's opponent theory* by stating that the optic nerves should somehow resemble the motor nerves in that they either rest or become excited and do not exhibit antagonistic behaviour.

4.21 Ramón y Cajal

Santiago Ramón y Cajal (1852–1934) was born in Petilla de Aragón, Navarre, Spain. He was a neuroscientist, pathologist, and histologist specialising in neuroanatomy and the central nervous system, with original investigations of the microscopic structure of the brain that made him a pioneer (actually the father) of neuroscience.

It was the year 1887 that Ramón y Cajal was awarded a professorship in Barcelona, where he got accustomed to Golgi's staining method that made it possible to visualise neural tissue.[31] He worked and even improved this method, as it became central to his work. This unlocked the potential to investigate the elusive (at that period) structure of the central nervous system. The result was an extensive mass of detailed hand-made drawings of neural tissue that depicted the *arborisations* of neural cells, covering many species and parts of neural systems.

Since 1880 he had been publishing important scientific works, among which his 1899 *Textura del Sistema Nervioso del Hombre y de los Vertebrados* (*Textbook on the nervous system of man and the vertebrates*) (Ramón y Cajal, 1899) is considered to be his Opus Magnus. Ramón y Cajal realised that to unravel the mysteries of the nervous system, and particularly the brain, is to always look at the big picture, the system as a whole. In addition, he supported the theory that a fundamental characteristic of any brain is the presence of organised neural circuitry (Llinás, 2003). His trend towards the integration into a global framework is particularly evident in the *Textura del Sistema Nervioso del Hombre y de los Vertebrados*. Among the most important proposals was Cajal's suggestion that the organisation of the central nervous system was the result of natural optimisation, by which the system saves space, time and material (*el ahorro del espacio, el ahorro del material, el ahorro del tiempo*). A compact presentation of his work can be found in his 1894 *The Croonian lecture-La fine structure des centres nerveux* (Ramón y Cajal, 1894a), whereas other important publications include his 1894 French edition *Les nouvelles idées sur la structure du système nerveux chez l'homme et chez les vertébrés* (*The new ideas on the structure of the nervous system in humans and vertebrates*) (Ramón y Cajal, 1894b) and the 1909 and 1911, also French editions of the *Histologie du système nerveux de l'homme & des vertébrés*, Tome I & II (*Histology of the nervous system of man & vertebrates*, Volumes I & II) (Ramón y Cajal, 1909, 1911). Cajal published a large volume of scientific articles on the fine structure of the nervous system in both French and Spanish. Of particular interest for the purposes of this treatise is his work towards the understanding of the structure and function of the nervous system that is relating to vision.

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[31] At that time, there were no staining techniques usable for the study of the nervous tissue. Staining techniques use special substances to cause the increase of the contrast among various tissue types, largely useful for optical inspection under the microscope. Golgi, around 1873, discovered a method suitable for staining nervous tissue in black, which he called *la reazione nera* (black reaction), which is typically called Golgi's method or Golgi's staining. This was a major breakthrough in neuroscience that made neural tissue visualisation possible under the microscope.

In the 1894 *Croonian Lecture* (Ramón y Cajal, 1894a), Cajal provided a detailed diagram of the retina of the human eye and a very brief and compact definition of the retina,

> On peut, malgré sa complication, considerer la rétine comme un ganglion nerveux formé par trois rangées de neurones ou de corpus–cules nerveux; la première rangée renferme les cônes et les bâtonnets avec leurs prolongements descendants formant la couche des grains externes; la seconde est constitutée par les cellules bipolaires, et la troisième est due à la réunion des corpuscules ganglionnaires. Ces trois séries d'éléments s'articulent au niveau des couches dites moléculaires ou réticulaires et internes.

He tried to simplify the description of the complicated structure by stating that the retina can be considered as a nervous ganglion formed by three rows of neurons or nervous corpuscles, including the cones and the rods with their descending extensions forming the layer of the outer grains, the layer of the bipolar cells, and the layer of the union of the ganglionic corpuscles. Figure 4.48 shows a graphical representation of a part of the human retina, based on the original drawing published as Fig. 4 in Ramón y Cajal (1894a). In this representation, according to Cajal's description, *A* denotes the cones from the region of the fovea centralis, *B* denotes the outer grains of this region, *C* denotes the articulation between bipolar cells and cones, *D* denotes the articulation between bipolar cells and ganglion cells. In addition, *a* and *b* are the cones and rods from various regions of the retina, *e* are the bipolar cells intended for cones, *d* are the bipolar cells connected with rods, *e* are the ganglion cells, *f* are spongioblasts, *g* denotes the centrifugal fibre, *h* the optic nerve, *i* the terminate arborisations of optical fibres in geniculate bodies, *j* are the cells that receive the visual impression and *m* are the cells from which the centrifugal fibres probably originate.

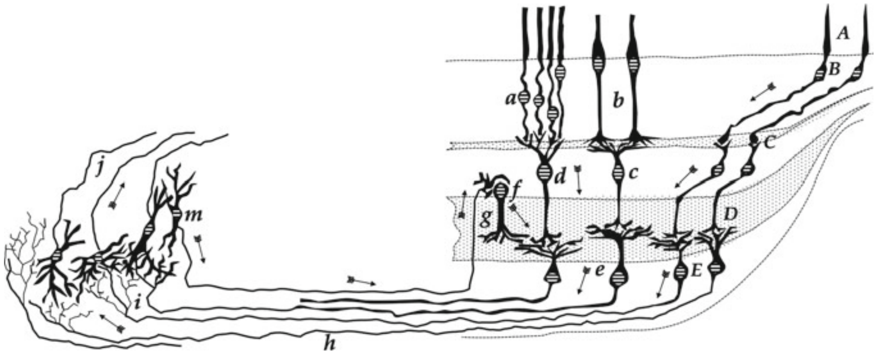

Fig. 4.48 Reproduction of Ramón y Cajal's schematic diagram for all the cells in the visual system

In the 1894 French edition *Les nouvelles idées sur la structure du système nerveux chez l'homme et chez les vertébrés* (Ramón y Cajal, 1894b), there is a whole chapter dedicate to the retina (VI.–RÉTINE). At the beginning of the chapter, Cajal makes it clear that the nervous elements of the retina are arranged in seven layers (counting the limiting membranes and the pigmentary zone),

1. Rods and cones
2. The outer grains or bodies of visual cells
3. The outer plexiform or molecular layer
4. Internal grains
5. The internal plexiform or molecular layer
6. Ganglion cells
7. Fibres of the optic nerve

All these elements are supported and isolated by large cells directed back and forth, from the outer surface of the retina to the area of cones and rods, cells that have been called Müller's fibres or retinal epithelial cells. Figure 4.49 shows a graphical representation of Cajal's figure for the retina of a mammal. These layers are apparent in this drawing which denotes *A* as the layer of cones and rods, *B* as the body of visual cells (outer grains), *C* as the outer plexiform layer, *E* as the layer of bipolar cells (internal grains), *F* as the internal plexiform layer, *G* as the layer of ganglion cells and *H* as the layer of optic nerve fibres; *a* denotes the rods, *b* the cones, *c*

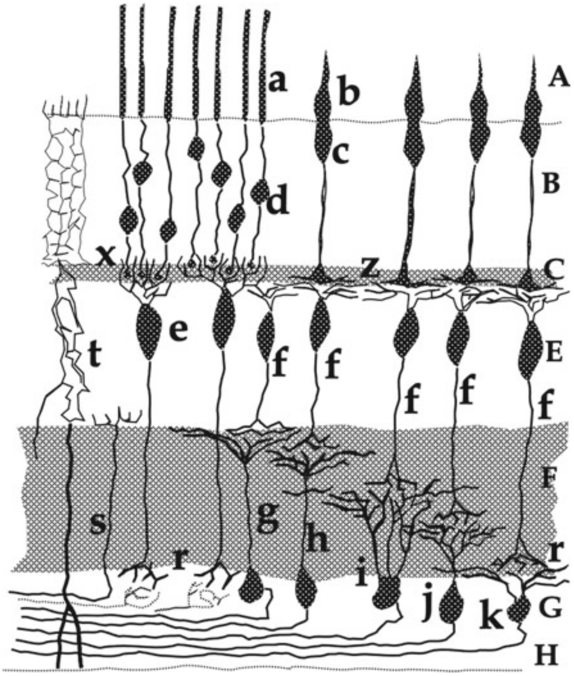

Fig. 4.49 Reproduction of Ramón y Cajal's cross section of the retina of a mammal

the body of the cone cell, *d* the body of the rod cell, *e* the bipolar cells for rods, *f* the bipolar cells for cones, *g*, *h*, *i*, *j*, *k* the branched ganglion cells in the various stages of the internal plexiform zone, *r* the lower arborisation of bipolar rod cells, in connection with the ganglion cells, *r* the lower arborisation of bipolar cells for cones, *t* the Müller cells or epithelial, *x* the contact between the rods and their bipolar cells, *z* the contact between the cones and their bipolar cells and *s* the centrifugal nerve fibre.

After presenting in detail the fine structure of the retina, Cajal proceeded with a description of its function. He was convinced that rods and cones collect light and two distinct pathways carry the information of those receptors, one for the colourless luminous intensity by the rods and one of the colour information by the cones. The visual impressions begin with the cones and rods and the signals that arise are transmitted by neural axis cylinders and are distributed by arborisations of nerve fibres. There are multiple connections among cones, bipolar cells and ganglion cells, with only an exception in the central fovea, where each cone is in contact with a single bipolar, which in turn is in contact with a limited protoplasmic arborisation of ganglionic corpuscles. Cajal connects this anatomical characteristic with the increased visual acuity in central vision. On the other hand, horizontal cells appear to bridge distinct regions of the retina including cones and rods.

Further details in the histological findings of the retina and the visual system can also be found in the 1909 and 1911 French editions *Histologie du système nerveux de l'homme & des vertébrés*, Tome I & II (*Histology of the nervous system of man & vertebrates*, Volumes I & II) (Ramón y Cajal, 1909, 1911) and particularly in Volume II.

Ramón y Cajal and Camillo Golgi (Italian biologist, 1843–1926, the inventor of the homonymous staining method) received the Nobel Prize in Physiology or Medicine in 1906.

4.22 Stephen Polyak

Stephen Lucian Polyak (1889–1955) was born Stjepan Lucian Poljak in Đurđevac (or Gjurgjevac), Austria-Hungarian Empire, later Croatia. After an adventurous early life, in 1928 he permanently moved to the United States and became a Professor of Neurology and Neuroanatomy in California and Chicago. Polyak is considered to be one of the most prominent neuroanatomists of the 20th century. Among his numerous contributions in the field, he gave a new interpretation of the basic visual processes by intensively studying the retina and revealing the role of the various cells in the retina organisation and their connectivity (Polyak, 1941, 1949, 1970). It should be noted that till that time, the prevailing theory was that rods and cones alone are responsible for colour vision (Triarhou, 2007).

Polyak was critical of the fact that theories of vision at that period focused on cones and rods alone as the building blocks of vision. He showed the retina as a complete complex instrument in which every type of cell plays its particular role in

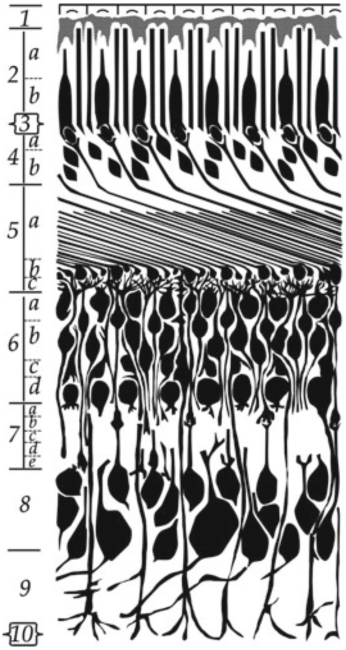

Fig. 4.50 Reproduction of Polyak's schematic presentation of a vertical section of a human retina near the fovea

the composition of the signal that is transmitted to the brain. A brief and compact account of his theory can be found in Polyak (1949). He was particularly fond of using the Golgi staining method to attain and study the neuronal morphology of the retina. A visualisation of a part of the human retina near the fovea is shown in the Fig. 4.50, a graphical reconstruction of Polyak's original figure. The drawing reveals the basic structural elements of the human retina, which consists of layers (in numbers) and sub-layers (in letters), numbered as follows: (1) pigment layer, (2) rods and cones, (3) outer limiting membrane (4) outer nuclear layer, (5) outer plexiform layer, (6) inner nuclear layer, (7) inner plexiform layer, (8) ganglion cell layer, (9) optic nerve fibres and (10) inner limiting membrane. The top of the diagram represents the layers in the outer region of the eye. Polyak reminded that all available evidence showed that only layer (2), the rods and cones, seem to be responsible for sensing light and creating a response that propagates to the brain as nerve impulses.

Polyak further examined the central region of the retina, the central fovea, which is depicted in a graphical representation of his original figure in Fig. 4.51; in the central fovea, the cones are practically directly exposed to lights, whereas the other layers are displaced. He hypothesised that the specific structure along with the density and size of the cones in this region (and also the absence of rods) results in acute central colour vision.

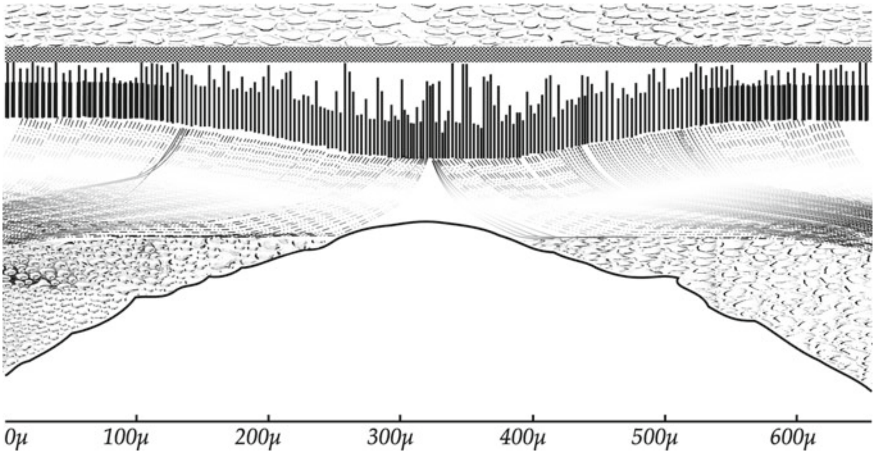

Fig. 4.51 Reproduction of Polyak's structure of the human central fovea

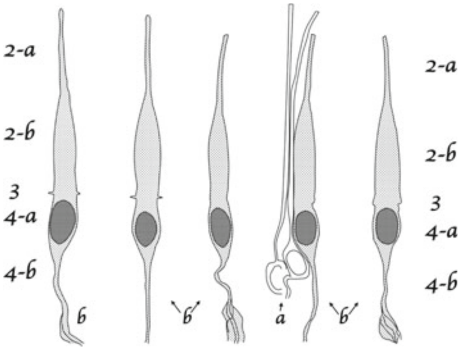

Fig. 4.52 Reproduction of Polyak's design of rods and cones from the central are of the Simian retina

> All these various considerations force us to recognise that the only functional role of the central fovea, at any rate in Man and other Primates, is the elimination of the dioptrical impediments to the passage of the central pencil of incident light rays.

In his study, Polyak also included a figure of cones and rods alone, as graphically represented in Fig. 4.52, based on his original drawing. This is a figure of cones and rods from the central area of the Simian retina, just outside of the central (rod-less) fovea (the layer numbering is the same as in previous figures). Light enters through the bottom of this drawing and the response travels from the top. In his analysis of cones and rods, he supported, with anatomical evidence, the theory of these two totally distinct receptors.

Polyak insisted though that the other layers in the retina also play an important role in transforming the signals that travel from the receptors to the brain. He studied the bipolar cells, which he categorised into five types, and also the ganglion cells, which he also categorised into five types. He was particularly interested in understanding the interrelations of all the retinal neurons in modulating the receptors' response and understood that most theories of his time were "largely invented" (his own words). He focused his anatomical work on revealing the way the various cells connect with each other and concluded that it was time

> ...to begin to analyse and to interpret functionally the anatomical structures of the retina in terms other than rods and cones.

He affirmed the theory that cones are responsible for the perception of colour, although he was not certain of the origin of the differentiation in the spectral response that makes colour perception possible. He wondered if there was a structural or purely chemical differentiation but was not able to provide evidence from one or the other. Nevertheless, he was convinced that bipolar and ganglion cells must play an important role in colour vision.

> ...the cone – precisely because of its all-embracing universality and structural-chemical homogeneity – is not capable of performing the role of a chromatic analyser. The cone merely furnishes a dynamical 'material' for other structures of the visual system to work with.
>
> ...
>
> Plainly, the bipolar and ganglion cells must in some way be the carriers of the process by which the global cone excitation is transformed and directed into one or the other channel, according to the spectral position of the stimulus, its intensity, and other qualities.
>
> ...
>
> What arrives in the centre are the impulses originating in the cones but in many ways modified by the intervening neurons.

Polyak provided excellent illustrations of the human retina, drawings of highly educative value and built a more stable basis for further research on this rather challenging topic. In the 1950s Polyak published a highly detailed analysis of the vertebrate visual system, including facts about its origin, structure, and function, in a large volume of around 1500 pages (Polyak, 1957).

4.23 Erwin Schrödinger

Erwin Rudolf Josef Alexander Schrödinger (1887–1961) was born in Vienna, Austria-Hungary. He was a physicist whose pioneering work in physics led to the development of what is known as the *Schrödinger equation* that describes how a wave function of a system changes dynamically over time. Although he is mostly considered a significant contributor to the development of modern physics, he has done considerable work on light and colour perception. He followed the steps of the great innovators in the field of the psychology of colour perception, Newton, Helmholtz and Maxwell. Some of his published works have been translated into English and were made accessible to a wider audience, like MacAdam (1970); Niall (2017). Most of his work on light and colour vision are included in the voluptuous 1920 publications, the *Grundlinien einer Theorie der Farbenmetrik im Tagessehen* (or an *Outline of a Theory of Colour Measurement for Daylight Vision*), published in three parts (Schrödinger, 1920a, b, c). In 1925 he also published the work *Über das Verhältnis der Vierfarben zür Dreifarbentheorie*) (*About the relationship between the four-colour theory and the three-colour theory*) (Schrödinger, 1925, 1994), in which he treated the relations between the most prominent colour theories.

Schrödinger in Part I of the *Outline* (Schrödinger, 1920a) analysed topics like the concepts of light and colour, light and colour addition and the dimensionality of colour perception. Regarding the concepts of light and colour, he made clear that the normal and most precisely quantifiable way to create colours is to have light strike upon an eye. He defined colour as a group of identical-looking lights, departing from the most common designation, by which lights of the same colour can produce very different impressions under different circumstances.

> Wir entfernen uns damit ein wenig von dem gewöhnlichen Sprachgebrauch und zwar insofern, als Lichter gleicher Farbe (nach unserer Terminologie) unter verschiedenen Umständen sehr verschiedene Eindrücke auf das Auge hervorbringen konnen, so daß sie zuweilen sogar mit verschiedenen Farbnamen belegt werden.

He tried in this way to provide an objective definition of *colour*, based on its physical composition, departing from the subjective nature that may originate from the function of adaptation. In addition, he provided a physical definition for a spectral light, as a light whose function of the wavelength $f(\lambda)$ differs from zero only in a very small range of wavelengths and called the corresponding colour a *spectral colour*. Schrödinger provided a range for spectral colours, spanning roughly between 475 nm, and 630 nm in the scale of the wavelength. What is unique for these colours is that they obey his definition for $f(\lambda)$ and a group for a spectral colour includes a single member. Any mixture of lights of the same hue is expected to be less saturated (more whitish, pale) in comparison to a spectral light.

> Als Spekfrallicht bezeichnen wir ein Licht, dessen $f(\lambda)$ nur in einem sehr kleinen λ-Bereich von Null verschieden ist, und die betreffende Farbe als Spekfralfarbe. Die meisten Spektralfarben (von etwa $\lambda = 475$ bis $\lambda = 630$) sind nun dadurch ausgezeichnet, daß sie sich überhaupt nur auf diese eine Art herstellen lassen —die Lichtergruppe umfaßt nur das eine Licht— wenn man davon absieht, daß bei genügend klein gewähltem λ-Bereich die Verteilung der Energie innerhalb dises Bereiches willkürlich ist, weil hinreichend benachbarte Wellenlängen sich in ihrer Wirkung auf das Auge nicht unterscheiden. Es gibt wohl Mischlichter vom gleichen Farbton, sie, erscheiden aber gegen das Spektrallicht immer etwas weißlich (weniger gesättigt).

Regarding the *dimensionality* of colour vision, Schrödinger made three basic statements that clearly support the trichromacy theory, but also emphasise the subjective nature of colour perception, as the dimensionality varies for three distinct cases.

- For normal colour vision (trichromats): there are linearly independent colour triples; four colours are always linearly dependent.
- For partially colour-blind people (dichromates): there are linearly independent pairs of colours; three colours are always linearly dependent.
- For totally colour-blind people (monochromats): every two colours are linearly dependent.

> A. Für Farbentuchtige (Trichromaten): Es gibt linear unabhängige Farbentripel. Vier Farben sind stets linear abhängig.
> B. Für partiell Farbenblinde (Dichromaten): Es gibt linear unabhängige Farbenpaar. Drei Farben sind stets linear abhängig.
> C. Für total Farbenblinde (Monochromaten): Je zwei Farben sind linear abhängig. Diese Aussagen bedeuten nichts anderes, als daß die Farbenmannigfaltigkeit für diese Personen drei bzw. zwei.

He supported the tridimensionality of the colour space manifold by pointing out that (a) any two spectral colours differ in more than one quantity, their intensity, in a way that any attempt to match them only by changing the intensity is insufficient, thus pairs of spectral colours are linearly independent; (b) any binary mixtures of spectral colours cannot be matched by any other single spectral colour with changes in intensity only, thus triplets are linearly independent.

In Part II of the *Outline* (Schrödinger, 1920b), Schrödinger focused on the representation of the colour space manifold, which he called the *envelope of the spectral cone*. A reproduction of his 1920 graph is shown in Fig. 4.53. The graph shows the spectral cone as a curve (*Spektralkurve*) and its intersection with an inclined plane (denoted as *Schnitt mit einer Ebene*). In this representation, he commented on the

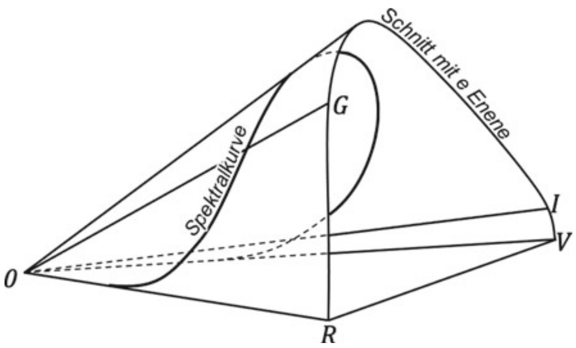

Fig. 4.53 Reproduction of Schrödinger's envelope of the spectral cone

prominent position of white, which is the colour of sunlight. On one hand, he diminished its significance by stating that it is purely the radiation of a black body of about 7000° C, and on the other, he supported its significance, as the evolutionary reason for colour vision. It is no surprise that the solar radiation peaks in the range of light that is visible. This is a very important statement that shown how the understanding of the development and the function of vision started to get a more clear 'shape' at the beginning of the 20th century.

> Denn fast ausschließlich unter der Einwirkung dieses Lichtgemisches ist unser Auge entstanden, hat es sich entwickelt und seine gegenwärtige Funktionsform angenommen. Daß dieses Lichtgemisch für die so entstandene Farbwahrnehmung eine ausgezeichnete, zwischen den möglichen Extremen vermittelnde Rolle spielt, ist nicht verwunderlich; ist doch, nebenbei bemerkt, auch für die merkwürdige Koinzidenz des Energiemaximums der Sonne mit dem Helligkeitsmaximum in einem Spektrum von konstanter Energie die einzige ungezwungene Erklärung die phylogenetische; an der Stelle des Energiemaximums und zu beiden Seiten desselben war die Entwicklung hoher Lichtempfindlichkeit sozusagen am rentabelsten, wenn es auf möglichst deutliche Wahrnehmung der Gegenstände auch bei schwacher Beleuchtung ankam.

Furthermore, Schrödinger in an attempt to achieve a theoretical determination of the coordinate system for the colour space, provided another important graph, that of colour-mixture curves of the interference (uniformly dispersed) spectrum of sunlight, in relative coordinates across the visible spectrum. This corresponds to the case in which light is determined by only its numerical function of wavelength ($f(\lambda)$). By using trichromacy and by trying to calculate the colour vector to be assigned to sunlight white purely mathematically, he produced these curves, a reproduction of which is provided in Fig. 4.54; the graph shows the normalised colour-mixture curves of the spectrum of sunlight with respect to three primary

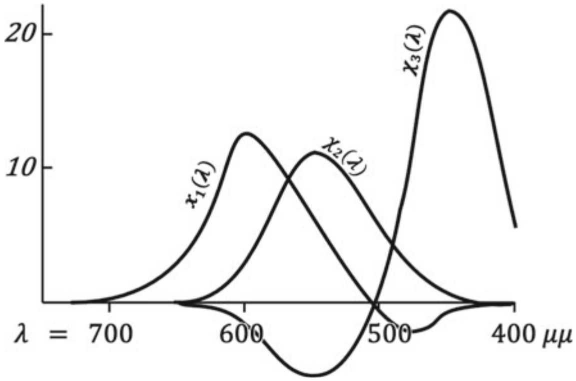

Fig. 4.54 Reproduction of Schrödinger's colour-mixture curves of the interference spectrum of sunlight, in relative coordinates across the visible spectrum

colours (or calibration lights), which he arbitrarily chose to be the red at the far end of the spectrum, the green at $\lambda = 505$ nm, and violet at the other far end of the visible spectrum. Since the idea was to produce the sunlight white, the proportion of these three colour-mixture curves is meant to produce white (the colour qualitatively resembling sunlight).[32] The curves are normalised in such a way that they have the same area between them and the abscissa.

In Schrödinger's analysis, supposing light as a function of wavelength $f(\lambda)$ then the three colour coordinates of that light (given a reference light $\phi(\lambda)$) can be expressed as

$$\int \frac{f(\lambda)x_i(\lambda)}{\phi(\lambda)}\mathrm{d}\lambda, \quad i = 1, 2, 3 \tag{4.15}$$

where $i = 1, 2, 3$ are the three dimensions of the colour space, x_i is the ith colour mixture function. Schrödinger remarked that only the relative illumination is necessary (f/ϕ) for the computations. For purely spectral colour light sources this ratio is 1, thus, the coordinates that result are just the averages of each of the three integrals of the colour mixture functions.

$$\frac{1}{C}\int x_i(\lambda)\mathrm{d}\lambda, \quad i = 1, 2, 3 \tag{4.16}$$

where C is the width of the wavelength interval.

In Part III of the *Outline* (Schrödinger, 1920c), Schrödinger provided advanced colour measurement concepts using strict mathematical notation, including measures of perceptual dissimilarity, an analysis of the concept of brightness and the sensitivity to colour dissimilarity, the introduction of the concept of geodesics in the estimation of colour dissimilarities on his colour space. In his understanding, *colour theories emerge as expressions of concepts that attempt to agree with the sensation.*

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[32] This graph was derived from the *elementary sensation curves* published by A. König and C. Dieterici, Zeitschrift für Psychologie und Physiologie der Sinnesorgane 4., P. 241ff., 1892.

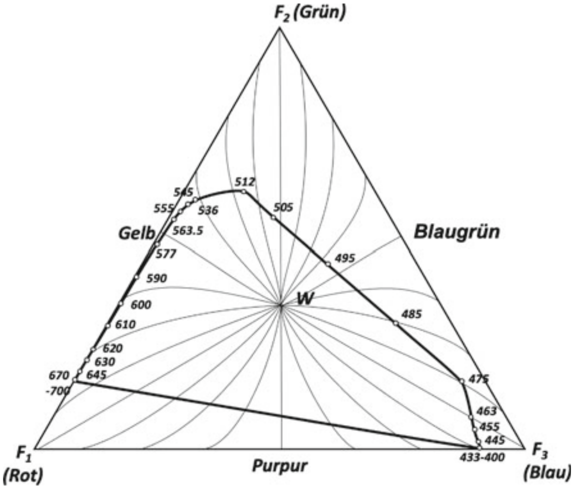

Fig. 4.55 Reproduction of Schrödinger's colour triangle with colour space geodesics to white and constant-hue loci

> ...ich betone ausdrücklich, daß es sich dabei nicht um eine “Folgerung aus der Theorie" handelt, die über Empfindungen natürlich überhaupt keine Aussage macht, sondern nur um den Versuch, einen Begriff, dessen bisherige quantitative Fassung zu äußerlich ist und sich mit der Empfindung nicht deckt, anders und tiefer zu fassen und so, daß er sich womöglich besser mit der Empfindung decke.

He stated that the colours that lie on the shortest path to white are equal in hue.

> Als farbtongleich mit einer gegebener Farbe hätte man dann also jene Farben zu definieren, die auf der kürzesten Linie zum Weiß liegen.

Figure 4.55 is a reproduction of the 1920 graph (Fig. 6) that Schrödinger provided to depict a colour triangle and the pencil of geodetic lines through the white point, in which colours of the same brightness are considered. In this representation, only the connections to the primary colours and their complementaries are rectilinear, and only in these cases would the mixture with white be constant in hue. For all intermediate colours, the hues of the mixtures with white would appear shifted towards the predominant primary colour. A colour near the white point has its equivalent in hue along the edge of the triangle, which can be located by progressing along a curve of the pencil.

> Figure 6 zeigt das Büschel geodätischer Linien durch den Weißpunkt in einem Farbendreieck, in welchem wir uns gleich-

> helle Farben lokalisiert denken wollen. Geradlinig sind nur die Verbindungen zu den Grundfarben und ihren Gegenfarben, nur in diesen Fällen wären die Weißmischungen konstant im Farbton. Von allen Zwischenfarben erscheinen die Weißmischungen im Farbton verschoben gegen die überwiegende Grundfarbe, d.h. gegen diejenige Grundfarbe, welche darin —verglichen mit dem Mischungsverhältnis der drei Grundfarben, das farblos erscheint— am stärksten vertreten ist. Denn man findet ja nach Annahme zu einer Farbe in der Nähe des Weißpunkts die farbtongleichen am Rande des Dreiecks, indem man längs einer Kurve des Büschels fortschreitet, und das Büschel zieht sich in einem dreistrahligen Stern gegen die Grundfarbenpunkte zusammen.

4.24 Ludwig Wittgenstein

Ludwig Wittgenstein (1889-1951) was born in Vienna, Austria-Hungary who worked in logic, the philosophy of mathematics, language and the mind. His most important publication was the *Tractatus Logico-Philosophicus* (*Treatise on Logic and Philosophy*), published in 1921 in a form of a collection of quotes (Wittgenstein, 1922). Wittgenstein was concerned with accurate symbolism like it is expressed in the relation of one fact (such as a sentence) to another so that it can be a symbol for that other. By analysing language and logic and by exploring the limitations of human thinking, *Wittgenstein defined language to be the limit to the expression of any thought*. His teacher, the world-known mathematician and philosopher Bertrand Russell (1872–1970) described Wittgenstein *as perhaps the most perfect example of genius*, thus it is more than important to know of Wittgenstein's research in his own words.

Wittgenstein used the term 'picture' (or image) in a broader sense for mappings in general, although the consequences in human perception insofar this treatise is concerned are important to understand. In paragraphs 2.1–2.1512 of the original text in German[33] he states that we make to ourselves pictures of facts. The picture presents the facts in logical space, the existence and non-existence of atomic facts. The picture is a model of reality. To the objects correspond in the picture the elements of the picture. The elements of the picture stand, in the picture, for the objects. The picture consists in the fact that its elements are combined with one another in a definite way. The picture is a fact. That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another. This connection of the elements of the picture is called its structure, and the possibility of this structure is called the form of representation of the picture. The form of representation is the possibility that the things are combined with

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[33] The complete text with the English translation can be found online @ https://archive.org/details/tractatuslogicop1971witt.

one another as are the elements of the picture. Thus the picture is linked with reality; it reaches up to it. It is like a scale applied to reality.

> Wir machen uns Bilder der Tatsachen. Das Bildstellt die Sachlage im logischen Raume, das Bestehen und Nichtbestehen von Sachverhalten vor. Das Bild ist ein Modell der Wirklichkeit. Den Gegenstaenden entsprechen im Bilde die Elemente des Bildes. Die Elemente des Bildes vertreten im Bild die Gegenstande. Das Bild besteht darin, dass sich seine Elemente in bestimmter Art und Weise zu einander verhalten. Das Bild ist eine Tatsache. Dass sich die Elemente des Bildes in bestimmter Art und Weise zu einander verhalten stellt vor, dass sich die Sachen so zu einander verhalten. Dieser Zusammenhang der Elemente des Bildes heisse seine Struktur und ihre Moeglichkeit seine Form der Abbildung. Die Form der Abbildung ist die Moeglichkeit, dass sich die Dinge so zu einander verhalten, wie die Elemente des Bildes. Das Bild ist so mit der Wirklichkeit verknuepft; es reicht bis zu ihr. Es its wie ein Masstab an die Wirklichkeit angelegt.

In paragraphs 2.161–2.171 of the original text Wittgenstein states in the picture and the pictured there must be something identical so that the one can be a picture of the other at all. What the picture must have in common with reality in order to be able to represent it after its manner-rightly or falsely-is its form of representation. The picture can represent every reality whose form it has. The spatial picture, everything spatial, the coloured, everything coloured, etc.

> In Bild und Abgebildetem muss etwas identisch sein, damit das eine ueberhaupt ein Bild des anderen sein kann. Was das Bild mit der Wirklichkeit gemein haben muss, um sie auf seine Art und Weise richtig Oder falschabbilden zu koennen, ist seine Form der Abbildung. Das Bild kann jede Wirklichkeit abbilden, deren Form es hat. Das raeumliche Bild alles Raeumliche, das farbige alles Farbige, etc.

Among the publications of Wittgenstein that were edited and published posthumously was a collection of *Bemerkungen über die Farben* (*Remarks on Colour*) (Wittgenstein, 1977), which is of particular interest for this treatise. In paragraph I.22, he states that we do not want to establish a theory of colour (neither a physiological one nor a psychological one), but rather the logic of colour concepts. And this accomplishes what people have often unjustly expected of a theory.

> Wir wollen keine Theorie der Farben finder (weder eine physiologische, noch sine psychologische), sondern die Logik der Farbbegriffe. Und diese leistet, was man sick oft mit Unrecht von einer Theorie erwartet hat.

It is clear in this collection of remarks that Wittgenstein was familiar with other colour theories, like the one Goethe derived, and also knew Runge's ideas about colour. The text, nevertheless, largely reflects his own way of philosophical thinking. Wittgenstein seems to have been intrigued by 'transparent' colours, the colour of transparent media like coloured glasses. He was positive that coloured glasses work like filters, taking away the colours of any objects behind them and changing their appearance according to their own colour. He was particularly concerned about white and argued that *there could not be a transparent white*. He asserted that *opaqueness is not a property of the white colour*. He seems to have accepted a *four-colour model based on blue, yellow, red and green*. In Part III of the Remarks, Wittgenstein poses some interesting questions and answers regarding the knowledge and perception of colour. For example, in III.4 he wonders whether a red is lighter than a blue and if this is a matter of experience. He shows a deep understanding of concepts like saturation in saying (in III.14) that a saturated X colour is an impression of colour in a particular surrounding and comparable to its 'transparent' version. In paragraph III.52 he states that we only need six colour words to communicate about colours (which supports his adoption of the four+two colour model). In the same paragraph, he adopts Runge's view on not being possible to communicate using self-opposing colour phrases, like reddish-green or yellowish-blue. At the same time, in III.61, he marks a core question to always have in mind, which asks "how do people learn the meaning of colour names". Later in the text, in III.106 he reminds that "the logic of the concept of colour is just much more complicated than it might seem". His philosophical inclination to the study of this subject is emphatically illustrated in paragraph III.120, where he remarks on the case of colour blindness,

> Do normally sighted people and colour-blind people have the same concept of colour-blindness? And yet the colour-blind person understands the statement "I am colour-blind", and its negation as well. A colour-blind person not merely can't learn to use our colour words, he can't learn to use the word colour-blind exactly as a normal person does. He cannot for example always determine colour-blindness in cases where the normal-sighted can.

4.25 Gestalt Psychology

In any historical account of colour theory, a note should also be made about a particular movement that came from a mixture of psychological and physiological studies, the movement of *Gestalt* theory.[34] Gestalt is a significant paradigm shift in psychology, born in Germany at the beginning of the 20th century. It was *Christian von Ehrenfels* (1859–1932), an Austrian philosopher, who gave this movement its name in *Über Gestaltqualitäten* (*The Attributes of Form*), his most important work, published in 1890 (Ehrenfels, 1890). The term Gestalt cannot be literally translated into English and is usually interpreted as ‘structure’, or ‘totality’, or ‘configuration’, or even ‘organised unity’. Gestalt proposed a paradigm shift to the dominant psychology of that period and made significant contributions to cognitive psychology, which was built around the idea that *the whole is more than the sum of its parts*. This phrase seems to originate in Aristotle’s *Μϵτά τα φυσικά* (*Metaphysics*), in which[35] is stated that

> Περὶ δὲ τῆς ἀπορίας τῆς εἰρημένης περί τε τοὺς ὁρισμοὺς καὶ περὶ τοὺς ἀριθμούς, τί αἴτιον τοῦ ἓν εἶναι; πάντων γὰρ ὅσα πλείω μέρη ἔχει καὶ μὴ ἔστιν οἷον σωρὸς τὸ πᾶν ἀλλ᾽ ἔστι τι τὸ ὅλον παρὰ τὰ μόρια, ἔστι τι αἴτιον, ἐπεὶ καὶ ἐν τοῖς σώμασι τοῖς μὲν ἁφὴ αἰτία τοῦ ἓν εἶναι τοῖς δὲ γλισχρότης ἢ τι πάθος ἕτερον τοιοῦτον.

This translates to

> With regard to the difficulty which we have described in connexion with definitions and numbers, what is the cause of the unification? *In all things which have a plurality of parts, and which are not a total aggregate but a whole of some sort distinct from the parts, there is some cause*; inasmuch as even in bodies sometimes contact is the cause of their unity, and sometimes viscosity or some other such quality.

Among the main characteristics in the Gestalt theory, one may distinguish

- The complexity of the human mind cannot be reduced.
- Mental representations do not correspond completely with those that exist in reality; people construct them by themselves.
- Through perception, people are able to acquire knowledge of the world, interact with it and connect with others.
- Gestalt theory focuses on visual perception.

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[34] Among numerous sources about Gestalt theory there is a specific website dedicated to it @ http://www.gestalttheory.net/ and belongs to the *International Society for Gestalt Theory and its applications*.

[35] The ancient Greek text and the English translation of 1933 by Hugh Tredennick was used, found online @ https://archive.org/details/in.ernet.dli.2015.185284/mode/2up, Book 8, (par. 1045a.1), par. VI, pp. 420–423.

According to Gestalt, people tend to mentally connect and integrate contiguous objects, connect elements to a single object or group, fill the gaps in shapes, perceive similar objects as having the same form by abstraction, and simplify the representations.

Historically, it is generally accepted that there are three founders of Gestalt, *Wolfgang Köhler*, *Max Wertheimer* and *Kurt Koffka*. *Wolfgang Köhler's* (1887–1967) main contribution was the formal introduction of *learning by discovery* and the basic notion that this process is active and dynamic (Köhler, 1925, 1847).

The key phrase of Gestalt (the whole is more than the sum of its parts) is attributed to him. The *phi-phenomenon* or apparent movement is *Max Wertheimer's* (1860–1943) most revolutionary discovery (Wertheimer, 1912). Simply described, according to this phenomenon, movement is perceived when a succession of images is presented, the basic concept for cinema. His most important works in Gestalt are included in the two volumes of the *Untersuchungen zur Lehre von der Gestalt* (*Investigations on Gestalt Theory*) (Wertheimer, 1923, 1922), in which he outlines the general theoretical context and the laws of organisation in perceptual forms. *Kurt Koffka* (1886–1941) contributed in a variety of fields, including memory, learning and perception. He also applied Gestalt to fields such as child psychology (Koffka, 1924). He emphasised the need to consider mental processes from a holistic point of view. He also helped Wertheimer in his research on the apparent movement by becoming involved as a subject. His main contribution to the understanding of Gestalt is included in his 1935 *Principles of Gestalt Psychology* Koffka (1935).

Gestalt soon became a new *school of thought*, which viewed human behaviour and perception as a complete whole. It led to major contributions in explaining some complex processes of sensation and, particularly, perception, focusing on the notion that humans perceive the world by viewing things in totality or from a holistic perspective.

4.26 John Guild

John Guild was a British scientist who worked at the National Physical Laboratory (NPL) in Teddington, England. His main work was in the development of a wide variety of optical instruments and techniques (Carter, 1991). Guild's contribution to the 20th century (and onward) colour technology is significant, and his work has been discussed in various publications and presentations by other researchers (Carter, 1991; Fairman et al., 1997). Two of his papers can be found in MacAdam's collection of colour science sources (MacAdam, 1970), namely *Some problems of visual perception* Guild (1970b) and *Quantitative data in visual problems* (Guild, 1970a).

Guild's contribution was mostly marked by his work towards the formulation of a common standard for colour space representation, the *CIE 1931* standard.[36] The relevant work along with tables with his measurements can be found in his 1931 publication on *The colorimetric property of the spectrum* (Guild, 1931), which he presented as work he had done two-three years before publication, and in which he included and integrated more experiments conducted by William David Wright (1929). A detailed description of the CIE colourimetric standard was published by him and Thomas Smith (1931), including all the functions, data and resolutions adopted. Guild was the NPL Representative in the CIE discussions towards the standardisation and served at the CIE Secretariat Committee on Colourimetry for 1928–1931.

Motivation for Guild's work was the need for standardisation in colour science at the beginning of the 20th century; this included the definition of the *standard observer*, the *standard human vision fundamentals* and *colour matching functions*, along with the *standard colour space* for a variety of applications. He was seriously involved in what was known as the colour matching experiments, in order to derive a concrete and mathematically expressed colour space for an objective description of colours. In these experiments, he used the standard white light defined by the National Physical Laboratory (the NPL Standard White Light). The spectral distribution of this light is shown in Fig. 4.56, reproduced from Guild's Table I in

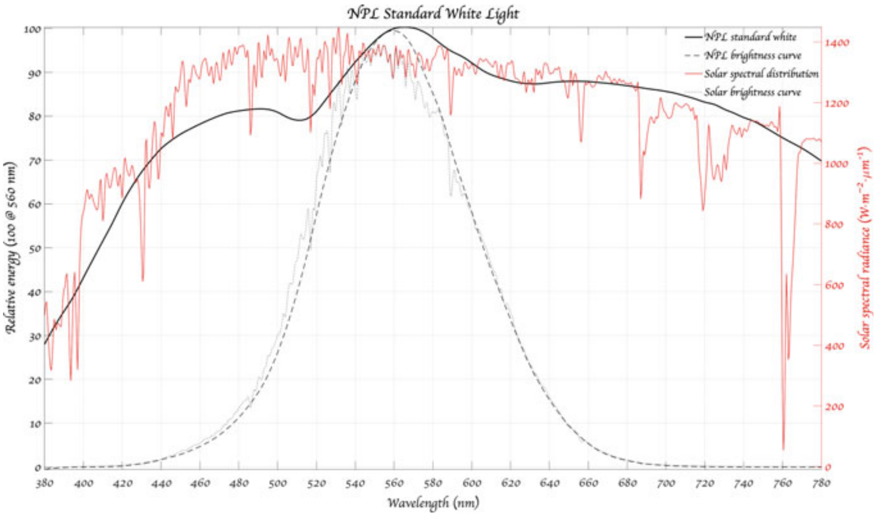

Fig. 4.56 Energy distribution of the NPL Standard White Light

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[36] CIE stands for Commission Internationale de l'Echlairage (International Commission on Illumination). As stated on CIE's website, the commission "is devoted to worldwide cooperation and the exchange of information on all matters relating to the science and art of light and lighting, colour and vision, photobiology and image technology.".

Guild (1931)[37]; the luminous efficacy data (the normalised response of the eye to luminance) and the perceived brightness of the white light are also shown from the original data Guild provided, along with the solar radiation energy distribution on the Earth's surface and the perceived brightness of the Sun, from modern high-resolution data. As it was already known, one simply needs to multiply the spectral distribution of a light source by the luminous efficacy of the eye to get the perceived brightness.

The goal of the original experiments was to create mixtures of three primary spectral (monochromatic) colours in order to match a given colour and derive the human vision colour matching functions. Supposing a linear colour system (adopting Grassmann's laws) in which any spectral colour Q_λ (or, as Guild named it, *colourimetric quality of a monochromatic stimulus*) is a linear combination of primaries R, G, B,[38]

$$Q_\lambda = aR + bG + cB \tag{4.17}$$

where R, G, B are the primaries, which Guild set to 700, 546.1 and 435.8 nm, the NPL standard primaries,[39] perceived as red (at the limit of vision), green and violet-blue respectively, as shown in an approximate representation in Fig. 4.57[40]; a, b, c are the trichromatic coefficients, which may be estimated by getting the contribution of each primary guided by instrument adjustment factors and the convention that $a + b + c = 1$.

Overall, the setup of the experiment and the outcomes is graphically shown in Fig. 4.58 and the details will be laid out in the following paragraphs.

First, the experiment takes into account each observer's sensation of white, which becomes the reference for the adjustment of the experimentation instrument (this is some equivalent to white matching or white balancing). For example, if for an observer the white is attained by 30 units of R, 60 units of G and 20 units of B, then, since only equal amounts of primaries should be mixed to attain white, R should be scaled by $a_w = 2$, G by $b_w = 1$ and B by $c_w = 3$. These factors characterise the colour matching of the specific observer, thus they are applied to any colour matching experiment that the observer participates in. If, for example, the

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[37] Solar radiation data were adopted from The National Renewable Energy Laboratory of the U.S. Department of Energy. The '2000 ASTM Standard Extraterrestrial Spectrum Reference E-490-00' where found at https://www.nrel.gov/grid/solar-resource/spectra-astm-e490.html and directly downloaded through https://www.nrel.gov/grid/solar-resource/assets/data/e490_00a_amo.xls. The 'Reference Air Mass 1.5 Spectra' data were found at https://www.nrel.gov/grid/solar-resource/spectra-am1.5.html and directly downloaded through https://www.nrel.gov/grid/solar-resource/assets/data/astmg173.xls.

[38] By that time, R, G, B was the standard representation of the set of the three primary colours, even though it was known that any other independent colour triplets could be used interchangeably.

[39] In subsequent experiments, Guild transformed the standard primaries into *working primaries* by using (4.17) and the results of the initial experiments, and by considering R, G, B to be the working primaries and Q_λ the standard primaries. Inverting this system of linear equations he attained the working primaries for the subsequent experiments.

[40] Any triangle that connects primaries encloses the complete gamut of colours that can be produced by those primaries assuming the additive system. The white triangle is showing the locations of the NPL primaries. For comparison, the much later standard showing the sRGB triangle is also drawn in the diagram.

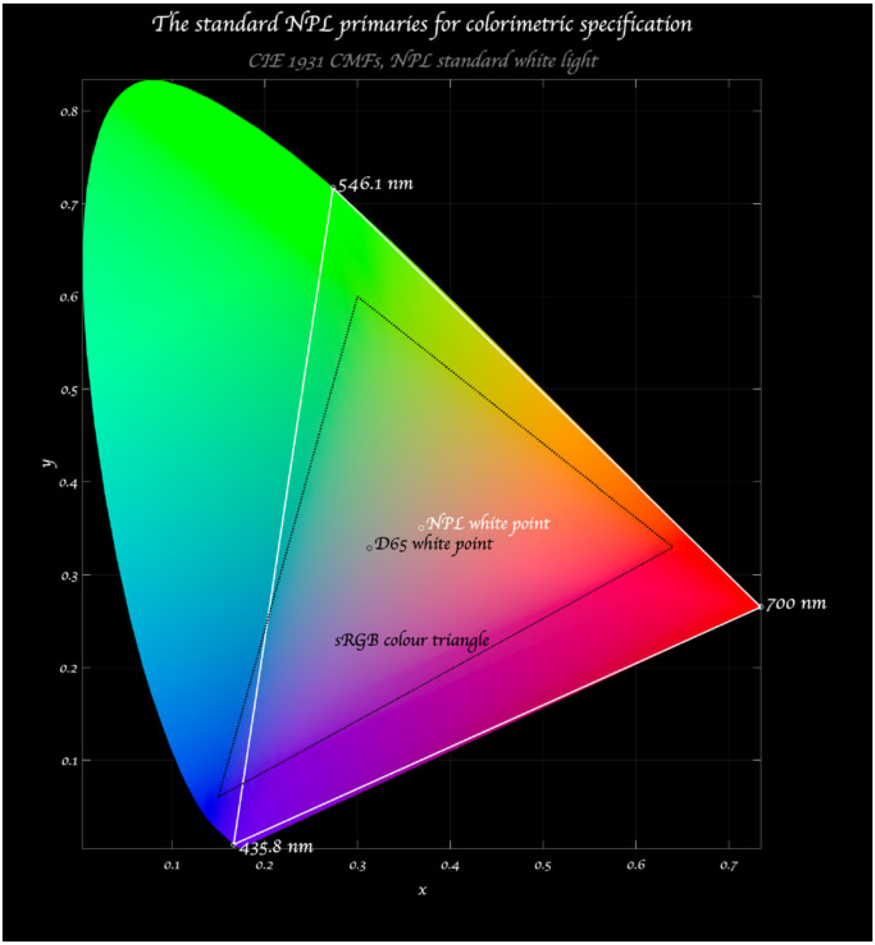

Fig. 4.57 A representation of the NPL primary colours for colourimetry on the CIE colour space that Guild helped develop

observer matches a colour with {40, 20, 20} units for the primaries, then these values are scaled with the factors {2, 1, 3} and become {80, 20, 60} which add to 160; taking the appropriate ratios, the coefficients are estimated as $a = 80/160 = 0.5$, $b = 20/160 = 0.125$ and $c = 60/160 = 0.375$. It is easy to understand that for any triplet that corresponds to a different colour Q_λ, different coefficients are estimated. These measurements describe the *chrominance* of a coloured light but say nothing about its *luminance*. This is why he also included *luminosity matching* in the experiments, which completed the picture about the information extracted by the visual system for a coloured light, by providing three more parameters, the *luminosity factors* L_R, L_G and L_B. By white balancing the chromaticities and using the three luminosity factors he was able to create a single luminosity factor L_λ. To arrive at a

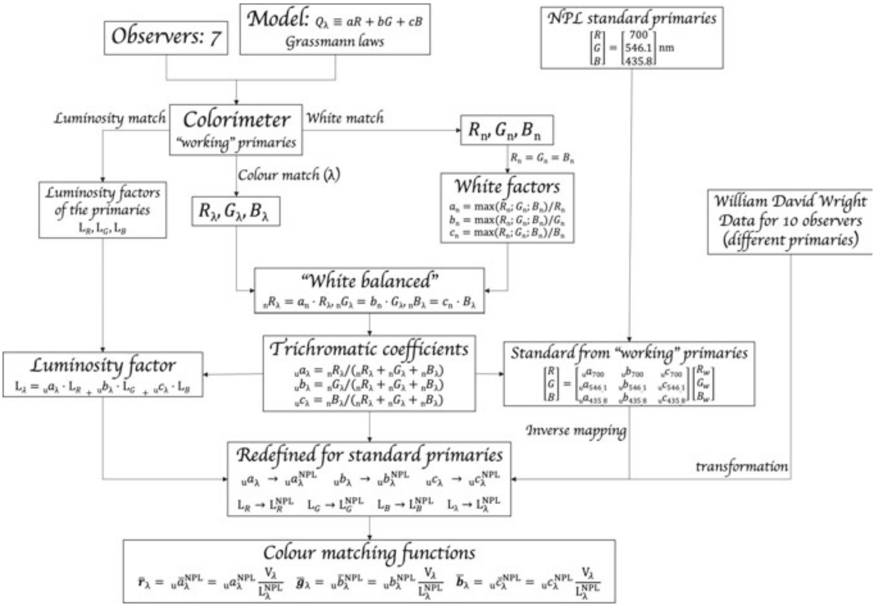

Fig. 4.58 Graphical representation of Guild's experiment

result that would be universal and independent of the three primary colours, he compensated his results for the NPL standard primaries, since he knew the primaries in his colourimeter were different. Thus he arrived at the unit functions of wavelength for the chrominance and luminosity factors ${}_ua_\lambda^{\mathrm{NPL}}$, ${}_ub_\lambda^{\mathrm{NPL}}$, ${}_uc_\lambda^{\mathrm{NPL}}$ and $\mathrm{L}_\lambda^{\mathrm{NPL}}$, such that

$$\begin{aligned} Q_\lambda &= {}_ua_\lambda^{\mathrm{NPL}} R + {}_ub_\lambda^{\mathrm{NPL}} G + {}_uc_\lambda^{\mathrm{NPL}} B \\ {}_u\mathrm{L}_\lambda &= {}_ua_\lambda^{\mathrm{NPL}} \mathrm{L}_R + {}_ub_\lambda^{\mathrm{NPL}} \mathrm{L}_G + {}_uc_\lambda^{\mathrm{NPL}} \mathrm{L}_B \\ {}_ua_\lambda^{\mathrm{NPL}} &+ {}_ub_\lambda^{\mathrm{NPL}} + {}_uc_\lambda^{\mathrm{NPL}} = 1 \end{aligned} \tag{4.18}$$

After experimenting with filters, he concluded to the luminosity factors $\mathrm{L}_R = 1.0$, $\mathrm{L}_G = 2.858$, $\mathrm{L}_B = 0.169$, to be used as relative factors. Then he recalculated the values to compensate for inconsistencies in the results and concluded to $\mathrm{L}_R = 1.0$, $\mathrm{L}_G = 4.39$, $\mathrm{L}_B = 0.048$,[41] for which he included the results for the standard observer in Table IV.

By repeating the experiment for the whole range of the visible spectrum, the whole variety of the coefficients was exposed, giving rise to specific response functions. Guild did that experiment with seven observers and incorporated the experimental results by Wright (1929) for another ten observers and represented the average results in tables and graphs. The average results give specific values for the *normal eye*, the standard observer and how the colour specification should be carried out. In Fig. 4.59 these results are shown in terms of three graphs, which were created

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[41] These luminosity factors of the primaries are expressed as a ratio and not as absolute values; they should be considered as $\mathrm{L}_R : \mathrm{L}_G : \mathrm{L}_B = 1 : 4.39 : 0.048$.

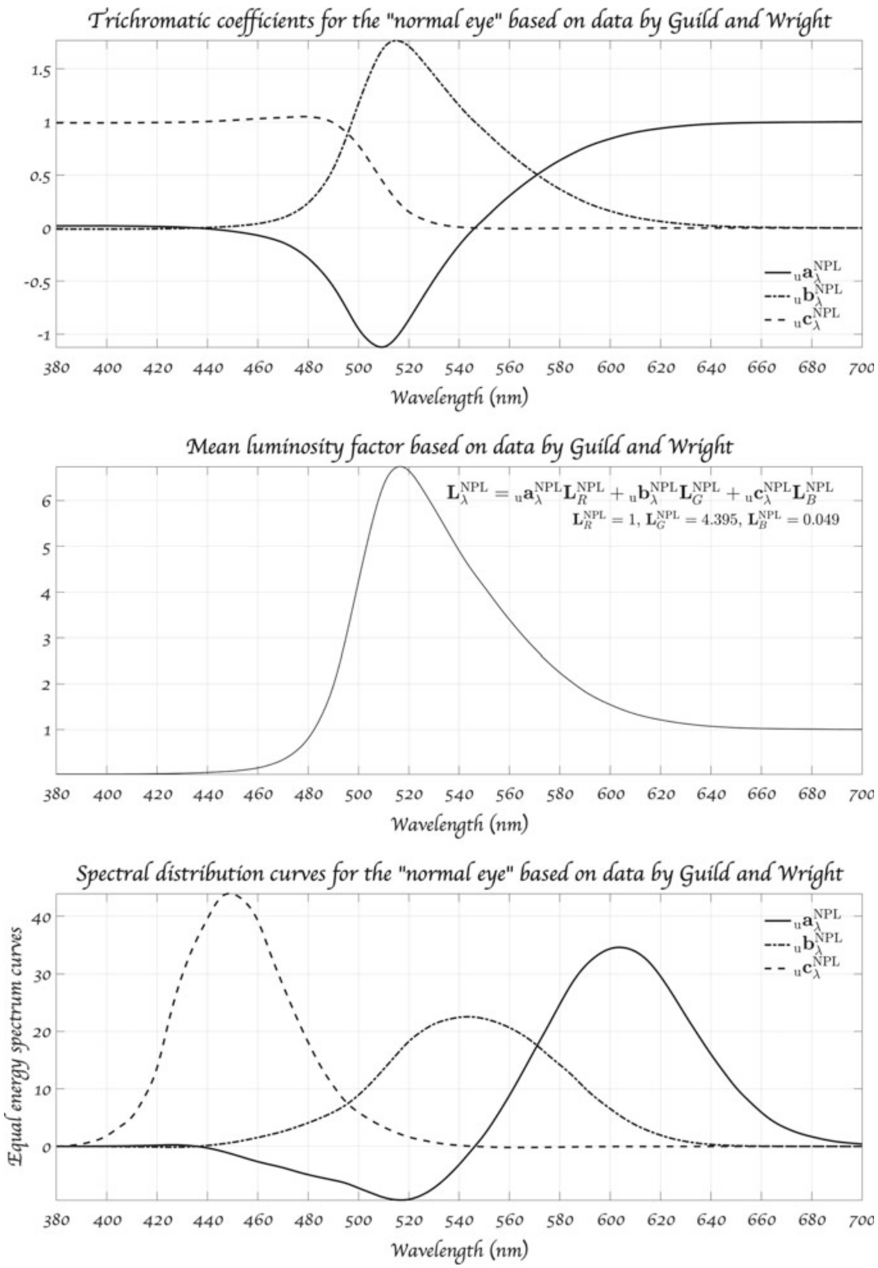
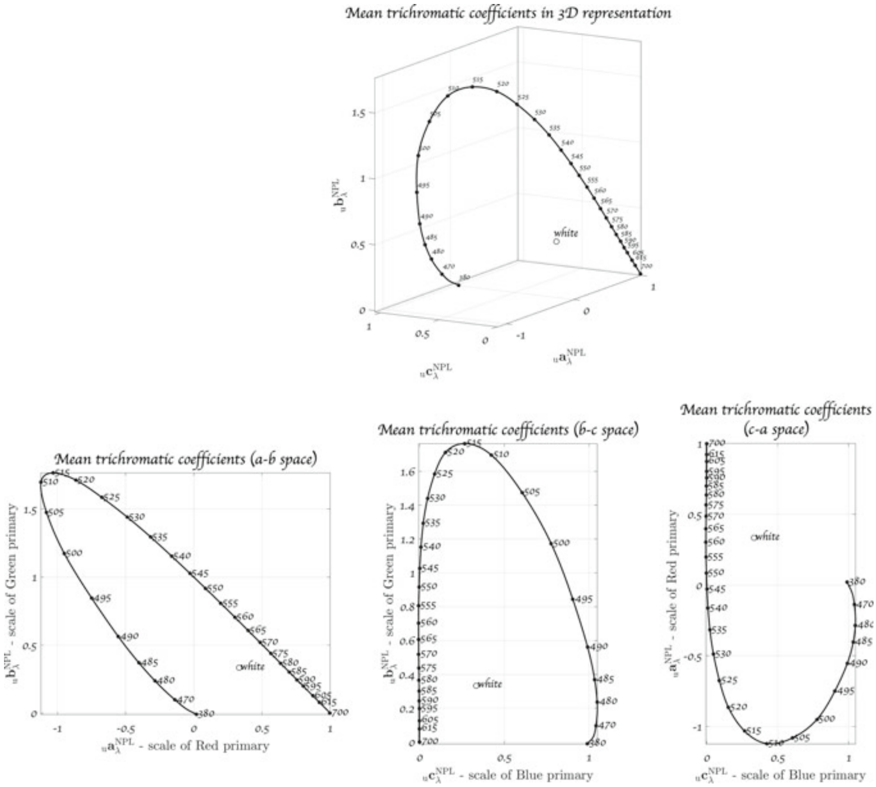


Fig. 4.59 Specification of the standard observer by the Guild-Wright experiments


Mean trichromatic coefficients in 3D representation

Mean trichromatic coefficients (a-b space)

Mean trichromatic coefficients (b-c space)

Mean trichromatic coefficients (c-a space)

Fig. 4.60 Chromaticity diagrams by the Guild-Wright data

by interpolating the values from the original data. The first graph is, for the first time, the picture of the standard observer, which shows the standard trichromatic coefficients of the *normal eye* (${}_u a_\lambda^{NPL}$, ${}_u b_\lambda^{NPL}$, ${}_u c_\lambda^{NPL}$), whereas the second graph shows the mean luminosity factor (L_λ^{NPL}) for the complete visible spectrum. The third graph shows the spectral distribution curves of the primaries for the standard observer that are produced by assuming an equal energy spectrum of an arbitrary value of 100 at all wavelengths.

Figure 4.60 show the resulting chromaticity graphs from the Guild-Wright experiments, regarding the standard observer and for the primaries at 700, 546.1 and 435.8 nm (NPL white light). The three-dimensional graph on the top shows the actual shape of the chromaticity space in three dimensions. The three two-dimensional graphs show the same space in the possible two-coefficient combinations. The graphs only show the spectral colours, marked by their corresponding wavelength and the location of the white. All colours are within the concave space that the spectral curves denote and by imagining a line connecting the 380 nm end with the 700 nm end (the so-called line of purples).

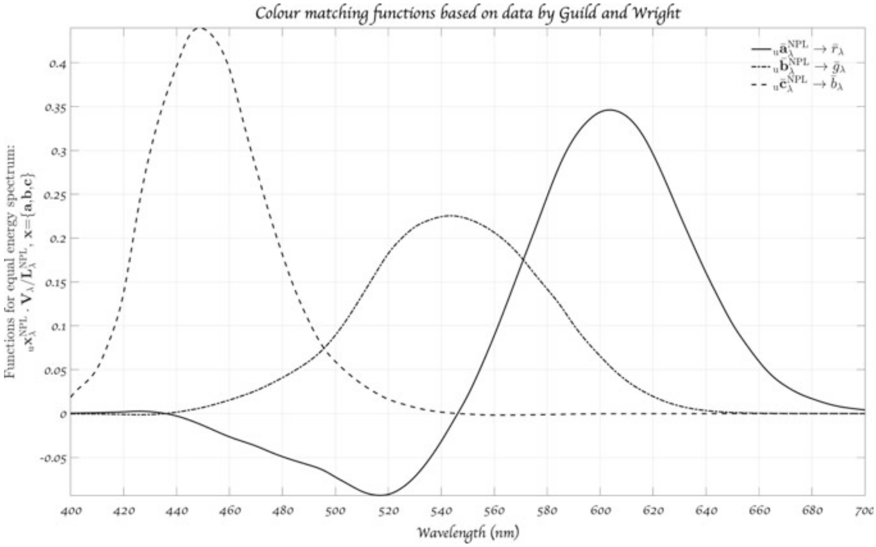


Fig. 4.61 Colour matching functions for the NPL primaries by the Guild-Wright data

The most useful colour matching functions can be produced by multiplying the trichromatic coefficients by the ratio of the luminous efficacy (shown in Fig. 4.56) to the luminosity factor,

$$\begin{aligned}
\bar{r}_\lambda &= {}_\mathrm{u}\bar{a}_\lambda^{\mathrm{NPL}} = {}_\mathrm{u}a_\lambda^{\mathrm{NPL}} \frac{\mathrm{V}_\lambda}{{}_\mathrm{u}\mathrm{L}_\lambda^{NPL}} \\
\bar{g}_\lambda &= {}_\mathrm{u}\bar{b}_\lambda^{\mathrm{NPL}} = {}_\mathrm{u}b_\lambda^{\mathrm{NPL}} \frac{\mathrm{V}_\lambda}{{}_\mathrm{u}\mathrm{L}_\lambda^{NPL}} \\
\bar{b}_\lambda &= {}_\mathrm{u}\bar{c}_\lambda^{\mathrm{NPL}} = {}_\mathrm{u}c_\lambda^{\mathrm{NPL}} \frac{\mathrm{V}_\lambda}{{}_\mathrm{u}\mathrm{L}_\lambda^{NPL}}
\end{aligned} \tag{4.19}$$

The colour matching functions that result are shown in Fig. 4.61.

In a paper most probably published some time after the establishment of the CIE 1931 standard, Guild with T. Smith (both members of the CIE Committee during that period) laid out a more concrete and corrected version of his original data and publication (Smith & Guild, 1931). Smith and Guild presented the general context, the corrected data and the resolutions included in the standard. Initially, they distinguished between the notion of colour as twofold; on one side it is to be considered *a subjective visual sensation* and on the other *an objective attribute of a physical stimulus* and its colour-matching relation to other stimuli. In addition, they presented a definitive view of the basic colour theory at that time, which stated that *colours are expressed in a linear trichromatic system using three primaries that correspond to*

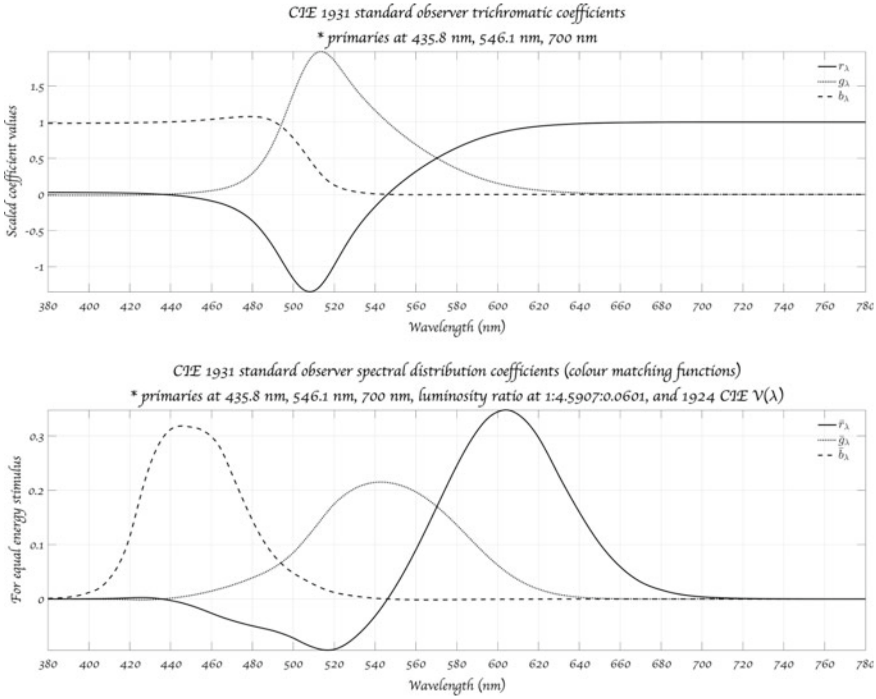


Fig. 4.62 The definition of the standard observer in CIE 1931 standard

linearly independent colour sensations. The presentation of the CIE 1931 standard was based on the description of the *five resolutions* that constituted it.

Resolution 1 defined the notion of the standard observer as defined by Guild's previously presented data and slightly corrected for better accuracy. Those data defined the CIE rgb colour space[42] (as trichromatic coefficients and colour matching functions) and the corresponding data were included in Table I of Smith & Guild (1931). The NPL standard white light was used as a reference with primaries at 435.8, 546.1 and 700 nm and the relative luminosities were fixed at the ratio of 1:4.5907:0.0601. Figure 4.62 shows a graphical representation of the data by Smith and Guild regarding the CIE 1931 standard observer. The first graph shows the r_λ, g_λ, b_λ trichromatic coefficients, whereas the second graphs shows the colour matching functions $\bar{r}_\lambda$, $\bar{g}_\lambda$, $\bar{b}_\lambda$.

Resolution 2 defined the standard illuminants to be used for colour applications. These illuminants were simply referenced as *A*, *B* and *C* and corresponded to a gas lamp at 2848 K for reference *A*, the same lamp with liquid filters composed of particular copper and cobalt sulphate solutions for reference *B*, and the same lamp with different liquid filters again based on copper and cobalt sulphate solu-

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[42] The red-green-blue (rgb) are used by convention to correspond to the perceived colours of the used primaries. Other primaries could easily result in another colour space.

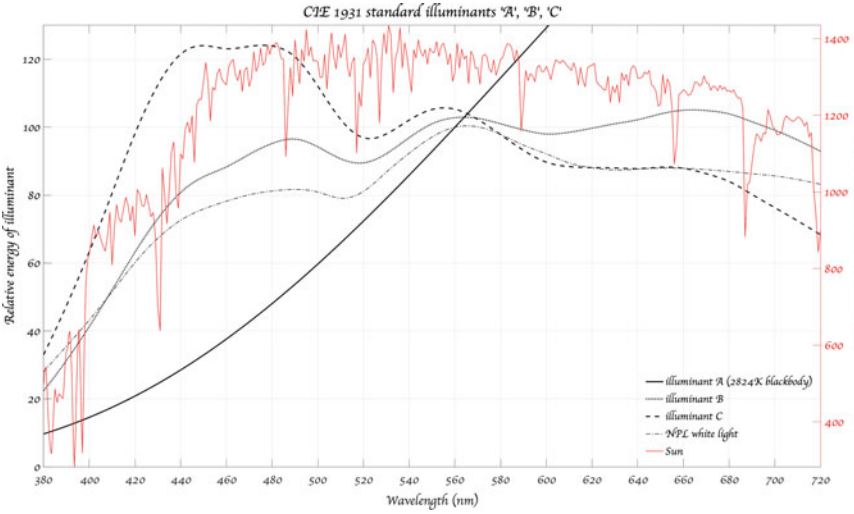

Fig. 4.63 The spectral power distribution of the CIE 1931 standard illuminants

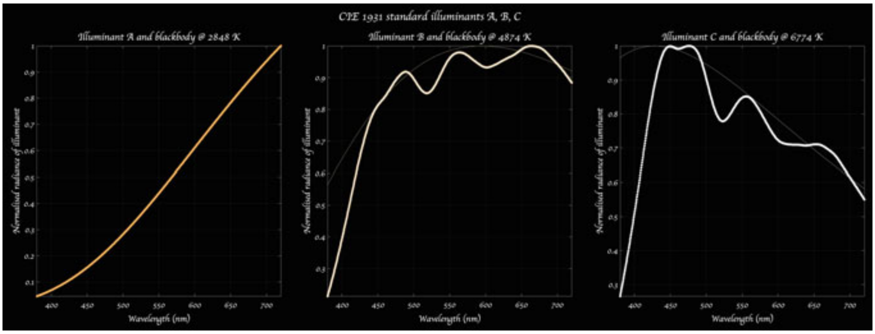

Fig. 4.64 Approximation of the radiation of the standard illuminants by black body radiation

tions for reference *C*. Tables II and V of Smith & Guild (1931) included the relative energy spectral distribution of the illuminants for practical applications. In the analysis presented in the paper, it was specified that the illuminant *B* corresponds to a blackbody at a temperature of 4800 K (currently estimated at 4874 K) and that the illuminant *C* corresponds to a blackbody at 6500 K (currently estimated at 6774 K). In addition, the previously used NPL standard white light was mostly similar to illuminant *C*. Figure 4.63 depicts the relative spectral power distribution of the three standard illuminants in the visible spectrum. For comparison, the graph includes the distribution of the NPL standard white light and the solar radiation at sea level (right-side y-axis). In addition, Fig. 4.64 shows the spectral power distribution of the standard illuminants in comparison to approximating blackbody radiation, depicted in a colour corresponding to the matching colour temperature.

Resolution 3 defined the conditions and setup for the colourimetric measurements of reflective and opaque media.

Resolution 4 defined that colourimetry is based on trichromacy and any colour should be expressed as a linear combination of three independent scales.

Resolution 5 defined the CIE 1931 XYZ colour space, as a projection of the original RGB space to a space where (a) no negative values are allowed and (b) one of the three variables corresponds to the perceived luminance. To fulfil these conditions, the X, Y and Z variables correspond to *fictional primaries*, totally outside the perceivable colour gamut. This process is presented both in terms of an analysis of the rationale and the method and is supported by detailed data in Tables III and IV of Smith & Guild (1931). This is one of the most analysed parts of the work done for the CIE 1931 standard. Originally, Smith & Guild (1931) presented this projection using a two-dimensional representation of the colour space, and particularly the blue-green representation. Smith and Guild emphasised that the estimates for the definition of the new x, y, z axes were not strict, as only the *alychne* was a mathematically and conceptually established notion. As defined by Schrödinger (1994), the alychne is the line of zero perceived luminance. Using the *luminosity factor* defined in (4.18), this translates to having

$$r + g\mathrm{L}_G + b\mathrm{L}_B = 0 \tag{4.20}$$

in which the luminosity factors of the primaries are normalised by the factor of the first primary ('R' by convention), corresponding to 1:4.5907:0.0601 (as stated in Resolution 1). As the colour matching function for the second primary ('G' by convention) is extremely close to the general luminosity sensation (luminous efficacy), the variable to be used for the side closer to that primary was Y coordinates. Thus the alychne should include the X and Z coordinates. Figure 4.65 graphically depicts the $b - g$ colour space, as Smith and Guild also did, and the transformation to the $z - y$ colour space, whereas Fig. 4.66 presents the resulting $\bar{x}$, $\bar{y}$, $\bar{z}$ colour matching functions based on the estimate of that xyz space, as described in the following paragraphs.

To solve this for the case of the $b - g$ space that Smith and Guild used, the equation of the XZ line is estimated by

$$\begin{aligned} s_{XZ} &= \frac{\mathrm{L}_R - \mathrm{L}_B}{\mathrm{L}_G - \mathrm{L}_R} \\ k &= \frac{(r_0 - 1)\mathrm{L}_R + g_0\mathrm{L}_G + b_0\mathrm{L}_B}{\mathrm{L}_G - \mathrm{L}_R} \\ g &= s \cdot b + k \end{aligned} \tag{4.21}$$

where s_{XZ} is the slope of the line, k is the displacement factor and (r_0, g_0, b_0) are the chromaticity coordinates of the B primary, which corresponds, according to the Guild data, to (0.0272, −0.0115, 0.9843). To determine the XY and ZY line equations there is no strict method according to Smith and Guild. They just reference

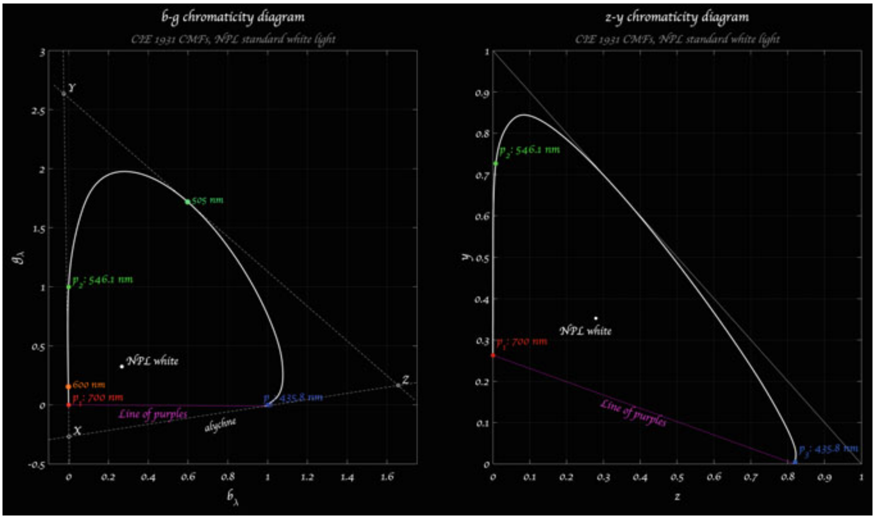

Fig. 4.65 Derivation of the $z - y$ colour space from the $b - g$ colour space

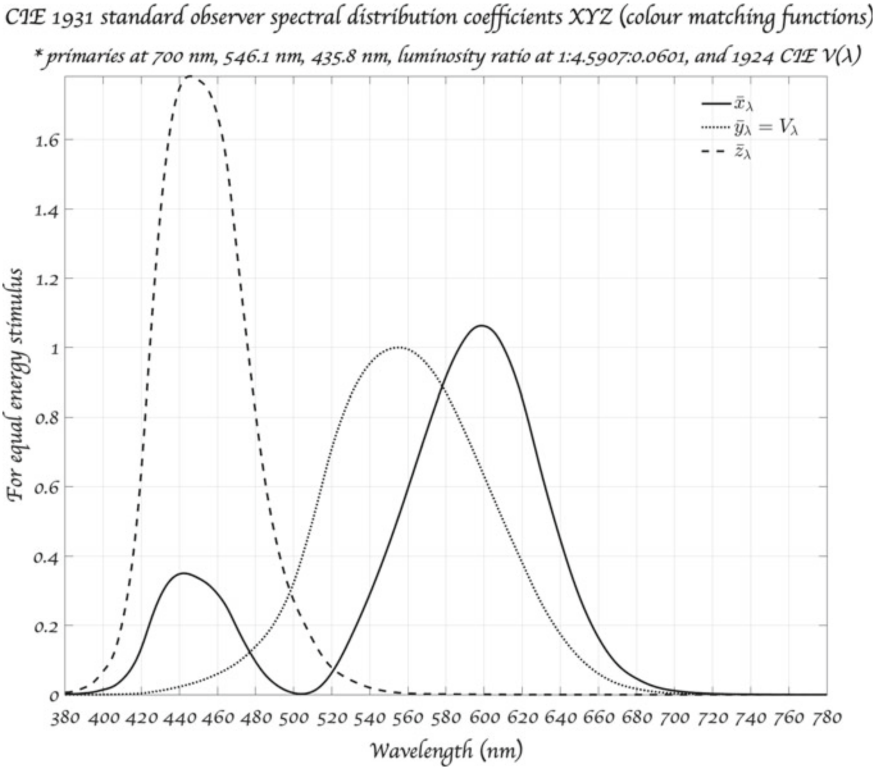

Fig. 4.66 Estimated colour matching functions from Smith and Guild data

suitable selections that fit the needs of applications and still fulfil the two basic prerequisites stated above. They state that the XY line should be tangent to the spectral locus at the location of the R primary. Thus it is difficult to proceed and replicate their results. Having also in mind the fact that the Tables they presented are relatively sparse and there is noise in the measurements, one may resort to interpolation techniques and suggestions by other researchers (Fairman et al., 1997; Wold & Valberg, 1999; Service, 2016). For example, Fairman et al. (1997) suggest to find the tangent to the spectral locus at the position of 670 nm. In the case of the experiments of the author, the interpolated original data where still noisy at that location and thus the 600 nm location was selected. The corresponding XY line equation can be expressed using the derivative of the locus at the selected location,

$$\begin{aligned} s_{XY} &= \frac{\mathrm{d}g_{[600\mathrm{nm}]}}{\mathrm{d}b_{[600\mathrm{nm}]}} \\ g &= s_{XY} \cdot b \end{aligned} \tag{4.22}$$

s_{XY} is the slope of the XY line. The same strategy is used to find the ZY line, which, according to Smith and Guild *should pass near the spectral locus in a direction that secures satisfactory metrical properties*. Again following Fairman et al. (1997), the selected locus point is at 505 nm and the corresponding ZY line equation can be expressed using the derivative of the locus at the selected location,

$$\begin{aligned} s_{ZY} &= \frac{\mathrm{d}g_{[505\mathrm{nm}]}}{\mathrm{d}b_{[505\mathrm{nm}]}} \\ g &= s_{ZY} \cdot (b - b_{[505\mathrm{nm}]}) + g_{[505\mathrm{nm}]} \end{aligned} \tag{4.23}$$

where s_{ZY} is the slope of the ZY line. The intersection of the three lines result the locations of the X, Y and Z primaries in the RGB space as

$$XYZ = \begin{bmatrix} 1.2661 & -1.6116 & -0.81979 \\ -0.26852 & 2.6351 & 0.16421 \\ 0.0023934 & -0.023487 & 1.6556 \end{bmatrix} \tag{4.24}$$

where the columns of the XYZ matrix are the coordinates X, Y and Z primaries in the RGB space. To transform to the fictional XYZ space assuming unitary vectors XY, XZ, ZY one should simply find the inverse of XYZ matrix and normalise it. This results a transformation matrix T which can be used to project the RGB colour space to the new XYZ space.

$$T = \begin{bmatrix} 0.4879 & 0.30029 & 0.21181 \\ 0.17413 & 0.82109 & 0.0047813 \\ 0.0000 & 0.0088345 & 0.99117 \end{bmatrix} \tag{4.25}$$

The transformation matrix that Smith and Guild presented is slightly different due to their different approximation.

$$T = \begin{bmatrix} 0.49000 & 0.31000 & 0.20000 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00000 & 0.01000 & 0.99000 \end{bmatrix} \tag{4.26}$$

The transformation from RGB to XYZ space can be based on the formula suggested by Fairman et al. (1997),

$$\begin{aligned} rgb &= \begin{bmatrix} r_\lambda \\ g_\lambda \\ b_\lambda \end{bmatrix} \\ s &= \sum_{i=1,2,3} T_{i,1} \cdot r_\lambda + \sum_{i=1,2,3} T_{i,2} \cdot g_\lambda + \sum_{i=1,2,3} T_{i,3} \cdot b_\lambda \\ x &= T_1 \cdot rgb \oslash s \\ y &= T_2 \cdot rgb \oslash s \\ z &= T_3 \cdot rgb \oslash s \end{aligned} \tag{4.27}$$

where $T_{i,j}$ is any i, j element of the transformation matrix and T_i is any i row vector of the matrix. The $\oslash$ operator denotes the element-wise division. Applying this transformation to the RGB data Smith and Guild provided the XYZ data obtained are shown in the $x - y$ diagram in Fig. 4.65. A more familiar and common representation is that of the $r - g$ space transformation to $x - y$ space, as shown in Fig. 4.67, following the same principle as in the case of the $b - g$ space. Smith and Guild provided extensive data in Tables III and IV in 1 nm intervals.

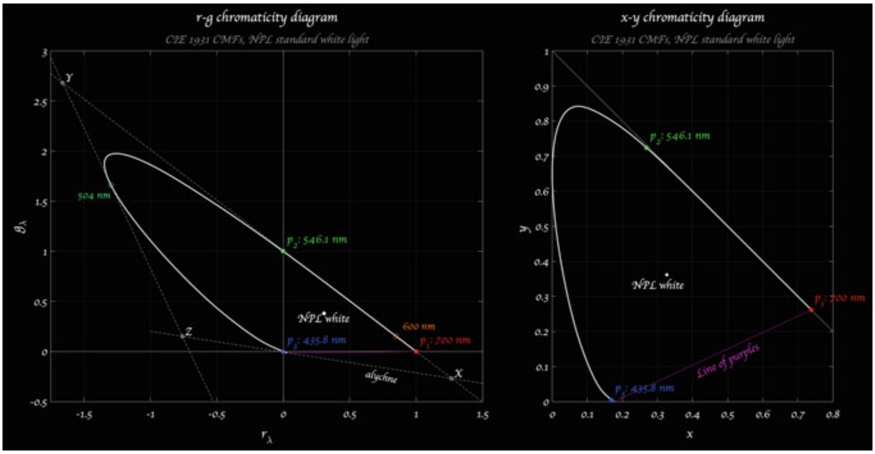

Fig. 4.67 Derivation of the $x - y$ colour space from the $r - g$ colour space

Guild in his 1970 work *Some problems of visual perception* found in MacAdam's collection (Guild, 1970b), presented a critical view of the current approaches and even the semantics of colourimetry and visual perception theories. He emphatically clarified that *in any colour matching experiment, the observer need never operate more than three independent controls, as, in general, three are necessary, and in no circumstances are more than three required.* Then he tried to also clarify the stages involved in a process of radiation reception, in general, which also applies to human vision. Thus he recognised four stages, including three objectives (1–3) and one subjective (4).

- Stage 1: the receptor, which interacts directly with radiation and determines the spectral sensitivity of the overall sensing system
- Stage 2: the coupler, which is the interface that transmits intensity-based messages to the next stages
- Stage 3: the indicator, which transforms the initial detection event message to a representation adequate for a higher level recognition
- Stage 4: the cognizer, which is the part of the system that *becomes aware* of the state of stimulation of the receptor.

By applying this general concept to vision, Guild concluded that *there are three, and only three, independent reception systems in simultaneous operation in human vision*. Consequently, he formulated a general hypothesis that *there could be any number, not less than three, of different types of receptor, and there could be central connections of any number of different modalities, not less than three*. He also proposed three other alternative theories, by which either the number of modalities or the number of the receptors may vary in any combination of those. Furthermore, he analysed those hypotheses in terms of logic based on evolutionary biology to consider some of them more improbable than others, but due to lack of knowledge in the mechanics of the human vision at the receptor level, he was unable to support one or another.