Maximizing Amount of Transferred Traffic for Battery Powered Mobiles

Eitan Altman[1,2]([✉]), Ghilas Ferrat[1,2], and Mandar Datar[1,2]

[1] INRIA Sophia Antipolis - Méditerranée, Valbonne, France
eitan.altman@sophia.inria.fr
[2] CERI/LIA, University of Avignon, Avignon, France

Abstract. There is a fast growing demand for mobile telephones. These rely on batteries to provide the power needed for transmission and for reception (up and downlink communications). Considering uplink, we analyse how the characteristics of the battery affect the amount of information that one can draw out from the terminal. We focus in particular on the impact of the charge in the battery on the internal resistance which grows as the battery depletes.

1 Introduction

In the design of power control algorithms, often one takes into account the level of depletion of the battery. When the battery is almost empty, a power saving mode is often applied. While this allows to extend the battery life, it is of interest to compare the benefits of this algorithm with one obtained from mathematical formulation of the problem as an optimisation one. In this paper we propose such a model that takes into account dynamic behavior of the battery and the fact that the internal battery resistance changes as a function of the battery's charge. Indeed, it is stated in that

"Li-ion has higher resistance at full charge and at end of discharge with a big flat low resistance area in the middle. Alkaline, carbon-zinc and most primary batteries have a relatively high internal resistance, and this limits their use to low-current applications such as flashlights, remote controls, portable entertainment devices and kitchen clocks. As these batteries deplete, the resistance increases further". We shall use the latter behavior in our modeling.

Our goal is to combine the dynamic behavior of the battery as function of its charge with capacity limits on transmission throughput from information theory to obtain limits on the amount of data that can be transmitted by mobiles that are powered with a battery.

2 Model

We assume that a fully charged battery has F Coulombs The figures in [1] reference suggests that the battery internal resistance R is linear decreasing in the battery's charge c and is thus given by

$$R(t) = R_0 - \rho c(t)$$

for some constants ρ and R_0. The current i satisfies

$$\frac{dR(t)}{dt} = \rho i(t)$$

since $dc/dt = -i$.

We model the battery by a source of V volts with the internal resistance of R. Another resitor r is then connected in series to the two other elements and it represents the terminal equipment (TE).

$$i = \frac{V}{R + r}$$

Thus

$$\frac{dR(t)}{dt} = \frac{\rho V}{R(t) + r}$$

3   Analysis and Results

The solution of this differential equation is

$$R(t) = \sqrt{2\rho t V + (r + R_0)^2} - r$$

for some constants ρ and R_0. We could easily verify that $R(0) = R_0$ and that $\frac{dR(t)}{t} = \frac{\rho V}{R(t)+r}$. And so, $i(t) = \frac{V}{\sqrt{2\rho t V + (r+R_0)^2}}$.

We can then compute the time $T(x)$ the battery takes to discharge from F to x as the solution of $F - x = \int_0^{T(x)} i(t)dt$. We finally get the closed form expression of $T(x) = \frac{1}{2V}(F - x)[2(r + R_0) + \rho(F - x)]$. The power spent at the TE is given by

$$P(t) = ri(t)^2 = \frac{rV^2}{(r + R_0)^2 + 2\rho V t}$$

Assume that all this power is transmitted and that the channel gain to the base station is h. Then assuming a single user and that the throughput Θ is given by Shannon capacity, we have for some constants W and N,

$$\Theta(t) = W \ln(1 + \frac{P(t)h}{N}) = W \ln(1 + \frac{rhV^2}{N((r + R_0)^2 + 2\rho V t)})$$

and the total amount of data that can be transmitted is $\int_0^{T(0)} \Theta dt$, denoted TA.

In practice a terminal has to use some constant power Δ for its electronic circuit which reduces the amount of data transferred.

$$TA(0) = \int_0^{T(0)} \Theta(t)dt - \Delta T(0) = \int_0^{T(0)} W \ln(1 + \frac{rhV^2}{N((r + R_0)^2 + 2\rho V t)})dt - \Delta T(0)$$

$$= W[\int_0^{T(0)} \ln[N(r + R_0)^2 + 2N\rho V t + rhV^2]dt - \int_0^{T(0)} \ln[N(r + R_0)^2 + 2N\rho V t]dt] - \Delta T(0)$$

$$(1)$$

Using $\frac{d}{dx}\frac{1}{a}((ax+b)\ln(ax+b)-(ax+b))=\ln(ax+b)$ and computing the TA, we get that

$$
\begin{aligned}
TA(0) \; = W &\left(\frac{1}{2N\rho V}N(r+R_0)^2 \ln\left(1-\frac{2N\rho VT(0)rhV^2}{(rhV^2+N(r+R_0)^2)(N(r+R_0)^2+2N\rho VT(0))}\right)\right. \\
&+ \frac{1}{2N\rho V}rhV^2 \ln\left(1+\frac{2N\rho VT(0)}{rhV^2+N(r+R_0)^2}\right) \\
&\left.+ T(0)\ln\left(1+\frac{rhV^2}{2N\rho T(0)+N(r+R_0)^2}\right)\right) - \Delta T(0)
\end{aligned}
$$

4   Maximization of the Average Throughput

Previously we had a closed form expression of the time $T(x)$ that the battery takes to discharge from an initial charge F to a x level.

$$
T(x) = \frac{(F-x)[2(r+R_0)+\rho(F-x)]}{2V}
$$

$$
TA(x) = \int_0^{T(x)} (W\ln(1+\frac{rhV^2}{N(r+R_0)^2+2\rho NVt})-\Delta)dt
$$

We also had a closed form expression of the total amount of data transmitted until we reach a x level of battery, $TA(x)$. Here the objective is to maximize the average throughput before charging the device, denoted $AP(x)$. Mathematically, the program can be written as

$$
\max_{x\in[0;F]} \; AP(x) = \frac{TA(x)-TC(x)}{T(x)} \tag{2}
$$

Let $\mathcal{L}(x,\lambda) = \frac{TA(x)-TC(x)}{T(x)}+\lambda(F-x)$ be the Lagrangian of the program. Solving it leads to

$$
\begin{cases} \frac{\partial}{\partial x}\mathcal{L}(x,\lambda) = 0 \\ \frac{\partial}{\partial \lambda}\mathcal{L}(x,\lambda) = 0 \end{cases}
$$

We consider a case in which the total cost of charging the device is the same whether the battery is empty or almost full, denoted $TC(x)=\gamma$.

$$
\Longleftrightarrow \quad \begin{cases} \frac{T'(x)\left(\Theta(T(x)-\Delta-TC'(x))T(x)-(TA(x)-TC(x))\right)}{T(x)^2} - \lambda = 0 \\ F-x = 0 \end{cases}
$$

Considering the case of a border solution where $\lambda > 0$, we have $x^* = F$. Meaning that an agent charges the device, every time his battery full. In the case of an internal solution, where $\lambda = 0$, the first equation leads to either $T'(x) = 0$ or $\Theta(T(x))T(x)-\Delta T(x)-TA(x)+\gamma = 0$. The corresponding solution to $T'(x) = 0$ is $x = F + \frac{r+R_0}{\rho} > F$ therefore impossible considering the program. Using the software $Maple(Maplesoft)$ for the second equation, the time necessary to reach an optimal level of battery before charging the device $T(x^*)$ given by,

$$T(x^*) = \frac{e^{Root}}{2\rho NV}$$

Where *Root* is the solution of the following equation in terms of z,

$$
\begin{aligned}
2W(N(r+R_0)^2 + 2\rho NV)\ln(rhV^2 + N(r+R_0)^2) \\
+ W(e^z - 4\rho NV)\ln(rhV^2 + N(r+R_0)^2 + e^z) \\
- 2W(N(r+R_0)^2 + e^z)\ln(rhV^2 + e^z) + 2WN(r+R_0)^2 z \\
- We^z \ln(N(r+R_0)^2 + e^z) + 2(Wz - \Delta)e^z \\
- 2WN(r+R_0)^2 \ln(N(r+R_0)^2) + 2\gamma \rho NV = 0
\end{aligned}
$$

In order to compute the internal solution x^*, the inverse function of $T(x)$ must be computed, and finally, the optimal level of battery is given by,

$$x^* = \frac{\rho F + r + R_0}{\rho} - \frac{\sqrt{e^{Root} + N(r+R_0)^2}}{\rho\sqrt{N}}$$

This solution has sense since it belongs to $[0; F]$ and that the agent charges his device only once his battery is at a lower level than the full level.

5   Numerical Applications

We have looked up real values for our various parameters, which make sense. We consider a battery life of 24 h, to simplify future computations. In general, a mobile phone's battery is a source of ca. $3,7$ V, has an initial internal resistance of ca. 105 mΩ, and when fully charged, has 7200 Coulombs. The second resistor has often a resistance of ca. 20.9 Ω (Fig. 1).
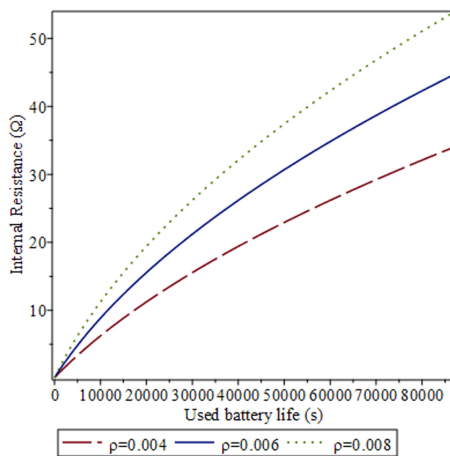

Fig. 1. Internal Resistance as a function of the remaining battery life

As stated in our model, a high level of charge means the internal resistance is low. We consider ρ as the restriction coefficient of the battery. As ρ is small as the power delivered by the battery is less constrained, the battery encounters less restrictions in delivering power, as shown in Fig. 2. In general, the restriction coefficient is ca. 0.006 (between 0.004 and 0.008) for most mobile phone battery's.
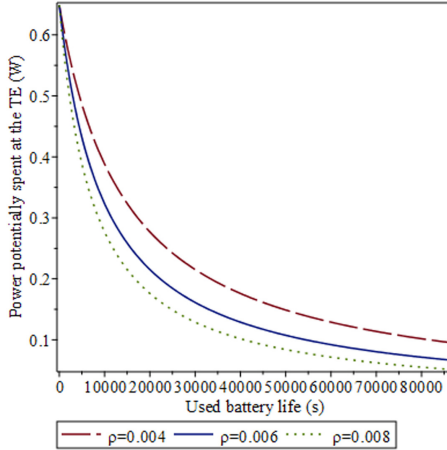

Fig. 2. Power potentially spent as a function of the remaining battery life
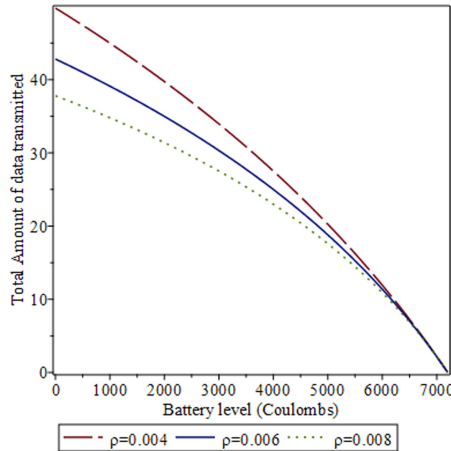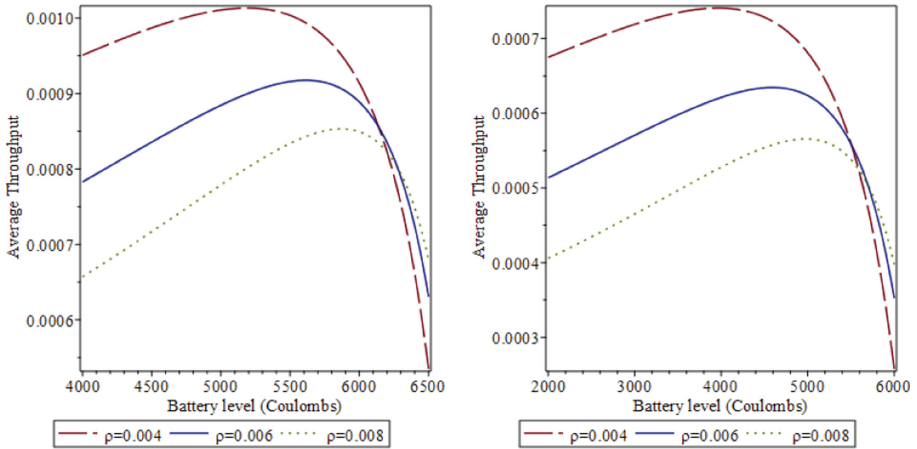

Fig. 3. Total amount of data transmitted as a function of the level of battery

Similarly, the total amount of data that can be transmitted in a full cycle of depletion is higher for smaller restriction coefficients, as shown in Fig. 3. And so, in a full cycle of depletion, the total amount of data that can be transmitted (i.e. TA(0)) is 49.78 for $\rho = 0.004$, 42.82 for $\rho = 0.006$, and 37.80 for $\rho = 0.008$.

Furthermore, assuming that recharging the battery is costly. We have optimized the average throughput over the level of remaining charge in the battery at which recharging would begin. We have assumed that the cost of charging the device was the same whether the battery was empty, almost empty, almost full or full. Here, with no loss of generality, we suppose that the cost is around 10 to 20% of the total amount of data that can be transmitted.

The figure on the left (respectively right) represents the average throughput, with a cost of 10% (resp. 20%) of the total amount of data that can be transmitted. We observe that, all other things being equal, a higher cost of charging drives the optimal battery level at which charging should begin to a lower level and so, leads to less cycles of charge.

6   Rising Internal Resistance

It is of interest to compare the amount of data that can be transmitted considering a constant resistance and a resistance as a function of the remaining battery life, like in our model. In [1] and [2], it is suggested that the battery's internal resistance R is an increasing function of used battery life and that a high level of resistance causes heating of the battery and less power can be spent, which ultimately leads to reducing the battery's life. As the battery's life is reduced, the total amount of data that can be transmitted decreases, the phenomenon through which rising internal resistance reduces performance of the device. As to show the phenomenon of rising internal resistance, we have plotted the difference between the throughput we would have if there was no rising effect and the average flow of data transmitted. Mathematically,

$$\int_0^{7200} \Theta(0) - \frac{TA(x)}{T(x)} dx.$$

In our model with our data, as shown in Fig. 4, we notice that the throughput with a constant resistance (i.e. $\int_0^{7200} \Theta(0)dx$) is of approximately 14 arbitrary
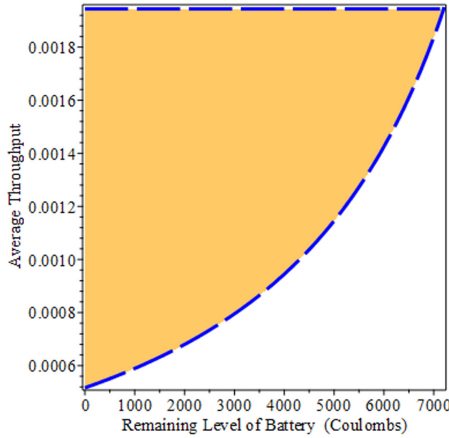
Fig. 4. Effect of Rising Internal Resistance

units and the average flow of data transmitted (i.e. $\int_0^{7200} \frac{TA(x)}{T(x)} dx$) is of approximately 7.01 in the same units. This difference is the physical phenomenon of rising internal resistance.

7   Conclusions

We have derived closed form solutions to the amount of information that a battery powered terminal can deliver. We made simplifying modelling assumptions, namely that the antenna can be represented as a resistor. In practice, it may be represented as an impedance that may vary in the frequency used. Also note that we did not model Alternative Current powers and the losses due to Direct Current/Alternative Current transformation. The next thing to do is to add a cost function that would penalize using the battery as a function of its charge so as to penalize operating it when the internal capacity of the battery is large. Indeed, as R increases, the efficiency of the battery decreases: most of its remaining power is spent on heating the battery as it is spent on R. Assuming that recharging the battery is costly, we have optimized over the level x of remaining charge in the battery at which the charging would begin so as to minimize the cost per cycle of depletion and recharging. We also plan to investigate interference and competition that may arise when more that one terminal is connected with a common base station. This is the objective of our future work.

References

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