Modeling Practices to Design Computer Simulators for Trainees' and Mentors' Education

Fabien Emprin

1 Introduction

Our work focuses on the simulation of human interactions for the education of mathematics teachers. We are working on three types of interactions: teacher–students in a classroom teaching situation (Emprin & Sabra, 2019), teacher–teacher in a mentoring situation, students–students in a geometric work situation (Emprin & Petitfour, 2020). Our goal, in this chapter, is to show how the work on these simulators can be upstream of the work on AI-aided educational working spaces and is articulated with the usual work without the digital tools (classroom observation), but also with digital training (use of simulators in training) and with the production and analysis of data on learning. For us, the development of AI-aided educational working spaces poses several problems: the nature of the aids that these tools can offer (on which aspects of teaching, to assist which parts of the practices), that of the use by teachers of such environments (digital technologies are struggling to integrate effectively into practices (Abboud & Emprin, 2009) and what follows, i.e., teachers preparation in such tools.

Our human interaction simulators belongs to the artificial intelligence tools in that it is an expert system that operate with pre-established rules, but our goal is to also show how they can feed deep learning algorithms. There are two main lines of work: first, human interaction simulators can be used to define and verify models that would be useful, by reducing the quantities of data to be processed; second, they generate data to power AI-aided environment.

In the first part of this chapter, we rapidly develop theoretical frameworks that guide the work of modeling human interactions in education and teacher preparation.

F. Emprin (✉)
Université de Reims Champagne Ardenne, CEREP, IREM de Reims, Reims, France
e-mail: fabien.emprin@univ-reims.fr

P. R. Richard et al. (eds.), *Mathematics Education in the Age of Artificial Intelligence*, Mathematics Education in the Digital Era 17,
https://doi.org/10.1007/978-3-030-86909-0_14

These frameworks allow us to define teaching and training practices and to make choices for building computer models.

In a second part of this paper, we develop a model of the mentoring situation that we implement in a simulation software. This Mentoring Dialogue Simulator (MDS) is used in teacher training in France and Russia (Galiakberova et al., 2020).

The third part is dedicated to the analysis of the exploitation of a simulator to generate data that can be exploited with AI-based algorithms. For this, we take the example of the Computer Classroom Simulator (CCS). Indeed, it offers a fully controlled situation and reproducible activity. This type of tool can easily be put online or on a Learning Management System (LMS) and thus proposed to a large number of users who themselves can make several tests. The data generated are therefore quantitatively very important and comparable (since the simulated situation is the same for all). This is what we have done with the CCS by developing an online tool associated with a collection of information on the user (questionnaire that the user could fill in). The models built can then evolve according to the information collected automatically but also during training sessions.

At the end of this chapter, we discuss the use and the limits of this work for an AI-Aided environment.

2 Theoretical Framework

In a training simulator (MDS or CCS), several levels of practice are involved: that of the students facing the Dynamic Geometry Software (DGS), that of the teacher in the classroom, that of the instructor using the simulator in training. These different Layers are nested like Russian nested Matryoshkas dolls. Each layer requires the mobilization of specific theoretical frameworks: student learning, teacher practices, teacher training practices.

2.1 Choices About Student Learning

What all our simulators have in common is that they all address the issue of teaching geometry with DGS). This decision was made to limit the didactic theoretical knowledge at stake and because this is a field in which many research studies have been carried out (Laborde, 2001, Sträßer, 2001, Baccaglini-Frank & Mariotti, 2010; Jones, 2000), allowing us to anticipate the difficulties encountered by students or teachers and thus to model them. Moreover, geometry is a fundamental field of knowledge, as Sherard (1981) point out and it is a necessity for the development of both individuals and our society:

> Geometry has not died because it is essential to many other human activities and because it is so deeply embodied in how humans think. With the introduction of computers with rich graphical capacities and the recognition of multiple ways of learning, our current situation

> offers an unprecedented opportunity for geometers and those who work visually (Whitely, 1999, p. 1).

The question of the teaching of geometry has moreover been the subject of several international studies such as Mammana and Villani (1998) or working groups in international conferences as CERME[1] (Jones et al., 2017).

We focus on the possibilities offered by the movement of figures in the process of conjecture and demonstration. When a drawing is made in the software using the appropriate properties, this drawing resists the dragging of its base points: a square constructed with perpendicular isometric diagonals intersecting in their middle remains a square whatever the points dragged, conversely if the properties are not used it will deform and become rectangles or simply quadrilateral. This led us to place the teaching situations in the simulators for students over 10 years old. In particular, we use two ideas of the model developed by Baccaglini-Frank and Mariotti (2010), based on the model of Arzarello et al. (1998, 2002).

First, DGS may become a potential bridge between two worlds, i.e., the mathematical world of Euclidean geometry and the phenomenological world of experience, which we will refer to as the spatial-geometry world. Therefore, DGS provides teachers with new insights and tools to overcome students' difficulties.

Second, the dragging mode in DGS can be seen as an instrument (Rabardel, 2002) that can change students' approach to the proving process.

> Dragging can help the user interpret the exploration in terms of logical dependency, in the following way […] Such use of dragging may develop through a process of instrumental genesis (Rabardel, 2002). If we consider dragging to be an artefact and place a user in the context of solving a problem (task), it is possible to identify specific utilization schemes associated with dragging (dragging utilization schemes, in the following 'dragging schemes') […] (Baccaglini-Frank & Mariotti, 2010, p. 228).

With regard to the proving process, we rely on the work of Arsac (1987) and Richard (2004), which highlight that the transition from the conjecture phase to the demonstration phase is complex, and on the distinction between argumentation and proof (Balacheff, 1998).

As a result, the classroom sessions that will be at the core of the simulators should highlight these elements. We are now interested in how to look at teaching practices.

2.2 *Twofold Approach to Define and Model Teaching Practices*

Robert and Rogalski (2005) built a theoretical framework for defining and analyzing teaching practices. This approach, which combines didactical and psychological perspectives, is known as the twofold approach didactic and ergonomic (TADE).

[1] Congress of the European Society for Research in Mathematics Education.

It examines practices on the basis of the following five components: a cognitive-epistemological dimension, i.e., the organization of tasks and the mathematics that is actually taught; a mediation–interaction dimension, i.e., interactions through verbal or written and direct or indirect communication; a personal dimension, i.e., whether the teacher regards mathematics as a science or merely as a subject for teaching and learning processes; a social dimension, i.e., a teacher works in a teacher's collective within which not everything is possible; and an institutional dimension, i.e., teachers are subject to academic programs and institutional injunctions.

This twofold approach to analyzing teachers' practices allows us to create a model of such practices, taking both the didactic and professional aspects of the work into account. It allows us to consider the content to be taught.

In a previous paper (Emprin, 2007, 2009), we demonstrated that the training of teachers to teach mathematics with digital technologies in France deals with very few of the dimensions referred to above. In the case of the proposed simulator, we have chosen to model the teacher's responses on four of the five components (we do not discuss the institutional component). Indeed, these four components make it possible to question the teacher's didactic choices in relation to the chosen theoretical framework (uses of the DGS for conjecturing and demonstrating) through his interactions with the pupils, his choice of learning situations, his way of seeing (from a personal point of view but also as part of a community of teachers) mathematics and in particular the place of demonstration.

In this work, the place of digital tools is important. Teachers' educators use simulation software, the students use a DGS. We need to be able to question the relationship between humans and digital tools.

2.3 Instrumental Genesis

The instrumental approach (Rabardel, 2002) is also used to analyze teacher activity when a digital artifact is used. Dragging is also an instrument for teachers, and schemes of use can be determined and implemented in a simulator.

Adopting the instrumental approach allows us to analyze the instrumental genesis of the pupils in relation to the tasks proposed by the teacher. We can then implement trajectories of these geneses in the simulator. We regard the instrument as being constructed by the teacher from an artifact, which is in this case a DGS.

The scheme of use is a central concept in the instrumental approach, especially with a view to understand the role of the artifact as a mediating tool in the construction of knowledge. The various components of the scheme have been identified as follows (Vergnaud, 1998): the goal (or subgoal) of the action, the teacher's rules of action for achieving the goal, the operating invariants that justify the rules of action, and the possibilities of inferences that allow the teacher to enrich his operational invariants according to the lived experience. According to Vergnaud (1998, p. 238), the role of mediator is the most important work of the teacher in class:

> [the teacher] starts out of class where he begins to organize a complex process of implementation with several constraints, conditions and variables from one class situation to another. It continues in the classroom with a large number of decisions and mediations in a very limited time.

Vergnaud (ibid) proposes referring to the various components of the schemes to analyze these decisions and the acts of mediation. The operating invariants reflect a form of appropriation of knowledge by the teacher.

Using this approach, the virtual teacher's answers result from the analysis of the operating invariants of practices.

We combine these theoretical frameworks with specific frameworks depending on the nature of the simulation. For example, to create a mentoring dialogue simulator we have to characterize the different postures of the mentor. The role of the mentor is particularly important at a time when the question of the appropriation by teachers of AI-assisted environments arises in the same way as for other digital tools.

3 How to Create a Model of Mentor–Teacher Interactions?

The mentoring dialogue can cover several practical aspects, for example, classroom management, learning plans design, organization of the session, and proposed teaching–learning processes. All these various aspects can be treated a priori in a simulator of human interactions; however, we have chosen to focus on processes related to the learning and teaching of mathematics. We based our work on two main assumptions:

A first founding assumption is that by making mentors aware of the dilemmas and possible roles in mentoring dialogues, they will be more efficient and able to make choices. The theoretical frameworks presented in this section allow us to design an artifact that enables an academic or professional supervisor (and even a peer) to approach a dialogue to analyze and question its practice.

Our second assumption is that going beyond the teacher-centered dialogues to bring them to student learning allows the tutored teacher to question and develop his practices. Specifically, "student teacher supervision, with few exceptions, remains a teacher-centered enterprise. Typical supervision policies, procedures, and forms focus on teachers' observable behaviors." (Paris & Gespass, 2001, p. 398). It is therefore a question of considering teaching practices in several of their dimensions, in particular the cognitive-epistemological dimension. To achieve this, the simulator must make it possible for the instrumental genesis of the DGS to be questioned at two levels: that of the student and that of the teacher.

Thus, designing a Mentoring Dialogue Simulator (MDS) requires the following three elements: excerpts from a classroom session conducted by the virtual teacher, a model of current virtual teacher practices (twofold approach), and a model of the mentoring relationship. The user of the simulator will be able to view or read transcripts of the session, make choices on conducting the mentoring dialogue, and receive feedback of his choice (see Fig. 1).

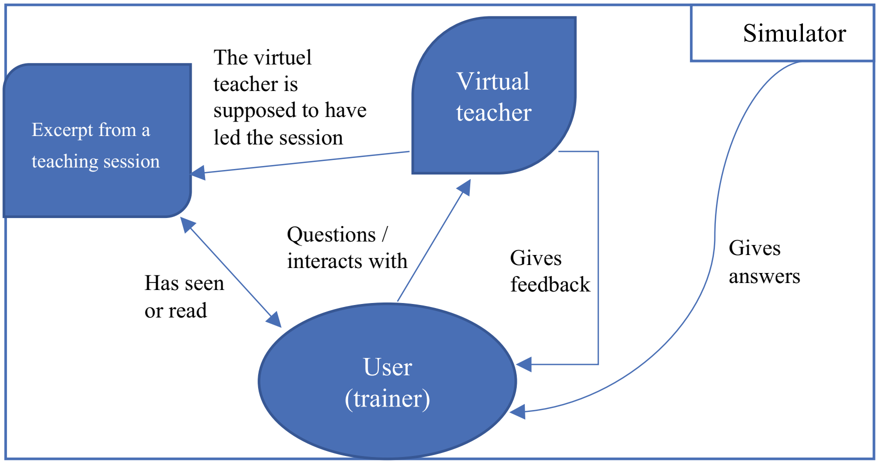

Fig. 1 Model of interactions in mentoring dialogue simulator

This MDS is designed to be used at the core of a face-to-face or training device for online trainers.

What makes our research method specific is the triple use of theoretical frameworks which make it possible to analyze the real practices observed, to design models and systems of interactions in simulators and to analyze the simulated practices of users.

We now present the choices and the design process for such a simulation.

3.1 From the Design of a Mentoring Dialogue Simulator (MDS) to a More General Model

3.1.1 Choice of Software

To simulate a dialogue of the analysis of practices first requires the existence of a teaching situation, which is the starting point of the process. This situation provides the basis for the cognitive-epistemological dimension of the practice. We must therefore provide elements that can be discussed in the interview; these are video clips showing the work of the student and verbal interactions between the teacher and the student.

Our goal is to encourage a reflection on the actions taken as well as reflexivity on the practices; we therefore opted for a part-scale simulator (Pastré, 2005). Unlike full-scale simulators, which aim to reproduce reality as faithfully as possible, part scale simulators focus on a specific part of the subject's activity. In simulated time—as opposed to in real time—it is possible to exchange views and think about an action before actually taking it. It is also possible to link several simulations quickly to test several hypotheses. To consider several practice dimensions, including social and personal dimensions, we decided to use a simulator environment with realistic conversational agents that can simulate emotions and use speech synthesis. With this choice, we dissociated ourselves from other existing simulators which use avatars with a comic/drawing appearance. This allowed us to enrich the system of interacting with the user and to increase the level of possible feedback. Studies on Embodied Conversational Agents (ECA) (Krämer & Bente, 2010) demonstrate that they have promising potential for improving the motivation and learning of users. However, a detailed discussion of these effects is beyond the scope of this chapter.

To create simulations of human interactions, we opted to use the Virtual Training Suite software (VTS—© Serious factory—Suresnes, France), which enables the generation of a realistic 3D environment with ECA to incorporate multimedia content, to build a rich scenario and to follow the answers of the users in the form of a score. Entirely customizable, the software manages the scenario as a graph. The overall scenario is divided into scenes that are depicted graphically (Fig. 2).

Within each scene, word bubbles provide elements of interaction (choice of verbal interaction, answers to a multiple-choice question, or clicking on the screen), and the branches that result are therefore the different decision paths (Fig. 3).

The scenario, once compiled, can be exported to different formats, including the web, SCORM (for LMS), iOS (iPad and iPhone), Android, Mac OS, or Windows, or deposited on a Serious Factory learning platform (the company that developed the software). We do not go into technical design details here but rather describe the conceptual aspects embedded in the simulation. For the construction of the MDS, we

Fig. 2 Cutting the scenario in scenes

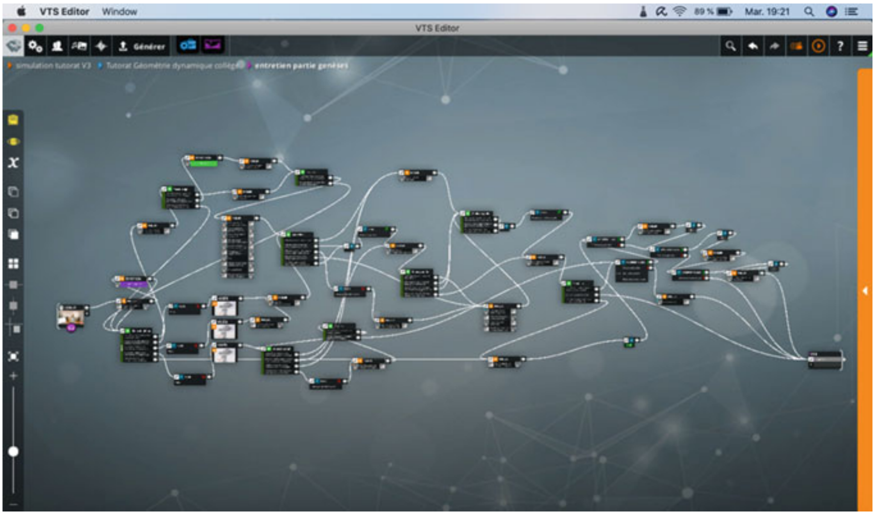

Fig. 3 Scenario graph of a scene

started from the theoretical framework MERIDs' Model and mentoring dilemmas of Brau-Antony and Mieusset (2013), with the use of didactic knowledge.

3.1.2 Combining MERID's Model and Mentoring Dilemmas

In a literature study (Hennissen et al., 2008, 2017), five key aspects were identified.

> As an answer to the first research question, from the selected empirical studies, five key aspects of mentoring dialogues emerged, which are often the focus of research: content of the dialogue, mentor teachers' style and supervisory skills, mentor teachers' input, time aspects of the dialogue and phases in the dialogue.

The authors extract/highlight three of these five aspects.

> In answer to the third research question, empirical evidence in the selected studies indicates that three key aspects connect with distinctive mentor teachers' behavior in mentoring dialogues: style/supervisory skills, input and time aspects. These three aspects are plausible candidates to constitute a conceptual framework. We connected these key aspects in the MERID model, which identifies four mentor teacher roles during mentoring dialogues: imperator, initiator, advisor, and encourager.

Mattéï-Mieusset (2013), for her part, identifies the following four dilemmas associated with this particular facet: transmitting the job or reflecting to allow the trainee to build his response; spotting the mistakes and successes of the trainee or helping them to emerge; supporting the trainee or evaluating him or her, guiding, imposing a framework, providing the trainee with tools, or leaving him or her free to make his own choices.

We combine these two frameworks which constitutes a complete theoretical framework to model the mentor/teacher relationship.

We described the didactic choices and instrumental genesis issues of the DGS implemented in the classroom scenario used in the following section. We then implemented the software in such a way as to bring out this knowledge and the knowledge of both didactic and mentoring activities in the user's possible answers and in the feedback of the software. The MDS is therefore designed in such a way that the user becomes aware of the facets and dilemmas related to the practice analysis interview activity and may identify knowledge that highlights the role of technology in the learning process.

3.1.3 Classroom Session Used as a Basis for Simulated Mentoring Dialogue

Our objectives in terms of didactic content led us to choose to focus on the use of geometry software for students aged 14–15. The students must conjecture about a property and then demonstrate it such that a triangle's three perpendicular bisectors meet at a point O, known as the circumcenter, which is also the center of the triangle's circumcircle:

> In the software. Draw a triangle. Name it ABC. Draw the perpendicular bisectors of each sides. What can you conjecture? Let O be the point of intersection of two of the perpendicular bisectors. Draw the circle with center O and radius [OA]. What can you conjecture? Then demonstrate the property.

We videotaped several teachers implementing this lesson and we transcribed some characteristic interactions of instrumental genesis problems related to DGS. From this material, we made four video clips, each of which presents a characteristic time of the lesson. We have chosen to focus on the use of DGS as part of both a conjecture and a demonstration. We rely on the research literature to identify the knowledge that could be transmitted during a training session. One key point emerges in the case of dynamic geometry, namely, the robustness of the construction of objects (Baccaglini-Frank & Mariotti, 2010; Laborde, 2000; Restrepo, 2008). The space in which students work is relevant because it brings a new system of constraints. The correct construction is one that is resistant to dragging. Moreover, this system of constraints requires an assimilation by the student that can be analyzed in terms of instrumental genesis (Rabardel, 2002).

The first video illustrates the screen of a student who built the middle point of a segment in the DGS environment by measuring the length of the segment and then placing a point on the segment and moving it to obtain the desired length (half the length of the segment). The teacher asks the student to drag the midpoint, which the student does. The resulting exchange is as follows:

Teacher (T): So, what's going on?

Student (S): It is no longer in the middle.

T: What's the problem there?

S: There's no problem.

T: So how can you say it's in the middle? You will say, "Yes, I measured and I placed it in the middle." But is the drawing something that is stable?

S: No.

T: So, is that the way to go?

Students use correct mathematical knowledge. The distance from one side of the segment to the middle is half the length of the segment, so this construction is not resistant to dragging. The teacher's intervention does not allow the student to understand the didactic rules specific to the DGS. The teacher asks the student if the drawing is stable but does not refer to the rule that "to be correct, a construction must withstand all movements." We also notice that the instrumental geneses of the DGS are poor for this student; he does not allow himself to move the point and does not use the primitive constructions implemented in the software to build resistant points. The instrumental genesis of the teacher is also weak; he does not use the software as a learning environment with feedback.

In a second excerpt, the teacher asks a student to provide a definition of the perpendicular bisector. The students answer is "a straight line that passes through the middle of a segment and is perpendicular." The teacher then begins an exchange based on the fact that the student did not use the correct article, stating that the student should not have used "a" but rather "the". This choice is debatable because it results in the student questioning an issue that is not directly related to the task at hand, even if the teacher's point is correct.

In the third video clip, when a group has correctly drawn the three perpendicular bisectors, the teacher then asks them to move a vertex. First, the student is worried that he will "have everything wrong," and then he asks himself why one of the three perpendicular bisectors does not move. The student cannot conclude anything other than the fact that there is a "base" in the triangle that results in the mediator of the base not moving. The interesting aspect here is that this example illustrates how point shifts on a resistant drawing can lead students to identify phenomena that allow them to gain a better understanding of mathematical properties. It demonstrates, once again, that the instrumental genesis of both the student and the teacher are too weak to benefit from work on DGS. The challenge of the interview will be to help the teacher not only realize the potential of the DGS but also incorporate the understanding under which he was operating. It's about following what Sträßer has highlighted: "There is no change in geometry learning and teaching without additional 'costs' in terms of investment of time and concepts to understand the software!" (Sträßer, 2001, p. 331).

The last clip extract concerns the end of the lesson, the moment when the students debate their ideas. It is a very typical situation because only the teacher speaks. He makes students guess words by means of the topaz effect (Brousseau, 1997, 2006). A question posed to students is progressively rephrased up to the point where the expected answer is ultimately generated: "If you have three lines, what is the word used to designate the point? The point of … / con-course, and we say lines are … // con-current. So, here you see again the difficulty of vocabulary control." The challenge is understanding why the teacher decided to share this information without interacting with the students. Is he aware of this lack? Is it a question of the length of the session? What drove him to make that choice? Could it have been social material constraints? What are the alternatives and their consequences?

We now describe the link between these classroom sessions and the system of interaction with the ECA.

3.1.4 Design of the Interview

The four video excerpts allow us to question each one a theoretical knowledge of didactics of mathematics. These four excerpts also allow us to define four entities (the scenes of the software) that make up the scenario (Fig. 4).

In each of the scenes, the mentor must therefore lead the virtual teacher to question himself on a specific problem (instrumental genesis) and help him acquire the targeted didactic knowledge. In the fifth and last scene, he must say, for each of the points, if he thinks he has succeeded, and the virtual teacher will answer him if he has indeed evolved.

In the scenes, the user (mentor) must interacting with the ECA. For this he chooses from a list the interventions and the actions he wants to perform. An example of such a list can be seen on the left in Fig. 4, which is a screenshot.

The context of the interview was chosen to correspond to the usual practices of mentors, who have indicated that they liked to settle down in a quiet place with their trainee (Brau-Antony & Mieusset, 2013).

The questions chosen by the user and the answers of the ECA are then heard. The ECA is animate and can express different feelings both by voice and by means of facial expressions, as presented in Fig. 5.

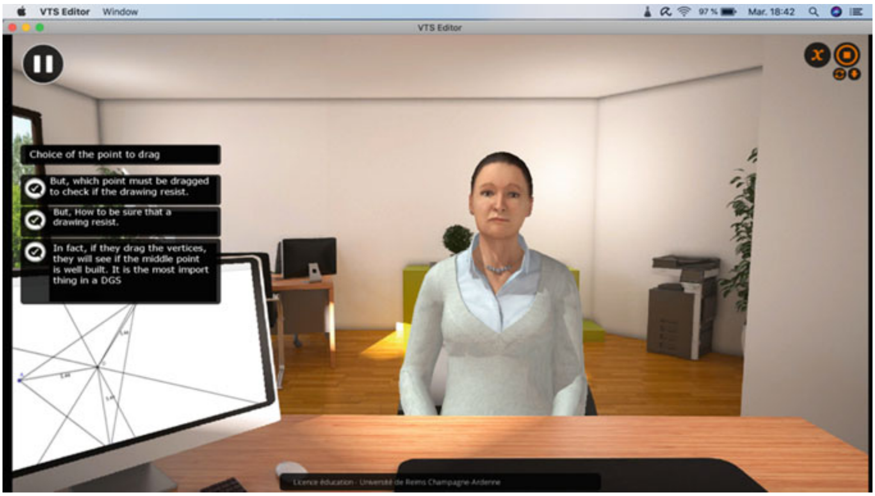

Fig. 4 Screenshot of the MDS

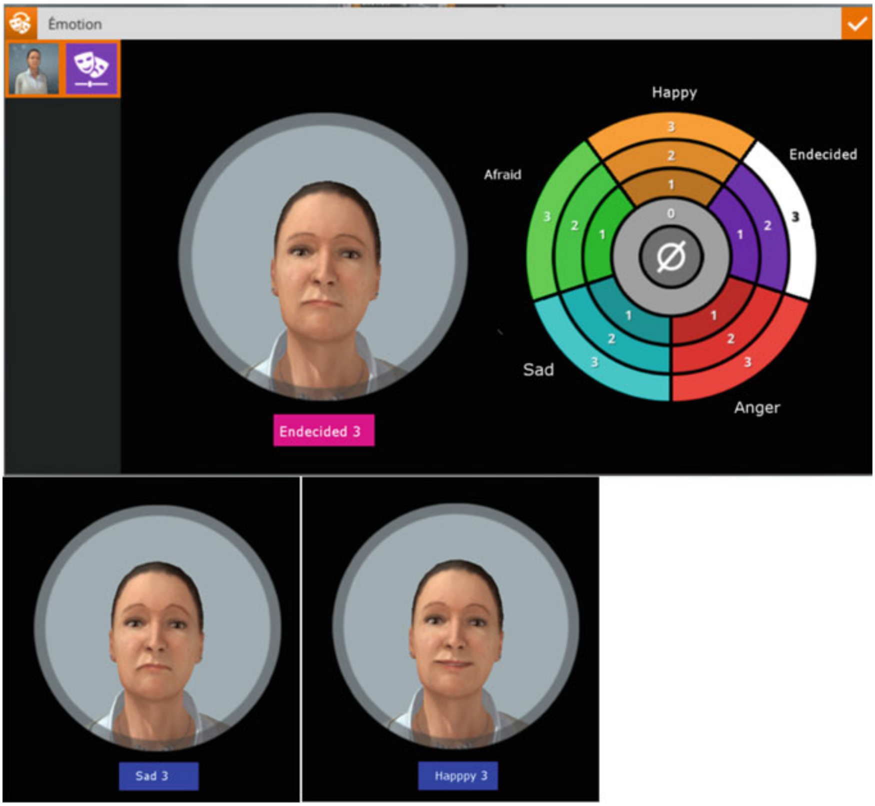

Fig. 5 Facial expressions in the VTS software

3.1.5 Choice of Interactions

The MDS simulator has been designed to allow users (i.e. future mentors) to become aware of the dilemmas and the different roles he can take in dialogues. At the end of each simulation the software must position it in relation to these different roles. To this end, each choice in the list (each possible interaction with the ECA) was assigned a weight in the form of a score for each of the axis of Fig. 7. For example, the axis "non-directive—directive" is graduated from 0 to 4. 0 for non-directive, 1 for undirective, 2 for a little bit directive, and 3 for directive. To be realistic, the sentences chosen have been extracted or inspired from the body of interviews by Mattéï-Mieusset (2013).

The intervention, "I propose that we play the video of your session to look at it together. You can stop whenever you want to, at a moment that seems interesting to you" is likely to result in the representation of the teacher emerging from a nondirective and reactive position without guiding the trainee too much. This choice therefore receives 0 on the “non-directive—directive” axis, 0 on the “reactive—active” axis, 0 on the axis (super teacher—evaluate) and 0 on the axis (leave free choice—guide).

The following intervention, however, is more guiding and comes from a nondirective and active position: “I will show you excerpt of your session and I’ll ask you questions about your choices.” This choice therefore receives 1 on the “non-directive—directive” axis, 3 on the “reactive—active” axis, 1 on the axis (support teacher—evaluate) and 3 on the axis (leave free choice—guide).

The following final intervention is made from an active and directive position and with a choice of guidance: “I can tell you what I saw; in fact, you asked your students the definition of the perpendicular bisector, and they told you ‘we build a mediator’; you explained to them that one does not say ‘a’ but ‘the’ perpendicular bisector.” This choice therefore receives 3 on the “non-directive—directive” axis, 3 on the “reactive—active” axis, 3 on the axis (support teacher—evaluate) and 3 on the axis (leave free choice—guide). All the choices made during the simulation add up and allow the user to position himself on each of the axes. For more clarity, the user receives a percentage.

The choices given to the user provide the virtual teacher to question his practices and improve his didactic knowledge, such as the following:

It is clear to me that the students who move a vertex of a triangle and wonder why one of the mediators does not move are not sufficiently familiar with either the figure or the operation of the DGS. This means that you did not spend enough time on the construction because you were guessing at something that is not at all clear to the students. If you decide to make the students work something out, you have to go to the end of the process.

An alternative element is as follows: “*First of all, I noticed that you are acting in the student’s place. Some of your students do not understand the resistance of objects. They do not understand it as a criterion for checking the drawing. You do not do enough to precisely move the basic points of the drawing precisely as criteria. Finally, your last interventions on the uniqueness of the mediator changed the subject while the students were fully occupied with the construction.*” Depending on the fact that these interactions more or less allow the teacher to evolve, they are given a score, which is not visible to the user. It is used in the last scene of the scenario.

3.1.6 Feedbacks

In this last scene, the user has to answer questions about the effects of his dialogue on the virtual teacher. He must state whether he believe that his virtual trainee has identified different elements that could be at stake during the dialogue, as illustrated in the screenshot in Fig. 6. The trainee answers by stating whether he has actually identified the various elements.

The user can thus compare what he thinks he has worked and what has actually worked.

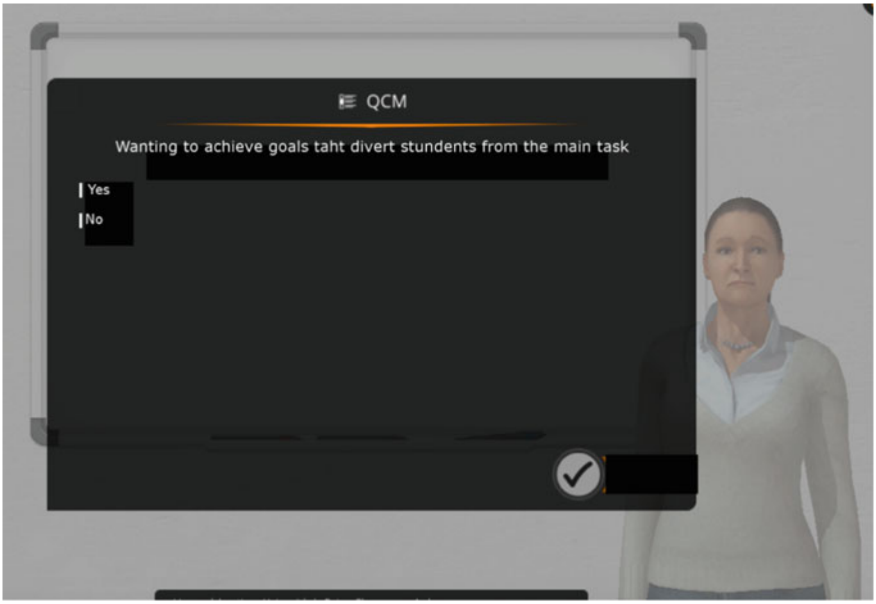

Fig. 6 Final questionnaire—determine what the ECA has identified or not

Once the work is complete, the software sends a final feedback. The user therefore receives his profile in the form of a percentage positioning on each of the axes of the model in Fig. 7. For example, the extract in Fig. 7, the user is on the transmit side of the scale (19%), which is also on the side of "point out the errors," "guide," and "evaluate." The last score concerns the appropriation of the didactic contents possible in the session. The score of 15 out of a possible 53 indicates that the trainee only worked on certain didactic concepts during this interview.

This positioning result from the score obtained in the MDS. Each choice can be interpreted according to the context, but that is precisely what makes this work interesting. In training sessions, this positioning can then be used and discussed with the mentor's educator.

Indeed, the simulator is not designed to be used alone but rather as the core of a training device, thereby allowing a collective reflection on the contents transmitted during the practice analysis period and the postures and management of the dialogue. These interpretations allow teachers to engage in a discussion during a training session. Such exchanges will relate to the institutional, personal, and social components of the practices. Our choice is therefore to generate discussions that truly involve these components and that therefore require trainees to formulate personal elements related to their teaching contexts, their representation of the profession, mathematics, etc.

All the work on the simulator, which is, let us recall it, an expert system and not a deep learning algorithms makes it possible to test theoretical models of the

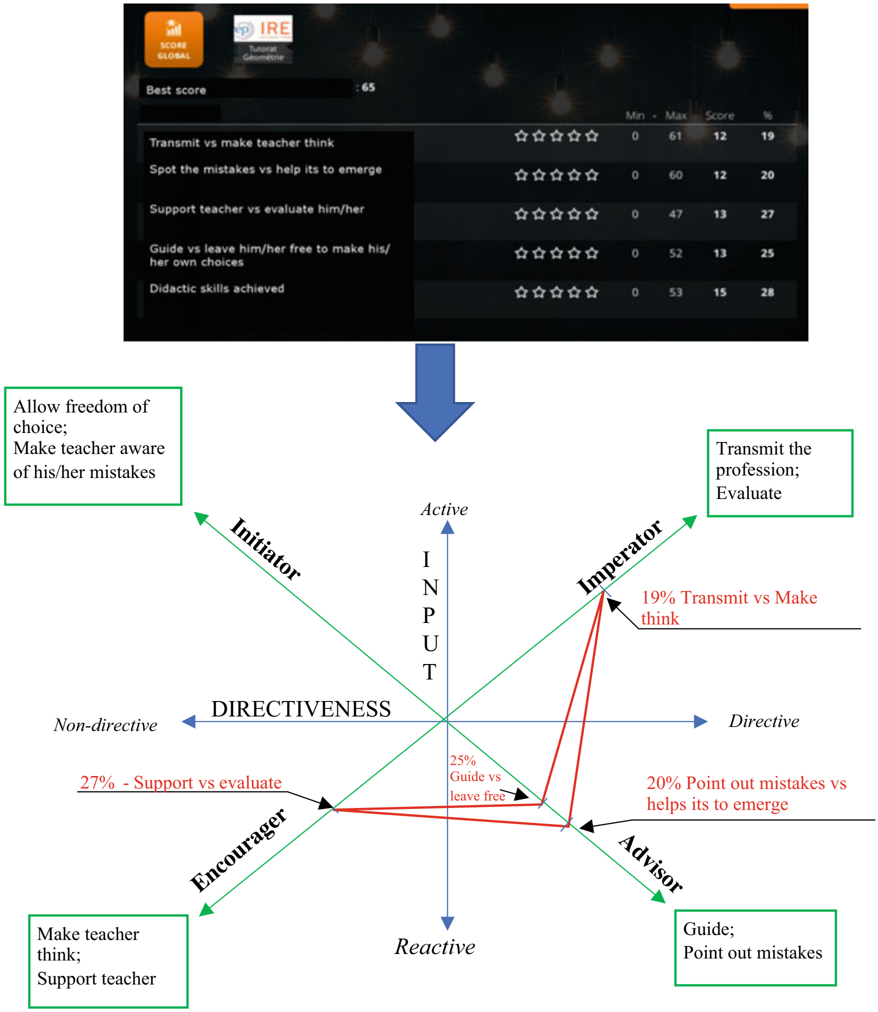

Fig. 7 Final diagram: positioning the user the theoretical model combining MERID'S model et Mattei-Mieusset's Dilemmas

mentoring activity and to check that it works, and is acceptable. The data collected are therefore qualitative but essential in our opinion to then consider systems using deep learning for example. In the last section, we will come back to the prospects of using these models. In the following section, we will look at how another simulator can be used to collect massive data.

4 Collecting Data in a Classroom Simulator

The Computer Classroom Simulator (CCS) is used to educate student (trainee) by way of “teaching” virtual student. The teacher has predetermined choices and they see the effect of their choices on student behavior. We have designed a simulator that simulates reactions centered on didactics, i.e., on the teaching-learning process. The user receives several types of feedback about the student: his mathematical work, his answers when questioned and his level of attention (represented by a VU meter). The simulated situation is a problem of conjecture in dynamic geometry software, we do not describe it in detail here, but it is presented in Emprin and Sabra (2019). To model this interaction system, we use the double approach (Robert & Rogalski, 2005) and the instrumental approach (Rabardel, 2002) presented in the theoretical part. We then operated by observing classroom situations and compiling the teacher’s choices and the students’ responses using the division of the activity into cognitive, meditative, social, personal, and institutional components as well as instrumental mediations. We then made statistics to recreate a situation consistent with the choice averages. The virtual teaching situation is designed accordingly to the Theory of Didactical Situation TDS (Brousseau, 1997, 2006). We implemented the CCS with the data collected and by replicating the observed behaviors.

4.1 Highlights in the Implementation Analysis

So, there are two kinds of data collected: didactic and what I will call pedagogical (in the common sense). The first concerns the problem that the student will have to solve: construction of a figure in the DGS or not (the file is provided partially or totally to the student) and the role of the drawing in the conjecture, the role of the phases of conjecture, proof, institutionalization. Here, the TDS makes it possible to measure the distance between what is proposed and the concepts of devolution, didactic situation, a-didactic phase (of a situation), antagonistic environment, institutionalization, time management of learning. The pedagogical aspects are those that are independent of the teaching situation in mathematics: choice of the conditions for giving the instruction (computers on or off), choice of the modalities of interaction with the students (collective, individual), time management from the point of view of the students’ attention.

4.2 Highlights in the Design of the CSS Simulator

The first choice is to program in the simulator, teachers’ decisions that have been observed in practice or in the preparations. It conditions the nature of the simulation: it simulates ordinary practices in the sense of those studied by Coulange (2012). This choice was discussed during the summer school of didactics in 2017 (Emprin et al., 2017): would it not be better to propose to the simulator user richer choices, based on didactic knowledge and therefore that he would not have thought of? It seems to me that we then have two types of classroom simulators: one that allows us to question ordinary practices and their effects, and the other that allows us to test new, richer practices.

The procedures and difficulties observed in the students were quantified and programmed in the software based on the a priori analysis. Thus, the number of students who succeeded in making a resistant construction in the DGS, in conjecturing from a right or wrong construction, in mobilizing or formulating the property of equality of the lengths of the diagonals of the rectangle corresponds to the observed proportion. The effects on student learning were quantified at three stages: at the end of the session by analyzing the students' answers to the question: are the lengths equal? But also, by the arguments written or formulated during the pooling, and finally by questioning the students a week later to find out what they had retained from the situation. The models that are programmed in CCS are based on observations: students who do not have a correct (resistant) figure are unable to conjecture. Even with a correct figure, the proof is difficult to produce if the students are not used (or encouraged) to use the DGS tools: moving points, measuring tools, adding extra segments. Moreover, in the absence of formal institutionalization during or after the session, even students who have found correct arguments remain with the superficial aspects of the problem. The only thing they retain is the answer.

The teacher's pedagogical choices are also programmed with their effects on the students. Thus, when the teacher is talking, and the students are doing something else (turning on the computers for example) only 1/3 of the students are able to repeat what has been said. On the other hand, when one of the students manages to say the sentence again, even those who did not listen the first time have understood and are then able to rephrase when they are invited to do so. Teachers who favor individual interactions spend much more time than those who interact with the group, but the latter interactions have more time consuming than individual interactions. Finally, some choices are also based on pedagogical hypotheses confirmed by numerous observations: students will not behave in the simulator when they do not understand or are unable to do the task, or when they have completed the work and have no new instructions.

The teacher's relationship to mathematics (personal component of practices in particular) is also questioned in this simulation. What is the relationship of mathematics to reality or what are the logical principles governing demonstration in mathematics? Indeed, in this problem the student must move from perception or measurement (whether paper and pencil or computer) to the demonstration of a property. Euclidean geometry, characterized by Euclid's axioms (Peyrard, 1804) corresponds to a mathematization of the real, whereas Hilbert's axiomatic (1900) gives geometry a status independent of the real. The training can thus refer to mathematical questioning. Numerous works such as those of Bkouche (1997) allow us to question and clarify teaching choices. Indeed, in simulated lesson, the question arises of the place of drawing and the figure, in particular the status of what is drawn in a DGS, of its role in the process of proof: between aid to intuition and obstacle to demonstration.

All of these elements make the simulator an artefact of the training situation that allows access to didactic concepts.

4.3 Main Concepts from the Didactics at Stake

A posteriori analysis of the simulator's design based on the observations allows us to question during a training situation several didactic aspects:

When the teachers finish the simulation session, the software output indicates that little learning has been achieved by the students. This is realistic compared to the weight of a single situation in the overall learning: what can a class really learn in 55 min? Nevertheless, this goes against the representations of teachers who declare themselves frustrated by this result. This may be indicative of their representation of learning (personal component) or of computer play. In the games, they know or serious games there is a way to win, an optimal strategy, in a simulator some choices are better than others but there is no 100% winning strategy.

Nevertheless, there are strategies that promote learning among virtual students. They are of two types: the existence of a real phase of institutionalization and a sufficient time of confrontation with the correctly constructed figure in the DGS to conjecturing, searching. With an erroneous figure or without a sufficient time, the confrontation of the student in the milieu (in the sense of the TDS) would not be effective.

Moreover, the programming of the simulation prevents the teacher from constructing the knowledge text alone. That is to say, he can only write on the blackboard what the students have said. This choice does not come from classrooms observations but is based on the researcher's desire to question the teacher's role in the institutionalization phases thanks to the simulation.

The didactic concepts of sub-figure (Mercier, 2018), drawing vs. figure (Duval, 1993; Laborde, 1985) and proof are fully evident in the simulation in that they allow for the analysis of students' difficulties or procedures: difficulties in locating rectangles in the drawing and in seeing that the second diagonal is a radius of the circle.

The status of the drawing for the students, in relation to their instrumental genesis of the DGS, remains very close to that of the paper/pencil. While they are, in fact, confronted with a class of drawings with the same properties, many of them only use it as a single representative of the figure.

Finally, the question of what forms the basis of proof in mathematics can be addressed (Mariotti, 2000) even though the software provides only a satisfactory answer. Using the measurement tool, the software indicates that the lengths are equal. Some virtual students are programmed not to offer a demonstration as soon as they have an answer given by the software.

This simulator is designed, based on the observation of practice and without introducing random parameters. Each simulated student will always react in the same way to the same series of choices. This makes it possible to obtain a perfectly controlled situation, strictly identical between each user. The only parameter which therefore varies is the user. It therefore makes it possible to collect information on teaching practices independently of the teaching–learning situation.

4.4 Collecting Data on Teachers' Practices

We have been putting online a version of the simulator for more than a year. It was associated with a form asking users to specify their gender, whether or not they are teachers, at what level they teach, their seniority, their perceived level of ability with digital technologies. Once this questionnaire was completed, the user had the choice to accept the sending of the data resulting from his simulation to the researchers. The chronological choices of the user in the simulation were collected as well as the synthesis of the students' learning (did they do the right thing? Did they learn the desired mathematical property? What do they remember after one week?).

Two hundred and sixty registered users carried out and submitted simulations which made it possible to collect 420 completed simulations. This set of data allow us to link choices with users' parameters.

We have not yet been able to use this data with deep learning algorithms for lack of a suitable partnership. We simply used statistical tools on a variety of variables (descriptive statistics, inferential statistics and multiple correspondence factor analysis) to find out whether statistical correlations emerged. The first interesting result of this correlation research is that there is none that is statistically significant. Neither age, nor gender, nor country, seniority in teaching or the perception of their level of familiarity with technology appears to be linked to the choices made. On the other hand, two main scenario models emerge. The first, where the teacher tries to have the geometric figures correctly constructed in the dynamic geometry software but does not manage to work correctly on the conjecture and the second during which the user passes very quickly on the construction to work on the conjecture. It is a central point of the training to make the user realize that the situation has two distinct objectives which are difficult to carry out simultaneously. The statistical data confirm that the simulation clearly highlights this aspect. We should now further explore the data to identify if other elements may appear, in particular elements which would not have been anticipated during the programming of the software. In addition, the data processing could also highlight actions that would be predictive of the final assessment. Finally, the processing of this data could make it possible to return personalized feedback.

5 Discussion

The two simulators that we have presented are aimed at helping teachers and mentors acquire knowledge and change their practices, more specifically regarding the use of a DGS to teach geometry.

According to Haenlein and Kaplan (2019) the initial work on AI is to be found in expert systems such as ELIZA and the General Problem Solver. These pieces of software consisted of a set of rules and a series of nodes "if–then" like our simulators.

Our approach goes in the same direction as the evolution of AI and our hypothesis is that it is an important step that allows us to lay the foundations for developing more complex systems. Indeed, the work we do advances the models of the teachers' or mentor's activity. This highlights the usual practices of teachers, the difficulties of these to develop learning situations using the potential of DGS. The experiments that we have conducted on the different simulators seem to us to be indispensable steps to consolidate the theoretical bases necessary to categorize and process data that could be collected more massively on the teaching practice.

As far as teaching practices are concerned, a first problem arises in order to move to tools using deep learning algorithms: access to data. In fact, accessing teaching practices and student learning even if we stay with the observable part of the activity, such as what the teacher says or what students respond to an exercise, requires collecting this information by filming classroom sessions. And even if we had hundreds of hours of filmed classes (or mentoring interviews) to see what students do based on what the teacher says, we would have to digitize and process this information. The models we experimented with would allow us to categorize this information and define observables. They make it possible to limit the complexity of the information to be collected.

Our goal in this chapter was to highlight two aspects that simulators can help: reduce the amount of data to be processed and the generate data usable by the AI. The models that we propose allow us to focus the data collection on the didactic dimensions of the teaching activity. For the second aspect, the quantities of data available (more than 30,000 teachers' choices associated with the simulated time in the session) or the possibility of deploying these tools on Learning Management Systems like Moodle thanks to the Scorm format opens up interesting possibilities.

There is a situation where this information on teaching practices is already digitized: this is the case of distance learning. Indeed, the exchanges between teacher and students as well as the student's activity pass through a course platform. The requested tasks and the students' results are therefore already in the form of computerized data. A first work could be based on these data, which are called learning analytics.

We have also shown that simulators can provide data that could be processed. If they are easily accessible and carried out in a completely controlled environment where the only parameter that changes is the teacher, they have a major drawback: they are not real teaching practices but simulated teaching practices. Nevertheless, it seems to us that these treatments could give information in a way 'in vitro' that could then be verified in reality. Thus, if we understand the difference between the choices on simulated and real practices, we can identify if some results, found on simulated practices, can be true in reality.

We have, in this chapter, focused our reflection on how simulators can be positioned upstream of devices using AI. However, a promising avenue seems to us to use them downstream of the AI. Indeed, deep learning algorithms that could be used to help learning will need to be tested at several levels: that of acceptability by teachers or trainers and that of effectiveness for students. The simulators can then play an important role to avoid testing, the first time, on real students with real teachers. This follows the adage that prevails for the use of simulators in the training of health personnel (e.g., doctors, dentists, nurses): "never the first time on a real (living) patient." These simulators also highlight the points of difficulty in the practices that AI-aided working spaces could attempt to resolve: the management of the assistance provided to students during the construction and conjecture phases, and the organization of the final debate phase in particular.

Simulators, as we have indicated, can integrate into LMSs such as Moodle. They thus make it possible to create new workspaces for practice, at least a certain form of practice. These new spaces offer perspectives for sharing between trainers, for collecting and processing learning data. This work is particularly the subject of the current HYPE 13[2] project. The simulators can also create new mediations between students and mathematics as in the geometrical drawing made by an embodied conversational agent.[3]

Moreover, if programs are able to offer teaching aids and to assist teaching, it will be necessary for teachers to be able to appropriate them, to understand the results provided, and also to know what their level of validity is. Simulators will then be valuable tools to interface between artificial and human intelligence.

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[2] The HyPE-13 project—Hybridizing and Sharing Teachings funded by the National Research Agency (ANR) is led by the University of Pau and the Pays de l'Adour (UPPA).

[3] See this simulator at https://fabien-emprin.pagesperso-orange.fr/actioninstr/.

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