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Investors' Adaptation to Climate Change: A Temporal Portfolio Choice Model with Diminishing Climate Duration Hazard

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1 Introduction

There is no denying that earth's climate is changing. This shift has prompted organizations as well as governments around the globe to take action toward climate change mitigation and adaptation. According to the Intergovernmental Panel on Climate Change (IPCC), "climate change mitigation" is the term used to describe the efforts aimed at reducing carbon emissions and greenhouse gases, whereas "climate change adaptation" refers to adjustments in natural or human systems in response to actual stimuli or expected stimuli and their effects (IPCC, 2001); the latter moderates harm or exploits beneficial opportunities.

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In financial markets, climate adaptation can be thought of as a temporal process that describes the progress in investors' awareness of current climate change risks as well as their beliefs and perceptions about future risks. This is consistent with the aspect of human adjustment in the IPCC definition. As a temporal process, climate adaptation can be characterized by two key features: time (duration) and risk (climate uncertainty). So, a successful adaptation plan should entail both an increase in investors' awareness and portfolio strategies that guarantee a perception that climate risks are decreasing over time.

This chapter proposes an asset pricing model that describes uncertainty (financial and climate risks) in investment decision-making as a temporal process over the duration of financial portfolios. The premise of modern portfolio theory (Markowitz, 1952, 1959) and Fahmy's (2020) recent extension of this theory are the foundation for this study's proposed framework. The novelty of the approach lies in describing investors' adaptation to climate change as a temporal process, in which investors' perception of climate hazards shapes their preferences and decisions over the duration of their financial portfolios. The proposed model has several advantages. First, the optimal solution of the model provides a parametric formula of climate duration hazard risk, which can be easily estimated empirically. Second, the model is flexible enough to be used by practitioners when projecting various climate risk scenarios and factoring climate risks into their portfolios. Third, the analytical results of the model yield a set of recommendations for a sustainable climate adaptation process. These recommendations are important to participants, regulators, and policy makers alike; such recommendations hopefully present a step forward toward winning the battle against climate change.

The chapter is organized as follows. Section 2 describes climate adaptation as a temporal process. Section 3 discusses investors' behavior over time and their perception regarding future climate risks. Section 4 introduces the model in question and discusses the main results. Section 5 demonstrates the empirical applicability of the model. Finally, Section 6 concludes and provides policy recommendations in line with the proposed framework.

2 Adaptation as a Temporal Process

Winning the battle against climate change is a monumental task that requires global collaboration. Stern (2007) offers an estimate of the financing needed to mitigate the effects of climate change in the range of US$200 billion and US$1000 billion. A report by the World Bank Group in 2008 echoes Stern's findings by suggesting that at least tens of billions of dollars are needed every year to finance the cost of adaptation to global warming. Raising this staggering amount of funds is clearly beyond the means of governments' limited budgets. Therefore, private investment in climate mitigation and adaptation projects is urgently needed, and on a large scale, this mobilization of funds can be achieved only through global financial markets with innovative solutions across asset classes (Reichelt, 2010). Although some organizations have already provided creative solutions that have attracted investors' interest in climate-related investments and have caused a rapid growth in green markets, investors are still unenthusiastic about investing in green instruments. This lack of enthusiasm is mainly attributed to investors' lack of knowledge about the potential impact of climate change on various asset classes (Shen et al., 2019) and/or investors' belief that green investment is more of a moral choice than a reward (Riedl & Smeets, 2017; Walley & Whitehead, 1994).

There is, however, evidence indicating a rise in the awareness of climate change amid climate-related events. For instance, Fahmy (2022) shows that admitting clean energy as an asset class on its own in portfolio construction is rewarding especially after the Paris Agreement. Choi et al. (2020) examine the impact of abnormally high temperature on investors' beliefs about climate change. The authors document a rise in the awareness of climate change at the time of those events. By further examining trading volume and stock markets' returns during the times of these events, the authors report that retail investors tend to overreact to climate events by selling carbon-intensive stocks, and carbon-intensive firms are perceived to under-perform firms with low emissions. Alok et al. (2019) find that institutional investors overreact to large climatic disasters that happen close to them. Moreover, in a recent survey on global institutional investors' perceptions of climate risks, Krueger et al. (2020) find that investors regard these risks to be important despite ranking them below financial, operational, and other types of risks.

Much of the extant literature suggests that investors' awareness, especially after the Paris Agreement (Fahmy, 2022) and the increased occurrence of climate crises around the globe, is on the rise. Climate awareness, however, is not enough to win the battle against climate change. Investors need assurance that green investments are rewarding and that temporal climate risks in green portfolios (i.e., portfolios that contain green instruments) are decreasing over time. Achieving the former objective is more related to mitigation, whereas the latter objective relates to adaptation. Although this analysis focuses on the second objective, it is worth noting that there is a natural risk/reward interconnection between the two objectives. Successful mitigation policies that stimulate investments in clean energy must have the potential to make investors perceive lower future or expected climate risks. On the other hand, successful adaptation policies that have the potential to alter investors' perception regarding future climate risks will, over time, enhance awareness and attract more green investments.

3 Investors' Temporal Behavior

In financial markets, processes known as asset allocation and portfolio construction form the basis of investment decisions. These processes are founded on Harry Markowitz's (1952, 1959) seminal work on portfolio theory. It postulates that, given a target expected rate of return (mean) on a financial portfolio, a risk-averse investor, who is facing a choice set X that consists of n risky assets, allocates their wealth over n assets to minimize the risk (variance) of the portfolio. Markowitz's mean-variance (MV) portfolio theory yields a vector of optimal asset weights that minimizes the risk of a portfolio of n assets.

The previous MV optimization, or some variant of it, for example, Black and Litterman (1992) model, is the process that is commonly used by fund and portfolio managers in the asset allocation phase in the portfolio management process (PMP).[1] In this phase, subject to the investment strategy of the fund, the total wealth of all investors in the fund is allocated on a number of asset classes (e.g., fixed income, domestic equity, foreign equity, commodities, real estate, and derivatives). Once the optimal investment weight in each class is determined, a process of security selection begins within each individual asset class such that the expected rate of return on the portfolio is maximized or the risk is minimized.

It is worth noting that the above-mentioned MV optimization is static in the sense that it is executed on the set X at a reference point in time. This reference point is equivalent to a trading time $t = 0$, that is, the time of constructing the portfolio before the actual trading takes place. As time progresses from point $t = 0$, investors' reactions to various types of uncertainty (e.g., global events, financial news, and other cognitive and behavioral biases) impact their temporal allocations and choices. This, in turn, could cause some investors to revisit their portfolios sooner than later for rebalancing. This dynamic process of continuously rebalancing or revisiting the portfolio over time is known as the dynamic portfolio duration problem. Fahmy (2020) provides a solution to this problem (i.e., an optimal time to revise/rebalance a portfolio under the assumption of uncertainty) via their time extension of the MV portfolio theory. In particular, by adding a time-choice set T to the set of monetary outcomes X and by modeling the investor's choice over the extended set $X \times T$, the author extends the MV portfolio theory and derives an analytical expression in which optimal portfolio duration is explicitly expressed as a function of different types of uncertainty. This explicit connection between time and uncertainty is what distinguishes Fahmy's (2020) model from other studies on portfolio selection under uncertain time-horizon (e.g., Blanchet-Scalliet et al., 2008; Brennan, 1998; Hakansson, 1969, 1971; Martellini & Urošević, 2006; Merton, 1971; Richard, 1975; Yaari, 1965).

The present chapter uses a modified generalization of Fahmy's (2020) model, in which duration hazard of climate change is added as an

[1] The PMP consists of three stages: planning, allocation, and performance evaluation. Asset allocation and portfolio construction take place in the second stage of the PMP.

additional source of uncertainty in the portfolio duration problem. The focus is on applying this generalized framework to climate adaptation. This framework is particularly suitable here since it yields analytical results that quantify the hazard of climate duration. The following section introduces the model and deduces the main results.

4 A Simple Two-Period, Risk and Reward Asset Pricing Model with Climate Duration Hazard

This section proposes a simple two-period, risk and reward asset pricing model that accounts for time and uncertainty. The proposed model, founded on Markowitz's MV portfolio theory (1952, 1959) and a generalization of Fahmy's (2020) MV-time extension, makes the distinction between the present and the future by separating the portfolio/investment decision into two sequential optimal decisions: an allocation decision on the space of monetary outcomes, X, and a duration decision on the time space, T. At time $t = 0$ (i.e., before trading takes place), the investor chooses an optimal allocation of assets on X that minimizes the variance (risk) of the portfolio. As time progresses from zero (i.e., as $t > 0$), the chosen allocation is subject to different types of uncertainty (including climate hazard). The investor chooses an optimal portfolio duration such that a utility of time function that represents the investor's preference, $U(t)$, is maximized. It is worth noting that the allocation decision executed on X at time $t = 0$ represents the certainty of the present. On the other hand, the optimal time to revise, rebalance, or even exit the market after trading represents the future uncertainty. A decision to exit the market amid an unexpected event with global implications, such as the recent COVID-19 pandemic, is due to the investor's perception that the portfolio duration hazard is increasing over time. This perception, which is usually fueled by intensive news coverage of the event, is what prompts rational investors to make irrational exit decisions amid global events or financial crises. Focusing on climate change, this chapter posits that if one can alter investors' beliefs to perceive a decreasing climate duration hazard in their financial portfolios, then a successful adaptation policy is guaranteed.

4.1 The Portfolio Allocation Problem on *X*

Consider constructing a portfolio p that consists of a number of n assets, which includes green securities. Let the time of constructing p be $t = 0$; in other words, assume that the time dimension T is absent for now. Denote the weight of asset i in portfolio p by w_i and the investor's level of wealth at time t by y_t. Therefore, y_0 is the initial level of wealth that the investor wishes to allocate on the n assets forming portfolio p.[2] Assume two periods: period 0 representing the present and period 1 representing the future. Notice that, in the previous setup, the choice set X is a set of monetary outcomes. At time $t = 0$, the monetary outcome y_0, which is an element of X, is an allocation of the initial wealth on a number n of risky assets such that the weight of asset i in this allocation is w_i and $\sum_i^n w_i = 1$. Following the premise of the MV portfolio theory, consider a risk-averse investor with a strictly concave utility of wealth function, $u(y)$, on X.[3] The investor's objective is to find the best allocation that maximizes the expected utility of future wealth. More formally, the investor solves the following problem:

$$\text{choose } w_1,\ldots,w_n \text{ in order to } \textit{maximize } Eu\left(y_1\right). \qquad (1)$$

Let the price of asset i in period t be P_{it}, for $i = 1, 2, \ldots, n$, and $t = 0, 1$. Notice that P_{i0} is the price of asset i at period $t = 0$, that is, it is the current or known price of the asset. In practice, this price is the end-of-day closing price of an asset in a financial exchange. Notice also that P_{i1} is the

[2] An investor could be a retail trader or an institutional investor, that is, a fund or portfolio manager.

[3] This chapter follows the convention of treating wealth as a commodity with an increasing total utility but diminishing in value added utility, that is, the added utility per additional increase in wealth is diminishing. This is known as the law of diminishing marginal utility of wealth. Mathematically, this means that $u(y)$ is an increasing function in wealth y, that is, the first derivative $u' > 0$, and diminishing in value added, that is, the second derivative is strictly negative; $u'' < 0$. This is the mathematical condition that guarantees the strict concavity of the utility of wealth.

asset's price at the beginning of period 1, that is, the future uncertain price of the asset. The symbol "tilde" makes the distinction between known and uncertain variables through its placement above the variable, which indicates that it is random. Thus, the rate of return on asset i is

$$\tilde{r}_i = \frac{\tilde{P}_{i1} - P_{i0}}{P_{i0}}, \; i = 1, \ldots, n. \tag{2}$$

The choice on the set X is described as follows. At time $t = 0$, the investor allocates their wealth y_0 over the n assets by purchasing a_i units of asset i at period 0 prices. Subsequently,

$$y_0 = a_1 P_{10} + a_2 P_{20} + \cdots + a_n P_{n0} = \sum_{i=1}^{n} a_i P_{i0}. \tag{3}$$

Notice that a negative (positive) a_i signifies selling (buying) some units of asset i. If the portfolio is constructed for the first time, then all the a_i terms will be positive. A rebalancing of an existing portfolio implies a mix of positive (long position) and negative (short position) a_i terms. Therefore, the weight of asset i, w_i, can be defined as

$$w_i = \frac{a_i P_{i0}}{y_0}, \; i = 1, 2, \ldots, n. \tag{4}$$

The rate of return on the portfolio p is, by definition, the weighted sum of the rates of return on the n assets forming it; that is,

$$\tilde{r}_p = w_1 \tilde{r}_1 + w_2 \tilde{r}_2 + \cdots + w_n \tilde{r}_n = \sum_{i=1}^{n} w_i \tilde{r}_i. \tag{5}$$

The mean of the portfolio, or the expected rate of return on portfolio p, denoted by μ_p, is the weighted sum of the expected returns of the individual assets forming the portfolio; that is,

$$\mu_p = E\tilde{r}_p = w_1 \times E\tilde{r}_1 + w_2 \times E\tilde{r}_2 + \cdots + w_n \times E\tilde{r}_n = \sum_{i=1}^{n} w_i \times E\tilde{r}_i. \tag{6}$$

The variance of portfolio p's return, denoted by $?_p^2$, is a weighted function of the individual variances of the n risky assets forming the portfolio and their pairwise covariances:

$$\sigma_p^2 = \mathrm{var}\left(\tilde{r}_p\right) = w_1^2 \times \mathrm{var}\left(\tilde{r}_1\right) + w_2^2 \times \mathrm{var}\left(\tilde{r}_2\right) + \cdots + w_n^2 \times \mathrm{var}\left(\tilde{r}_n\right) + 2w_i w_j \mathrm{cov}\left(\tilde{r}_i, \tilde{r}_j\right), \tag{7}$$

for all $i \neq j$. The investor's wealth in the future, that is, in period 1, y_1, is the number of units per assets purchased times period 1's price; that is,

$$\tilde{y}_1 = a_1 \tilde{P}_{11} + a_1 \tilde{P}_{21} + \cdots + a_n \tilde{P}_{n1} = \sum_{i=1}^{n} a_i \tilde{P}_{i1}. \tag{8}$$

Equation (8) can be re-written as

$$\tilde{y}_1 = \sum\nolimits_{i=1}^{n} a_i \left(\tilde{P}_{i1} - P_{i0}\right) + \underbrace{\sum_{i=1}^{n} a_i P_{i0}}_{=y_0}, \tag{9}$$

where the second term on the right-hand side is the initial wealth in Eq. (3). Multiplying the first term on the right-hand side of Eq. (9) by $\frac{P_{i0}}{P_{i0}}$ gives

$$\tilde{y}_1 = \sum_{i=1}^{n} \frac{a_i P_{i0}}{1} \times \frac{\left(\tilde{P}_{i1} - P_{i0}\right)}{P_{i0}} + y_0 \tag{10}$$

$$= y_0\left(1+\sum_{i=1}^{n}\underbrace{\frac{a_i P_{i0}}{y_0}}_{=w_i \text{ from}(4)}\times\underbrace{\frac{\left(\tilde{P}_{i1}-P_{i0}\right)}{P_{i0}}}_{=\tilde{r}_i \text{ from }(2)}\right)=y_0\left(\underbrace{1+\sum_{i=1}^{n}w_i\times\tilde{r}}_{=\tilde{r}_p \text{ from }(5)}\right)=y_0\left(1+\tilde{r}_p\right).$$

Equation (10) states that the future uncertain level of wealth is the current wealth grown by the portfolio return. Applying the expectation operator and the variance operator to y_1 in Eq. (10) yields, respectively, the expected value (mean) and variance of the future uncertain level of wealth as functions of the mean and variance of the portfolio p; that is,

$$E\tilde{y}_1 = y_0\left(1+\underbrace{E\tilde{r}_p}_{=\mu_p}\right), \tag{11}$$

and

$$\begin{aligned}\operatorname{var}\left(\tilde{y}_1\right) &= \underbrace{\operatorname{var}\left(y_0\right)}_{=0}+\operatorname{var}\left(y_0\tilde{r}_p\right)+\text{zero cross covariances}\\ &= y_0\underbrace{\operatorname{var}\left(\tilde{r}_p\right)}_{=\sigma_p^2}.\end{aligned} \tag{12}$$

By taking a second-order Taylor approximation of u about $E\tilde{y}_1$, one can show that the investor problem in Eq. (1) is equivalent to minimizing the variance of the portfolio in Eq. (12). To demonstrate this, let G be the Taylor approximation and notice that

$$\begin{aligned}u\left(\tilde{y}_1\right)\approx G &= u\left(E\tilde{y}_1\right)+u'\left(E\tilde{y}_1\right)\left(\tilde{y}_1-E\tilde{y}_1\right)\\ &+\frac{1}{2}u''\left(E\tilde{y}_1\right)\left(\tilde{y}_1-E\tilde{y}_1\right)^2+R,\end{aligned} \tag{13}$$

where $R=\sum_{i=3}^{\infty}\frac{1}{i!}u^i\left(E\tilde{y}_1\right)\left(\tilde{y}_1-E\tilde{y}_1\right)^i$ is the remainder of the approximation. Applying the expectation operator on both sides of Eq. (13) yields

$$Eu(\tilde{y}_1) = u(E\tilde{y}_1) + \frac{1}{2}u''(E\tilde{y}_1) \times \underbrace{\operatorname{var}(\tilde{y}_1)}_{=y_0\,\sigma_p^2 \text{ from}(12)} + ER. \tag{14}$$

Equation (14) shows that the investor's expected utility of wealth is a function of $E\tilde{y}_1$ and $\operatorname{var}(\tilde{y}_1)$, the mean and variance of period 1's wealth. As previously mentioned, from Eqs. (11) and (12), both $E\tilde{y}_1$ and $\operatorname{var}(\tilde{y}_1)$ are, in turn, functions of the mean and variance of portfolio p respectively. In other words, Eq. (14) establishes the link between expected utility of wealth and the portfolio mean and variance. In particular, Levy and Markowitz (1979) show that, provided that ER goes to zero, a strictly concave utility of wealth function ($u'' < 0$) guarantees that maximizing the investor's objective in Eq. (1) is equivalent to minimizing the variance of the portfolio; that is,

$$\max Eu(\tilde{y}_1) \text{ if and only if} \min \sigma_p^2. \tag{15}$$

This is evident since $u'' < 0$ guarantees that the second term on the right-hand side of Eq. (14) is negative. Therefore, minimizing this term, that is, minimizing σ_p^2, is equivalent to maximizing $Eu(\tilde{y}_1)$. The assumption that the utility of wealth is strictly concave is sensible since the concavity of u implies risk aversion. The restriction that the remainder goes to zero in Eq. (14) to ensure the equivalence in Eq. (15), however, is not a simple one. A choice of a strictly concave utility function, for example, $u = \ln y$, might not ensure that ER goes to zero. It is worth noting that a quadratic utility function is increasing, concave, and has a zero remainder under Taylor's approximation. However, when entertaining this function, there will be a point of satiation beyond which utility decreases.[4]

Luckily, in the present model, the previous technical concerns will have no impact on the model and its analytical results. This is true since the proposed model focuses on the progress of the initial allocation over time regardless of the form of the utility of wealth function on X. In

[4] This concern led to the alternative of putting a distributional restriction on the rates of return to achieve the equivalence in Eq. (15).

particular, the present model assumes that at time $t = 0$, a risk-averse investor with a strictly concave utility of wealth function, $u(y)$, chooses optimal portfolio weights such that $?_p^2$ is minimized. The utility of the resulting allocation at time 0, that is, $u(y_0)$, is assumed to be constant:

$$u(y_0) = m, \tag{16}$$

where m is the level of satisfaction from owning the portfolio before the actual trading takes place.

4.2 The Portfolio Duration Problem on *T*

This section discusses the temporal progression of the initial allocation $u(y_0) = m$ over time. The assumption of a risk-averse investor with a strictly concave utility of wealth u still holds to ensure optimal initial allocation on X. In particular, given an initial allocation y_0 at time $t = 0$ that minimizes the variance (risk) of the constructed portfolio, the investor at $t > 0$ is solving the following problem: choosing an optimal portfolio duration, t^*, such that a utility of time function, $U(t)$, is maximized. Notice that the utility of time function $U(t)$ on the T space is different from the utility of wealth function $u(y)$ on the X space. The former corresponds to duration choices, whereas the latter corresponds to monetary/allocation choices. Per the previous sub-section, the investor has already solved the allocation problem at time $t = 0$. In order to solve the duration problem, one must define $U(t)$.

Under a set of conventional preference axioms, Fahmy (2020) shows the existence of a unique utility of time function that represents the investor preference over the portfolio duration that takes the following form:

$$U(t) = M + (m - M)e^{-\theta t}, \theta > 0, \tag{17}$$

where $m = u(y_0)$ is the lower bound of U, that is, it is the value derived from the allocation of the initial level of wealth at time $t = 0$. Notice that when $t = 0$ in Eq. (17),

$$U(0) = m = u(y_0). \tag{18}$$

That is, both functions, u and U, agree at time $t = 0$. The parameter M is an upper bound of U. It is the maximum amount of satisfaction that corresponds to a maximum portfolio duration time t. This assumption is imposed for the mathematical tractability of the function, and it has no impact on the results of the model. The utility level corresponding to that time is $U(t) = M$. Thus, the function in Eq. (17) is bounded from below by m and from above by M, as Fig. 11.1 shows, where $m = 2$, $M = 10$, and $\theta = 0.8$ (left panel) or $\theta = 2$ (right panel). Parameter θ is what governs the rate of decay of U. Notice that since $U(\tilde{t}) > U(t)$ for all $0 < t < \tilde{t}$, then dU/dt is strictly positive; that is, the marginal utility of duration is positive. Moreover, the second derivative with respect to time is negative, that is, the function is strictly concave. This means that the marginal utility of portfolio duration is diminishing over time. In other words, the marginal utility that is gained from an increase in portfolio duration by one period in the short-run is higher than the same increase in duration in the long-run. Since the magnitude of the additional increase in utility per additional period is governed by parameter θ, an investor with large θ derives higher value of one increment increase in time in the short-run and, subsequently, reaches maximum utility faster than an investor with a lower θ. These behaviors can be observed from the shape of the utility function in Eq. (17) under different parameterization of θ as Fig. 11.1 shows, where the left panel depicts a utility function with low $\theta = 0.8$ and the right panel corresponds to a larger $\theta = 2$. The behavior of the former

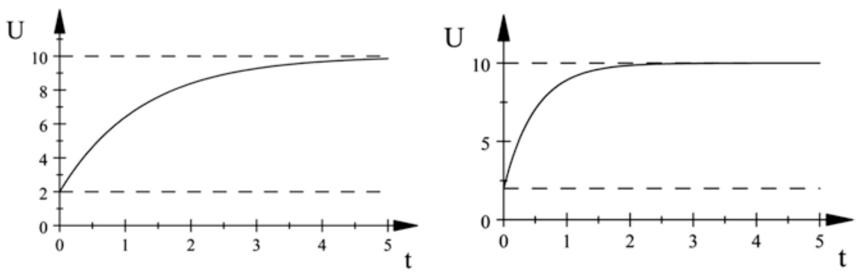

Fig. 11.1 The utility of time function

belongs to rational or institutional investors, who tend to react less to financial news and market events, whereas the latter belongs to irrational investors who tend to overreact.

The previous discussion suggests that parameter θ captures the degree of investors' overreaction to market conditions. By solving the investor duration problem, Fahmy (2020) shows that optimal portfolio duration is inversely related to the degree of overreaction. In particular, by taking a Taylor approximation of U about an expected duration time, Et, and solving for the optimal time, t^*, that maximizes this approximation, the author was able to derive the following optimal duration decision rule:

$$t^* = Et + \frac{1}{\theta}, \text{ or equivalently, } t^* - Et = \frac{1}{\theta}. \tag{19}$$

Figure 11.2 depicts the second-order Taylor approximations of the utility functions in Fig. 11.1 about an expected duration time $Et = 2$ weeks. Notice here that Et is the investor's own expectation regarding the time to revise or rebalance the portfolio. The distance between the optimal duration and this expectation is inversely related to parameter θ, which captures the degree of overreaction to market conditions. The rationale behind this inverse relation is that the lower the overreaction, the more likely that the investor will be to revise the portfolio in the long-run. This behavior is consistent with institutional investors, and it is well documented in the literature that long-term duration strategies are more profitable for rational investors under perfect information (Fahmy, 2020;

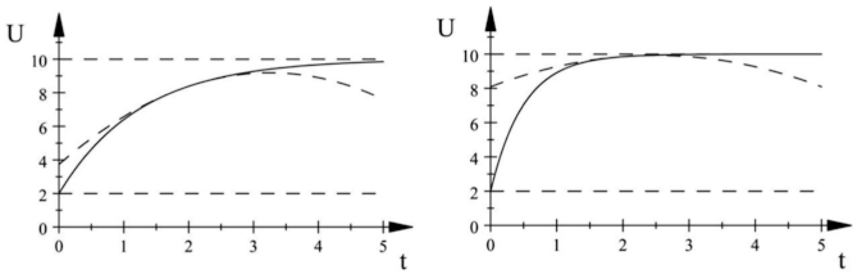

Fig. 11.2 Second-order Taylor approximations of the utility functions

Jegadeesh & Titman, 1993). On the other hand, irrational or retail investors with large θ who tend to overreact to financial news are more likely to revise/rebalance the portfolio in the short-run.

The result in Eq. (19) departs from most specifications in the literature on portfolio selection under uncertain time-horizon in its explicit treatment of the non-money attribute of the decision-making process, that is, Et (duration) and θ (uncertainty). That being said, two remarks regarding the empirical applicability of the result in Eq. (19) and how it fits in the present framework are in order: first, in practice, the investor's own expectation of portfolio duration, Et, is usually the investor or the fund manager's predetermined duration strategy, which could be short term (one or two weeks) or long term (twelve or more weeks). In an empirical analysis, it is sensible to assume a given expected duration a priori and to maximize $U(t)$ locally around it. Second, it is important to note that the degree of overreaction is intrinsic to the investor and is prone to cognitive and behavioral biases. In other words, it does not capture the risk of the event per se; rather, it captures the investor's attitude and reaction toward it. Most investors are risk averse. However, the degree of risk aversion and overreaction to financial news and events vary from one investor to another. In summation, although the result in Eq. (19) is useful in explaining various phenomena in financial markets (Fahmy, 2020), it does not capture climate uncertainty. The next sub-section shows how to modify this result to account for climate risk.

4.3 Climate Duration Hazard

I propose to match $U(t)$, the utility of time function in Eq. (17) over all possible values of t, with a posterior duration distribution $F(t|$ data) over future unknown states.[5] Since U is a monotonic increasing function in its argument (t in the present model), then a cumulative probability distribution function F may be convenient to describe utility.

If one entertains the frequentist approach in statistics and thinks of the portfolio duration problem as an experiment that is repeated in different

[5] This matching of utility with a distribution function is not new to the literature. The underlying theory of this matching approach is treated in Novick and Lindley (1979). An application of this theory on education is presented in a companion paper (Novick & Lindley, 1978).

states such that the trials of the experiment are the rebalancing/revising of the portfolio over time, then one could think of duration time as a random variable, denoted by d, with a density function $f(t)$ and a distribution function $F(t)$ over the set $(0, t]$. The reason the support of the distribution function F is a left-open right-closed set is to guarantee the right continuity of the distribution function F. This technical assumption is imposed to ensure that F is an adequate distribution function and that it will not impact the results of the model. In practice, two suitable distributions are commonly used to describe duration data: the exponential distribution, which is defined as

$$F(t) = 1 - \mathrm{e}^{-\theta t}, \tag{20}$$

and the Weibull distribution, defined as

$$F(t) = 1 - \mathrm{e}^{-\theta t^{\alpha}}. \tag{21}$$

The former distribution is characterized by parameter $\theta > 0$, whereas the latter is characterized by $\theta > 0$ and $\alpha > 0$.

The survival function, $S(t)$, of the portfolio duration is the probability of its survival beyond time t; that is,

$$S(t) = \mathrm{Prob}(d \geq t) = 1 - F(t), \tag{22}$$

where $F(t)$ is the portfolio duration distribution. The hazard function, $h(t)$, is the likelihood that the portfolio revision/rebalancing or the market exit will be completed at time t, which is conditional on the portfolio surviving or lasting up till that time; that is,

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{d}{dt}F(t)}{1 - F(t)}, \tag{23}$$

Per Eq. (23) and the definitions of F in (20) and (21), if portfolio duration, d, follows an exponential distribution, then the hazard function, h, is constant and equal to parameter θ. On the other hand, under the Weibull distribution, the hazard function exhibits duration dependence:

$$h(t) = \theta \alpha t^{\alpha - 1}, \tag{24}$$

for the Weibull distribution with parameters $\alpha > 0$ and $\theta > 0$. Notice that the hazard function of the Weibull distribution is increasing in duration if $\alpha > 1$, decreasing in duration if $\alpha < 1$, and constant (as well as equal to the exponential case) if $\alpha = 1$. The parameter α is known as the hazard rate parameter, and it governs the behavior of the hazard function in Eq. (24).

If the lower bound of U in Eq. (18) is zero, that is, $m = u(y_0) = U(0) = 0$, and the upper bound $M = U(t) = 1$, then the utility function $U(t) = M + (m - M)e^{-\theta t}$ becomes $U = 1 - e^{-\theta t}$. Together the previous two restrictions, with the monotonicity of U over the range $(0, t]$, guarantee that U satisfies the requirements of a distribution function in general and matches the exponential distribution function $F(t) = 1 - e^{-\theta t}$ in Eq. (20).[6] Moreover, since the Weibull distribution, $F(t) = 1 - e^{-\theta t^{\alpha}}$, is just a monotone transformation of the exponential distribution, $F(t) = 1 - e^{-\theta t}$, then it follows immediately that the function,

$$V = M + (m - M)e^{-\theta t^{\alpha}}, \alpha > 0, \theta > 0, \tag{25}$$

is a positive monotonic transformation of Fahmy's (2020) utility of time function $U = M + (m - M)e^{-\theta t}$, and V also represents the investor's preference over time.

Figure 11.3 plots the monotonic transformation function V in Eq. (25) for different values of the hazard rate parameter α using the same parameterization of U in Eq. (17), namely the upper bound $M = 10$, the lower bound $m = 2$, and the overreaction parameter $\theta = 0.8$. The thick bold line depicts V with a large hazard rate $\alpha = 8 > 1$. Notice how the

[6] The restrictions that the lower bound is 0 and the upper bound is 1 are not restrictive and can be thought of as rescaling of the utility function. Furthermore, this rescaling will not affect the analytical results of the model.

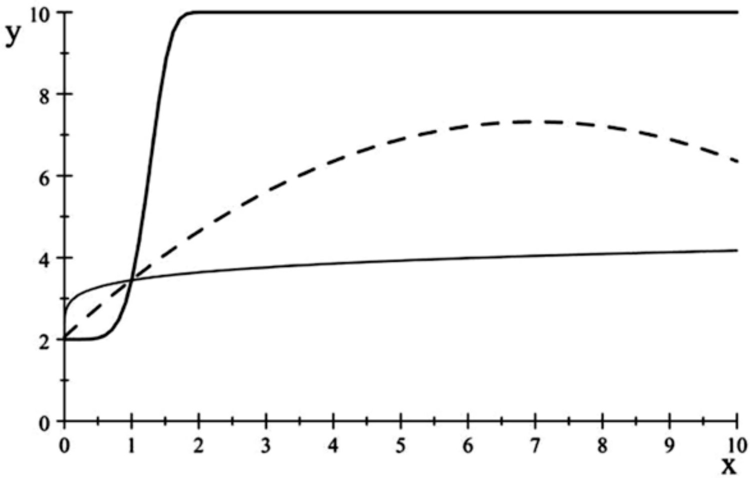

Fig. 11.3 A plot of the utility function *V, where the y-axis represents V(t) and the x-axis represents time t.*

trajectory of the function reaches its upper bound M. This swift swing over a short period of time is indicative of the high hazard rate perceived by the investor. This perception is what prompts the investor to re-visit the portfolio or even exit the market in the short-run during a global event. The dashed line in Fig. 11.3 corresponds to a utility function V with a neutral hazard rate $\alpha = 1$. This is in fact the exact same utility function that is plotted in Fig. 11.1 (left panel). Notice that when $\alpha = 1$, V is equivalent to the original utility function U that does not account for duration hazard. Finally, the thin solid line in Fig. 11.3 corresponds to the utility function V with a low hazard rate $0 < \alpha = 0.2 < 1$. Notice how this function progresses slowly to its upper bound. The "slow" pace is due to the fact that the investor perceives the diminishing duration hazard, which in turn prompts her to choose a long-term duration strategy rather than a short-term one.

The three versions of utility function V in Fig. 11.3 intersect at $t = 1$. Consider the possibility of a global climate event at time $t = 0$. The three curves in Fig. 11.3 depict three different scenarios in the short-run, that is, at a portfolio duration $0 < t < 1$, and in the long-run, at duration $t > 1$.

In the short-run, it is clear from integrating the area under each V curve that the total utility for investors who perceive climate hazard to be decreasing over time, that is, those investors with a utility of time function V with $0 < \alpha < 1$ (solid thin line in Fig. 11.3) is higher than others who believe that the hazard is neutral (dashed line) or increasing (solid thick line) over time. The same conclusion may apply to the long-run scenario, when $t > 1$. This is true since the utility functions with neutral and increasing hazard rates will reach the upper bound M faster than the decreasing duration hazard function (solid thin curve). The previous analysis reveals that a decreasing climate duration hazard perception is more rewarding for investors.[7] If policy makers, regulators, and key players in financial markets embrace the task of designing sound policies and strategies that guarantee a diminishing climate hazard over time, investors will be keener to hold green instruments. Moreover, this approach has the advantage of maintaining stability in financial markets by altering the attitude of investors who tend to panic and overreact in the short-run because of a climate event; this helps them to hold their positions rather than exiting the market. This preference reversal is crucial in reducing disruptions in financial markets that are mainly caused by retail investors. I illustrate this preference reversal in the following section.

To conclude this section, I solve the portfolio duration problem using the proposed monotonic transformation utility function V in Eq. (25) that accounts for climate duration hazard. Proceeding in the exact same way as in Sub-sect. 4.2, it is clear that taking a second-order Taylor approximation V about Et and solving for the optimal portfolio duration time that maximizes this approximation yield the following expression:

$$t^{*} = Et + \frac{ET}{\alpha\theta Et + \left(1-\alpha\right)}, \tag{26}$$

where α is the duration hazard rate and everything else is represented in Eq. (19). Equation (26) states that the optimal portfolio duration is a function of three types of uncertainty, namely, the investor's own expected

[7] Recall that the investor in the present framework is maximizing $U(t)$ around an expected duration time Et.

duration, *Et*; the investor's degree of overreaction to news, θ; and the hazard rate of duration, α, which may represent the climate duration hazard in the proposed model.

5 Empirical Analysis

Consider a risk-averse investor with an extremely low overreaction parameter $\theta = 0.02$. In practice, this could refer to institutional investors, mutual fund managers, or sovereign wealth managers. These types of investors usually prefer to adopt long-term duration strategies. Therefore, a reasonable mandate for these funds is to revisit the portfolio for rebalancing every two or three quarters. Ultimately, taking an average $Et = 32$ weeks target long-term duration is reasonable for parameterizing the problem. Consider an existing MV optimal portfolio that contains green instruments. Assume a major climate event at time $t = 0$. The investor's reaction to this event depends on the way they perceive its degree of hazard over time. If the investor believes that the climate duration hazard is increasing over time, that is, if $\alpha > 1$ in the utility of time function in Eq. (26), the likely reaction would be a revising/rebalancing of the portfolio sooner than later. When α is "high" or when its level is magnified by news coverage and social media, it is possible for the investor to exit the market in the short-run, that is, within days of the climate event. If many investors opt to exit, a major sell-off could significantly disrupt financial markets. The major sell-off that stock markets worldwide have recently witnessed amid the recent COVID-19 pandemic is a key example. It is easy to capture this behavior if one sets $\alpha = 5.6 > 1$ in Eq. (26). This results in

$$t^* = Et + \frac{ET}{\alpha\theta Et + (1-\alpha)} = 32 + \frac{32}{5.6 \times 0.02 \times 32 + (1-5.6)} \tag{27}$$
$$= 0.5 \text{ weeks.}$$

Ultimately, a rational investor with an expected long-term duration strategy $Et = 32$ weeks might exit the market within a few days of a major climate event due to the perception of increasing climate duration hazard.

On the other hand, if measures are taken to assure investors that the duration hazard of climate events is decreasing over time, then long-term (profitable) strategies are likely to be entertained. To see this, set $\alpha = 0.2$ in Eq. (27) and compute the optimal duration given the same intrinsic parameters, namely, expected time $Et = 32$ and overreaction $\theta = 0.02$. This gives a long-term duration strategy of 66 weeks instead of 0.5 weeks:

$$t^{*} = Et + \frac{ET}{\alpha\theta Et + (1-\alpha)} = 32 + \frac{32}{0.2 \times 0.02 \times 32 + (1-0.2)} \tag{28}$$
$$= 66 \text{ weeks.}$$

This analysis demonstrates that successful climate adaptation over time relies on enhancing investors' awareness of climate change risks and assuring them that these risks are diminishing over time. Therefore, per the following section, portfolio strategies and policies that guarantee a perception that climate risk is decreasing over time are essential for successful adaptation.

6 Concluding Remarks and Policy Recommendations

The temporal model that this chapter proposes provides a measure of climate duration hazard in the optimal solution of the portfolio duration problem. The analytical result of the proposed model (Eq. 26) reveals that investors exhibit increasing impatience and tend to revise, rebalance, or even exit the market in the short-run amid a major climate event due to the perception that climate hazard is increasing over time. This behavior has negative implications on financial markets in general as well as on climate mitigation and adaptation efforts in particular. Investors who believe that climate hazards are rising over time tend to be skeptical about investments in clean energy and/or mitigation or adaptation projects. Reversing this belief is, therefore, at the core of climate adaptation.

While altering investors' perceptions of climate risks is the key to sustainable climate adaptation, it is not an easy task. It requires the collective collaboration of politicians, policy makers, regulators, and practitioners in the finance industry. In particular, in order to achieve this incredible goal, politicians should put climate mitigation and adaptation policies at the top of their agenda. International organizations and governments around the world should increase their efforts toward implementing policies that stimulate investments in renewable clean energy that reduces the emissions of greenhouse gases, for example, feed-in tariff policies (Bürer & Wüstenhagen, 2009; Hofman & Huisman, 2012). Global institutions such as the World Bank are already working on creating more effective green solutions across asset classes. These solutions have been mainly focused on the fixed income class of assets, for example, green bonds, cool bonds, and eco notes (Reichelt, 2010). Innovative solutions that create more awareness in other asset classes are needed (Fahmy, 2022). Many investors are not aware of the carbon footprint and the climate impact of the companies in their portfolios. Few investors who hold oil and gas stocks in their portfolios are conscious of the risk they face with respect to those companies' stranded assets (Anderson et al., 2016). Despite the unanimous agreement on climate change following the Paris Agreement, climate risk remains unpriced by the market, and thus, future uncertainty about climate risk remains an increasingly important risk factor for investors—particularly long-term investors. CEOs of private companies should increase their efforts to reduce the carbon footprints of their products and, more importantly, to provide investors with clear signals and transparent rules with respect to how this reduction is to be achieved. Fund and portfolio managers should focus on factoring climate risks in their portfolios and design hedging policies that aim to lower the risk exposure to climate events without compromising the rewards of the portfolios.

In conclusion, the collective efforts of all of the above-mentioned players in financial markets will have the potential of inversely impacting the perception of increasing climate hazard over time. This preference reversal will, ultimately, lead to a successful climate adaptation and more sustainability over time.

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