Isotropic and Kinematic Hardening Laws: Plastic Behavior of Mild Steel Under Shear Tests

Olfa Daghfas[✉], Amna Znaidi, and Rachid Nasri

Laboratory of Applied Mechanics and Engineering LR-MAI, University Tunis El Manar, ENIT
BP37-Le belvédère, 1002 Tunis, Tunisia
`Amna.znaidi@ipeiem.utm.tn, Rachid.Nasri@enit.rnu.tn`

Abstract. The principal objective of this work is to model the behavior of a mild steel material from the experimental data using simple and cyclic shear tests on samples taken from different loading directions. The implemented model will use criterial yield functions taking into account equivalent constituents anisotropic coefficients and shape coefficient with, initially, isotropic hardening laws. These functions have been used successfully in proportional loading as simple shear tests. Secondly, the kinematic hardening law is established for complex loading such as cyclic shear tests. For complex testing, two behavior laws (linear kinematic Prager law and nonlinear kinematic Armstrong–Frederick law) are introduced. Then, an identification strategy will be implemented with regard to several hypotheses to identify the parameters of the proposed model. These parameters are determined, with isotropic hardening in simple shear tests and with the combined hardening in cyclic shear tests. From experimental data, a selection is performed to pick out the furthest suitable hardening laws (isotropic law and kinematic law) in order to model the behavior of the mild steel. Finally, a smoothing with experimental hardening shear curves reveals the adequate description of the strong anisotropy of studied material using the plastic behavior model.

Keywords: Identification · Shear tests · Hardening laws · Kinematic hardening

1 Introduction

Knowledge of plastic material behavior is critical to modeling the shaping of metals. The description of an elastoplastic behavior law requires at least the definition of three components: a plastic model, a hardening law (isotropic, kinematic and mixed) and evolution law (Znaidi et al. 2016; Daghfas et al. 2017; Rym et al. 2020a).

The implementation of the complex model is based on the plactic Barlat criterion (Barlat et al. 1991) which takes into consideration the anisotropy, the shape coefficient and three isotropic hardening laws: SWIFT law, Ludwick law and Voce law (Olfa 2019a; Daghfas 2019b) associated with normality. When the isotropic hardening is unable to predict strong anisotropy as well as other characteristics of anisotropy, kinematic hardening (linear and non linear) was interposed and extensivelly used for this cause. Indeed, several kinematic hardening criterions exist in literature wherein Prager (1949) suggested the linear type of kinematic hardening. Furthermore, Armstrong and Frederick

(1966) improved the Prager model by introducing a non linear kinematic hardening term to estimate the complex deformation characteristics. Subsequently, the latest criterion has been enhanced by Lemaitre and Chaboche (2001) to better predict the bauschinger effect. The recent formulations of kinematic hardening, as revised by Chaboche (2009) and recent proposals eg, Feigenbaum and Dafalias (2007) and Barlat et al (2011) are powerful enough to capture different events, in particular for cyclic deformations.

To study the effect of the yield criterion, the criterion YIELD 91 (Barlat and Brem 1991) was written with an isotropic hardening compared with experimental results of (Gahbiche 2006) and with a mixed hardening (kinematic-isotropic) (Olfa et al. 2019a). In this work, kinematic hardening laws are described by a stress tensor product, a translation of the threshold surface. An identification of the model parameters is developed using an experimental database applied to mild steel sheets FeP04. A study is conducted on identification of hardening curves. The model parameters are determined, on the one hand with isotropic hardening in case of simple shear tests and the other hand with the combined hardening in case of cyclic shear tests which are compared to experimental data.

2 Plastic Behavior Model

Our model is defined by:

– A yield criterion:

$$f(\mathbf{q}, \alpha) = \alpha_c(\mathbf{q}) - \sigma_s(\alpha) \leq 0 \ \mathbf{q} = A : \left(\boldsymbol{\sigma}^D - \mathbf{X}\right) \tag{1}$$

A: linear transformation tensor; $\sigma_s(\alpha)$: stress tensor deviator;
X kinematic variable in tensoriel form associated with plastic deformation.
$\sigma_s(\alpha)$: function that represents the isotropic hardening; α: scalar variable for isotropic hardening.

The plasticity criterion used is the Barlat Yield 91 (Barlat and Brem 1991) which the equivalent stress σ_c is defined as follows:

$$\sigma_c(\mathbf{q}) = \left(\left|q_1 - q_2\right|^m + \left|q_2 - q_3\right|^m + \left|q_1 - q_3\right|^m\right)^{1/m} \tag{2}$$

The identification procedure is simplified by considering a plane stress state combined with two hardenings according to the applied loading:

Hypothesis 1: In monotonic loading, the isotropic hardening is selected where $\mathbf{X} = 0$.
Hypothesis 2: In cyclic loading, the nonlinear kinematic hardening law (Armstrong - Frederick) is expressed as follows:

$$\dot{X}_{ij} = C_x \dot{\varepsilon}_{ij}^p - C_{sat} X_{ij} \dot{p} \tag{3}$$

Where C_x kinematic hardening rate, C_{sat} saturation value, $\dot{\varepsilon}_{ij}^p$, $\dot{p}$: equivalent and cumulative plastic deformation rates.

Isotropic hardening law (Daghfas et al. 2017) (Olfa et al. 2019a; Daghfas 2019b) (Rym et al. 2020a; Harbaoui 2020b) used is given by.

The Swift law:

$$\sigma_s(\alpha) = k(\varepsilon_0 + \varepsilon)^n \tag{4}$$

The Ludwick law:

$$\sigma_s(\varepsilon^p) = \sigma_0 + K(\varepsilon^p)^n \tag{5}$$

The Voce law:

$$\sigma_s(\varepsilon^p) = \sigma_y(1 - \alpha \exp(\beta \varepsilon^p)) \tag{6}$$

– A plastic flow law:

$$\dot{\varepsilon}_p = \dot{\alpha} \frac{\partial f}{\partial q}; \dot{\alpha} \geq 0; \dot{\alpha} f = 0; \dot{\alpha} \dot{f} = 0 \tag{7}$$

The off-axis angle noted by ψ presents the angle variation of different loading directions with respect to the rolling direction.

3 Results and Discussions

3.1 Experimental Results

– Simple shear curves

Firstly, uniaxial shear experiments were performed in seven directions compared to the rolling direction such as 0°, 15°, 30°, 45°, 60°, 75° and 90°. Secondly, cyclic shear experiments (load-unload cycle) are conducted at rolling direction.

Figure 1 (a) represents the experimental hardening curves in seven loading directions. Figure 1 (b) represents cyclic hardening curve in the rolling direction.

The experimental results of off-axis shear (Fig. 1(a)) show that this material has a pronounced anisotropy for shear strain but low enough for the yield stress. In this sense, we find that mild steel material is appropriate for deep drawing.

According to hardening curves presented in Fig. 1 (a), a shear anisotropy is more marked in terms of deformation, particularly for $\Psi = 45°$. The mechanical response in the rolling ($\Psi = 0°$) and transverse ($\Psi = 90°$) directions are not equivalent for the uniaxial strain.

In addition, these experimental curves for seven orientations exhibit a strong anisotropy in mild steel, particularly from the point of view of shear strength and ductility in different loading directions.

Considering strength and ductility, mild steel material has high mechanical properties in the 45° direction compared to rolling and transverse loading directions.

Moreover, the anisotropy of this material is shown by another indicator which is strain hardening rate.
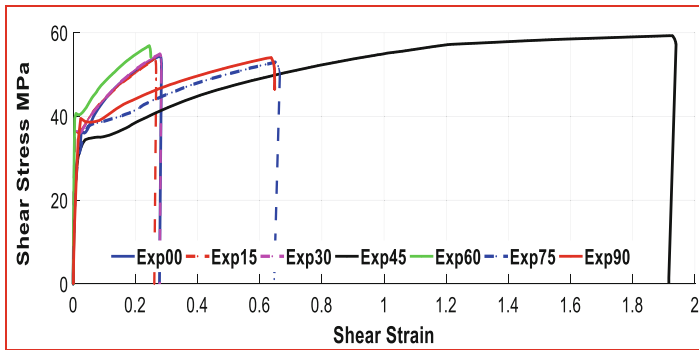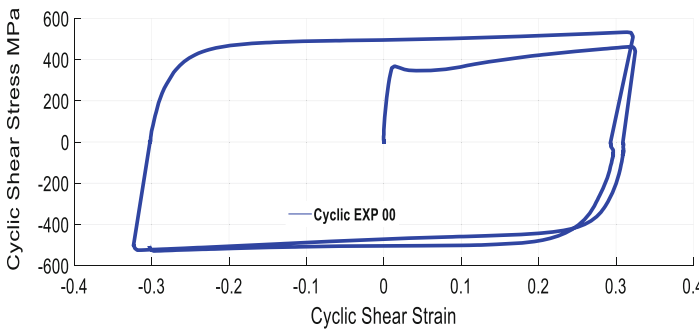
Fig. 1. (a) Experimental simple shear (b) Experimental cyclic shear

The identification procedure is performed on mild steel FeP04. The uniaxial shear results are firstly analyzed to determine the characteristics of the isotropic hardening and secondly to identify the shape factor and parameters of Barlat criterion.

– Cyclic shear curves

Shear tests with a load-unload cycle have been carried out to evaluate the intervention of kinematic hardening. Thereby, shear stress is applied up to a well-defined value of shear strain then the load direction is reversed up to fracture of specimens.

However, different specimens are subjected to a 0.3% deformation in rolling direction before unloading (Fig. 1(b)). Figure 1(b) shows the Baushinger effect which is the alteration anisotropy of elastic limit of a material. In other words, it is an asymmetry of the elastic limit in unloading compared to its loading value. It is shown the shift of the limit domain to the positive stress values. This phenomenon is essential for understanding the fatigue, and the deterioration of the materials performance under reverse loading (Olfa et al. 2019a).
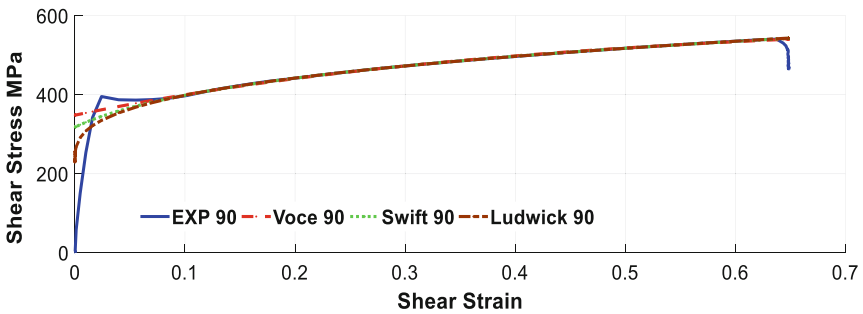
3.2 First Identification Step

Three laws are chosen for smoothing the experimental curves, which accurately described the behavior of the material. For this identification step, we choose the Swift law, the Ludwick law and the Voce law.
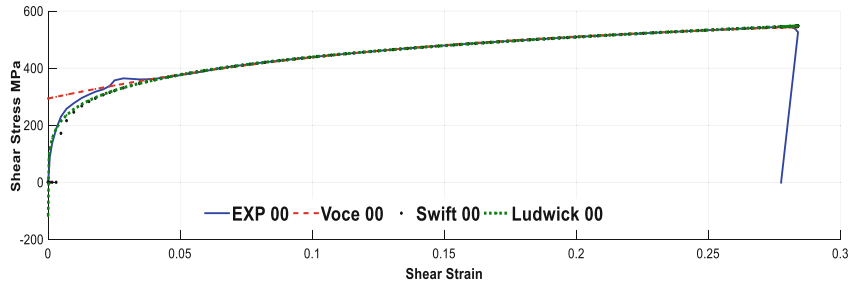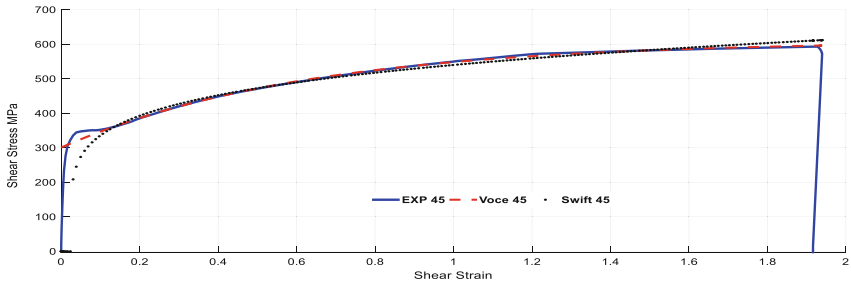
Fig. 2. Identification of hardening curves **of** shear tests at different values of Ψ (a) = 0°(b) Ψ = 45°(c) Ψ = 90°

Figures 2 show the shear curves according to different directions relative to the rolling direction. This shows that our strategy is well validated for the identification of parameters of SWIFT law that is well suited to mild steels.

3.3 Second Identification Step

Using plastic Barlat model and respecting the assumptions, the identification of the steel sheet is equivalent to choosing the coefficients of behavior model by reducing the quadratic gap between the experimental and numerical hardening curves. The parameters f, g, h, m and n relating to Barlat model are presented in Table 1.

Table 1. Identified anisotropy parameters and form parameter m

f	g	h	n	m
0.3847	0.2092	0.3035	0.7967	7.8417

3.4 Identification of Anisotropy Parameters with Mixed Hardening

The isotropic hardening does not exist alone, it is still physically accompanied by a minimum Baushinger effect, hence the frequent presence of a non linear kinematic hardening (NLKH) in addition to the isotropic hardening (IH).

To improve this strategy, we introduce the hypothesis 2 (kinematic hardening) in addition to the hypothesis 1(the isotropic hardening). The Bauschinger effect is expressed by the kinematic hardening rate C_x.

In addition, there is mainly a great coupling between the parameters describing the isotropic hardening and those characterizing the kinematic hardening. All behavioral model parameters are defined in the following table.

Table 2. Identified parameter sets of kinematic identification

f	g	h	n	m	Cx
0.5507	0.3973	0.4910	1.4510	9.9869	0.1960

From these identified values (see Table 2), Fig. 3 shows the identification of cyclic hardening curve using the non linear kinematic hardening.

The mixed isotropic and kinematic hardening identifies very well the anisotropic behavior of mild steel material standpoint hardening curve in rolling direction.
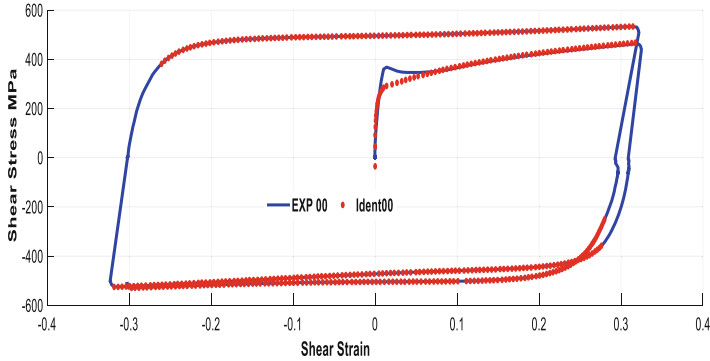
Fig. 3. Identification of cyclic hardening curve

4 Conclusion

Our identification strategy focused on the anisotropy parameters of material relative to model Barlat, the isotropic hardening of SWIFT law and the non linear kinematic Armstrong and Frederick hardening law. Thus, plastic behavior model with six parameters is well identified. The identified hardening curves are in well compromise compared to the experimental. Experimental data show that this material has a pronounced anisotropy for the shear strain but low enough for the yield stress. In this sense, we find that mild steel material is appropriate for deep drawing. This model has the advantage of being simple since it involves only internal variable tensor. It does not account for all aspects of the anisotropic behavior of metals. The second hardening law is modeled by non linear Armstrong–Frederick that is well appropriate for modeling the behavior of mild steel in cyclic loading. It justified a good agreement to the transition from the elastic domain to the plastic one through the reverse loading direction.

References

Barlat, F., Brem, D.L.J.: A six components yield function for anisotropic materials. Int. J. Plast. **7**, 693–712 (1991)

Armstrong, P.J., Frederick, C.O.: A mathematical representation of the multiaxial Bushinger effect. CEGB report R.B/B/N 731 (1966)

Lemaitre, J., Chaboche, J.L.: Mechanics of solid materials. 2nd and 3rd cycles. Dunod, Paris (2001)

Chabobe, J.L.: On the calibration of the Chaboche hardening model and a modified hardening rule for uniaxial ratcheting prediction. Int. J. Solids Struct. **46**, 3009–3017 (2009)

Feigenbaum, H.P., Dafalias, Y.F.: Directional distortional hardening in metal plasticity within thermodynamics. Int. J. Solids Struct. **44**, 7526–7542 (2007)

Barlat, F., Gracio, C.J., Gyu, L., Edgar, F.R., Vincze, G.: An alternative to kinematic hardening in classical plasticity. Int. J. Plast. **27**, 1309–1327 (2011)

Gahbiche, A.: Experimental characterization of stamping metal sheets, application to the identification of constitutive laws. Doctoral thesis of the university of Monastir, National School of Engineers of Monastir (2006)

Znaidi, A., Daghfas, O., Gahbiche, A., et al.: Identification strategy of anisotropic behavior laws: application to thin sheets of A5. J. Theoret. Appl. Mech. **54**, 1147–1156 (2016)

Olfa, D., Amna, Z., Amen, G., Rachid, N.: Plastic behavior of 2024–T3 under uniaxial shear tests. In: Haddar, M., Chaari, F., Benamara, A., Chouchane, M., Karra, C., Aifaoui, N. (eds.) Design and Modeling of Mechanical Systems—III, pp. 1039–1049. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-66697-6_102

Olfa, D., Amna, Z., Amen, G., Rachid, N.: Identification of the anisotropic behavior of an aluminum alloy subjected to simple and cyclic shear tests. J. Mech. Eng. Sci. **233**(3), 911–927 (2019a). https://doi.org/10.1177/0954406218762947

Daghfas, O., Znaidi, A., Nasri, R.: Plastic behavior laws of aluminum aerospace alloys: experimental and numerical study. J. Chin. Soc. Mech. Eng. **40**(5), 523–532 (2019b)

Rym, H., Olfa, D., Amna, Z.: Strategy for identification of HCP structure materials: study of Ti–6Al–4V under tensile and compressive load conditions. Arch. Appl. Mech. (2020a). https://doi.org/10.1007/s00419-020-01690-7

Harbaoui, R., Daghfas, O., Znaidi, A.: Mechanical behavior of materials with a compact hexagonal structure obtained by an advanced identification strategy of HCP material, AZ31B-H24. Frattura ed Integrità Strutturale **53**, 295–305 (2020b). https://doi.org/10.3221/IGF-ESIS.53.23

Prager, W.: Recent developments in the mathematical theory of plasticity. J. Appl. Phys. **20**(3), 235–241 (1949)