Revisiting Peirce's Rules of Transformation for Euler-Venn Diagrams

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Abstract. Charles S. Peirce introduced in 1903 a set a transformation rules for Euler-Venn diagrams. This innovation contrasted with earlier practices where logicians rather extracted the desired information by a simple 'glance' at their diagrams. Also, Peirce's set of rules was the starting point of Sun-Joo Shin's more recent systems which, in turn, inspired most subsequent modern diagrammatic systems. Despite their significance, these rules got little attention from both diagram and Peirce scholars. In this paper, we revisit Peirce's rules of transformation and discuss the extent to which they 'survived' in modern diagrammatic systems. We will specifically consider their clarity and completeness to assess Peirce's assumption that some of his rules may be simplified while others may have been overlooked.

Keywords: Peirce · Euler-venn diagram · Rules of transformation

1 Introduction

Charles S. Peirce made significant contributions to logic diagrams. In addition to his work on the theory of diagrams, it is known that he improved the Euler-Venn's scheme and that he designed a fascinating system of Existential Graphs. His manuscripts continue to reveal remarkable advances, such as his recently rediscovered inclusion diagrams [5,15]. In this paper, we discuss one of Peirce's major, yet seldom noticed, innovations: his rules of transformation for Eulerian diagrams found in his manuscript 'On logical graphs' (1903), generally known as MS 479 [12][1]. This set of rules is historically significant for at least two reasons.

[1] Unfortunately, manuscript MS 479 has still not been properly published. It has only been partially reproduced and poorly edited in Peirce's *Collected Papers* [13]. This transcription, on which was based Shin's account, should be used with extreme caution. The manuscript is also not reproduced in Ahti-Veikko Pietarinen's edition of Peirce's existential graphs [15], but additional text and variants are included [14]. Apparently, Peirce intended to include his manuscript as a chapter in a volume of *Logical Tracts* [14, p. 72]. The original manuscript MS 479 is freely accessible on the Peirce Archive repository (https://rs.cms.hu-berlin.de/peircearchive/pages/search.php). The page numbers we indicate for MS 479 are the file titles in the Peirce Archive.

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First, it contrasts with earlier work on logic diagrams where no such formal rules were provided. Sun-Joo Shin argued that "Peirce was probably the first person that discussed the rules of transformation in a diagrammatic system" [18, p. 24]. Indeed, Peirce's predecessors generally invited their readers to detect the conclusion of an argument by a simple "glance" at the diagram that represents its premises [20, p. 15][2]. Peirce rather provided rules in accordance with which the diagram of the premises is to be transformed to produce the diagram of the conclusion from which the conclusion can be read off[3]. By introducing his rules, Peirce was "attempting to massage Euler diagrams into something that would possess more of the character of a logical *language* than a *diagram* or a *picture*" [14, p. 95]. As such, Peirce opened the way to a formal view of diagrams [9].

Second, Peirce's rules played a crucial role in the shaping of modern diagrammatic systems. Indeed, Shin's work, which is commonly regarded as the primary inspiration for subsequent systems [19], was itself based on the rules that Peirce has enumerated almost a century earlier [18, p. 28]. This legacy of Peirce is almost ironical when it is reminded that Peirce himself did not think highly of his Eulerian diagrams, sketched his rules rather loosely and did not believe the system to have the potential for significant growth [12] (see also [14, p. 84]). The formidable development of diagrammatic logic in recent decades does not support Peirce's scepticism, but it demonstrates the importance of his pioneering work on transformation rules in diagrammatic reasoning.

Despite their significance, Peirce's transformation rules attracted little attention, except for Shin's account [18, pp. 28–40]. Peirce scholars are justifiably more interested in Peirce's true *chef d'oeuvre*, his Existential graphs [2][4], while modern diagram scholars understandably discover those rules mainly through Shin's account (which does not reproduce Peirce's original formulations). In this paper, we revisit Peirce's rules of transformation and discuss the extent to which they 'survived' in modern diagrammatic systems. We will specifically consider their clarity and completeness to assess Peirce's assumption that some of his rules may be simplified while some others may have been overlooked [12]. For the purpose, we first review and discuss each of Peirce's six rules. To ease the reading of the paper, we discuss rules-1 to 3 in Sect. 2, rule-4 and its variations in Sect. 3 and rules-5 to 6 in Sect. 4. Finally we compare in Sect. 5 Peirce's set of rules with some modern diagrammatic systems, namely Shin's systems Venn-I and Venn-II [18] and the more recent system Venn$_{i_n}$ [4].

[2] Lewis Carroll is a remarkable exception here. See [7,8]. A comparison of Carroll's rules with those of Peirce is found in [10].

[3] Peirce explained that he used rules "in the sense in which we speak of the "rules" of algebra; that is, as a permission under strictly defined condition" [12]. In his entry on 'Symbolic Logic', published a year earlier, Peirce defined a rule as "a permission under certain circumstances to make a certain transformation" [11, p. 450].

[4] Peirce's mature Eulerian diagrams and Existential graphs were developed at the same time and share several features, including the formulation of transformation rules. But they differ significantly in their purpose: Eulerian diagrams served mainly for logical calculus while Existential graphs were designed for logical analysis. Roughly, calculus aims at carrying reasonings while analysis investigates them. On the opposition between calculus and analysis, see [3,11, p. 450].

2 Rules 1 to 3

Rule 1: "Any entire sign of assertion (i.e. a cross, zero or connected body of crosses and zeros) can be erased" [12, p. 042][5].

This particular rule basically helps us to erase certain information from a given conjunction of information. Using rule-1 we can get the diagram in Fig. 2 from the diagram in Fig. 1 by removing zero, cross and the connected body of crosses from the regions (M − S − P), ((M ∩ S) − P) and ((S − M − P) ∪ ((S ∩ P)− M)) respectively.

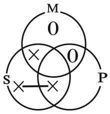

Fig. 1. .

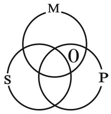

Fig. 2. .

Rule 2: "Any sign of assertion can receive any assertion" [12, p. 043].

Using rule-2 we can introduce new pieces of information in the form of disjunction. For example, the cross in the region (S − P) and the zero in the region (P − S) both receives cross to get the diagram in Fig. 4 from the diagram in Fig. 3.

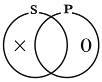

Fig. 3. .

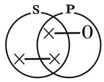

Fig. 4. .

Rule 3: "Any assertion which could permissively be written if there were no other assertion can be written at any time, detachedly" [12, p. 043].

Although there is no doubt that both rule-1 and rule-2 are quite intuitive, rule-3 does not turn out as such. How do we know which assertion is 'permissible' to be written in a diagram? Peirce never explained this particular rule nor gave any kind of example right after stating this rule[6]. But later, while showing how one can obtain conclusion in syllogistic reasoning using these six rules, Peirce mentioned that using rule-3 we can unify two diagrams [12, p. 046]. Now suppose we have the following two diagrams, Fig. 5 and Fig. 6.

The regions ((M ∩ P) − S) and ((P ∩ S) − M) in Fig. 5 are both blank. We don't know whether these two regions are empty or non empty and thus neither zero nor cross is permissible to be written in these regions. But if we consider the diagram in Fig. 6 together with the diagram in Fig. 5 then we have the new information 'Something are both M and P but not S and anything that

[5] Here cross and zero represents non-emptiness and emptiness of a region respectively. The connected lines between any of these symbols represent their disjunction

[6] This rule was written by Peirce on the margins of his manuscript, without further explanation. It seems to have been added later, as shown by the renumbering of the following rule.

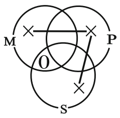

Fig. 5. .

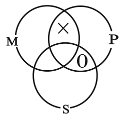

Fig. 6. .

is both P and S is also M' in our premise from the Fig. 6. Taking these two diagrams together we know which assertions are permissible in the regions ((M ∩ P) − S) and ((P ∩ S) − M) of the diagram in Fig. 5. Thus by using rule-3 we can introduce cross and zero in the regions ((M ∩ P) − S) and ((P ∩ S) − M) respectively and get the diagram in Fig. 7.

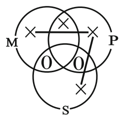

Fig. 7. .

Shin understood rule-3 in a similar manner and later used it as the basis of her 'unification rule'. However, rule-3 might not be merely a unification rule for diagrams. This rule let us introduce any new piece of information in a diagram if we have prior knowledge about it. For example, throughout MS 479 Peirce has mentioned that "nothing exists" is an absurd assertion [12, p. 034]. So we can take the universe to be always non-empty. Having this prior knowledge about the universe, take any diagram, say the diagram in Fig. 8. Now by using rule-3, we can introduce a connected body of crosses in Fig. 8 such that each region of the diagram has a cross of this connected body. The resulting diagram is in Fig. 9.

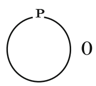

Fig. 8. .

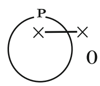

Fig. 9. .

3 Rule 4

In this section, in a manner similar to Shin's exposition, Peirce's original rule-4 is divided for convenience into several sub-rules that are discusses here separately.

Rule 4: (i) "In the same compartment repetitions of the same sign, whether mutually attached or detached, are equivalent to one writing of it" [12, p. 043].

So both the diagrams in Fig. 10 and Fig. 11 are equivalent to the diagram in Fig. 12.

(ii) "Two different signs in the same compartment"

(a) "if attached to one another are equivalent to no sign at all and may be erased or inserted" [12, p. 043].

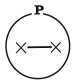

Fig. 10. .

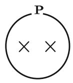

Fig. 11. .

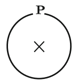

Fig. 12. .

The diagrams in Fig. 13 and Fig. 14 are equivalent.

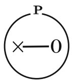

Fig. 13. .

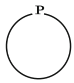

Fig. 14. .

(b) "But if they are detached from one another, they constitute an absurdity" [12, p. 043] (see Fig. 15).

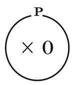

Fig. 15. .

Point to be noted that all the above conditions are based on the presupposition that signs are not connected with any other signs in other compartments.

For the case where the same signs exists separately in the same compartment, the clause (i) of rule-4 is superfluous. Because, using rule-1 we can erase the extra sign and get the diagram in Fig. 12 from the diagram in Fig. 11. Again, if we have the diagram in Fig. 12, then we have the information that 'P is non-empty'. So by using rule-3 we can have the diagram in Fig. 11 from the diagram in Fig. 12. When the same signs exists mutually attached in the same compartment, we also do not need clause (i) of rule-4 to get the diagram in Fig. 10 from the diagram in Fig. 12. It can be done using rule-2. But we need rule-4(i) to get the diagram in Fig. 12 from the diagram in Fig. 10. A question might arise here – we have the information that 'P is non-empty or P is non-empty' (Fig. 10) and we know that 'P is non-empty' (Fig. 12) is always derivable from this information then why not use rule-3 to get the diagram in Fig. 12 from the diagram in Fig. 10? It is because, rule-3 alone lets us introduce certain information about which we have prior knowledge. It does not let us deduce anything from that information. So even if we know that 'P is non-empty or P is non-empty' (Fig. 10), we can not use rule-3 to derive 'P is non-empty' and get the diagram in Fig. 12.

Even if we need rule-4(i), it still needs modification. The condition that 'signs are not connected with any other signs in the other compartment' makes rule-4(i) incapable to get certain syntactically different looking diagrams which represents the same information. For example, the diagrams in Fig. 16 and Fig. 17 represents the same information but we cannot get the diagram in Fig. 17 from the diagram in Fig. 16 by using rule-4(i) unless we drop the condition 'signs are not connected with any other signs in the other compartment'.

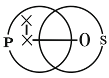

Fig. 16. .

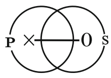

Fig. 17. .

In plain sight rule-4(ii-a) is also not needed here as we have the information that any region in diagram is 'either empty or non-empty' and thus by using rule-3 we get the diagram in Fig. 13 from the diagram in Fig. 14. The converse can be done by using rule-1[7]. But we need rule-4(ii-a) to get the diagram in Fig. 19 from the diagram in Fig. 18 as we cannot 'derive' the information '(S − P) is non-empty' from the information 'either (P ∩ S) is empty or (P ∩ S) is non-empty or (S − P) is non-empty' using rule-3. But again, similar to rule-4(i), the condition 'signs are not connected with any other signs in the other compartment' prevents us from doing so. The converse can be done using rule-2.

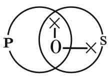

Fig. 18. .

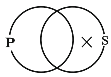

Fig. 19. .

The rule-4(ii-b) seems more of a definition than a rule as it ended abruptly saying that "If they are detached from one another, they constitute an absurdity". In Peirce's Existential graphs, we know that empty oval is considered as "constantly false proposition or absurdity" [2, p. 219] i.e. it is considered to be a 'contradiction'(see [2,14,16]). Also, in existential graphs within a cut anything can be inserted [17, p. 647], in other words anything follows from contradictions. So, classical explosion rule was always present in Peirce's diagrammatic systems. In rule-4(ii-b), Peirce meant classical explosion by saying that "they constitute an absurdity" and everything follows from it. But, although presented in practice in Peirce's Existential graphs [17, p. 647] it is not mentioned explicitly here. So a modification regarding this rule is needed.

Rules-4(i) and (ii) were criticized by Shin due the usage of the words 'equivalence' and 'absurdity'. Shin claimed that "By analyzing clause (i) and (ii) of this rule, I will show that Peirce does not make a clear distinction between syntax and semantics either. This confusion leads him to several problematic treatments of diagrams" [18, p. 30] and Shin believed that "this reveals Peirce's lack of a distinction between syntax and semantics" [18, p. 35].

According to Shin, in rule-4(i) and (ii-a), Peirce used the word 'equivalent' to actually represent 'semantically equivalent' diagrams, not 'syntactically equivalent' diagrams. The main base for this argument, as shown by Shin [18, p. 31], is that the following diagrams in Fig. 20 will be considered 'equivalent' by rule-4(i) and (ii-a). But, although these diagrams represent the same facts, they are syntactically different looking.

[7] Shin also proposed to use rule 1 to get Fig. 12 and Fig. 14 from Fig. 11 and Fig. 13 respectively. She also proposed to use rule 2 to get Fig. 10 from Fig. 12.

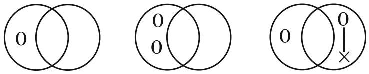

Fig. 20. .

Shin also pointed out the following three problems that occur when 'semantic equivalence' is taken into consideration.

(1) The possibility of semantically equivalent diagrams having a different syntactic form was not considered when Peirce criticized Lambert for having two different looking diagrams for the two equivalent proposition "Some A are B" (Fig. 21)[1] and "Some B are A" (Fig. 22)[12,18, p. 38].

Fig. 21. .

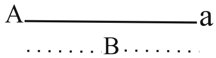

Fig. 22. .

(2) A deductive system, which is both sound and complete, lets us deduce a formula 'α' from a set of formula Γ if and only if α is a semantic consequence of Γ. So if α is semantically equivalent to some formula β then we can deduce α from β and vice-versa. But taking this assumption as a rule will make the existence of a deductive system unnecessary [18, p. 31].
(3) Peirce "did not have an accurate semantics to support his use of "equivalence" in a proper way" [18, p. 31]

Shin's criticisms can be disputed if we are reminded that 'Equivalence' is not always semantic since it can also be syntactic. Two different looking diagrams, say D and D$^{'}$, can be 'syntactically equivalent' if there is a rule that lets us get D$^{'}$ from D and vice-versa. The diagrams in Fig. 10, Fig. 11 and Fig. 12 all represent the same information i.e. 'P is non-empty' and rule-4(i) lets us get the diagrams from each other. Similar situation happens for the diagrams in Fig. 13 and Fig. 14. In [12, p. 041], Peirce already mentioned that two opposite signs, which are connected together in the same region, should annul each other and be equivalent to no sign at all. Then why did he need to construct rule-4(ii-a) that's says the same thing? The reason is that Peirce was trying to construct a rule that will let us get two syntactically different looking diagrams, which represents the same fact, from each other. That's why words like "equivalent to one writing of it" and "may be erased or inserted" was used respectively in rule-4(i) and (ii-a). It is true that we cannot just introduce a rule that would say that 'if formula α is a semantic consequence of a set of formulas Γ, then we can deduce α from Γ', but it is permissible, and even desirable, to have a rule that lets us get two equivalent but syntactically different looking diagrams from each other? In Shin's own system, she had the rule of splitting sequence where the diagram D_2 could be deduced from D_1 and both diagrams represent the same fact (see Fig. 23 [18, p. 123]).

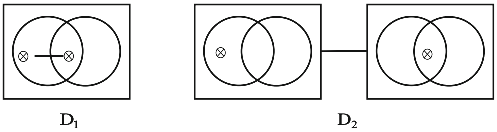

Fig. 23. .

One may reasonably argue that Peirce's opposition to Lambert's was motivated by the absence of rules in the latter to go from one diagram to it's semantically equivalent diagram. Shin's next objection that Peirce did not have accurate semantics didn't stand here as the notion 'equivalence' is used here 'syntactically'.

For rule-4(ii-*b*) Shin's objection was mainly regarding the notion of 'absurdity'. According to Shin there are two possible interpretations of 'absurdity'.

(1) "One is to take this phrase to mean that we should not be allowed to draw a diagram with two different kinds of signs in the same compartment. If so, this system has no way to represent a contradiction" [18, p. 32]. We argued earlier that this was not Peirce's interpretation of 'absurdity'.
(2) "The other interpretation of clause (ii-2), which seems to be more plausible, is that a diagram with "o" and "x" in the same compartment means absurdity. According to this interpretation, clause (ii-2) does not tell us how to transform a given diagram, but explains what assertion is made if a diagram has more than one character in a certain way. When we recall that these rules are stipulated to tell us what we are permitted to do in manipulating diagrams, it is rather puzzling why Peirce had to explain what a diagram means under these rules. What assertion is made in a diagram belongs to semantics, whereas the transformation rules belong to syntax. This clearly reveals Peirce's lack of a distinction between syntax and semantics" [18, p. 32]. This is again a same problem as we have dealt for the notion 'equivalence'. Peirce mentioned that whenever cross and zero exists detachedly in the same compartment it leads to absurdity way before introducing his rule of transformation. There was no need for him to again write it as a rule here. Also, in existential graphs, transformation rules are presented in a very much syntactic point of view, a fact that rather suggests an understanding of semantics and syntax. Yet, it is true that rule-4(ii-*b*) is not properly written and needs modification.

Rule 4: (iii) "If two contrary signs are written in the same compartments, the one being attached to certain others, P, and the other to certain others, Q, it is permitted to attach P to Q and to erase the contrary signs" [12, p. 043].

Using rule-4(iii) we can remove the contrary signs when it is in a disjunctive form. Peirce has given the following example where, by using the rule-4(iii), the diagram in Fig. 25 is obtained from the diagram in Fig. 24 [12, p. 044].

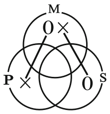

Fig. 24. .

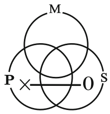

Fig. 25. .

For rule-4(iii), Shin pointed out that if we have the following argument

“*All S is P or some S is P.*
No S is P.
Therefore, there is no S”[18, p. 33].

Then we have the diagrams in Fig. 26 where “first diagram in the following represents what the two premises convey. One of the o’s in the first diagram is not attached to any other sign. Accordingly, the antecedent of clause (iii) is not satisfied. However, if we allow P (in clause (iii)) to be an empty sign, then we get the second diagram from the first one. After that, we need to add the second premise, “No S is P,” to the second diagram. This is how we get the third diagram, which represents the conclusion of the previous syllogism. In order to get the rightmost diagram (which we want to get), we need to represent the second premise twice” [18, p. 33].

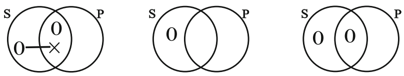

Fig. 26. .

If we follow the examples given by Peirce in MS 479 in [12, pp. 046–047] we will find that in the above case where two contradictory signs, ‘0’ and ‘×’, are in the same region ‘S ∩ P’ but only ‘×’ is connected with another sign in ‘S − P’, we only erase the ‘×’ and get the third diagram instead of the second one by applying rule-4(iii) to the first diagram. But again this condition, where one of the contrary signs is not connected with some other sign in some other region, is not mentioned precisely in rule-4(iii) and this rule needs to be modified.

4 Rules 5 to 6

Rule 5: “Any Area-boundary, representing a term can be erased, provided that if, in doing so, two compartments are thrown together containing independent zeros, those zeros be connected, while if there be a zero on one side of the boundary to be erased which is thrown into a compartment containing no independent zero, the zero and its whole connex be erased” [12, p. 044].

Rule-5 is a similar to rule-1, where we can erase information. By eliminating the curve M from the diagram in Fig. 27, we obtain the diagram in Fig. 28. Since, by the rule-4(i) the diagram in Fig. 28 is equivalent to the diagram in Fig. 29, the final diagram obtained after eliminating curve M is in Fig. 29.

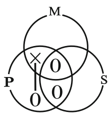

Fig. 27. .

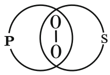

Fig. 28. .

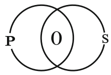

Fig. 29. .

Rule 6: "Any new Term-boundary can be inserted; and if it cuts every compartment already present, any interpretation desired may be assigned to it. Only where the new boundary passes through a compartment containing a cross the new boundary must pass through the cross, or what is the same thing a second cross connected with that already there must be drawn and the new boundary must pass between them, regardless of what else is connected with the cross. If the new boundary passes through a compartment containing a zero, it will be permissible to insert a detached duplicate of the whole connex of that zero, so that one zero shall be on one side and the other on the other side of the new boundary" [12, p. 044].

By Introducing the curve M, we get the diagram in Fig. 31 from the diagram in Fig. 30.

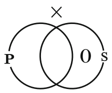

Fig. 30. .

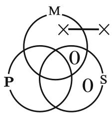

Fig. 31. .

Rule-6 let us introduce a closed curve without changing the given information. In this particular rule Peirce mentioned what is to be happen to a cross or a zero after introducing a closed curve and there were several examples also. But in all of them there were no connected body of cross and zero together (see Fig. 32).

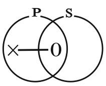

Fig. 32. .

So if we introduce the curve M in Fig. 32 which one of the following figures (Fig. 33 to Fig. 36) will we get as a result?

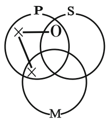

Fig. 33. A new cross was introduced and connected with the old one without taking the connected zero in consideration

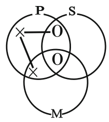

Fig. 34. A new cross has been added with the old connex of the cross and only a single zero, without it's connex, has been duplicated

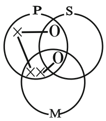

Fig. 35. Whole connex of the zero has been duplicated

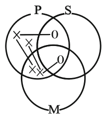

Fig. 36. Connex of zero has been duplicated

Now both Fig. 34 and Fig. 35 are not a valid transformation from Fig. 32. By valid transformation we meant the validity notion used in Shin [18] or Venn_{i_n} [4]. So we cannot get this two diagrams by using rule-6. Both Fig. 33 and Fig. 36 are valid. But the final figure that we will get from Fig. 32, by using rule-6, is Fig. 36. Figure 33 has been discarded since it doesn't precisely follow the conditions mentioned in the rule. While we introduce a new curve, a region having zero, is divided into two parts and each part should have a single detached zero. But this has not been done in this figure. Now in Fig. 36, everything mentioned in rule-6 has been followed. A new cross has been introduced and connected with the old connex of cross. Connex of zero has been duplicated− in this case we see two crosses for the connex of zero in the region $((P - S - M) \cup ((P \cap M) - S) \cup (P \cap S \cap M))$. This is because, while duplicating connex of zero, we find that the region containing cross has been divided in two parts. So two crosses have been introduced instead of one (like we did in Fig. 35).

Shin criticizes rule-5 and rule-6 saying that these rules do not exhaust all the possible cases as nothing about the existing '$\times$' or the connected bodies of $\times$'s has been mentioned in them. Although this is true for rule-5 it is not so for rule-6. In rule-6 all the possible cases have been discussed. For rule-5, Peirce gave several examples in [12, pp. 046–047]. By examining these examples, we can say that after eliminating a curve, a cross or a connected body of crosses remains in the same region. But again it is not mentioned explicitly in the rule[8].

[8] Additional difficulties may appear when the number of closed curves increases, if the diagrams are not simple or reducible. Peirce occasionally used Venn diagrams for more than 3 curves. Some examples are found in [15]. On the construction of diagrams for n number of curves, see [6].

5 Comparison with Modern Diagrammatic Systems

We previously alluded to modern systems Venn I, Venn II and Venn$_{i_n}$ which are all based on Peirce's extended version of Venn diagrams [4,18]. Before proceeding, we need to mention that there are two more diagrammatic objects in Venn$_{i_n}$, 'names of individuals' and 'absence of individuals' (see [4]). If we exclude these objects then Venn$_{i_n}$ is similar to Shin's Venn-II system. From here onward, to ease the comparison with Peirce's rules, whenever we refer to Venn$_{i_n}$ we exclude the diagrammatic objects 'names of individuals' and 'absence of individuals' and anything regarding them. The main differences between these systems and Peirce's system are given in Table 1.

Table 1. Differences between the four systems

Primitive symbols	Peirce	Venn-I	Venn-II	Venn$_{i_n}$
Universe	Sheet of drawing	Rectangle □	Rectangle □	Rectangle □
Predicate	Closed curve ○	Closed curve ○	Closed curve ○	Closed curve ○
Emptiness	0	Shading	Shading	Shading
Non-emptiness	$\times$	$\otimes$	$\otimes$	x
Disjunction	——— connecting 0's or $\times$'s or 0's and $\times$'s	——— connecting only $\otimes$'s	——— connecting $\otimes$'s or connecting two diagrams	——— connecting x's or connecting two diagrams

For simplicity, here onward we are going to use 'x' to represent 'non-emptiness' in Venn-I and Venn-II system also. Since the connecting line of Peirce (———) also connects diagrams in Venn-II and Venn$_{i_n}$, we have a new type of diagrams called compound diagrams (type-III diagrams for Venn$_{i_n}$ system [4]) where each of its components are called atomic diagrams (type-I or type-II diagrams for Venn$_{i_n}$ system. If a diagram consists of a single curve in a rectangle it is called a type-I diagram. If there are more than one curve then it is called a type-II diagram [4]). For example, the diagram in Fig. 37 is a compound diagram which represents the information 'Either All A are B and Some B are not A or Some A are not B and No A is B'. For Peirce's system we can represent this type of information of 'disjunctions of conjunctions' form by just converting the form into 'conjunction of disjunction'. So the corresponding diagram for Fig. 37 in Peirce's system is shown in Fig. 38. Generally, Peirce did not used any such compound diagrams but he did proposed an alternative way of representing a diagram when we deal with a complex form of information (see [12, p. 052]). There are no compound diagrams in the system Venn-I.

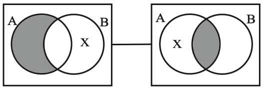

Fig. 37. .

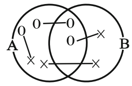

Fig. 38. .

In the previous section we mentioned that rule-4 and rule-5 need modifications. Now suppose, we modify these rules accordingly, i.e.

(1) we remove the condition 'signs are not connected with any other signs in the other compartment' from rule-4(i) and (ii-*a*).
(2) For rule-4(ii-*b*), we add the condition that if 'it constitutes absurdity then anything follows'.
(3) For rule-4(iii), we add the condition that if two contrary signs are in the same compartment and only one of them is attached to some other sign, say R, in another region, then it is permitted to erase only the attached contrary sign and to keep the sign R as it is.
(4) For rule-5, after erasure of curve, the cross or connected body of cross will remain in the same position.

After this kind of modification it can be shown that Peirce's rules are adequate to perform any kind of transformations that are permitted in the other three systems. Table 2 shows which of Peirce's rules are analogous to the rules of the three systems.

For example, using types I-II diagrams, suppose we have the following two diagrams D_1 and D_2 (see Fig. 39). Now we get the diagram D_2 from D_1 by eliminating the x-node of the x-sequence in the region $(((A \cap B) - C) \cup ((A \cap C) - B) \cup (C - A - B) \cup ((B \cap C) - A))$ that falls in the shaded region $((A \cap C) - B)$ of the diagram D_1.

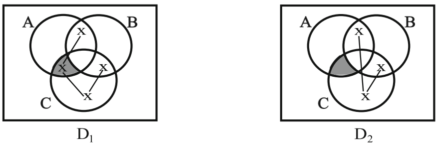

Fig. 39. .

In Peirce's system we get a similar transformation by using rule-4(iii) (see Fig. 40).

Table 2. Comparison of transformation rules among four systems

Types of Diagrams	Venn-I	Venn-II	Venn$_{i_n}$	Peirce
Type-I/II	The rule of erasure for Closed Curves	The rule of erasure for Closed Curves	Elimination Rules for Closed Curves	Rule-5
	The rule of erasure for Shading	The rule of erasure for Shading	Elimination Rules for Shading	Rule-1
	The rule of erasure for $\otimes$ or sequence of $\otimes$'s	The rule of erasure for $\otimes$ or sequence of $\otimes$'s	Elimination Rules for x or sequence of x's	Rule-1
	The rule of erasure of part of $\otimes$-sequence	The rule of erasure of part of $\otimes$-sequence	Elimination Rules for part of x-sequence	Rule-4(iii)
	The rule of spreading $\otimes$'s	The rule of spreading $\otimes$'s	Extension Rules for x-sequence	Rule-2
	The rule of introduction of basic regions (Closed Curves)	The rule of introduction of basic regions (Closed Curves)	Introduction Rules for Closed Curves	Rule-6
	The rule of conflicting information (Classical Explosion)	The rule of conflicting information (Classical Explosion)	Inconsistency Rules (Classical Explosion)	Rule-4(ii-b)
	The rule of unification of diagrams	The rule of unification of diagrams	Unification Rules	Rule-3
	N.A. As the universe can be either empty or non-empty	N.A. As the universe can be either empty or non-empty	Introduction Rules for x's	Rule-3
Type-III[a]	N.A. As there is no type-III diagrams	The rule of connecting diagram	Extension Rules for Diagrams	Rule-2
		The rule of splitting $\otimes$'s	Rules of Splitting Sequences	Not required here[b]
		The rule of the excluded middle	Rule of Excluded Middle	Rule-3
		The rule of conflicting information (Classical Explosion)for type-III diagram	Inconsistency Rules (Classical Explosion) for type-III diagram	Rule-4(iii)

[a]By rules for type-III diagrams, we only mean the rules using which type-I/II diagrams produce a type-III diagram.
[b]The rule of splitting sequences basically gives an equivalent type-III diagram of a type-I/II diagram. There is no change of information while using this rule. Thus it is not needed in Peirce's system, where we have only type-I or type-II diagrams.

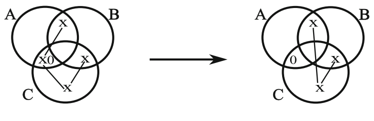

Fig. 40. .

Thus rule-4(iii) is analogous to the rule of erasure of part of $\otimes$-sequence in Venn-I and Venn-II or the elimination rule for part of x-sequence for the system Venn$_{i_n}$.

When we use type-III diagrams, we need an additional rule 'the rule of construction'. When dealing with such transformations in Peirce's case we need all together rule-4(i), rule-4(ii-a) and rule-2. For example, consider the following type-III diagrams, $D_1 - D_2$ and $D_3 - D_4$, in Fig. 41 and Fig. 42 respectively.

We get the diagram $D_3 - D_4$ from the diagram $D_1 - D_2$ through the following transformations (Fig. 43 to Fig. 45).

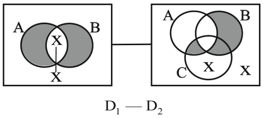

Fig. 41. .

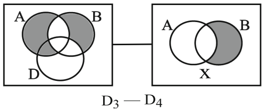

Fig. 42. .

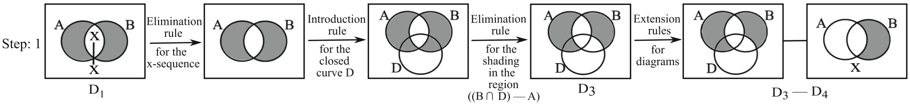

Fig. 43. .

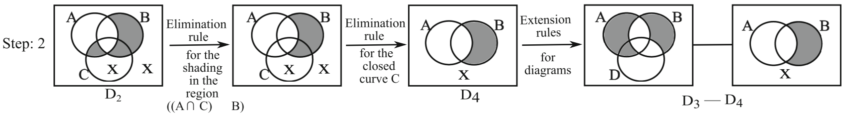

Fig. 44. .

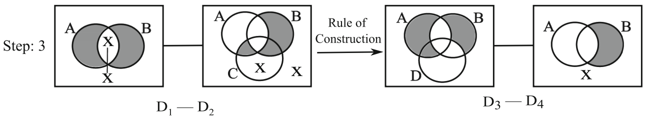

Fig. 45. .

In Peirce's system, by transforming the information from the form of 'disjunction of conjunction' to the form of 'conjunction of disjunction', we get corresponding diagrams of $D_1 - D_2$ and $D_3 - D_4$ in the Fig. 46 and Fig. 47 respectively. We get the diagram D_6 from the diagram D_5 by using the rules shown in Fig. 48.

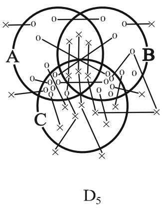

Fig. 46. .

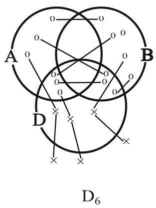

Fig. 47. .

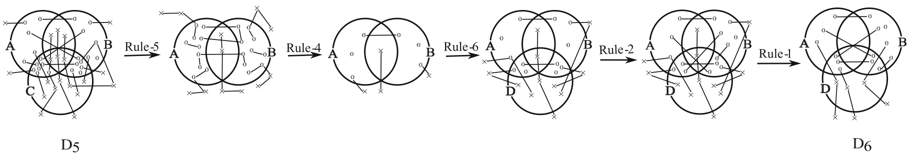

Fig. 48. .

6 Conclusion

In this paper, we exposed and discussed Peirce's set of rules for the transformation of Euler-Venn diagrams. We invoked its historical importance, then we identified the uses and shortcomings of each rule. We also considered the extent to which they 'survived' in modern diagrammatic systems.

Peirce himself conjectured that some of the rules may be simplified and some rules may have been overlooked. Such a task was more recently undertaken by Shin who argued that "(1) Some of the rules need to be clarified. (2) We need more rules to make this system complete. (3) Some semantic terminology (equivalence or absurdity) is used without clarification" [18, p. 35]. Our work partly corroborates Peirce's intuition and Shin's criticism. However, we demonstrate that only minor modifications are required. Moreover, such modifications were already implemented by Peirce in hi usage of the rules in the many examples that he provided. These examples were absent from Shin's account which was primarily based on the abridged transcription of manuscript MS 479 included in Peirce's *Collected Papers* [13].

A look at the original manuscript allowed us to return Peirce's original formulations and the modifications that his examples have suggested to him. Finally, we argued that slightly modified Peirce's rules are adequate to perform any kind of transformations that can be done to diagrams in the modern diagrammatic systems Venn-I, Venn-II and Venn$_{i_n}$.

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