Dynamic Stability of the Plate at the Second Resonance with Creep Taken into Account

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Abstract. The problem of determining the side regions of dynamic stability of a rectangular plate, the material of which obeys the linear creep law, is considered. The solution of the differential equation of vibrations of a plate loaded with constant and variable periodic loads in the plane of the plate is presented as a series with separated variables, which satisfies the conditions for fixing the plate. As a result of using the Bubnov-Galerkin method, differential equations of the third order with variable periodic coefficients are obtained for finding functions that depend on time. To construct even domains of dynamic instability, a solution in the form of a trigonometric series was used. As a result, a system of equations was compiled for the coefficients of the series. To calculate the even regions of instability of the plate, an equation of critical frequencies was obtained in the form of a system determinant. This equation allows us to construct the boundaries of the regions of instability. The study of the influence of the relaxation time and the ratio of long-term and instantaneous moduli of elasticity on the position of the second region of dynamic instability has been carried out.

Keyword: Rectangular plate dynamic stability creep

1 Problem Setting

We load a rectangular plate in its plane with variable compressive forces evenly distributed along the edges

$$N_x = N_{x0} + N_{xt}\, cos\theta t,\ \ N_y = N_{y0} + N_{yt}\, cos\theta t, \tag{1}$$

In [1] the problem of finding side regions of dynamic instability for a plate was solved, the material of which obeyed the hereditary law of deformation. There are materials that obey other laws of viscoelasticity used in the work [2–5]. In this work, we will describe the properties of the material by a linear creep law, which for a plane stress state has the form [6]:

$$\sigma_x + n\dot{\sigma}_x = \frac{\mathrm{H}}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) + \frac{\mathrm{E}n}{1-\nu^2}\left(\dot{\varepsilon}_x + \nu\dot{\varepsilon}_y\right), ,$$
$$\sigma_y + n\dot{\sigma}_y = \frac{H}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) + \frac{En}{1-\nu^2}\left(\dot{\varepsilon}_y + \nu\dot{\varepsilon}_x\right), ,$$

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$$\tau_{xy} + n\dot{\tau}_{xy} = \frac{H}{2(1+\nu)}\gamma_{xy} + \frac{En}{2(1+\nu)}\dot{\gamma}_{xy}, \tag{2}$$

where n is the relaxation time, H is the long-term modulus of elasticity, E is the instantaneous modulus of elasticity.

The differential equation of vibrations of a plate, the material properties of which are described by law Eq. (2), has the form [7]:

$$\frac{H}{E}D\nabla^2\nabla^2 w + Dn\frac{\partial}{\partial t}\nabla^2\nabla^2 w + m\frac{\partial^2 w}{\partial t^2} + nm\frac{\partial^3 w}{\partial t^3} + (N_{x0} + N_{xt}\cos\theta t)\frac{\partial^2 w}{\partial x^2} +$$
$$\left(N_{y0} + N_{yt}\cos\theta t\right)\frac{\partial^2 w}{\partial y^2} + (-N_{xt}\theta\sin\theta t)n\frac{\partial^2 w}{\partial x^2} + (N_{x0} + N_{xt}\cos\theta t)\frac{\partial^3 w}{\partial x^2\partial t}n$$
$$+\left(-N_{yt}\theta\sin\theta t\right)n\frac{\partial^2 w}{\partial y^2} + \left(N_{y0} + N_{yt}\cos\theta t\right)\frac{\partial^3 w}{\partial y^2\partial t}n = 0, \tag{3}$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$ - instantaneous cylindrical stiffness of the plate.

The article [7] considered the problem of finding the boundaries of the regions of instability, for which Eq. (3) has periodic solutions with a period of $2T$, including the main regions of dynamic instability. In this paper, we consider the problem of finding solutions with period T. This will allow us to construct even regions of dynamic instability.

2 Solution Technique

The solution to Eq. (3) колебаний пластины will be sought by the Bubnov-Galerkin method, separating the variables in the form [8]:

$$w = \sum f_i(t)X_i(x)Y_i(y) \tag{4}$$

where $f_i(t)$ - sought time functions, $X_i(x)$ and $Y_i(y)$- functions that satisfy the boundary conditions along the edges of the rectangular contour with respect to w. Substituting the solution in the form of a series with separated variables Eq. (4) into Eq. (3) of the plate vibrations and using the Bubnov-Galerkin method, we obtain differential equations with variable coefficients, containing derivatives of the third order in time t:

$$\dddot{f} + \frac{1}{n}\ddot{f} + \Omega^2(1 - 2\mu\ \ cos\theta t)\dot{f} + \frac{\Omega^2}{n}(\xi - 2\mu\ cos\theta t + 2\mu n\theta\ sin\theta t)f = 0. \tag{5}$$

where Ω is the frequency of natural vibrations of a plate loaded with constant components of longitudinal forces N_{x0} and N_{y0}, $\mu\mu$ - excitation factor, ξ - dimensionless parameter depending on the ratio of instantaneous and long-term elastic modulus;

$$\Omega^2 = \omega^2\left(1 - \frac{N_{x0}N_{2*} + N_{y0}N_{1*}}{N_{1*}N_{2*}}\right),$$
$$\mu = \frac{1}{2}\frac{N_{xt}N_{2*} + N_{yt}N_{1*}}{N_{1*}N_{2*} - N_{x0}N_{2*} - N_{y0}N_{1*}}, \xi = \frac{\frac{H}{E}N_{1*}N_{2*} - N_{x0}N_{2*} - N_{y0}N_{1*}}{N_{1*}N_{2*} - N_{x0}N_{2*} - N_{y0}N_{1*}}$$

N_{1*} and N_{2*} - critical values of efforts N_{x0} and N_{y0} at their independent static action, ω-natural vibration frequency of an unloaded plate,

$$N_{1*} = \frac{I_1}{I_3}D,\ N_{2*} = \frac{I_1}{I_4}D,$$

$$I_1 = \int_0^a \int_0^b \left(\frac{d^4X}{dx^4}Y + 2\frac{d^2X}{dx^2}\frac{d^2Y}{dy^2} + \frac{d^4Y}{dy^4}X\right)XYdxdy,\ I_2 = \int_0^a \int_0^b X^2Y^2dxdy,$$

$$I_3 = -\int_0^a \int_0^b X\frac{d^2X}{dx^2}Y^2dxdy, I_4 = -\int_0^a \int_0^b X^2\frac{d^2Y}{dy^2}Ydxdy,\ \omega^2 = \frac{DI_1}{mI_2}.$$

In this problem, it is necessary to introduce the dimensionless time $\tau = \theta t$. Then Eq. (5) for the time-dependent function will take the form:

$$f''' + \frac{1}{n\theta}f'' + \frac{\Omega^2}{\theta^2}(1 - 2\mu\cos\tau)f' + \frac{\Omega^2}{n\theta^2}(\xi - 2\mu\cos\tau + 2\mu n\theta\sin\tau)f = 0. \tag{6}$$

where the prime denotes the derivative with respect to the dimensionless time τ. There are four dimensionless coefficients in the equation $\mu, \xi, \alpha = \frac{1}{n\theta}, \phi = \frac{\Omega^2}{\theta^2}$. The solution of Eq. (6) for the function f (t) will depend on the values of these coefficients. Note that these equations have unboundedly increasing solutions for some ratios of these coefficients, which occupy entire regions on the parameter plane $\mu, \frac{\theta}{2\Omega}$. One of the central tasks of the theory of dynamic stability is to determine the position of regions of dynamic instability [8]. Regions of unboundedly increasing solutions are separated from stability regions by periodic solutions with periods T and $2T$. Two solutions of the same period limit the region of instability, two solutions of different periods - stability area. Questions related to the determination of regions of dynamic instability with a period of 2T, including the main region, are considered in the work [7]. We obtain even stability regions if we seek the solution of Eq. (6) for the time function in the form of a series with the period T:

$$f(t) = \sum_{k=2,4,6}^{\infty}\left(a_k \sin\frac{k\tau}{2} + b_k\cos\frac{k\tau}{2}\right) + b_0 \tag{7}$$

Substitute series Eq. (7) into the differential equation for the time function:

$$\sum_{k=2,4}^{\infty}\left(-a_k\frac{k^3}{8}cos\frac{k\tau}{2} + b_k\frac{k^3}{8}sin\frac{k\tau}{2}\right) + \alpha\sum_{k=2,4}^{\infty}\left(-a_k\frac{k^2}{4}sin\frac{k\tau}{2} - b_k\frac{k^2}{4}cos\frac{k\tau}{2}\right)$$
$$+ \varphi(1 - 2\mu cos\tau)\sum_{k=2,4}^{\infty}\left(a_k\frac{k}{2}cos\frac{k\tau}{2} - b_k\frac{k}{2}sin\frac{k\tau}{2}\right)$$
$$+ \alpha\varphi\left(\xi - 2\mu cos\tau + \frac{2\mu}{\alpha}\sin\tau\right)\left[b_0 + \sum_{k=2,4}^{\infty}\left(a_k sin\frac{k\tau}{2} + b_k cos\frac{k\tau}{2}\right)\right] = 0. \tag{8}$$

Transforming Eq. (8) of vibrations of the plate and equating the coefficients for the same trigonometric functions sin $\frac{k\tau}{2}$ and cos $\frac{k\tau}{2}$, we obtain a system of linear equations with respect to b_0, a_k and b_k:

$$\begin{gathered}
a_1\alpha\left(-\frac{1}{4}+\varphi\xi+\varphi\mu\right)-\alpha\varphi\mu a_3-\frac{1}{2}b_1\left(-\frac{1}{4}+\varphi-\varphi\mu\right)+\frac{1}{2}\varphi\mu b_3=0,\\
\frac{1}{2}a_1\left(-\frac{1}{4}+\varphi+\varphi\mu\right)+\alpha b_1\left(-\frac{1}{4}+\varphi\xi-\varphi\mu\right)-\frac{1}{2}\varphi\mu a_3-\alpha\varphi\mu b_3=0,\\
\alpha\left(-\frac{k^2}{4}+\varphi\xi\right)a_k-\frac{k}{2}b_k\left(-\frac{k^2}{4}+\varphi\right)-\alpha\varphi\mu(a_{k+2}+a_{k-2})+\varphi\mu\frac{k}{2}(b_{k+2}-b_{k-2})=0,\\
\frac{k}{2}a_k\left(-\frac{k^2}{4}+\varphi\right)+\alpha b_k\left(-\frac{k^2}{4}+\varphi\xi\right)-\varphi\mu\frac{k}{2}(a_{k+2}+a_{k-2})-\alpha\varphi\mu(b_{k+2}+b_{k-2})=0,\\
(k=2,4,6...)
\end{gathered} \tag{9}$$

Received a homogeneous system of linear equations. Let us compose the determinant from the coefficients of the unknowns b_0, a_k and b_k, equate it to zero:

$$\begin{vmatrix}
\alpha\phi\xi & -\mu\alpha\phi & 0 & 0 & 0 & ...\\
2\mu\alpha\phi & \alpha(1-\phi\xi) & 1-\phi & \mu\alpha\phi & \mu\phi & ...\\
2\mu\phi & 1-\phi & -(1-\phi\xi) & \mu\phi & -\mu\alpha\phi & ...\\
0 & \mu\alpha\phi & 2\mu\phi & \alpha(4-\phi\xi) & 2(4-\xi) & ...\\
0 & 2\mu\phi & -\mu\alpha\phi & 2(4-\phi) & -\alpha(4-\phi\xi) & ...\\
... & ... & ... & ... & ... & ...
\end{vmatrix}=0. \tag{10}$$

Received the equation of critical frequencies. By critical frequencies we mean the frequencies of the external load θ_* corresponding to the boundaries of the instability regions. Using this equation, we construct even domains of dynamic instability lying near the frequencies $\theta_*=\frac{2\Omega}{k}(k=2,4,6...)$.

Replace the coefficient ϕ with $p^2=\frac{1}{4\phi}$, α transform as follows: $\alpha=\frac{1}{n\theta}=\frac{r}{p}$, where $r=\frac{1}{2\Omega n}$, then, instead of Eq. (10) of critical frequencies, we have:

$$\begin{vmatrix}
\xi & -\mu & 0 & 0 & 0 & ...\\
2\frac{r}{p}\mu & \frac{r}{p}\left(4p^2-\xi\right) & 4p^2-1 & \frac{r}{p}\mu & \mu & ...\\
2\mu & 4p^2-1 & -\frac{r}{p}\left(4p^2-\xi\right) & \mu & -\frac{r}{p}\mu & ...\\
0 & \frac{r}{p}\mu & 2\mu & \frac{r}{p}\left(16p^2-\xi\right) & 2\left(16p^2-1\right) & ...\\
0 & 2\mu & -\frac{r}{p}\mu & 2\left(16p^2-1\right) & -2\left(16p^2-\xi\right) & ...\\
... & ... & ... & ... & ... & ...
\end{vmatrix}=0. \tag{11}$$

3 Results

To construct the second region of instability, the third approximation was considered, that is, the determinant of the seventh order was used. Equation (11) of critical frequencies includes coefficients r depending on the relaxation time, and ξ, taking into account the influence of instant E and long-term H elastic moduli. The values of these coefficients affect the position of the dynamic regions of instability. To study the influence of the coefficients on the position of the regions of dynamic instability, we construct the regions of dynamic instability at constant values ξ and varying from zero to infinity r. In (see Fig. 1) shows the results of calculating the regions of instability at $\xi = 0.5$. It can be seen from the figure that the boundaries of the region with increasing r first move away from the p axis, and then begin to approach. Simultaneously the graphs move to the axis μ. This suggests that the minimum value of the excitation coefficient first increases, then decreases again. This means that with an increase in the minimum excitation coefficient, the smallest amplitudes of the periodic forces N_{xt} and N_{yt} increase, in this case, the frequency θ of the acting forces decreases. A similar movement of areas can be seen in (see Fig. 2), where the value $\xi = 0.8$. In this case, the boundaries of the regions of instability are located rather close to each other. In both cases, there is a point of intersection of all boundaries of the regions of dynamic instability. With an increase in the value, the coordinate of the intersection point decreased from 0.37 to 0.29, p increased from 0.38 to 0.454. At $r = 0$, for an infinitely long relaxation time, the boundary of the dynamic instability regions has a point of tangency with the p axis at 0.5 at the accepted values ξ. In this case, the material works as elastic with an instantaneous modulus of elasticity E. For an infinitely large value, the point of tangency with the p-axis for a larger value ξ For an infinitely large value, the point of tangency with the p-axis for more is higher is higher. The plate material works as elastic with a long modulus of elasticity H. Comparing the boundaries of the dynamic instability regions on the graphs (see Fig. 1

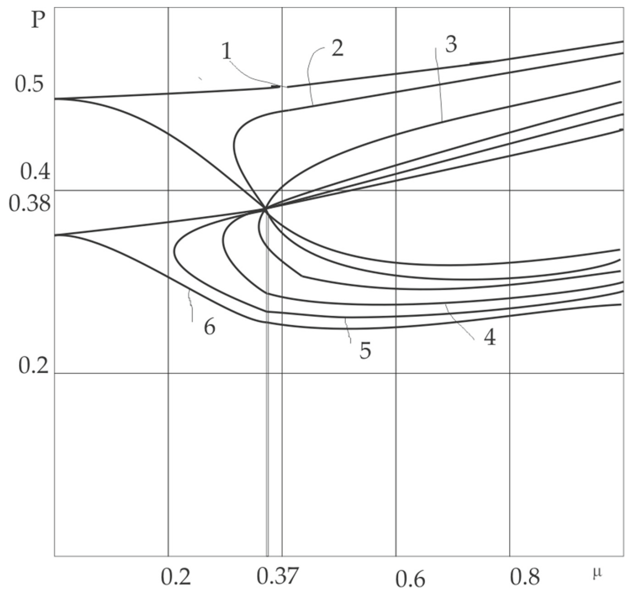

Fig. 1. The boundaries of the regions of dynamic instability at $\xi = 0.5$: r = 0; 2; r = 0.2; 3. r = 0.5; 4. r = 1; 5. r = 2; 6. r = ∞.

et Fig. 2) with the same r, excluding $r = 0$ and infinitely large values, it should be noted that a decrease ξ and, therefore, the ratio of the elastic moduli leads to an increase in the minimum value of the excitation coefficient and reducing the frequency of the buckling load.

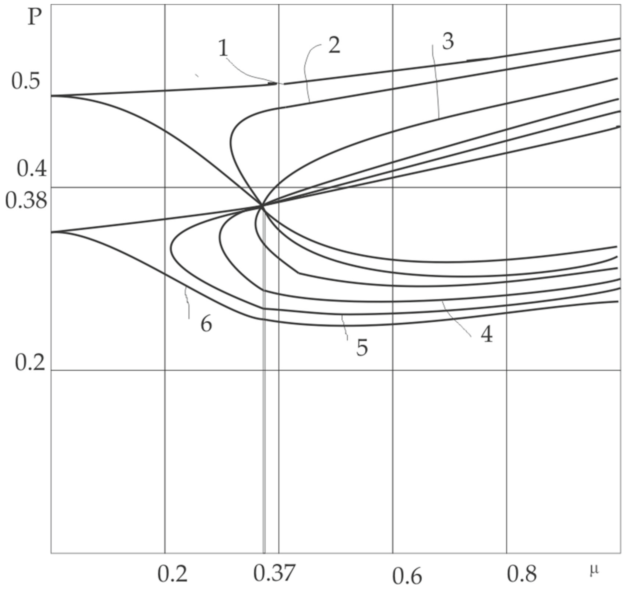

Fig. 2. The boundaries of the regions of dynamic instability at ξ = 0.8: r = 0; 2. r = 0.2; 3. r = 0.5; 4. r = 1; 5. r = ∞.

4 Conclusions

An equation of critical frequencies for even regions of dynamic instability for a rectangular plate, the material of which obeys a linear creep law, is obtained in this work. The influence of the relaxation time and the ratio of the instantaneous and long-term elastic moduli on the position of the second region of dynamic non-stability are investigated. It was revealed that an increase in the parameter r at constant leads ξ to the fact that the minimum value of the excitation coefficient first increases, then decreases again. Consequently, with an increase in the minimum excitation coefficient, the smallest amplitudes of the periodic components of the forces Nxt and Nyt increase, in this case, the frequency θ of the acting forces decreases. In addition, at the same, excluding $r = 0$ and infinitely large values, a decrease ξ and, therefore, the ratio of the elastic moduli leads to an increase in the minimum value of the excitation coefficient μ and a decrease in the frequency of the load causing the loss of stability.

It should be noted that when solving the problem posed in the work, you can use other dependencies to describe the viscoelastic properties of the material. For example, use the results presented in [9, 10].

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