A Periodic Inventory Model for Perishable Items with General Lifetime

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Abstract. We study a perishable inventory system controlled by a (T, S) periodic review order up to level policy. Items have a general lifetime distribution and the lead time is assumed to be constant. The demands arrive according to a Poisson process with rate λ. The customer is impatient. If he is not served, he will leave the queuing systems. Using the approximate solution of the steady-state probabilities, we are able to obtain the cost components' analytical expression. Next, we compare the analytical results of our model with the simulation results. Finally, we perform a sensitivity analysis to evaluate the effect of the lifetime variation and cost parameters on the optimal cost and stock level S. Our model is an extension of an exponential distribution case. With the general lifetime distribution, we are able to have total flexibility for setting the lifetime variability. This approximation is closer to reality, especially in random environment conditions. Obtained results, from the analytical approximation model, are close to those of the optimal policy with an average accuracy of 7%.

Keywords: Perishable items · General lifetime · Periodic review

1 Introduction

Nowadays, the management of perishable products such as food, blood, and pharmaceutical products, has become one of the major global issues in terms of cost optimization. A new study done by the Food and Agriculture Organization (FAO) indicates that 13.8% of the total produced food in the world is lost [1]. Product perishability is also one of the critical problems in the healthcare field, especially that most of the pharmaceutical goods and all blood products have a limited lifetime. The use of an outdated product in this field puts human life at risk, which makes the problem very serious. The AABB members estimate that the total number of outdated blood components, in blood centers and hospitals in 2013, was 932,000 units. In fact, 4.64% of all processed blood components are perished this year [2].

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In today's competitive global market, it is mandatory for companies managing perishable products to reduce these losses in order to remain competitive in their industrial fields. The root cause for this intriguing problem can be explained by the stochastic nature of demand, the limitation and the variability of lifetime, and the high customer service requirement. Due to the importance of this issue, perishable inventory has received significant attention from researchers and the scientific community [3, 4].

In the absence of an optimal policy to manage this type of goods, our challenge is to find an analytical approximation that considers the real faced constraints. By doing so, we will be able to make a balance between satisfying customer and decreasing product perishability. So, the question is: How to deal with perishable products when facing several constraints and get an analytical approach sufficiently close to the real system that can evaluate the system performance before making a decision.

This work tackles the above-described problem by investigating the case of a perishable periodic inventory model with general lifetime distribution and proposing an analytical solution to the steady-state probabilities and the cost components. This model can be considered more realistic reflecting the stochastic nature of the lifetime and the demand. The main contribution of this paper is the extension of the analytical approximation given by Kouki [5] to the model in which the lifetime has a general distribution. In addition, since most blood platelets are managed by periodic inventory control, our model can be used as an alternative to existing models, where blood life is typically assumed to be either fixed or exponentially distributed with an average of time units.

The remainder of the paper is organized as follows. Section 2 introduces the relevant literature review of perishable inventory management. Section 3 presents the proposed inventory model. In Sect. 4, we compare the obtained analytical results to the simulation results and we provide a sensitivity analysis to evaluate the impact of different parameters (lifetime variability, Lead time, …) on the inventory average and the operating cost. Finally, Sect. 5 draws conclusions and suggests potential further research directions.

2 Literature Review

Perishable inventory management is becoming increasingly complex. Products are becoming perishable due to various factors such as product characteristics, nature of demand, and changing customer expectations. Due to the importance of this problem, researchers are focusing on the optimal policy identification.

The literature concerning perishable inventory systems can be classified into various classes depending on whether the inventory is controlled continuously or periodically, products' lifetime is assumed to be constant or random, the lead time is positive or instantaneous, and whether demand is deterministic or stochastic. Nahmias [6] classifies perishable inventory management in two basic categories: Periodic or Continuous Review. Under the continuous review system, the inventory level is known and tracked continuously, requiring more resources and investment. This case is not detailed in this work. We refer readers to [3].

However, the periodic review is inexpensive since inventory evaluation takes place at specific times. These models can be classified according to the lifetime characteristics: fixed and random lifetime. For more details, Nahmias [6] and Goyal [7] presented an interesting review of perishable inventory management based on this classification.

Considering the fixed lifetime models', the first analysis of optimal policies was given by Van Zyl (1964) [6]. But until now, the adoption of the classic periodic replenishment policy for perishable items with a fixed shelf life, a deterministic lead time, and fixed order costs is very difficult to found [8].

But the assumption of a deterministic lifetime is not realistic especially when this type of goods can be deteriorated at any moment in the period of its lifetime due to random perturbation factors such as temperature, humidity, lightness, and packaging location. Actually, disregarding the stochastic nature of the lifetime can be a source of losses and costs. Kouki [5] gave a numerical investigation and showed that the ignorance of randomness of lifetime leads to higher costs.

The stochastic aspect can be illustrated by several probability distributions: Binomial, Uniform, Normal, Exponential, and General. Most works that deal with random lifetime use the exponential distribution, and the majority of these models were found under a continuous review policy [5, 8]. Under the periodic review, the first model that examined a perishable good with zero lead time and random lifetime, was given by Vaughan [9], where the lifetime had a Gamma distribution. Kouki [10] analyzed a periodic review inventory control system of a perishable product having random lifetime with exponential distribution. To cover more cases, Kouki [8] considers that item lifetimes follow an Erlang distribution. Another probability distribution model that covers most cases is the general distribution. It considers Exponential distribution, Gamma distribution, Normal Distribution, etc. The only one who used this distribution was Nahmias [11]. He assumed that successive deliveries would outdate in the same order of their arrival in inventory and showed that the structure of the optimal policy is the same as in the case where the lifetime is deterministic. This model considered that the lead time is zero, which is incoherent with reality [11].

The literature review of perishable inventory management is very rich, but the case of a periodic review with general lifetimes and positive lead times has not been investigated yet. The focus of this research will be the presentation of an analytical approximation model that is close to the optimal policy with acceptable accuracy.

3 Model Description

In the absence of the optimal policy to manage perishable products, we propose an approximate model with a periodic review (T, S) policy. The demand arrives following a Poisson distribution with rate λ and customers are impatient. The lifetime of each product has a general distribution F with means m. Each time an order is triggered, it arrives in stock after a constant lead time $L \leq T$. In this section, we describe the approach made to get the expression of the total cost performance $Z(T, S)$. First, we detail the steps made to get the steady-state probabilities P. Then, we calculate the different cost components. Finally, we present the total cost.

To compute the steady-state probabilities $P(i)$, of the inventory level i, $i = 0, \ldots S$, at $nt + L, n = 1, 2, \ldots$, we rely on the transient probabilities from state i to state j during t. This transition occurs either through a demand arrival or a perished product. So, to calculate these probabilities, we use the demand rate λ and the rate at which items perish, which we denote by $\gamma(n)$. Given that, there are n items in the stock. Movaghar [12] showed that for an $./M/1 + G$ queue:

$$\gamma(n) = \frac{n\Phi(n-1)}{\Phi(n)} - \lambda \tag{1}$$

where $\Phi(n) = \int_0^\infty \left(\int_0^x (1 - F(y))dy\right)^n e^{-\lambda x} dx$.

In this case, we are interested in calculating the transient probability from the state i to the state j during t time such that $nt + L \leq t < (n+1)t + L$. Regarding our model the inventory items' i can be modeled as an $./M/1 + G$ queue whose service rate is $\frac{1}{\lambda}$, whereas the "$+G$" represents the remaining lifetime, which follows the distribution F.

It should be noted that the actual lifetime for a given cycle T is, of course, different from the lifetime given by the distribution F because it may happen that, for a given cycle, some items remain in the stock when an order is received. So, the inventory age is constituted by multiple age categories. In fact, there are items whose lifetime is different from those having an age given by F. In our model, it is assumed that the remaining lifetime for any cycle T is always the same, and it is given by F. This is true if the lifetime follows an exponential distribution that has a memoryless property. For any other lifetime distribution, our model approximates the distribution of the true remaining lifetime by F.

The first step to get the transition probability from i to j is to write the Kolmogorov's Eqs. 2:

$$p'_{i,j}(t) = \begin{cases} -(\lambda + \gamma(i))p_{i,j}(t), i = j \\ -(\lambda + \gamma(j))p_{i,j}(t) + -(\lambda + \gamma(j+1))p_{i,j+1}(t), j < i \leq S \end{cases} \tag{2}$$

Using the Laplace transform, we obtain

$$\begin{cases} (z + \lambda + \gamma(i))p_{i,j}(z) = 0, i = j \\ (z + \lambda + \gamma(j))p_{i,j}(z) = (\lambda + \gamma(j+1))p_{i,j+1}(z), j < i \leq S \end{cases} \tag{3}$$

We can show by induction that the solution of the above equations is:

$$p_{i,j}(z) = \frac{1}{\lambda + \gamma(j)} \prod_{k=j}^{i} \frac{\lambda + \gamma(k)}{z + \lambda + \gamma(k)} \tag{4}$$

and the inverse of Laplace transform of the above equation gives the transition probability from any state i to j and time $t, nT + L \leq t < (n+1)T + L, n = 1, 2, \ldots$

$$p_{i,j}(t) = \left(\prod_{n=j+1}^{i}(\lambda+\gamma(n))\right)\sum_{k=j}^{i}\left((-1)^{i-j}\mathrm{e}^{-t(\lambda+\gamma(k))}\prod_{n=j,n\neq k}^{i}\frac{1}{\gamma(k)-\gamma(n)}\right) \tag{5}$$

We can now find the steady-state probability P using the relation:

$$P = P \times A \times B \tag{6}$$

where $A = p_{i,j}(L)_{0\leq i\leq S, 0\leq j\leq S}$ and $B = p_{i,j}(T-L)_{0\leq i\leq S, 0\leq j\leq S}$

Now we are ready to derive the cost components. The inventory average is given by:

$$\begin{aligned} E(I) &= \frac{1}{T}\sum_{i=1}^{S}\sum_{j=1}^{i}P(i)\int_0^T jp_{i,j}(t)dt \\ &= \frac{1}{T}\sum_{i=1}^{S}\sum_{j=1}^{i}\sum_{k=j}^{i}\left(jP(i)\prod_{n=j+1}^{i}(\lambda+\gamma(n))\right)\left(\frac{(-1)^{i-j}\left(1-\mathrm{e}^{-T(\lambda+\gamma(k))}\right)}{\lambda+\gamma(k)}\prod_{n=j,n\neq k}^{i}\frac{1}{\gamma(k)-\gamma(n)}\right) \end{aligned} \tag{7}$$

Similarly, the expected outdated quantity is:

$$\begin{aligned} E(O) &= \frac{1}{T}\sum_{i=1}^{S}\sum_{j=1}^{i}P(i)\int_0^T\gamma(j)p_{i,j}(t)dt \\ &= \frac{1}{T}\sum_{i=1}^{S}\sum_{j=1}^{i}\sum_{k=j}^{i}\left(\gamma(j)P(i)\prod_{n=j+1}^{i}(\lambda+\gamma(n))\right)\left(\frac{(-1)^{i-j}\left(1-\mathrm{e}^{-T(\lambda+\gamma(k))}\right)}{\lambda+\gamma(k)}\prod_{n=j,n\neq k}^{i}\frac{1}{\gamma(k)-\gamma(n)}\right), \end{aligned} \tag{8}$$

and finally, the expected lost sales can be written as:

$$\begin{aligned} E(S) &= \frac{\lambda P(0)}{T} + \frac{\lambda}{T}\sum_{i=1}^{S}\sum_{j=1}^{i}P(i)(\lambda+\gamma(1))\int_0^T(T-t)p_{i,j}(t)dt \\ &= \frac{\lambda P(0)}{T} + \frac{\lambda}{T}\sum_{i=1}^{S}\left(P(i)(\lambda+\gamma(1))\prod_{n=2}^{i}(\lambda+\gamma(n))\right)\sum_{k=1}^{i}\left(\frac{(-1)^{i-1}\left(-1+\mathrm{e}^{-T(\lambda+\gamma(k))}+T(\lambda+\gamma(k))\right)}{[\lambda+\gamma(k)]^2}\prod_{n=1,n\neq k}^{i}\frac{1}{\gamma(k)-\gamma(n)}\right) \end{aligned} \tag{9}$$

The total cost is:

$$Z(T,S) = \frac{K}{T} + hE(I) + wE(O) + bE(S) + c(\lambda + E(O) - E(S)) \tag{10}$$

Where K is the unit ordering cost, h is the holding cost per unit time, w is the outdated cost, b is the lost sales cost, and c is the purchasing cost.

4 Numerical Results

In this section, we conduct an extensive numerical study to identify the accuracy of our model's numerical results compared to the obtained simulation results using the simulation software Arena. In the second part, we conduct a sensitivity analysis to evaluate how the lifetime variability of perishable items affects the optimal cost and the system's performance. We also show the impact of the different cost parameters on the optimal total cost.

4.1 Comparison with the Simulation Model

In our numerical analysis, we assume that the lifetime follows a Gamma distribution with mean 3. The scale and shape parameters are given respectively from the intervals below: [2, 5, 10, 20, 50, 100, 150, 200, 300, 500, 1000, 2000, 5000] and [2, 5, 10, 20, 50, 100, 150, 200, 300, 500, 1000, 2000, 5000]. The demand rate is $\lambda = 2$, and the lead time is assumed to be constant, L = 1. 2500 MATLAB scenarios are done under different parameters settings: the unit ordering cost per order K belongs to the interval: [5,10,50,100], the unit holding cost per unit h is fixed to 1, the outdated cost W and the purchasing cost c are respectively equal to 5 or 10, the lost sales' cost b could be 50 or 100.

For the simulation model, we consider that the replication length is 10 000 units of time. This number is enough to get a representative result. After getting the two results, a comparative analysis of the obtained numerical results with simulation results is made to identify the accuracy between the two models. We consider that the gap % is the percentage of deviation between the analytical result given by our model TCa and simulation results TCs. The accuracy of the total cost is evaluated using the Eq. 11:

$$\text{gap\%} = 100 * \frac{TCs - TCa}{TCs} \tag{11}$$

Our model is exact when the lifetime follows an exponential distribution (alpha = 3 and beta = 1). The gap indicates that the optimal solutions obtained by both methods (MATLAB and Arena) are the same. The Table 1 presents the results of the proposed model and the simulation one for the 24 instances in the case of an exponential distribution.

Table 1. Comparison of the proposed model with the (T, S) simulation model – case of the exponential distribution

Instance	Cost Parameters					Proposed Model			Simulation model		
						T*	S*	Tca	T*sim	S*sim	TCs
	K	c	h	w	b						
1	5	5	1	5	50	0.5	10	62.503	0.5	10	62.503
2	5	5	1	10	50	0.5	9	70.624	0.5	9	70.624
3	5	10	1	5	50	0.5	9	94.516	0.5	9	94.516
4	5	5	1	10	100	0.5	11	76.440	0.5	11	76.440
5	5	5	1	5	100	0.5	11	66.781	0.5	11	66.781
6	5	10	1	10	100	0.5	10	109.981	0.5	10	109.981
7	10	5	1	5	50	1	13	69.873	1	13	69.873
8	10	5	1	10	50	1	12	79.896	1	12	79.896
9	10	10	1	5	50	1	12	103.602	1	12	103.602
10	10	5	1	10	100	0.5	11	86.440	0.5	11	86.440
11	10	5	1	5	100	1	15	75.258	1	15	75.258
12	10	10	1	10	100	0.5	10	119.981	0.5	10	119.981
13	50	5	1	5	50	2	20	100.783	2	20	100.783
14	50	5	1	10	50	1.5	15	113.058	1.5	15	113.058
15	50	10	1	5	50	1.5	15	136.401	1.5	15	136.401
16	50	5	1	10	100	1.5	18	122.773	1.5	18	122.773
17	50	5	1	5	100	1.5	19	107.783	1.5	19	107.783
18	50	10	1	10	100	1.5	17	160.801	1.5	17	160.801
19	100	5	1	5	50	2.5	24	124.263	2.5	24	124.263
20	100	5	1	10	50	2	19	139.841	2	19	139.841
21	100	10	1	5	50	2.5	21	162.645	2.5	21	162.645
22	100	5	1	10	100	2	23	152.050	2	23	152.050
23	100	5	1	5	100	2	24	134.132	2	24	134.132
24	100	10	1	10	100	2	21	192.314	2	21	192.314

Hence, we can conclude that our model is accurate, and guarantees global optimality in the case of an exponential distribution.

For the other cases, the mean gap between the simulation and the MATLAB result is 7%. Our obtained numerical results also indicate that in some conditions ($T \leq 1.5$, $S \leq 13$), the gap is equal to zero. Still, in some other situations, the analytical solution obtained by MATLAB is very close to that of the optimal solution obtained by Arena. For example, in the case of $T \leq 2$, the mean of gaps is 1%. The root cause of this gap can be explained by the approximation that we have done in our model. In reality, the remaining lifetime of articles is not the same. Our model doesn't consider the lifetime spent in the system, which represents the inventory age and the remaining lifetime. If $T \geq 2$, the risk of having different inventory age of product is even higher. To conclude, the smaller T is, the more the risk of having a different inventory age decrease. For the overall items, this risk can be reduced by having T < 2.

4.2 Sensitivity Analysis

In this section, we conduct a sensitivity analysis to evaluate the effect of the lifetime variability and the cost parameters on the optimal stock level and the optimal total cost.

We start by analyzing the impact of the lifetime variability, which is presented by a ratio CV. CV is defined as the ratio between the standard deviation and the expected value of the lifetime

$$CV = \frac{\sqrt{Alpha * Beta^2}}{Alpha * Beta} \tag{12}$$

Impact of the CV

We observe that the optimal cost increases with increasing lifetime variability. For the optimal stock level and the optimal period, we observe that the sensitivity level to CV is the highest when CV is above 0.007. They are stable when CV is between 0.01 and 0.06. For a CV value higher than 0.07, S* and T* decrease with CV. This can be explained that the higher of CV value is, the more the risk of perishability of the products increases, and the shorter the article's lifetime becomes, as shown in Fig. 1.

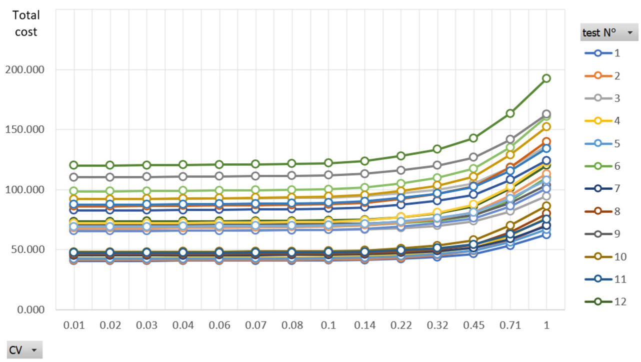

Fig. 1. Sensitivity analysis

Impact of Different Parameters

The main objective here is to evaluate the effect of the variation of the different parameters on the cost performance of our model. Firstly, considering the outdating cost, we have concluded that the higher the expiration/outdating cost is, the higher the total cost is. For the ordering cost K, it's obvious that the total cost increases with K, although the gap between the calculated total cost and simulation decreases. Next, we analyze the evolution of the total cost compared to the lost sales cost. As expected, the accuracy of the gap and the optimal cost increases with the lost-sales. This can be explained by the fact that to reduce the number of lost sales, the decision-makers have to buy more items. This will lead to more perished products and higher total cost. Finally, we analyze the behavior of the total cost with the purchasing cost. We conclude that the gap decreases when C increases, but the total cost behaves the same.

By using this model, the management team of an industrial perishable products unit can define the period and the inventory level that lead to the optimal cost based on the value of the different cost parameters. They can also choose the supplier using the impact of the ordering cost K in the total cost. The analysis of the effect of the different parameters variations to the total cost shows that the decision should be made after an analysis phase of all the cases that could appear (cost variations, lifetime variations, environment variations…). The use of this model can help decision makers to get the optimal solution.

5 Conclusion

In this paper, we consider a (T, S) perishable inventory system with lost sales. The lifetime of the product follows a general life distribution. Using the steady-state probabilities, we first derived the different analytical expressions of the total cost components. Next, we develop a MATLAB program that allows us to define the total optimal operating cost in a well-defined case. Finally, we conduct a numerical study to compare our analytical results with the simulation results. This allow us to conclude that our model is quite close to the real model with a mean accuracy gap of 7%.

This model could be extended considering (S,s) policy or a (T,S) multi-echelon model since in many sectors, the supply chain are more integrated, especially after the appearance of the industry 4.0. It is more likely to find inventory management systems that deal with perishable products in a multi-echelon chain rather than in a mono-echelon model. So, extending this work to periodic multi-echelon model would be a very interesting direction for future research.

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