Multi-period Multi-sourcing Supply Planning with Stochastic Lead-Times, Quantity-Dependent Pricing, and Delivery Flexibility Costs

Belgacem Bettayeb[1(✉)], Oussama Ben-Ammar[2], and Alexandre Dolgui[3]

[1] LINEACT CESI, EA 7527, CESI Lille, 8 boulevard Louis XIV, 59046 Lille, France
bbettayeb@cesi.fr

[2] Mines Saint-Etienne, University of Clermont Auvergne, CNRS UMR 6158 LIMOS, Centre CMP, Departement SFL, 13541 Gardanne, France
oussama.ben-ammar@emse.fr

[3] IMT Atlantique, LS2N, UMR-CNRS 6004, La Chantrerie, 4 rue Alfred Kastler, 44300 Nantes, France
alaxandre.dolgui@imt-atlantique.fr

Abstract. This work studies the problem of multi-period multi-sourcing supply planning with stochastic lead-times, quantity-dependent pricing, and delivery flexibility costs. We present a problem formulation that takes also into account holding and backlog costs and finite capacities of suppliers. The objective is to minimize the expected total cost while respecting suppliers' capacity constraints and satisfying customer demand. In this paper, the proposed stochastic integer linear program is detailed and the first results of experiments are presented and discussed.

Keywords: Supply planning · Multi-sourcing · Stochastic lead-times · Digressive pricing · Delivery flexibility

1 Introduction and Literature Review

In order to ensure their competitiveness, while guaranteeing a high service level to their customers, industrial companies need to optimize not only their production processes, but also other related downstream and upstream processes like replenishment, inventory, and transportation. Therefore, it is necessary to coordinate material, information and financial flows in an integrated manner to guarantee a competitive and profitable supply chain (SC) for all stakeholders [1]. However, because of its interdependent network structure, any incident or disturbance occurring in one of the elements of the chain can propagate and amplify, creating more effects which degrade the performance of the whole SC. In fact, incidents and disturbances are inherent in such a complex systems and

Published by Springer Nature Switzerland AG 2021
A. Dolgui et al. (Eds.): APMS 2021, IFIP AICT 630, pp. 511–518, 2021.
https://doi.org/10.1007/978-3-030-85874-2_54

are due to uncertainty, even the ignorance, of one or more influencing parameters of the system and the absence of countermeasures to predict and prevent them.

Controlling uncertainty and reducing its effects on Supply Chain performance has become, for several decades, a major concern for decision-makers and research communities in Supply Chain Management (SCM) [2,3]. This concern is reinforced by the recurrent observation of the vulnerability of SC to the disturbances generated by the uncertainty on some SC parameters [4,5]. Various sources of uncertainty have been identified and studied through SC risks analysis and many of them have been formalized and integrated into supply planning and inventory control models [6].

To reduce the effect of uncertainty in SC, several studies introduced safety stocks [7]. However, safety stocks are not all the time efficient and advantageous when the uncertainty is on lead times [8–10]. Other approaches based on stochastic optimization techniques have also been proposed in supply planning and inventory control [11–13]. The most of these models consider a one-period supply planning or multi-period supply planning with a constant demand, and independent and identically distributed lead-times. These models have some limitations from the perspective of optimization and economies of scale because they ignore and/or neglect the effect of dynamic parameters such as demand and capacity. However, multi-period supply planning with stochastic lead-times represents the difficulty to deal with order crossovers and the randomness characterising the quantities received within periods [8]. The problem of crossover is generally avoided by either ignoring them or building models that dissipate its effect or prevent it [14–16]. Another common practice to deal with uncertainty in SCM is multi-sourcing, which has the advantages of working with several competing suppliers. However, SC managers need to define adequate strategies to select and manage several suppliers. Concerning suppliers selection, several attributes are usually used, such as quality, price, delivery performance, etc. Although no unanimous ranking of the importance of these attributes exists, delivery performance is always identified as one of the three most important [17]. Another study in [18] indicates flexibility attribute as the most important overall, followed by cost and delivery performance.

The first suppliers selection techniques proposed in the literature are mostly based on mono- or multi-objective functions that are optimised within a static environment, where the decisions are made for a strategic level horizon. Since few decades, dynamic supplier selection (DSS) problems have emerged and different models have been proposed while considering a dynamic environment where one or several parameters vary over time, such as demand, capacity, prices, etc. The majority of DSS approaches seek to minimize an average total cost while finding the order quantities for selected suppliers [19,20]. For the case of stochastic lead-time with multiple suppliers, [21] developed a mathematical model of a single-item continuous review (s, Q) inventory policy. As in our model, orders replenishment can be split among several suppliers. The objective is to optimize the inventory policy parameters, namely the reorder level and quantity ordered to each supplier, while minimizing the expected total cost per time unit. [22] proposed a two-phase framework for supplier selection and order allocation with different possible transportation alternatives (TAs) per supplier. The proposed optimization model allocates a set of optimal order quantities to the selected suppliers for each time period in the planning horizon. Recently, [11] propose a mathematical formulation for the problem of dynamic supplier selection strategy in multi-period supply planning under stochastic lead-times. The authors propose a stochastic integer non-linear program (SINLP) aiming to optimize suppliers selection and planned lead-times while minimising the expected total cost.

The aim of this work is to study the problem of multi-period replenishment with multiple suppliers under stochastic lead-times and to study the effects of suppliers capacity limit, digressive pricing policy and delivery flexibility cost.

The structure of the remainder of this paper is as follows. Section 2, contains the description of the stochastic integer linear programming formulation of the problem. Then, we report and discuss the first experimental results in Sect. 3. Finally, principle conclusions from this work and future research directions are stated in the concluding section.

2 Problem Formulation

We consider the problem of multi-period replenishment planning of a system of single-product, single-buyer, and multiple-vendors. The demand of each period is known and can be ordered from one or several suppliers, each having a stochastic discrete lead-time defined by its probability mass function. Each supplier is also characterised by its capacity limit for each period and its own digressive pricing policy. The latter is applied to the whole quantity ordered over the planning horizon. We suppose that each period's not satisfied quantity is back-ordered and the equivalent backlogging cost is incurred to the buyer. The latter covers also the inventory holding cost. Note that backlog and inventory quantities are stochastic because of the randomness of suppliers lead-times and that we have no restrictive assumptions concerning orders' crossover nor the structure of the demand over the planning horizon. We consider the case where each demand can be split into small batches over different suppliers and/or periods (splitting) and that suppliers release separately the deliveries of different batches ordered at the same period via a supplementary cost for each batch (delivery flexibility cost).

For this problem formulation, we use the following notations for input data and decision variables:

- $\mathcal{T}$ ordered set of time periods indices of the planning horizon
- $\mathcal{S}$ ordered set of suppliers indices
- $\mathcal{I}_s$ ordered set of indices of quantity intervals defining supplier s pricing policy
- D_t demand of period t
- C_{st} capacity limit of supplier s at period t
- $[l_{si}, u_{si}]$ lower and upper limits of the i-th quantity interval of supplier s pricing policy
- c_{si} unit selling price of the i-th quantity interval of supplier s pricing policy

c_s^o ordering cost of supplier s
c^h unit inventory holding cost per time period
c^b unit backlogging cost per time period
$[L_s^-, L_s^+]$ range of possible discrete lead-time values of supplier s
$L_{s\tau t}^\omega$ actual lead time, in scenario ω, of the quantity released by supplier s at period τ to satisfy demand of period t
$F_s(.)$ cumulative distribution function of supplier s lead-time
$Q_{s\tau t}$ **integer decision variable** that gives the quantity to be ordered from supplier s at period τ to satisfy demand of period t
K_{si} **integer decision variable** that gives the total quantity to order from supplier s within the i-th interval of its pricing policy
Y_{si} **binary decision variable** indicating if the total ordered quantity from supplier s is within the i-th interval of its pricing policy
$Z_{s\tau t}$ **binary decision variable** indicating a non-zero quantity is ordered from supplier s at period τ to satisfy demand t

Before giving the problem formulation as a stochastic integer linear program (SILP) model integrating the uncertainty of lead-times and the notion of flexibility, let firstly introduce the following definitions.

Definition 1. *For all $s \in \mathcal{S}$ and $t, \tau, i \in \mathcal{T}$, let $\mathcal{M}_t$ be the set of indices of all ordered quantities $Q_{s\tau i}$ that can be involved in the calculation of the backlogging level at period t. It is defined as follows:*

$$\mathcal{M}_t = \{(s,\tau,i) \in \mathcal{S}\times\mathcal{T}^2 : t-L_s^+ +1 \leq \tau \leq t-L_s^- \text{ and } \tau+L_s^- \leq i \leq \tau+L_s^+\} \quad (1)$$

Corollary 1. *If the planned lead time of each supplier s is between L_-^s and L_+^s, the cardinality of $\mathcal{M}_t$ is equal to $\sum_{s\in\mathcal{S}}(L_+^s - L_-^s + 1) \times (L_+^s - L_-^s)$.*

Definition 2. *Let $\alpha_{s\tau i}^\omega = \mathbb{1}_{\{L_{s\tau i}^\omega \leq t-\tau\}} : (s,\tau,i) \in \mathcal{M}_t$ be a boolean variable that indicates for a given scenario ω if the quantity ordered from supplier s at period τ to satisfy the demand of period i arrives before period t:*

$$\alpha_{s\tau i}^\omega = \begin{cases} 1 & \text{if } \tau + L_{s\tau i}^\omega \leq t, \text{ with probability } F_s(t-\tau) \\ 0 & \text{if } \tau + L_{s\tau i}^\omega > t, \text{ with probability } 1 - F_s(t-\tau) \end{cases} \quad (2)$$

As $\alpha_{s\tau i}^\omega$ is binary for each triplet (s,τ,i), the number of possible scenarios is equal to $|\Omega_t| = 2^{|\mathcal{M}_t|}$. A given scenario ω is composed of a set of $\alpha_{s\tau i}^\omega$ for all $(s,\tau,i) \in \mathcal{M}_t$. This allows to define the set of all possible aggregated scenarios as follows:

$$\Omega_t = \left\{(\alpha_{s\tau i}^\omega)_{(s,\tau,i)\in\mathcal{M}_t} : w \in \{1,\ldots,2^{|\mathcal{M}_t|}\}\right\} \quad (3)$$

Each scenario $\omega \in \Omega_t$ has the probability of occurrence p_t^w defined in Eq. (4) below:

$$p_t^\omega = \prod_{(s,\tau,i)\in\mathcal{M}_t} (\alpha_{s\tau i}^\omega \times F_s(t-\tau) + (1-\alpha_{s\tau i}^\omega) \times (1 - F_s(t-\tau))) \qquad \forall\omega \in \Omega_t \quad (4)$$

where $\alpha_{s\tau i}^\omega \in \{0,1\}$ and $\sum_{\omega\in\Omega_t} p_t^\omega = 1$.

In the proposed model formulation, it is assumed that each demand can be split into several quantities that are ordered from different suppliers and/or at different periods. Delivery flexibility is also allowed via an additional cost, i.e. ordered batches from each supplier at a given period are released separately and have independent lead-times occurrences. This strategy can be formulated as the SILP given in Eqs. (5)–(16).

$$\text{SILP: } \min \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} p_t^{\omega}.\left(c^h I_{t\omega}^{+} + c^b I_{t\omega}^{-}\right) + \sum_{s\in\mathcal{S}}\Big(\sum_{j\in\mathcal{I}_s} c_{sj}.K_{sj} + \sum_{t\in\mathcal{T}}\sum_{\tau\in\mathcal{T}} c_s^o Z_{s\tau t}\Big) \quad (5)$$

s.t.

$$I_{t\omega}^{+} - I_{t\omega}^{-} = \sum_{s\in\mathcal{S}} \sum_{\tau=1}^{t-L_s^{+}} \sum_{i=\tau+L_s^{-}}^{\tau+L_s^{+}} Q_{s\tau i} + \sum_{(s,\tau,i)\in\mathcal{M}_t} \alpha_{s\tau i}^{\omega} Q_{s\tau i} - \sum_{\tau=1}^{t} D_\tau \qquad \forall t\in\mathcal{T}, \forall\omega\in\Omega_t \quad (6)$$

$$Q_{s\tau t} \leq D_t \qquad \forall s\in\mathcal{S}, \forall t,\tau\in\mathcal{T} \quad (7)$$

$$\sum_{t\in\mathcal{T}} Q_{s\tau t} \leq C_{s\tau} \qquad \forall s\in\mathcal{S}, \forall\tau\in\mathcal{T} \quad (8)$$

$$\sum_{s\in\mathcal{S}} \sum_{\tau=t-L_s^{+}}^{t-L_s^{-}} Q_{s\tau t} = D_t \qquad \forall t\in\mathcal{T} \quad (9)$$

$$\sum_{j\in Z_s} Y_{sj} \leq 1 \qquad \forall s\in\mathcal{S} \quad (10)$$

$$l_{sj}Y_{sj} - K_{sj} \leq 0 \qquad \forall s\in\mathcal{S}, \forall j\in\mathcal{I}_s \quad (11)$$

$$K_{sj} - u_{sj}Y_{sj} \leq 0 \qquad \forall s\in\mathcal{S}, \forall j\in\mathcal{I}_s \quad (12)$$

$$\sum_{j\in\mathcal{I}_s} K_{sj} - \sum_{t\in\mathcal{T}}\sum_{\tau\in\mathcal{T}} Q_{s\tau t} = 0 \qquad \forall s\in\mathcal{S} \quad (13)$$

$$\sum_{i\in\mathcal{T}} D_i.Z_{s\tau t} - Q_{s\tau t} \geq 0 \qquad \forall s\in\mathcal{S}, \forall t,\tau\in\mathcal{T} \quad (14)$$

$$Y_{sj}, Z_{s\tau t} \in \{0,1\} \qquad \forall s\in\mathcal{S}, \forall j\in\mathcal{I}_s, \forall t,\tau\in\mathcal{T} \quad (15)$$

$$I_{t\omega}^{-}, I_{t\omega}^{+}, K_{sj}, Q_{s\tau t} \in \mathbb{N} \qquad \forall s\in\mathcal{S}, \forall j\in\mathcal{I}_s, \forall t,\tau\in\mathcal{T} \quad (16)$$

In the SILP model described by Eqs. (5)–(16), we consider all possible aggregated scenarios (see Definition 2) and minimize the Expected Total Cost (ETC) that is composed of inventory, backlogging and purchasing costs, while determining which proportion of a given D_t is ordered from a given supplier s at a given period τ. Purchasing costs are dependent on the selected suppliers and the number of orders and related quantities.

Constraints (6) express the inventory level $I_{t\omega}$ at the end of each period t for each scenario ω. Constraints (7) mean that each quantity ordered from supplier

s at period τ to satisfy the demand of period t is less than D_t. Constraints (9) force the sum of quantity ordered to satisfy the demand of period t to be equal to D_t. It also guarantees the satisfaction of all demands. Constraints (10) to (13) allow to select the pricing level to apply by each supplier, dependently on the total ordered quantities. Constraint (14) ensures that $Z_{s\tau t}$ is equal to 1 if $Q_{s\tau t}$ is non-zero. Constraints (15) and (16) define the domains of decision variables.

3 Numerical Example and Discussion

The SILP model of the problem has been coded in C++ and solved using IBM ILOG CPLEX solver. The numerical example presented here concerns a test instance with 10-period planning horizon, 5 non-zero demands (see Table 1), and 3 suppliers. Inventory cost parameters are $c_h = 10$ and $c_b = 15$. Suppliers have constant capacities, with $C_{1,t} = 60$, $C_{2,t} = 50$ and $C_{3,t} = 100$ for all $t = 1, \ldots, 10$. Suppliers are characterised by their lead-times probability distributions given in Table 2a and their pricing policy parameters given in Table 2b. Here, the total number of scenarios is equal to 1024 (see Corollary 1). The optimal solution of the numerical example is shown in Table 3, where the three last rows give the optimal quantities to order from each supplier at each time period. One can see that, even if the third supplier has the highest price (75) and ordering cost (1000), it is solicited for three orders ($Q_{3,4,5} = 70$, $Q_{3,4,6} = 100$ and $Q_{3,8,9} = 80$) which represent 62.5% of the total demand. This proves that the selling price as well as the ordering cost are not the only levers for choosing a supplier. However, buying exclusively from a single supplier does not seem to be the best strategy for lowering prices and protecting against uncertainties. The model that we propose makes it possible to find a good compromise between the various costs associated to inventory, purchasing and ordering.

Table 1. Vector of demands

Periods	5	6	7	8	9
Demand	180	100	30	10	80

Table 2. Characteristics and parameters of suppliers

(a) Lead-times probability distributions

s		l: lead-time values			
		1	2	3	4
1	$\mathbb{P}(L^s = l)$	0.24	0.76	-	-
2	$\mathbb{P}(L^s = l)$	-	0.53	0.16	0.31
3	$\mathbb{P}(L^s = l)$	0.95	0.05	-	-

(b) Pricing policies parameters

s	Pricing levels						Ordering
	Level 1			Level 2			Cost
	l_{s1}	u_{s1}	c_{s1}	l_{s2}	u_{s2}	c_{s2}	c_s^o
1	1	20	69	21	500	65	800
2	1	30	67	31	500	65	700
3	1	500	75	-	-	-	1000

Table 3. Solution of the numerical example using the *ILP−WFNS* model.

t	1	2	3	4	5	6	7	8	9	10
D_t	-	-	-	-	180	100	30	10	80	-
$\mathbb{E}(I_t^+)$	-	-	-	-	31.3	14.6	4.8	-	-	-
$\mathbb{E}(I_t^-)$	-	-	-	-	8.9	8.9	7.7	-	4.0	-
$(Q_{1t5},\ldots,Q_{1t9})$	-	-	-	$(60,\ldots)$	-	-	-	-	-	-
$(Q_{2t5},\ldots,Q_{2t9})$	-	-	$(50,\ldots)$	$(\ldots,30,10,.)$	-	-	-	-	-	-
$(Q_{3t5},\ldots,Q_{3t9})$	-	-	-	$(70,100,\ldots)$	-	-	-	$(\ldots,80)$	-	-

ETC* = 74041.5; CPU time = 14.3 s

4 Conclusion

In this preliminary work, we propose a stochastic integer linear program (SILP) that minimises the expected total cost for the problem of multi-period multi-sourcing supply planning with stochastic lead-times, quantity-dependent pricing, and delivery flexibility costs. The results show the effectiveness of using multi-supplier strategy to cope with uncertainty of lead times. They also prove the relevance of considering other aspects related to suppliers, such as capacity, ordering costs and pricing policy. This approach could help decision maker to optimize its ordering policy. This work will be continued to focus on improving the model and its resolving approach in order to be able to study large and real-life sized instances. In fact, the weakness of the current model is its exponentially increasing number of scenarios with the number of suppliers and their ranges of lead-times distributions.

References

1. Stadtler, H.: Supply chain management and advanced planning–basics, overview and challenges. Eur. J. Oper. Res. **163**(3), 575–588 (2005)
2. Flynn, B.B., Koufteros, X., Lu, G.: On theory in supply chain uncertainty and its implications for supply chain integration. J. Supply Chain Manage. **52**(3), 3–27 (2016)
3. Simangunsong, E., Hendry, L.C., Stevenson, M.: Supply-chain uncertainty: a review and theoretical foundation for future research. Int. J. Production Res. **50**(16), 4493–4523 (2012)
4. Snyder, L.V., Atan, Z., Peng, P., Rong, Y., Schmitt, A.J., Sinsoysal, B.: Or/ms models for supply chain disruptions: a review. IIE Trans. **48**(2), 89–109 (2016)
5. Kleindorfer, P.R., Saad, G.H.: Managing disruption risks in supply chains. Prod. Oper. Manage. **14**(1), 53–68 (2005)
6. Brahimi, N., Absi, N., Dauzère-Pérès, S., Nordli, A.: Single-item dynamic lot-sizing problems: an updated survey. Eur. J. Oper. Res. **263**(3), 838–863 (2017)
7. Gonçalves, J.N., Carvalho, M.S., Cortez, P.: Operations research models and methods for safety stock determination: a review. Oper. Res. Perspect. **7**, 100164 (2020)

8. Ben-Ammar, O., Bettayeb, B., Dolgui, A.: Optimization of multi-period supply planning under stochastic lead times and a dynamic demand. Int. J. Prod. Econ. **218**, 106–117 (2019)
9. He, X.J., Kim, J.G., Hayya, J.C.: The cost of lead-time variability: the case of the exponential distribution. Int. J. Prod. Econ. **97**(2), 130–142 (2005)
10. Van Kampen, T.J., Van Donk, D.P., Van Der Zee, D.J.: Safety stock or safety lead time: coping with unreliability in demand and supply. Int. J. Prod. Res. **48**(24), 7463–7481 (2010)
11. Ben-Ammar, O., Bettayeb, B., Dolgui, A.: Mathematical model for dynamic suppliers' selection strategy in multi-period supply planning with lead-times uncertainty. IFAC-PapersOnLine **52**(13), 1040–1044 (2019)
12. Esmaeili-Najafabadi, E., Azad, N., Pourmohammadi, H., Nezhad, M.S.F.: Risk-averse outsourcing strategy in the presence of demand and supply uncertainties. Comput. Ind. Eng. **151**, 106906 (2021)
13. Ben-Ammar, O., Bettayeb, B., Dolgui, A.: Integrated production planning and quality control for linear production systems under uncertainties of cycle time and finished product quality. Int. J. Prod. Res. **58**(4), 1144–1160 (2020)
14. Jiang, R., Guan, Y.: An o (n2)-time algorithm for the stochastic uncapacitated lot-sizing problem with random lead times. Oper. Res. Lett. **39**(1), 74–77 (2011)
15. Huang, K., Küçükyavuz, S.: On stochastic lot-sizing problems with random lead times. Oper. Res. Lett. **36**(3), 303–308 (2008)
16. Riezebos, J.: Inventory order crossovers. Int. J. Prod. Econ. **104**(2), 666–675 (2006)
17. Verma, R., Pullman, M.E.: An analysis of the supplier selection process. Omega **26**(6), 739–750 (1998)
18. Van der Rhee, B., Verma, R., Plaschka, G.: Understanding trade-offs in the supplier selection process: the role of flexibility, delivery, and value-added services/support. Int. J. Prod. Econ. **120**(1), 30–41 (2009)
19. Ware, N.R., Singh, S., Banwet, D.: A mixed-integer non-linear program to model dynamic supplier selection problem. Exp. Syst. Appl. **41**(2), 671–678 (2014)
20. Ahmad, M.T., Mondal, S.: Dynamic supplier selection model under two-echelon supply network. Exp. Syst. Appl. **65**, 255–270 (2016)
21. Abginehchi, S., Farahani, R.Z.: Modeling and analysis for determining optimal suppliers under stochastic lead times. Appl. Math. Model. **34**(5), 1311–1328 (2010)
22. Songhori, M.J., Tavana, M., Azadeh, A., Khakbaz, M.H.: A supplier selection and order allocation model with multiple transportation alternatives. Int. J. Adv. Manuf. Technol. **52**(1–4), 365–376 (2011)