Chapter 1
The Scope of This Book

Abstract This book presents a history of mathematics between 1607 and 1865 in that part of North America which is the present United States of America (excluding Alaska), and this first chapter begins with some discussion of the meanings which could be given to the title of the book. During much of the seventeenth century the number of European-background settlers was always small in comparison with the number of Native American peoples, and the struggles by the settlers and the Indigenous inhabitants to survive meant that any desire to study higher forms of mathematics, or to conduct research in mathematics, was virtually non-existent. It was difficult for them even to provide ways and means by which young children could learn the Hindu-Arabic methods of counting or calculating. Products of technology like paper, slate, and ink were not readily available, and very few mathematics-knowledgeable teachers were available. The situation improved during the period 1700–1865, but even during the first half of the nineteenth century most young children did not have ready access to mathematics textbooks. In this introductory chapter, issues associated with the education of Native American children, and of children of indentured European-background workers and African-American slaves are also considered. Toward the end of the chapter, six research questions are stated, and summaries of what will be investigated in the remaining eight chapters of the book are given.

Keywords *Abbaco* sequence for arithmetic • *Abbaco* tradition • Counting systems • Decimal systems of counting • Definitions of mathematics • Glendon Lean • Hindu-Arabic numeration system • History of mathematics • History of mathematics education • Indigenous counting systems • Jamestown

> History, as nearly no one seems to know, is not merely something to be read. And it does not refer merely, or even principally, to the past. On the contrary, the great force of history comes from the fact that we carry it within us, are unconsciously controlled by it in many ways, and history is literally present in all that we do.
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> (Baldwin, 1998, pp. 722–723)

What is Mathematics?

The title of this book, "*Toward Mathematics for All: Reinterpreting History of Mathematics in North America 1607–1865,*" demands comment. First, this will be *a* reinterpretation of history rather than "*the* history." We hold that there is no such thing as a unique history of any discipline for any period of time. Whoever writes a history writes it from a particular vantage point. Historians "see" different things

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when they write about the same events and eras from different vantage points. That draws attention to the idea that a *history* is not a unique set of events occurring during a particular period of time, but rather an account of a set of events and relationships between events as seen, constructed, and interpreted by the person, or persons, writing the history. In that sense, all historical accounts must be subjective.

What is true of history is also true of mathematics (Stedall, 2012). Richard Courant's and Herbert Robbins's (1941) book, *What is Mathematics?* has been the subject not only of much praise by scholars but also of muted criticism (see, for example, Blank, 2001). It seems to us that some authors have colonized the meanings of "mathematics" and "mathematicians" whenever they have discussed histories of mathematics. For them, the word "mathematician" should be applied only to academics teaching "high-level mathematics" in colleges or to persons conducting high-level research which involves mathematical analysis in research institutions (Stedall, 2012). Correspondingly, some would prefer to reserve the word "mathematics" for high-level studies and research carried out by "mathematicians."

David Zitarelli (2019), in his recent book *A History of Mathematics in the United States and Canada*, addressed the meaning of "mathematician" directly when he wrote:

> What is a mathematician? A modern mathematician, after all, would chafe at the notion of someone who did not produce one mathematical work labeled in such a way. . . . Up to the time of the watershed year 1876, a mathematician in America was someone sufficiently steeped in the subject to be able to teach advanced parts of the subject and, moreover, to apply these topics to related fields. . . . However, I claim that David Rittenhouse is a mathematician by today's standards, because he published papers on mainstream mathematics that were entirely new to him. Other figures defy this easy distinction, such as Isaac Greenwood; even though he presented his own approach to topics novel to American students at the time, they were not original, and so I label him a mathematical practitioner. Generally, the only four individuals (up to 1876) I call mathematicians are Rittenhouse, Nathaniel Bowditch, Robert Adrain, and Benjamin Peirce. All others were mathematical practitioners. (p. 55)

Later in his book, Zitarelli (2019) distinguished between mathematical "enthusiasts," mathematical "practitioners," and "mathematicians" (p. 118). For example, he described Benjamin Franklin as a "mathematical enthusiast of the first rank" (p. 78) and commented that although "Franklin and Jefferson may have contributed little directly to mathematics, they certainly appreciated the subject's importance and took pride in their ability to apply it" (p. 79). This book will attempt to present a history of mathematics in North America (excluding Canada) for the 258-year period 1607–1865, and not a "history of mathematicians." That latter task has been well tackled by others (see, e.g., Bell, 1945; Roberts, 2019).

Unlike some commentators (e.g., Kline, 1972; Parshall, 2003; Smith & Ginsburg, 1934), we will not restrict the meaning of the word "mathematics" to the findings of "research" carried out by "mathematicians." From our perspective, 2- to 3-year-old children learning to quantify a collection of objects are engaged in a form of important mathematics; so too are 11-year-old children learning to measure angles with a protractor; so are 16-year-olds as they reflect on what it means to prove when they first meet the traditional *reductio ad absurdum* proof that there is no rational number which, when squared, equals 2; so too are 18-year-olds struggling to cope with the intricacies of elementary differential calculus; so too are adults who have left school but are attempting to work out the implications for their family finances of a mortgage reduction from 4 percent to 3.5 percent. And, of course, so too was Andrew Wiles as he attempted, ultimately successfully, to prove Fermat's Last Theorem (Stedall, 2012). This book is concerned with the history of mathematics in North America, as seen from the democratized perspective just outlined.

Although we can agree with David Zitarelli's (2019) definition of a mathematician as "someone who contributed an original piece of mathematics" (p. 118), we wonder what the word "original" means in that context. We do *not* agree with those who would think that school students studying "mathematics," or subjects like "arithmetic," "algebra," "trigonometry," "geometry," or "calculus," are not engaged in mathematics. A person playing a piano may not be a musician, but that person is engaged in making music. A person studying history may not be a historian but is nevertheless engaged with history. A middle-school school student coming to recognize the truth of the associative property for the multiplication of rational numbers would not normally be regarded as a mathematician but *is* engaged with mathematics.

In this book a *mathematical* task will be regarded as one which requires the use of calculations, or algebra (including functions, graphs), or formal logical reasoning, or geometry, or trigonometry, or limits, or calculus, or anything else commonly recognized as being "mathematical." Furthermore, mathematics can be either "pure" or "applied." Applied mathematics is to be associated with tasks which are concerned with developing and using mathematics to pose, model, and solve, and also to extend and generalize real-world-related problems—like, for example, in business, or surveying, or navigation, or astronomy (including space exploration), or, at the present time, with information technology.

This book offers a history of mathematics from a vantage point which includes mathematics formally investigated by research mathematicians, by "applied mathematicians," and by persons in families, in schools, in colleges, and in society in general who are attempting to "mathematize" problems that they want to solve. Although we have enjoyed, and profited from, reading David Zitarelli's (2019) *A History of Mathematics in the United States and Canada*, we recognize that David's concept of mathematics is very different from ours.

Karen Hunger Parshall (2003) had this to say about the "historiographical" point of view on the history of mathematics that Morris Kline embraced

> From the historiographical point of view that Kline adopted in his study, mathematical results merited inclusion in the historical narrative provided they formed a weight-bearing link in that great chain of mathematical ideas that stretches across time from the present to the past. (Notice here the direction of time's arrow!) For Kline, the history of mathematics is the story of how *contemporary* mathematical theories evolved; it is a technically oriented, intellectual history of ideas. This sort of historiographical framework suggests historical questions such as "How did X use Y's mathematical work to advance theory Z?" and "How did A do B without knowing C?" Answers to these and other questions provide important insights into the development of mathematical theory; or, to put it another way, the historiographical perspective that generates these kinds of questions illuminates important aspects of the history of mathematics. But do other crucial facets of that history remain obscure from the viewpoint?
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> (Parshall, 2003, pp. 114–115)

We plead guilty to narrowing the meaning of the symbols "in North America 1607–1865" in our title so that the words have a different meaning from what they usually have. Of course, Canada is part of North America, but in this book we conveniently confine "North America" to all parts of the present mainland United States of America (except Alaska) and recognize that the extent of the territory described varied during the period 1607–1865. It will never refer to any parts of what are now called Canada, Alaska, or Mexico. It will often refer to the colonial settlements largely on the eastern coast of North America which were outside of Canada (with Florida being included after 1822), and to the present mainland states of the United States of America.

The date 1607 has been chosen because it denotes the year when the first permanent European settlement in "North America" began. The early settlers had left behind the houses, castles, churches, schools, universities, systems of administration, and other cultural artifacts of their homelands to take on the challenges they found in Jamestown (Ames, 1957). The year 1865 was a less obvious choice as an upper bound. For us, it represents a time when a new meaning was being given to the word "mathematics" in the United States of America. More on that will be discussed in Chapters 4 through 6 of this book. Here it suffices to notice that 1607–1865 is a 258-year span of time that has as its upper bound a year that marks the end of the Civil War. Even in 1865 only a small proportion of children in the United States of America were given the opportunity to study formal mathematics beyond counting and the four operations on Hindu-Arabic numerals. In one sense, "mathematics for all" was a long way from being achieved—but, in another sense, a pathway toward it was being established, and the methods being used to create it, and the identities of those who would create it, would indelibly affect not only direction but also the terrain over which that pathway would go.

From a historiographical perspective, the most important difference in the history which will be presented in this book from other histories of mathematics is that it is intended to throw light on the discontinuities and challenges faced by *all* who have walked, or are now walking, or who would soon begin to walk, on the "mathematics-for-all" pathway. From that perspective, this history is written from an *education* vantage point. We recognize, though, that the perspective on history that we offer is a *beginning*—much more will need to be done.

Mathematics Studied in North America in the Seventeenth Century

In May 1607, 104 English males (mostly men, but a few youths) arrived in North America to start a settlement. They decided to establish several forts, which they called "Jamestown," in what is now the State of Virginia. Jamestown was named after King James I of England, and "Virginia" after the company which financed the venture (Egloff & Woodward, 1992; Wecter, 1937). There had been numerous earlier, failed, attempts by Europeans to establish footholds in this New World—for example, at St Augustine in today's Florida in 1565, and the Roanoke Colony in today's North Carolina in 1585—but the 1607 event would result in the first *permanent* British settlement being established in North America (Morison, 1971; Price, 2003). During the seventeenth century not only did the Jamestown settlement survive, but other "colonies" were established along the east coast (Ames, 1957), for example—in New Hampshire, Massachusetts Bay, Connecticut, Rhode Island, New York, New Jersey, Delaware, Pennsylvania, Maryland, Virginia, North Carolina, South Carolina, and Georgia.

The total number of European-background people—including indentured servants—living in the colonies grew to about 250,000 by the beginning of the eighteenth century with "the women and children comprising at least two-thirds of the population" (*Ames, 1957, p. 6). During the seventeenth century the number of Native Americans fell but the number of black slaves brought from Africa steadily increased (Berlin, 1998; Blackburn, 1997; Dexter, 1887; Guasco, 2014; Wareing, 1985; Wells, 1975). Most European-background families were engaged in a struggle to survive (Ames, 1957; University of Michigan, 1967)—coping with the heavy demands of clearing the land, building, planting, harvesting, trading, performing household chores, defending territory and buildings, and establishing churches, businesses and legal and administrative structures (Eggleston, 1888). Locally-appointed councils created and interpreted the rules by which different communities operated. The Church was important in all of the colonies, and participation in its establishment and forms of worship was an important societal expectation. Although schools were established, and often supported by locally-arranged mandatory taxation, attendance at these schools was irregular because the labor of all but the youngest of the children was needed to assist in the struggle to survive. It became common for boys to go to school in winter, but not at other times. Nevertheless, it was true that some of the European-background settlers had attended high-class educational institutions in their homelands, before moving to North America, and they wanted their children to receive a higher education—and that explains why several “Latin” grammar schools, and higher-education colleges were established (Andrews, 1912; Cremin, 1970; Cubberley, 1920; Dauben & Parshall, 2014).

In 1642, the Massachusetts Bay Colony passed the first law in the New World requiring children to be taught to read and write. In 1647, Massachusetts passed another law requiring all towns of 50 families to have an elementary school and every town of 100 families to have a “Latin” school (Cremin, 1970; Cubberley, 1920). But passing laws to make attendance at school compulsory for children in a certain age-group, and making those laws effective were two different things, and it was many years before schools were attended regularly by all children in European-background families. In almost all cases, Native American children, children of indentured servants, and children of African American slaves were not welcomed in the “public” schools. The politics associated with the decisions which created these situations has been treated extensively elsewhere (see, e.g., Cremin, 1970; Smith, 1947), and is not a subject of attention in this book.

The summary in the above paragraphs suggests why most of the early settlers did not regard the formal study of mathematics as a sensible thing for themselves or for their children. Certainly, some families wanted their children to be well educated, and that motivated the establishment of higher-level education institutions. But these were more the result of settlers wanting to ensure that there was a reasonable local supply of medical doctors, lawyers and, especially, clergymen than of any serious appreciation of the value of higher education. As in Europe, the thinking was that any decent institution of higher learning should focus on the classics—definitely Latin, also some Greek, and perhaps a little Hebrew, should be part of the intended curriculum. Also, school learning was to complement the family and church so far as religious teaching was concerned. In all European-background communities, learning to read the Bible was regarded as extremely important. By contrast, mathematics beyond, perhaps, knowing how to count and measure in local situations was seen, by most, as largely irrelevant. Any idea of offering courses involving high-level mathematics, or conducting and reporting mathematics research, was rarely contemplated.

Eggleston (1888) summarized the position of education in the British colonies around 1700 in the following way:

> The schools were few and generally poor. Boys, when taught at all, learned to read, write and “cast accounts.” Girls were taught even less. Many of the children born when the colonies were new grew up unable to write their names. There were few books at first, and no newspapers until after 1700. There was little to occupy the mind except the Sunday sermon. (p. 95)

For most European-background settlers there was neither time nor opportunity to pursue formal studies of any of algebra, geometry, or applied subjects like surveying, or navigation.

Estimates of the number of Native Americans already living, in 1607, in those parts which would become known as the "British colonies" have varied greatly—from 1 to 5 million. Whatever the number was, it fell as the seventeenth century progressed as a result of the introduction of devastating European diseases and race wars. The number of European-background persons grew from 104 at Jamestown in 1607 to about 250 thousand in the colonies in 1700 (Marshall, 2001; United States Census Bureau, 2004). For much of the seventeenth century, if not all of it, the number of Native Americans exceeded the number of European-background persons.

Terminology

We are not concerned, specifically, to provide extensive details in relation to the settlement of Jamestown in 1607. Rather "Jamestown" and "1607" will be used symbolically, denoting, respectively, that part of North America which today is part of the mainland of United States (not including Canada, or Alaska, or Hawaii, etc.), and the time when permanent settlement of Europeans in the New World (of "North America") first occurred. In this chapter we will be especially interested in the "mathematics" in this New World—not only the mathematics brought to the New World by the settlers, but also the forms of mathematics known and used by Native Americans at that time.

Our definition of the term "mathematics" for this chapter is inclusive in the sense that we are giving equal weight to mathematics and mathematics education. By the term "mathematics" we will include all aspects related to quantification, or counting, of discrete sets of objects, and ways of facilitating such quantification. It will also include methods of locating objects, and reasoning in space, and all aspects related to measurement of quantities, as well as to words and methods by which related concepts are defined and related, and the reasoning which permits theorems to be provided and proved.

We defined mathematics in this inclusive way in an attempt to make clear what we are investigating in this chapter. In the first half of the seventeenth century European educational institutions were still coming to grips with groundbreaking new mathematical ideas being put forward by mathematicians like the Frenchmen François Viète (1540–1603) and René Descartes (1596–1650), the Dutchman, Simon Stevin (1554–1620), the Scot, John Napier (1550–1617), and the Englishman, James Harriot (Struik, 2012). But such developments were a long way from the minds of most of the settlers in Jamestown or of other European-background settlers in what would become the British colonies. What mattered most for them was getting enough food and clothes in order to survive with dignity, and to establish peaceful relationships with local Native Americans.

In this book some attention will be given to the "spatial," "time," and "measurement" aspects of mathematics—the history of the development of these concepts, and how there are important cultural differences, has provided an ongoing agenda for researchers (see, e.g., Harris, 1981, 1991; Núñez & Cooperrider, 2013). Paul Libois, the radical Belgian mathematician and mathematics educator, referred to different kinds of geometrical spaces—a Euclidean space (x, y, z), a Galilean space (x, y, z, t) and other spaces like (x, y, z, t, p, T), where t denotes time, p pressure and T temperature. According to Libois (1951). the space of Euclid "was obtained through abstraction starting from (essentially) the consideration of solid bodies, imagined independently from time, and fixed with respect to an immovable body (the Earth)," but the other spaces were obtained from abstraction derived from real-life "optical, electrical and magnetic phenomena" (quoted from De Bock and Vanpaemel's (2019) translation, p. 15), and for Libois this suggested an educational approach for mathematics starting with naïve observations of "real" physical objects and proceeding via paths which involved increasing levels of abstraction. In other words, mathematics was not only what was arrived at through abstraction but included the path toward abstraction. The distinction is important in the history of mathematics in North America between 1607 and 1865, as deep thinkers like Benjamin Franklin, Thomas Jefferson, and Abraham Lincoln—persons not always regarded as mathematicians—consciously attempted to create abstract systems from realities, and then to apply those abstract systems to solve problems which confronted them. That will be discussed further in Chapter 8 of this book.

Indigenous Counting Systems and the Coming of the Hindu-Arabic Numeration System

Despite Tobias Dantzig's (1930) assertion to the contrary, there is considerable evidence that all well-formed groups of people have developed ways of counting (Bishop, 1988; Owens, Lean, Paraide & Muke, 2018; Silverman, 2006). That was obviously true in North America in the seventeenth century. More than a century ago, W. C. Eels (1913) reported that his research had revealed 306 different number systems employed by North American Indians and, of those, 146 were essentially decimal (i.e., base 10), 106 were essentially quinary or quinary decimal (i.e., base 5), 35 were vigesimal (i.e., base 20) or quinary-vigesimal, 15 were quaternary (i.e., essentially base 4), 3 were ternary (base 3), and 1 was octonary (base 8). Eels admitted that some of his classifications could have been wrong "due to inadequate data" (p. 293*n*).

Glendon Angove Lean's (1992) research, carried out between 1970 and 1990 in Papua New Guinea and Oceania, uncovered over 800 different languages and over 800 different counting systems (Owens et al., 2018)—many of which were still being used in villages in 1990. Although many of the counting systems documented by Lean (1992) were of the decimal variety, those decimal systems—originating in most cases from the number of fingers and thumbs on two hands—had subtle differences. Both Eels and Lean recognized that different base 10 structures existed—for example, in one structure "16" might be thought of, and expressed as, 10 + 5 + 1, and in another as 10 + 2 + 2 + 2, etc.

Eels (1913) and Lean (1992) found that there were many systems which employed bases other than 10, and there were also some “body-count” systems (with no “base”). Often systems had bases related to fingers and toes. For example, counting fingers and toes probably gave rise to vigesimal systems, and were often found—although not always—among groups which did not normally wear moccasins or other forms of “shoe” which covered feet. An interesting case came from the now-extinct Yuki language in California, which had an octal system because the speakers counted using the spaces between their fingers rather than the fingers themselves (Ascher, 1992). In 1752 a former William and Mary College mathematics professor, the Reverend Hugh Jones, argued that a base 8 number system was superior to decimal systems for arithmetical computations. His 47-page manuscript on that theme, *The Reasons and Rules and Uses of Octave Computation or Natural Arithmetic,* is now held in the British Museum.

Lean (1992) found that none of the indigenous counting systems that he identified had a name for “zero.” Specific numerical and linguistic treatments of fractions were not found either (although indigenous languages always included expressions for sharing, or splitting, etc., which thereby enabled what might be regarded as fraction concepts to be identified and discussed).

During the period 1607–1865 there were large groups of Native Americans to be found in many regions within North America—the Iroquois (including the Mohawks, Senacas, Oneida, Onondago, Cayoga), the Navajo, the Apache, the Cheyenne, the Sioux (including the Lakota, Dakota, and Nakota), the Hopi, the Seminoles, and the Commanches, were just a few of these groups. Each had its own language and its own counting system. The groups’ counting systems helped them to keep track of what they owned and what they measured, and to provide answers to practical issues arising from how they lived. Most did not know, or care about, the counting systems of others (Eels, 1913). Worksheets colorfully summarizing the number systems of 68 different indigenous groups in North America can be found at http://www.native-languages.org/numbers.htm.

It would be unhelpful to provide further details, here, for indigenous counting systems—that is not the main theme we are addressing. Rather, it is important to note that when the European “settlers” arrived in Jamestown in 1607 they brought with them another counting system—one which had not been used by any of the Native American societies up to that time. That system was the Hindu-Arabic numeration system, with its numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, and its ingenious place-value system for representing numbers greater than 9. It also had well-developed ways of predicting how many objects were in sets of objects by applying standard algorithms for “addition, subtraction, multiplication and division.” This system had initially been developed in India during the first seven centuries of the Common Era (CE) and had then been adopted and utilized across Arab nations before it found its way into European nations during the period 900–1200 CE (Danna, 2019; Ifrah, 2000; Menninger, 1969; Smith & Karpinski, 1911; Wardley & White, 2003; Høyrup, 2014). Its power had been displayed as it transformed local, national, and international commerce.

Historical perspective suggests that the Hindu-Arabic system of numeration was the most transformative mathematical development of all time, and the rapidity of its spread across India, then across Arab nations, and then across European nations testified to the recognition, by merchants in many parts of the "Old World," that it was a key to wealth and success (Danna, 2019; Høyrup, 2014; Ifrah, 2000). But before 1607 it was unknown to the Native American peoples. The invaders spoke various strange languages, but the leaders of the various Native American communities had no reason to suspect that over the next several hundred years there would be as much pressure, and sometimes more pressure, placed on them to change from their traditional counting systems to this "new" Hindu-Arabic numeration system as there would be for them to change from the languages that they used in everyday conversations.

Who Used the Hindu-Arabic Numeration System in North America, 1607–1699?

During the period 1607–1620 the only persons in North America to use the Hindu-Arabic numeration system would have been the settlers at Jamestown. Although we do not know how many of the original settlers were able to use the system freely, we do know that as the seventeenth century progressed more and more Europeans who knew how to use the system crossed the Atlantic and settled at various points along the East coast of North America. We also know that in 1635 the Boston Latin School was established in New England, and New College—which would become Harvard College—was established at nearby Newtown(e) (now Cambridge) in 1636. Although the early European education institutions did not give special attention to arithmetic—their focus was on community living and, for older children, on Latin—they provided basic education in religion and reading complemented by writing and a small amount of arithmetic. The forms of education which were implemented differed markedly from those in today's schools because there was very little paper or ink available, there were rarely any textbooks, there were no written examinations, and most teachers lacked sound understandings of what they were expected to teach. At the former New College, which became Harvard College in 1639, students from well-to-do families prepared to become lawyers, medicos and, especially, clergymen, and throughout much of the day students were expected to converse in Latin (Morison, 1935)—although English tended to be used for instruction in mathematics (Zitarelli. 2019).

During the seventeenth century most of the children of European-background free settlers would have been expected to learn to read, write, and say numerals expressed as combinations of some of the Hindu-Arabic numerals 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 (Finegan, 1917; Kilpatrick, 1912). Of one thing we can be certain: any forms of mathematics studied in the early "schools" would not have been known by more than a handful of indigenous persons living in Native American communities.

The *Abbaco* Sequence for Arithmetic

The "intended curriculum" for most mathematical programs in North American schools during the period from 1607 through 1865 derived from what has been called the *abbaco* sequence (Ellerton & Clements, 2012). That sequence was a well-ordered set of topics associated with courses in business arithmetic which had been standardized in European reckoning schools. For many years it was accepted by scholars that the *abbaco* sequence was initially developed in India, then further developed in Arabic nations, and finally translated into European city states—largely through Leonardo of Pisa's (Fibonacci's) *Liber Abbaci*, which was written around 1200 CE (see, e.g., Smith & Karpinski, 1911; Yeldham, 1926, 1936). In recent years however, Jens Høyrup (2005) has shown that between 900 and 1200 CE there were features of the *abbaco* tradition, and also aspects of algebra, already to be found in parts of Western Europe, and especially in Spain.

The *abbaco* sequence began with "numeration tables" which provided summaries of the Hindu-Arabic numeration base 10, place-value system. It then moved on to algorithms for the four operations (addition, subtraction, multiplication, and division) on whole numbers (Yeldham, 1936). Then came elementary measurement (including units) in which the Hindu-Arabic numerals were used to indicate measurements of amounts of quantities. Part of this measurement section was concerned with a topic known as "reduction." Then followed loss and gain, ratio and proportion (called the "rules of three"), currency exchange, equation of payments, barter, interest (simple and compound), tare and tret(t), discount, and brokerage. At the most advanced level would come topics like vulgar (i.e., "common") fractions, commission, alligation (i.e., the arithmetic of mixing quantities), fellowship (i.e., the arithmetic of partnerships), false position, progressions, involution and evolution, permutations and combinations, and mensuration.

The *abbaco* sequence usually did not include formal study of any of algebra, Euclidean geometry, or trigonometry, and only a small proportion of students, almost all of them from well-to-do families, ever got to study those branches of mathematics—usually in "high-class" grammar schools and colleges. For most students, however, the emphasis was on learning rules and cases in the *abbaco* sequence and on applying those to problems which might arise in business contexts (Ellerton & Clements, 2012, 2014).

In the seventeenth and eighteenth centuries, pre-college students rarely owned a textbook, and only a small proportion of them proceeded to the more advanced *abbaco* topics. In fact, only a few of the teachers had ever studied the more advanced topics themselves. The method of instruction was almost always consistent with what has been called the "cyphering tradition," by which most male students aged

10 years or more, and about 20 percent of female students in that age bracket, prepared handwritten "cyphering books" (see Chapter 3 of this book).

Differences in the Opportunity to Learn *Abbaco* Arithmetic in North America, 1607–1865

Mathematicians have always been interested in identifying and documenting the careers of females who were exceptionally gifted in mathematics—probably because many members of society have long questioned the idea, sometimes put forward, that mathematics should be regarded as a quintessential male subject. There has been much written about the contention that females, considered as a group of people, are not as talented as males in mathematics and the physical sciences, but are more talented than males in language studies, needlework and sewing (see e.g., Cohen, 1993; Harris, 1997; Patterson, 2012). But, of course, there are categories of people other than those distinguished by gender which, historically, have been associated with lower levels of participation, or lack of participation, in higher mathematics. One can think of class (working class versus upper class, etc.), region (rural versus urban), race differences, and so on.

This book is concerned with the history of mathematics in North America between 1607 and 1865. It will be assumed that the word "mathematics" embraces all aspects related to measurement of quantities, as well as spoken words and written symbols and other aspects of the language by which physical objects and concepts were defined, quantified, related, and communicated during the period under consideration. We also include "higher mathematics"—the kinds of mathematics studied in the upper echelons of departments of mathematics in colleges and universities, and also the kinds of mathematics that researchers investigate—within the ambit of our discussion. We will argue that historians investigating the development of mathematics in North America between 1607 and 1865 need to recognize that many groups of people within North America have always been, and continue to be, severely disadvantaged with respect to the opportunities that they have been given to study mathematics, and especially higher forms of mathematics. By contrast, certain other groups have been advantaged.

We begin by creating 16 subdivisions based on a subdivision in time (two periods, one between 1607 and 1699, and the other between 1700 and 1865); four subdivisions based on race and servitude (European-background persons who were not indentured servants, European-background persons who were indentured servants, Native Americans, and African-American persons); and two subdivisions based on gender (male or female).

In summary, the framework draws attention to:

- *Two subdivisions based on time:* we distinguish between the amount of participation in mathematics in North America (a) between 1607 and 1699, and (b) between 1700 and 1865.

- *Four subdivisions based on different groups studying, or teaching, or researching mathematics in the following categories*: (a) European-background persons who were not indentured servants; (b) European-background persons who were indentured servants; (c) Native Americans; and (d) African-Americans.
- *Two subdivisions based on gender*: (a) male persons and (b) female persons.

These subdivisions can give rise to $2 \times 4 \times 2 = 16$ distinguishable groups—for example, one might consider "the 1607–1699 group of Native American males," or "the 1700–1865 group of African-American females."

The reader might wonder whether the number of European-background whites who were indentured servants was sufficiently large to warrant their being separated into a unique category. The answer is definitely "Yes" (Chessman, 1965). Economic historians and economists have reported data indicating that the number of indentured servants increased in all 13 colonies in the seventeenth century (Galenson, 1984). There are data indicating that between the years 1630 and 1776, one-half to two-thirds of Caucasian immigrants to the 13 colonies came as indentured servants (Ames, 1957; Smith, 1947; Whaples, 1995).

Of the 16 groups, only 6 had significant percentages of persons—i.e., significantly more than 0%—who received a formal education that took account of more than a very elementary level in the *abbaco* sequence. Our *estimated* percentages of students in the 6 groups who received such an education are shown between parentheses at the end of each line in the following list:

1. 1607–1699 European-background males who were not indentured servants (40%);
2. 1607–1699 European-background males who were indentured servants (10%);
3. 1700–1865 European-background males who were not indentured servants (70%);
4. 1700–1865 European-background males who were indentured servants (30%);
5. 1700–1865 European-background females who were not indentured servants. (20%);
6. 1700–1865 European-background females who were indentured servants. (10%)

We emphasize that the percentages shown merely represent our estimates—research has not been done which would reveal the actual percentages. It was only in rare circumstances that a Native American or an African American person had the opportunity to study *abbaco*-type arithmetic in common schools at any time between 1607 and 1865. That fact needs to be recognized in any evaluation of Kamens and Benavot's (1991) claim, a claim repeated by Jeremy Kilpatrick (2014), that in

U.S. common schools, arithmetic was made a compulsory school subject by 1790. The meaning of "compulsory" in that assertion is problematic. In fact, although Kamens and Benavot acknowledged that there was no U.S. national curriculum for common schools in 1790, their analysis assumed that this was "not a serious drawback" (p. 171). We disagree. For example, they do not take account of the fact that throughout the whole of the period 1607–1865 most boys who attended schools in rural districts, did so in winter months only; furthermore, attendance rates and intended curricula differed from state to state. We find Kamens and Benavot's (1991) analysis of curricula in U.S. common schools of the seventeenth and eighteenth centuries seriously lacking in specific detail and their main conclusions highly questionable.

We estimate that only about 20 percent of all white European-background males living in North America during the seventeenth century had ever studied, or would study, *abbaco* arithmetic beyond the most elementary level, and that during the eighteenth century the corresponding percentage was about 35. During the seventeenth century much less than 10 percent of European-background females living in North America would have studied *abbaco*-arithmetic beyond the most elementary level, and during the eighteenth century the percentage was never likely to have risen to above 20 (Ellerton & Clements, 2012, 2014).

The remarkable thing is that even in the 1790s certainly less than 10 percent and probably well less than 5 percent of those belonging to all the other 10 categories, had ever studied arithmetic beyond the most elementary *abbaco* level. The fierce inequalities of educational opportunity which might be associated with that statement have never been adequately addressed by researchers in education, history, or mathematics.

The 16-subgroup structure for analysis outlined in the above paragraphs offers a basis for a research agenda so far as the history of mathematics and mathematics education in North America is concerned. Consider, for a moment, what other categories might be added (e.g., rural versus urban, North versus South, English-speaking versus non-English-speaking, students doing apprenticeships versus students still at day-school, students living in big cities versus those living in remote frontier regions). One might reflect, too, on the extent to which the situations would differ if we were wishing to provide a basis for comparing the histories of mathematics and mathematics education in Great Britain, or France, or Spain, or Germany, or The Netherlands, or, more generally, in Western Europe.

An examination of the conjectures we have just made should make it clear that we contend that during the seventeenth century relatively few people living in the 13 colonies studied mathematics beyond *abbaco* arithmetic or other elementary forms of "Western" mathematics. That was largely because most had neither the opportunity nor the desire to do so. We do not know how many would have liked to study *abbaco*-type mathematics in the various groups but were not given the opportunity to do so, but it is likely that that number would have been small. What is interesting is our conjecture that after 1607 the situation improved—if that is the right word—only slightly over the next 200 years. That conjecture is consistent with the summary presented by David Eugene Smith and Jekuthiel Ginsburg (1934). And, incidentally, a similar situation prevailed in Great Britain with respect to the mathematics education of the young—Howson and Rogers (2014) have reported that in 1824 less than 50 percent of those attending British schools were taught arithmetic.

It is not surprising, then, that by 1865, in North America, there was a massive problem facing anyone who did not have a European background and wanted to study mathematics, at any level (Drake, 1963). In the state of Virginia, for instance, there were more African-American slaves and their children than there were European-background persons who were not indentured servants, and hardly any of the African Americans had attended school (Drake, 1963; Wareing, 1985). Research is needed which establishes benchmarks and progressions in learning so far as participation in mathematics of different racial groups in North America is concerned. Compared with what prevailed in France and Germany, for example, and contrary to a claim made by Kamens and Benavot (1991) and accepted by Kilpatrick (2014), we believe that in 1865 the United States, as a nation, had a lot of "catching up" to do, at all levels of mathematics education (Kline, 1972; Parshall, 2003; Smith & Ginsburg, 1934). If our conjectures are reasonably accurate then there is no way we would expect that by 1900 more than a tiny proportion of North American mathematicians children" would reach the same level of research quality in mathematics as that reached by European mathematicians. In 1865, and even in 1900, most girls, Native Americans, and African-Americans (and working-class children, etc.) had much less opportunity than "corresponding children of the same age in some Western European nations to advance in any mathematical studies (Vickers, 2008).

The Main Aims for This Book

This book offers an overview of a history of mathematics in the 13 colonies during the colonial period and in the United States of America during the period 1776–1865. Throughout the book the word "mathematics" will be taken to mean mainly the Hindu-Arabic *abbaco* sequence for arithmetic if we are referring to children (up to the age of 15 years). For students, between 15 and 18 years, it will refer to more advanced topics in the Hindu-Arabic *abbaco* sequence, to measurement, and sometimes to algebra, trigonometry, geometry, and sometimes (though rarely) to calculus. At the college and research levels, we will be referring to the mathematics studied or taught or researched by students and teachers.

In Chapters 5 through 8 we will argue, like Parshall (2003) and Smith and Ginsburg (1934), that internationally-recognized research in mathematics by North American scholars did not appear until the early 1800s, and that there was not a great deal of this before 1865. One of the issues considered in this book is why it took so long for an internationally-recognized mathematics research sub-culture to appear in North America.

We shall assume that the terms *intended* mathematics curriculum, *implemented* mathematics curriculum, and *attained* mathematics, as introduced by Ian Westbury (1980), are well defined. The "intended curriculum" corresponds to the sequence of mathematical topics, and approaches, which schools, textbook authors, local education authorities, and teachers expect students to learn for a well-defined period (like, for example, over a period of one year, or over a period of, say, four years). It also includes the idea of preferred teaching methods of the schools, textbook authors, and teachers for delivering the intended content. By contrast, the "implemented curriculum" will refer to the content actually taught, and to the ways it was taught. The "attained curriculum" will refer to what the students learned and retained about the content of the implemented curriculum.

The Six Research Questions

We now state the following six main questions which will be investigated in this book:

1. What were the intended, implemented and attained mathematics curricula for young children (aged less than 10 years) in North America (a) during the period 1607–1820? and (b) the period 1820–1865?, and to what extent do the answers to those questions vary across North America, and in different groups of children (e.g., boys versus girls, European-background children versus Native American children, and European-background children versus African-American children)?
2. What were the intended, implemented and attained mathematics curricula for North American children aged between 10 and 15 years during (a) the seventeenth century, and (b) the period 1700–1865, and to what extent do the answers to those questions vary across North America, and across different groups?
3. What were the intended, implemented and attained mathematics curricula for North American pre-college children aged between about 15 and 18 years during (a) the seventeenth century, and (b) the period 1700–1865, and to what extent do the answers to those questions vary across North America, and across different groups?
4. What were the intended, implemented and attained mathematics curricula for North American college students during (a) the period 1607–1776? and (b) the period 1776–1865, and to what extent do the answers to those questions vary across North American colleges, and across different groups?
5. What perspectives on the purposes and status of mathematics in college curricula were held in the North American colonies during the period 1607–1865?

6. What are the implications of the answers to the first five questions (above) for those investigating the history of "higher" mathematics in North America? What future research is needed, and to what extent will it be feasible to conduct that research?

Research mathematicians reading this book might be disappointed with those six questions because only one of them—the fifth—refers, albeit indirectly—to the history of mathematics research in North America. We have worked from the perspective that "mathematics" encompasses more much than merely research in mathematics or the teaching of higher-level mathematics in advanced colleges. We believe that for the period between 1607 and 1865 the history of mathematics in North America should be as much concerned with the history of the development of structures by which people of all ages were enabled to learn mathematics—that is to say, with the history of mathematics education—as with changes in the mathematics which was studied or researched in higher-education institutions. That is not to say that serious research in mathematics did not take place in North America during the period 1607–1865. Identifying that research is regarded as something within the scope of this book.

One might ask why anybody should write a book on the history of mathematics in North America between 1607 and 1865? What use could such a history possibly be for today's readers? Is this book nothing more than an academic exercise? Well, no, we hope that this book will be important for those who want to gain an insight into why mathematics came to be identified, by so many, with white, male privilege. The quotation from James Baldwin (1998)—after the abstract and keywords at the start of this chapter—is relevant to what we are trying to say through the pages of this book. Please read Baldwin's statement again, now, and also, read it once more after you've finished reading Chapter 9, the last chapter of this book.

The Concept of "School" in this Book

Before moving on it will be useful to define the concept of "school" as it was used in North America during the seventeenth and eighteenth centuries. The word "school" will be taken to include "academies," "apprenticeship schools," "common schools," "dame schools," "evening schools," "grammar schools," "local schools," "private schools," "public schools," "subscription schools," and "writing schools" (Clements & Ellerton, 2015; Cremin, 1970, 1977), as well as more specialized establishments like "dance schools," "elocution schools," and "navigation schools' and "French ladies' colleges." A narrower interpretation of the word "school" than what is implied by that collection of terms is also relevant—so that any formal education environment in which at least one "teacher" regularly met with at least one "student," at an agreed place, for the purpose of helping the student(s) to learn facts, concepts, and skills, from at least one of reading, writing, or arithmetic, will be regarded as having been a school (Ellerton & Clements, 2012). This definition implies that for the purposes of this book a school did not need to offer formal tuition in any form of mathematics.

Higher-level colleges—such as King's College (now called Columbia University), Harvard, William and Mary, and Yale—will *not* be regarded as "schools." During the seventeenth and eighteenth centuries, and also during the early nineteenth century, such higher-level institutions were usually called "colleges" and were sharply distinguished from "schools."

Outline of Chapters in this Book

There are nine chapters. In this first, introductory, chapter we have provided necessary definitions, and offered conjectures which were intended to define a research agenda for scholars already investigating, or intending to investigate, the history of mathematics in North America. We also put forward six research questions which will be addressed and answered.

Chapter 2 will offer a summary of the mathematics studied by young children (aged less than 10 years) in North America during the period 1607–1865, and Chapter 3 will offer a summary of intended, implemented and attained mathematics curricula in North America during the same period for children aged between 10 and 16 years. Chapter 4 will do likewise, only with respect to those who proceeded as far as more advanced *abbaco* arithmetic topics, or for those who studied elementary forms of algebra, trigonometry, geometry (and perhaps applied topics like navigation and surveying) at the pre-college level. With each of Chapters 2, 3, and 4, findings will be linked to the conjectures we made after we defined 16 categories of people earlier in this chapter. Chapter 5 will be concerned with the introduction and development of algebra in curricula after 1607, and Chapter 6 will focus on creative *applied* mathematics-related developments which occurred and were reported by education establishments. Chapter 7 will address issues associated with college mathematics during the period 1607–1865, and Chapter 8 will identify persons who developed distinctive ways of looking at, and using, mathematics during the same period.

In the final chapter (Chapter 9), tentative answers will be given to each of the six research questions. The statements of these tentative answers will lead directly to a consideration of questions which might fruitfully be addressed by future researchers, and of difficulties that those carrying out such future research might be expected to experience.

We think of this book as representing our final words to those who will carry out needed research in the future. We hope the book will be rich in the sense that it will pass on to readers what we have learned over the past 15 to 20 years as we have researched the history of North American mathematics and mathematics education. At times it has been an exhilarating experience for us, chasing rare references, artifacts, and documents, reflecting on what others have written, and reporting the conclusions that we have reached. Some might think it is unfortunate that so few scholars have contributed to the enterprise, but a more positive view is that the field "is ripe unto harvest."

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