Impact of Outlier-Adjusted Lee–Carter Model on the Valuation of Life Annuities

Cem Yavrum and A. Sevtap Selcuk-Kestel

Abstract Annuity pricing is critical to the insurance companies for their financial liabilities. Companies aim to adjust the prices using a forecasting model that fits best to their historical data, which may have outliers influencing the model. Environmental conditions and extraordinary events such as a weak health system, an outbreak of war, and occurrence of pandemics like Spanish flu or Covid-19 may cause outliers resulting in misevaluation of mortality rates. These outliers should be taken into account to preserve the financial strength and liability of the life insurance industry. In this study, we aim to determine if there is an impact of mortality jumps in annuity pricing. We question the annuity price fluctuations among different countries and two models on country characteristics. Moreover, we show the annuity pricing on a portfolio for a more comprehensive assessment. To achieve this, a simulated diverse portfolio is created for the prices of four types of life annuities. Canada, Japan, and the United Kingdom as developed countries with high longevity risk, Russia and Bulgaria as emerging countries are considered. The results of this study prove the use of outlier-adjusted models for specific countries.

Keywords Mortality · Annuity pricing · Lee–Carter model · Outlier-adjusted

1 Introduction

Life expectancy has increased significantly over the last century due to improvements in medicine, technology, and awareness in health issues, especially for developed countries. For instance, from 1960 to 2010, the life expectancy at birth in Canada, Japan, and the United Kingdom (UK) are increased by 15%, 18%, and 13%, respec-

C. Yavrum · A. S. Selcuk-Kestel (✉)
Institute of Applied Mathematics, Actuarial Sciences, Middle East Technical University, Ankara, Turkey
e-mail: skestel@metu.edu.tr

C. Yavrum
e-mail: cyavrum@metu.edu.tr

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tively, whereas, in Bulgaria, this change is around 8.9% (Human Mortality Database 2021). This variation is remarkably low in Russia whose life expectancy at birth shows an increase of only 0.3% (from 68.70 to 68.92) (Human Mortality Database 2021). These differences between countries can be explained by the causes such as environmental conditions, economic crisis, low incomes, the weak healthcare system and some extraordinary events that are outbreak of war, occurrence of pandemics, and radical changes in economics or politics (Chang et al. 2016). As a consequence, mortality rates which are the main component of pricing annuities, life insurance, retirement payments may result in misleading predictions and evaluations to set up a decision-making, like critical policy decisions on the retirement and insurance systems. The insurance premiums and annuities, one of the most significant income items of the financial sector, are calculated using the estimated future mortality rates. Therefore, using future mortality estimates generated by the model that does not include all of these factors can impact the financial strength of the life insurance industry and the stability of the pension system of a country.

In addition to its variation by time and age, mortality trends may have outliers (jumps) at specific time dimensions and age points. Mortality jumps are rare, but their presence could alter the long-term mortality trends by triggering many unexpected deaths, thereby affecting future estimates. As Stracke and Heinen (2021) state, additional claims received from unexpected pandemic would cost nearly €5 billion (50% of the market's total annual gross profit) for the German insurance market, as not even being the worst scenario. Another example, the earthquake and tsunami that occurred in southern Asia in 2004 made nearly 130,000 people missing and killed 180,000 (Carpenter 2005). If such an event would occur in a more economically developed area, the life insurance industry would deal with handling catastrophic losses (Carpenter 2005).

In light of all these significant indicators, companies should model the dynamics of mortality over time and to achieve this, a number of stochastic mortality models are introduced in the literature. These models are divided into two of which the first one is paying attention to the force of mortality using continuous time processes, and the second one concentrates mortality rates directly by discrete-time processes. While some continuous time models are pointed out by Biffis (2005) and Cairns and Dowd (2006), Lee and Carter (1992) developed the most famous one originated by a new method for extrapolation of age patterns and trends in mortality. Despite some drawbacks, its simplicity and quick-applicability make the Lee–Carter model to be utilized by some census bureaus, such as Census Bureau of the United States (Hollmann et al. 2000).

Regardless of the mortality estimation method, the presence of mortality jumps could lead to influences both in the sample and partial autocorrelation functions, which may cause erroneous predictions in the model (Li and Chan 2007). The use of the inadequate model affects pricing and reserve allocation for life insurances and annuities. This situation poses big threats to the solvency and price competitiveness of life insurance companies. Hence, appropriate mortality forecasting models should be used to predict this situation (Brouhns et al. 2002). Lee–Carter model has been extended by Renshaw and Haberman (2003) and Brouhns et al. (2002), where

Renshaw and Haberman describe a method for modeling reduction factors using regression methods within the generalized linear modeling framework, Brouhns et al. use Poisson regression model to forecast age–sex-specific mortality rates. However, none of these approaches include possible random outliers in the mortality data. Lin and Cox (2008), Cox et al. (2006), Chen and Cox (2009) improve their models to allocate outlier locations using a discrete-time Markov chain, Poisson distribution and independent Bernoulli distribution, respectively. Li and Chan (2005) use an approach to take into account possible outliers based on the Lee–Carter model. They create outlier-adjusted time series used with Lee–Carter model by using the iteration process developed by Chen and Liu (1993) to determine the locations of outliers and adjust their effects. Chang et al. (2016) develop the iteration process to incorporate outlier effects. After that, Chen and Liu (1993) improve their process for the joint estimation of model parameters and outlier effects. The securitization of insurance industry is critical as its strength reflects the economic stability of the country. The Global Economy (TheGlobalEconomy.com 2021) indicates that the average of insurance company assets including annuities and pension plans in the world as part of GDP is 16.48% in 2016. It can be easily said that a decrease in this number would have an impact on country's economy on its own. For this reason, the mortality jumps influencing the prices of annuities and pension liabilities also become a consideration of economic robustness.

There are studies that include the above-mentioned approaches to determine the effect of models incorporate with mortality jumps on securitization (Chen and Cox 2009; Liu and Li 2015). However, the effect of mortality jumps on the prices of life annuities and their liability are not covered depth in the literature. Due to the influence of sudden jumps in mortality, we aim to capture the outliers and we employ Outlier-Adjusted Lee–Carter model (Chen and Liu 1993; Li and Chan 2007) to the mortality rates of selected countries whose history exposes epidemics, wars, civil turbulences. To depict the influence of economic welfare, we select Canada, the UK, Japan as developed and old-age-dominated populations, whereas Bulgaria and Russia as emerging countries. To investigate the impact of mortality jumps on annuity price fluctuations, we make comparisons based on two approaches: (i) Outlier-Adjusted Lee–Carter (OALC) model and (ii) Lee–Carter (LC) model. We expect these evaluations illustrate the striking impact of mortality jumps on insurance pricing. The findings illuminate that the mortality jump model should be considered, especially for the countries that have disadvantages in the field of migration, economy, individual incomes, healthcare system, and/or underwent many wars and diseases in its past.

The chapter is organized as follows. Section 2 shows the methodology of the models and performance criteria as well as the iteration cycle that needs to be applied during the process. Section 3 contains the implementation, results of the applied models and the difference of annuity prices between selected countries. Section 4 concludes the results with a brief discussion.

2 Outlier-Adjusted Lee–Carter Model

Lee–Carter model which defines long-term mortality forecasts based on a combination of standard time series and an approach to handle the age distribution of mortality, is given as

$$ln(m_{x,t}) = a_x + b_x\kappa_t + \epsilon_{x,t} \tag{1}$$

where $m_{x,t}$ is the age-specific central death rate for age x at time t, a_x stands for the age pattern of death rates, b_x is the age-specific reactions to the time-varying factor, κ_t indicates the mortality index in the year t, while $\epsilon_{x,t}$ is the error term that captures the age-specific influences not reflected in the model for age x and time t. It is a widely known fact that the model is overparameterized. To obtain a unique solution,

$$\sum_x b_x = 1 \quad \text{and} \quad \sum_t \kappa_t = 0 \tag{2}$$

whose implementation to the model enables the age pattern of death rates, a_x, becoming the average value of $ln(m_{x,t})$ over time. Two-stage estimation procedure obtains the unique solution. We then ensure that the number of deaths deduced from model and the actual number of deaths are equal to each other. Additionally, Box and Jenkin's approach is employed to generate an autoregressive integrated moving average (ARIMA) model for estimating the mortality index, κ_t.

2.1 Outlier Modeling and Adjustment

Lee–Carter model which is the base for implementation constitutes an outlier-adjusted model with the help of an iteration cycle. The outlier analysis consists of two issues at which the first one is the determination of the location of the outlier values that may exist in the mortality index and the second one is finding and adjusting the effects of outliers if any exists. For the first issue, the value of standardized statistics of outlier effects should be found in order to detect outliers (Chang et al. 2016). For the latter one, more complex approaches and processes should be applied to standardized statistics of outlier effects (Chen and Liu 1993). Furthermore, there are two types of problems encountered in the outlier detection and adjustment procedure (Chen and Liu 1993). These are (i) having an outlier in mortality data may cause an error in model selection, (ii) even the model is selected correctly, the effect of outliers can significantly influence the estimation of model parameters. Chen and Liu's approach partly solves the second problem, while the first one stays the same. Regardless of the above-mentioned shortcomings, we use this method in our analysis, as it is the newest one in the literature.

Let Z_t be an outlier-free time series following an ARIMA(p,d,q),

$$\phi(B)(1-B)^d Z_t = \theta(B)\alpha_t \tag{3}$$

where B is the backshift operator such that $B^s Z_t = Z_{t-s}$ and α_t represents white noise random variable with mean zero and constant variance σ^2.

Time series with outliers can be formed as outlier-free time series plus the effects of emergent outliers, $\Delta_t(T, w)$, where T and w are the location and the size of an outlier, respectively. Then, the series becomes,

$$Y_t = Z_t + \Delta_t(T, w) \tag{4}$$

Four types of outliers (Tsay 1988; Chen and Liu 1993; Li and Chan 2005) are Innovational Outlier (IO), Additive Outlier (AO), Temporary Change (TC), and Level Shift (LS). While an AO influences only a single observation, an IO affects all observations after T year with some decreasing pattern until the effect vanishes. This situation is slightly different for TC and LS. The effect of outlier remains the same influencing all observations after T year for LS and it decreases until to reach zero point by linearly for TC. It is stated commonly that a large portion of the outliers comprises of AO and IO (Chang et al. 2016). Since we focus on short time effects of mortality jumps that arise from extraordinary events affecting mortality rates for a short time, we consider these two intervention models, AO and IO, which are expressed as

$$\Delta_{t_{AO}}(T, w) = wD_t^T \tag{5}$$

$$\Delta_{t_{IO}}(T, w) = \frac{\theta(B)}{\phi(B)(1-B)^d} wD_t^T \tag{6}$$

respectively. Here, D_t^T is a binary variable having 1 in the presence of outliers at time T.

The outlier detection method (Chang et al. 2016) is grounded on the effects of outliers on the residuals of the model. The values of standardized statistics of outlier effects should be calculated to detect possible outliers. To achieve this, Z_t is expressed with its polynomial function of $\pi(B)$,

$$\pi(B) = \frac{\phi(B)(1-B)^d}{\theta(B)} = 1 - \pi_1 B_1 - \pi_2 B_2 - \cdots \tag{7}$$

where π_j weights of outliers found at location T, influencing the years after T so, $j \geq T$. While the distance between j and T increases, π_j becomes zero as the effect of an outlier at location T does not impact on distance mortality values.

Given

$$\pi(B)Z_t = \alpha_t, \tag{8}$$

the Eq. 4 becomes,

$$\hat{e}_t = \pi(B)Y_t \quad \text{for} \quad t = 1, 2, 3, \ldots \tag{9}$$

where $\hat{e}_t$ defines the residuals from the time series with outliers. In terms of considered outlier types, AO and IO, the residuals become

$$\hat{e}_{t_{AO}} = wD_t^T + \alpha_t \tag{10}$$

$$\hat{e}_{t_{IO}} = w\pi(B)D_t^T + \alpha_t \tag{11}$$

Equations 10 and 11 can be symbolized as a general time-series structure as follow:

$$\hat{e}_t = wd(l, t) + \alpha_t \tag{12}$$

where $l = (AO, IO)$; $d(l, t) = 0$ with $t < T$; $d(l, T) = 1$ for both types. When $k \geq 1$,

$$d(AO, T + k) = 0, \quad k = 1, 2, 3, \ldots \tag{13}$$

$$d(IO, T + k) = -\pi_k, \quad k = 1, 2, 3, \ldots \tag{14}$$

It is clear to reach the conclusion that the effect of an AO is contained only at a particular point T, whereas the effect of IO is dispersed through the time after at time point T. Consequently, from the least squares theory, the effect of an outlier at $t = t_1$ can be formed as

$$\hat{w}_{AO}(t_i) = \hat{e}_{t_i}, \quad i = 1, 2, 3, \ldots \tag{15}$$

$$\hat{w}_{IO}(t_i) = \frac{\sum_{t=t_1}^{t_n} \hat{e}_t d_{IO,t}}{\sum_{t=t_1}^{t_n} d_{IO,t}^2}, \quad i = 1, 2, 3, \ldots \tag{16}$$

where t_n indicates the last attainable age.

For locating possible outliers, we analyze the maximum value of the standardized statistics (Chang et al. 2016) as,

$$\hat{\tau}_{AO}(t_i) = \frac{\hat{w}_{AO}(t_i)}{\hat{\sigma}_\alpha}, \quad i = 1, 2, 3, \ldots, \tag{17}$$

$$\hat{\tau}_{IO}(t_i) = \frac{\hat{w}_{IO}(t_i)}{\hat{\sigma}_\alpha} \left(\sum_{t=t_1}^{n} d_{IO,t}^2 \right)^{1/2}, \quad i = 1, 2, 3, ... \tag{18}$$

Here, $\hat{\sigma}_a$ is the estimate of residual standard deviation. Hence, the possible location of an outlier can be determined when the standardized values exceed a chosen constant value of C.

In order to decide whether the outlier is a form of AO or IO when both of their effects are greater than C, we follow a simple rule that chooses the type of outlier whose effect is greater than the others (Society 2010). To achieve a high degree of sensitivity in locating the outliers and to be consistent with the literature (Chang et al. 2016), we take the value C as 3.0.

As next, the standard deviation of residuals should be calculated to reach the numerical value for the maximum of standardized statistics given by the Eqs. 17 and 18. To calculate residual standard deviation, the median absolute deviation is used. Given $\tilde{e}$ is the median of the estimated residuals,

$$\hat{\sigma}_\alpha = 1.483 \times \text{median}\{|\hat{e}_t - \tilde{e}|\} \tag{19}$$

is quantified to find the estimates of $\hat{\tau}_{AO}(t_i)$ and $\hat{\tau}_{IO}(t_i)$, $i = 1, 2, 3, \ldots$

Once the outlier locations are set, the outlier adjustment is needed to estimate new model parameters and hence the outlier effects. To accomplish this, an iteration cycle repeated until no more outliers are found, is needed. When the iteration stops, ultimate ARIMA and its parameters are used to forecast the mortality index, κ_t.

The iteration process whose algorithm is given in detail (Algorithm 1) starts with determining the order of the underlying ARIMA for κ_t in LC model. After this step, the residuals are needed as the next stage. Then, the coefficient of $\pi(B)$, outlier effects of AO and IO, and standard deviation of the residuals are evaluated. For determining whether there is an outlier in the series, the absolute value of standardized statistics for all time points are computed and compared with the pre-determined C value. This pre-determined value can be adjusted according to the strength of the model to locate the jumps. If none of the absolute values of the standardized statistics exceeds C value, then the series is taken as outlier-free or outlier-adjusted. Otherwise, the effects of outliers should be removed from the residuals. With the new residuals, the standard deviation is found again for the possibility of detecting new outliers. This process continues until no further outlier is found. The final ARIMA and its parameters are used to create an outlier-adjusted Lee–Carter model for the ultimate mortality index.

During the iteration process, the number of parameters and the variance of the model may change. Thus, they can have an impact on the accuracy of models. We measure the performance of the models using the Akaike Information Criterion (AIC). The ultimate ARIMA and its parameters are employed to forecast the mortality index. Afterward, the death probability of age x, $\hat{q}_x$, is calculated as

$$\hat{q}_x = \frac{\hat{m}_x}{1 + (1 - c_x)\hat{m}_x} \tag{20}$$

Here, $\hat{m}_x$ is the forecasted central death rate for age x, c_x is the average number of years lived within the age interval x and $x + 1$. As in the Human Mortality Database protocol (Human Mortality Database 2021), c_x is taken as 0.5 for all ages except zero. For the beginning age, the last observed numbers in the data for all ages are used.

Algorithm 1: Iteration Process

Step 1: Use Box and Jenkin's approach to identify the orders p,d,q.
Step 2: Compute the residuals of mortality index found from LC model.
Step 3: Calculate the coefficient of $\pi(B)$ and then, outlier effects of AO and IO accordingly.
Step 4: Evaluate the standard deviation of residuals obtained in *Step 2*.
Step 5: Compute standardized statistics for AO and IO for all time points, decide whether there is an outlier. Then, determine the type of outliers by comparing values with C.
Step 6: If no outlier is found in *Step 5*, then Stop: Assign the series is outlier-free or outlier-adjusted. Otherwise, remove the effects of outliers by defining new residuals for AO and IO.
Step 7: Re-calculate the standard deviation of residuals with adjusted residuals and go to *Step 5*. Repeat this cycle until no further outlier can be identified.
Step 8: After the locations of all possible outliers are found, remove the effects of outliers from the mortality index to calculate new mortality index.
Step 9: Go to *Step 1* using new series based on the mortality index and repeat the cycle until no further outlier is found.
Step 10: The final ARIMA and its parameters are used to create the ultimate forecasting model for mortality index.

3 Implementation of the Models

Even though human life is restricted and life expectancy has more or less a similar pattern, country, geographic location, economic development, climate, race, and political–social effects cause variations. The time influence is also an important factor in aging structure. We choose five countries of those two are located in Central Europe, one in North America, one in Central Asia, and one is in Far East. Along with their geographical differences, the UK and Japan do have longevity issues, though their developed economies. Nevertheless, Russia and Bulgaria are accounted as emerging countries. Except for Canada, all countries experience big influence of wars and epidemics. Russia and Bulgaria are mostly prone to political risks due to reformist changes in the twentieth century. In terms of social welfare, the UK, Canada, and Japan have long history in life and pension insurance and annuity products, whereas Russian and Bulgarian insurance markets are flourishing.

Mortality data is collected using Human Mortality Database (2021) with single age based (0–110). The summary of the data and related parameters are given in Table 1. Among the selected countries, the longest period belongs to Canada with 96 years of mortality rates, whereas the Russian rates goes back to only 56 years. The forecasts are projected till 2060.

Table 1 Specific properties of mortality data

			Annuity	
	Country	Years	Ages	Periods
Developed	Canada	1921–2016		
	The UK	1922–2016	0	5 Yrs
	Japan	1947–2017	30	10 Yrs
Emerging	Russia	1959–2014	70	30 Yrs
	Bulgaria	1947–2010		

3.1 Model Estimations

Using the steps explained in previous sections, we estimate the parameters of LC model for each country. The plots of estimated values of a_x, b_x, and κ_t are represented in Fig. 1. a_x, the age pattern of death rates; b_x, age-specific reaction time factor; κ_t, mortality index. Although there is no significant difference between countries based on age pattern of death rates, great differences are seen for b_x and κ_t. The plot of a_x shows that the average values of $ln(m_{x,t})$ over the years are quite similar between countries. However, every age reacts to mortality improvements differently, as seen in b_x plot, meaning that while mortality improvements benefit younger generation in Canada, they provide more benefit to adults in Russia. This pattern can be seen for Bulgarian data too. Bulgarian population reacts unequally to mortality improvements as the mortality index of younger age groups benefit much more than older age groups. The plot of κ_t clarifies that mortality rates decrease significantly in Canada, Japan, and the UK thus, create problems on longevity risk; Russia and Bulgaria have fluctuating mortality trends creating unstable mortality rates over the years.

The locations, types, and effects of outliers are detected using the iteration cycle for Russia and Canada. In Table 2, along with orders of ARIMA and its required parameters, the estimates of c_o and $\hat{\sigma}^2$ representing the constant value within the model and the estimation of the variation of κ_t, respectively, are summarized. The effect of outlier w and its standardized value (τ) are listed. When no outliers are detected consequently two times, the iteration process stops and enables us to reach outlier-adjusted mortality index.

For Russian data, the iteration starts with ARIMA(3,1,2) and reaches ARIMA(0,1,0) after 23 iterations. During those iterations, many outliers from both types are identified and their effects are removed from residuals and mortality index simultaneously. The type of outlier identified in the last observation of time-series cannot be distinguished between AO and IO, as emphasised by Chen and Liu (1993), as seen in Russian data. Another significant observation deduced from the table is that outlier-adjusted models have a smaller $\hat{\sigma}^2$ than starting models indicating that the outlier-adjusted models are superior to original models. A larger variance for κ_t affects the performance of the model, so it should be reduced in order to capture a better model. For Canadian data, the orders of ARIMA do not change even after 18

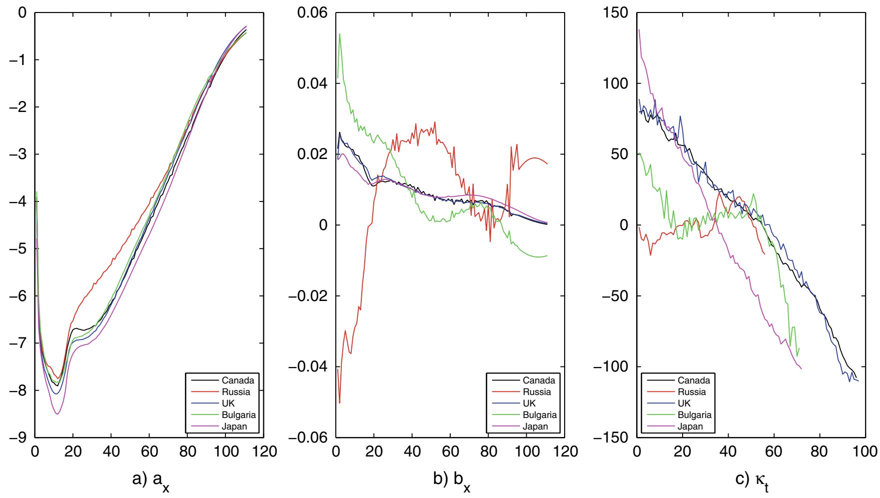

Fig. 1 Lee–Carter model parameter estimates

iterations but removing the effects of outliers help us have a smaller $\hat{\sigma}^2$ for the model. The same analyses on the UK, Japan, and Bulgaria are performed and presented in Table 3.

We determine that compared to Russia and Bulgaria, fewer outliers are found for the UK and Japan. For Japan, only one outlier with AO type is identified in time point two. After this effect is removed, the outlier-adjusted mortality index follows ARIMA(2,1,2). For the UK, the orders of ARIMA do not change where the variance of κ_t decreases significantly from 25.20 to 17.50 in four iterations. More drastic changes are observed for Bulgarian data. Six outliers which all of them are AO type, are found in nine iterations. Outlier-adjusted mortality index is constructed based on ARIMA(0,1,3) even though iteration starts with ARIMA(0,1,0). Although the ultimate models do not change for the UK and Canada, all countries depict variation so that the models generated from Lee–Carter do not reflect the historical data truly.

Although not all outliers identified in the iteration process, some of them may be associated with actual historical events. When the corresponding years of outliers are analyzed, it can be said that wars and economic crises play a major role in changing mortality rates thus, having outliers in mortality data. The Second World War affects the UK in 1940 and 1942, which is associated with mortality outliers. In addition to wars, some important accidents such as Chernobyl in 1986 in Ukraine (former USSR) influence more than one country increasing their mortality rates. After the dissolution of the USSR in 1990 causing separation into 15 different countries makes fluctuations in mortality rates between 1993 and 1996 for Russia as well. Moreover, the effects of economic crises can be seen in Canada between 1929 and 1939 identified as the Great Depression, in Russia between 2011 and 2014 having a sharp decrease in income and purchasing power that may be explanatory of increase in mortality

Table 2 Outlier detection for Russian and Canadian mortality

Country	ARIMA	Iteration	Parameters							Outliers			
			c_0	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	$\hat{\sigma}^2$	Time	Type	w	τ
Russia	(3,1,2)	1	−0.93	−0.01	−0.75	0.28	0.33	1.00	13.04	6	IO	−5.34	−3.10
										28	AO	−9.32	−3.31
										35	AO	11.52	4.09
		2	−0.09	−0.10	0.70	0.01	0.18	−0.63	18.83	6	IO	−5.95	−4.28
										7	AO	14.27	6.27
										53	AO	−6.84	−3.01
		3	−0.04	1.54	−0.94	0.22	−1.31	0.56	16.42	7	AO	7.04	3.00
		4	−0.52	0.14	−0.67	−0.05	−0.03	0.84	18.01	–	–	–	–
	(0,1,0)	5	−0.33	–	–	–	–	–	20.50	36	AO	17.65	4.23
		6	−0.33	–	–	–	–	–	18.26	–	–	–	–
	(0,1,0)	7	−0.33	–	–	–	–	–	18.26	37	AO	14.10	3.55
		8	−0.33	–	–	–	–	–	15.79	7	IO	−8.43	−3.09
		9	−0.33	–	–	–	–	–	14.76	7	IO	−8.38	−3.21
										8	AO	14.97	4.07
		10	−0.33	–	–	–	–	–	20.09	–	–	–	–
	(0,1,0)	11	−0.33	–	–	–	–	–	20.09	7	IO	11.75	4.20
										8	IO	−12.40	−4.43
		12	−0.33	–	–	–	–	–	23.16	8	IO	−18.53	−6.55
										9	AO	20.88	5.20
		13	−0.33	–	–	–	–	–	18.65	8	IO	−8.27	−3.07
										9	AO	18.65	4.88
		14	−0.33	–	–	–	–	–	29.45	–	–	–	–

(continued)

Table 2 (continued)

Country	ARIMA	Iteration	Parameters							Outliers			
			c_0	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	$\hat{\sigma}^2$	Time	Type	w	τ
	(2,1,2)	15	−0.25	0.38	−0.56	–	−0.86	1.00	20.78	8	AO	17.73	4.27
		16	−0.06	0.24	0.47	–	−0.07	−0.47	13.57	–	–	–	–
	(0,1,0)	17	−0.33	–	–	–	–	–	14.49	38	AO	9.20	3.03
										54	AO	−10.33	−3.40
		18	−0.33	–	–	–	–	–	15.31	–	–	–	–
	(0,1,0)	19	−0.33	–	–	–	–	–	15.31	54	IO	7.68	3.77
										55	AO	−15.30	−5.38
		20	−0.33	–	–	–	–	–	13.87	–	–	–	–
	(0,1,0)	21	−0.33	–	–	–	–	–	13.87	56	AO/IO	−9.86	−3.39
		22	−0.17	–	–	–	–	–	12.10	–	–	–	–
	(0,1,0)	**23**	**−0.17**	**–**	**–**	**–**	**–**	**–**	**12.10**	**–**	**–**	**–**	**–**
Canada	(0,1,0)	1	−1.90	–	–		–	–	5.38	6	AO	7.32	3.13
										17	IO	6.10	3.73
		2	−1.90	–	–	–	–	–	5.34	17	IO	6.27	4.09
										18	AO	−12.48	−5.66
		3	−1.90	–	–	–	–	–	6.47	–	–	–	–
	(0,1,0)	4	−1.90	–	–	–	–	–	6.47	17	IO	−9.07	−5.87
										18	AO	12.37	5.58
		5	−1.90	–	–	–	–	–	4.49	18	IO	6.92	4.35
		6	−1.90	–	–	–	–	–	5.77	18	IO	6.46	4.30
										19	AO	−11.70	−4.95
		7	−1.90	–	–	–	–	–	7.81	19	AO	−7.26	−3.09
		8	−1.90	–	–	–	–	–	9.80	–	–	–	–

(continued)

Table 2 (continued)

Country	ARIMA	Iteration	Parameters							Outliers			
			c_0	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	$\hat{\sigma}^2$	Time	Type	w	τ
	(1,1,0)	9	−2.49	−0.35	–	–	–	–	8.60	18	IO	−13.58	−7.57
										19	AO	12.30	5.40
		10	−1.79	0.06	–	–	–	–	4.71	19	AO	9.27	4.26
										20	AO	7.47	3.43
		11	−1.91	−0.01	–	–	–	–	5.98	9	IO	5.02	3.07
		12	−1.97	−0.04	–	–	–	–	5.75	11	AO	−6.62	−3.20
		13	−2.00	−0.06	–	–	–	–	6.13	–	–	–	–
	(0,1,0)	14	−1.90	–	–	–	–	–	6.15	19	IO	−7.62	−4.99
										20	AO	6.69	3.04
		15	−1.90	–	–	–	–	–	4.76	11	IO	4.50	3.05
										19	IO	−4.55	−3.08
										20	AO	7.83	3.69
		16	−1.90	–	–	–	–	–	5.56	11	IO	4.64	3.25
										12	AO	−9.58	−4.66
		17	−1.90	–	–	–	–	–	5.67	–	–	–	–
	(0,1,0)	**18**	**−0.32**	**–**	**–**	**–**	**–**	**–**	**4.13**	**–**	**–**	**–**	**–**

Table 3 Outlier detection for Briton, Japanese, and Bulgarian mortality

Country	ARIMA	Iteration	Parameters								Outliers			
			c_0	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	ϕ_3	$\hat{\sigma}^2$	Time	Type	w	τ
The UK	(0,1,1)	1	−2.09	–	–	–	−0.51	–	–	25.20	8	IO	13.28	3.16
											19	IO	17.14	4.08
		2	−2.09	–	–	–	−0.48	–	–	18.91	9	AO	−14.34	−3.01
											21	AO	−18.14	−3.81
		3	−2.09	–	–	–	−0.44	–	–	17.50	–	–	–	–
	(0,1,1)	**4**	**−2.09**	**–**	**–**	**–**	**−0.44**	**–**	**–**	**17.50**	**–**	**–**	**–**	**–**
Japan	(1,1,3)	1	−0.47	0.84	–	–	−1.24	0.16	0.08	9.67	2	AO	−11.61	−3.41
		2	−2.44	0.30	–	–	−0.47	−0.02	0.29	11.39	–	–	–	–
	(2,1,2)	**3**	**−1.46**	**1.20**	**−0.62**	**–**	**−1.53**	**1.00**	**–**	**8.46**	**–**	**–**	**–**	**–**
Bulgarian	(0,1,0)	1	−2.27	–	–	–	–	–	–	108.61	63	AO	−41.04	−4.10
		2	−2.27	–	–	–	–	–	–	102.30	–	–	–	–
	(1,1,2)	3	−1.29	0.58	–	–	−0.94	0.61	–	80.06	64	AO/IO	−26.97	−3.19
		4	−0.21	0.95	–	–	−1.37	0.55	–	64.44	53	AO	−24.11	−3.00
		5	−1.52	0.33	–	–	−0.79	0.68	–	66.49	21	AO	22.47	3.01
											62	AO	−23.68	−3.18
		6	−1.71	0.27	–	–	−0.65	0.69	–	66.13	–	–	–	–
	(3,1,0)	7	−1.49	−0.19	0.04	0.51	–	–	–	59.24	23	AO	21.43	3.02
		8	−1.51	−0.17	0.05	0.48	–	–	–	59.11	–	–	–	–
	(0,1,3)	**9**	**−2.08**	**–**	**–**	**–**	**−0.14**	**0.21**	**0.54**	**58.31**	**–**	**–**	**–**	**–**

Table 4 Model performance indicators for selected countries

	Values of AIC	
	LC	OALC
Canada	653.97	477.68
Russia	130.05	120.02
The UK	306.44	285.55
Japan	186.12	163.64
Bulgaria	298.76	280.17

rates. It is also interesting to note that Canada does not contain any outliers after 1940, whereas Russia has outliers identified in the twenty-first century explaining continuing unstable mortality trends.

The comparison of solely implementation of LC with OALC model is made using AIC statistics and the results are given in Table 4. The smaller AIC is achieved when the outlier influence is incorporated into LC model.

3.2 Forecasting the Mortality Rates

The mortality index is forecasted until 2060 for all countries using outlier-adjusted ARIMA with 95% confidence interval and presented in Fig. 2. Since κ_t for Russian and Bulgarian data have more fluctuations in their mortality rates, the confidence intervals pose wider bands. As expected for the UK, Japan, and Canada, the forecasted

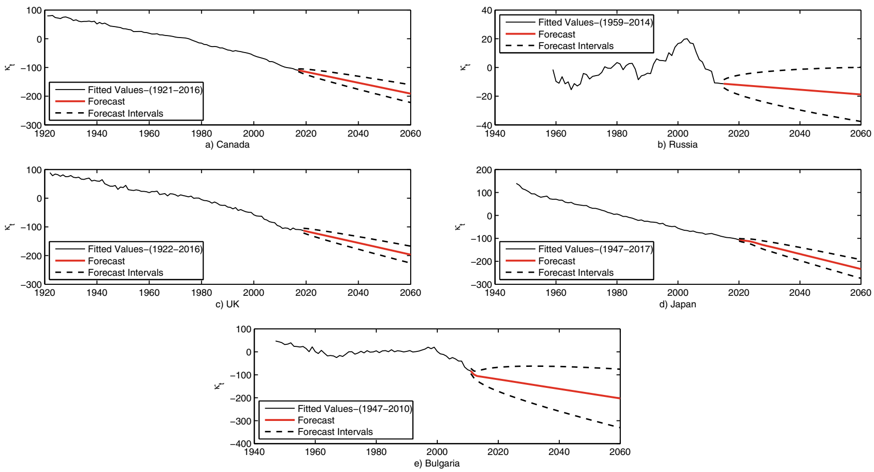

Fig. 2 Forecasted mortality index for selected countries

mortality index decreases smoothly till 2060, whereas we may observe some increase for Russia and Bulgaria as their confidence intervals cover an increasing part in the mortality index as well. Using the forecasted mortality index, OALC model enables us to estimate the survival probabilities. Thus, the expected life expectancy at birth can also be constructed based on survival probabilities.

3.3 Annuity Pricing and Portfolio Impacts

Life annuities are financial products sold by insurance companies paid annually or at different intervals beginning at a stated year. Annuities are generally purchased by investors who aim to provide a fixed income after their retirement. There are two perspectives in terms of annuities: (i) buyers that make regular payments to the insurance company in the period called accumulation and (ii) companies make payments to buyers. In addition, both parties may pay the sum of the annuity in advance so, subsequent payments are calculated accordingly. It is known that even slight fluctuation in price can affect both parties deeply. For the company which is responsible for hundreds of thousands of annuitant payments, a small change in annuity price can cause a huge deficit in its financial budget.

The prices of whole life annuities evaluated at the end of the year (due) for the specific ages 0, 30, and 70 in 2060 for both LC and OALC models are presented in Table 5. Here, the interest rate i is chosen an arbitrary value of 3%. It can be depicted that the life annuity prices are reduced with respect to age for both models and all countries. As longevity is an important consideration, Japan has the highest prices for all ages in both models followed by Canada and the UK. It is clear to notice that Russia has the lowest annuity prices for all ages compared to other countries, also indicating that Russia has high mortality rates. To illustrate the performance of OALC model, the price differences between LC and OALC are presented. The difference values are positive for all ages in Canada and Japan, whereas take negative values for the UK and Bulgaria. For Russia, the difference is positive for beginning age though it is negative for the age of 30 and 70, representing that Russian mortality contains serious fluctuations over the years. The maximum absolute change is achieved for Russia, followed by Japan.

For a more comprehensive assessment, we create a portfolio consist of 10,000 insureds randomly generated from a uniform distribution between ages 15 and 75. Then, the portfolio is used for four different annuity types: (i) 5 year term, (ii) 10 year term, (iii) 30 year term, and (iv) Whole life. The annuity prices are calculated for both LC and OALC models and represented as the sum of the portfolio given in Table 6. The price increases as the number of term increases. While the price difference of whole life between models for Russia is around 5%, this difference does not exceed 1% for other countries. For the UK, the performances of models are so close to each other as there are almost no differences in four annuity types. In larger and

Table 5 Prices of whole life annuities (due) in 2060

	Age	Whole life annuity price		
		LC	OALC	Difference (%)
Canada	0	31.59	31.66	0.22
	30	27.57	27.73	0.56
	70	14.48	14.77	2.01
The UK	0	31.53	31.52	−0.01
	30	27.38	27.38	−0.01
	70	13.95	13.95	−0.01
Japan	0	31.97	32.04	0.22
	30	28.46	29.39	5.02
	70	16.36	16.73	2.25
Russia	0	25.53	28.43	11.36
	30	25.36	24.13	−4.82
	70	10.08	9.59	−4.87
Bulgaria	0	30.84	30.81	−0.10
	30	25.60	25.53	−0.28
	70	10.44	10.37	−0.71

Table 6 Prices of life annuities (due) in 2060 for portfolio

Annuity type		Life annuity price				
		Canada	Russia	The UK	Japan	Bulgaria
5 year term	LC	55,461	54,750	54,190	55,641	50,500
	OALC	55,495	54,366	54,190	55,665	50,015
	Difference (%)	0.06	−0.70	0.00	0.04	−0.96
10 year term	LC	93,980	91,298	93,808	94,668	92,397
	OALC	94,111	90,061	93,806	94,762	92,269
	Difference (%)	0.14	−1.35	0.00	0.10	−0.14
30 year term	LC	190,730	174,350	189,280	196,450	177,210
	OALC	191,730	168,310	189,260	197,410	176,690
	Difference (%)	0.52	−3.46	−0.01	0.49	−0.29
Whole life	LC	235,410	205,380	232,350	248,130	208,430
	OALC	237,500	194,990	232,320	250,480	207,610
	Difference (%)	0.89	−5.06	−0.01	0.95	−0.39

more diverse portfolios, the difference between prices can be more pronounced. Nevertheless, Canadian, Briton, and Japanese mortality data can be considered to be less affected by the model modification.

4 Conclusion

The effect of possible outliers in the mortality data on the price of life annuities may vary according to race, geographic location, economic welfare, and demographic structures. For this reason, Canadian, Briton, and Japanese mortality representing developed countries, Russia and Bulgaria as emerging markets are selected. By such a comparison, we aim to reach a better explication of the differences in mortality behaviors between countries. Moreover, we also compare in terms of four different annuity types to depict the product influence. In order to check the accuracy and sensitivity on a portfolio, calculations are also made on a simulated group of 10,000 insureds whose ages are randomly assigned between 15 and 75.

This study exposes that using the outlier-adjusted model in forecasting mortality rates is critical to the annuity prices for the countries with outliers in their mortality rates. It is observed that Russia is the most affected country as the annuity price differences in all scenarios come up to be the largest compared to others. On the other hand, the differences do not exceed 1% for the UK, Japan, and Canada as developed countries facing longevity risk for their populations. It is shown that as an emerging market, Bulgaria does not show sensitivity to outliers. The political reforms in Russia can be taken as a consequence of sensitivity to outliers.

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