The Cobb-Douglas Production Function for an Exponential Model

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Abstract We investigate China's post-1978 economic data in terms of compatible Cobb-Douglas production functions exhibiting different properties for different periods of time. Our methodology is grounded in the fact that the Cobb-Douglas function can be derived under the assumption of exponential growth in production, labor, and capital. We show that it appears to be the case by employing R programming and the method of least squares. Each Cobb-Douglas function used to characterize the economic growth within the corresponding period of time is determined by specifying the values of the labor share from the available empirical data for the period in question. We conclude, therefore, that the Cobb-Douglas function can be employed to describe the growth in production for the periods 1978–1984, 1985–1991, 1992–2002, 2003–2009, and 2010–2017 each marked by specific events that impacted the Chinese economy.

Keywords China's economic growth · Cobb-Douglas production function · Data analysis · Exponential model · Invariants · Reforms

1 Introduction

China's economic growth, spurred by the launch of market-oriented policy reforms in 1978, has been nothing short of spectacular, propelling the country to the position of the world's largest economy (on a purchasing power parity basis). It must be noted,

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however, that although the Chinese economy has been expanding at a steady pace with real annual GDP growth averaging 9.5% through 2017, several notable events that occurred in the last 40 years have arguably impacted its remarkable expansion. First, in October of 1984 the policymakers had introduced the idea of commodity economy (a euphemism of "market economy" at the time) into practice that led to further liberalization of the Chinese economy. Next, in 1992 the reforms had become effectively irreversible, following the implications from Deng Xiaoping's southern tour. The next important event took place in December of 2001 when China became a member of the World Trade Organization. The effects of the WTO membership had further influenced China's economic growth. Finally, the year of 2009 had marked China's remarkable recovery from the Great Recession. Accordingly, we will investigate the economic data from the periods marked by aforementioned events, namely, 1978–1984, 1985–1991, 1992–2001, 2002–2009, 2010–2017.

To conduct our study, we will employ the Cobb-Douglas aggregate production function. Our choice stems from the empirical evidence available to us and analytical properties of this function that connects production and the corresponding impact factors (in most cases, capital and labor). More specifically, we view the Cobb-Douglas functions as a consequence of the vigorous growth in production, labor, and capital. It is normally possible to describe such a growth by exponential models that can be fit to the corresponding data (see Sato (1981), Sato and Ramachandran (2012), Smirnov and Wang (2020a), Smirnov and Wang (2020b) for more details and references). In effect, that is exactly what was done in 1928 by Charles Cobb and Paul Douglas themselves Cobb and Douglas (1928). Importantly, this observation also puts limitations on the applicability of the Cobb-Douglas function in growth models. It is our contention that only the data that can be approximated with exponentials can be accurately described by the Cobb-Douglas function. In what follows, we will demonstrate that China's economic growth in production, labor, and capital satisfies this requirement, that is the data for each of the five periods outlined above can be approximated by the corresponding exponential functions.

Therefore, our goal in this paper is to use the available empirical data to characterize the economic growth in post-1978 China during each of the five periods from the viewpoint of analytic properties of the Cobb-Douglas function. It should be mentioned that China's economic data has already been studied with the aid of the Cobb-Douglas function from different perspectives. Thus, for example, Chow and Li Chow and Li (2002) fitted a Cobb-Douglas function to the data from 1950 to 2010 under the assumption of constant return to scales, while Rawski Jefferson et al. (1992) argued that China's economic data enjoyed decreasing returns to scale during the period 1984–1987. However, in the latter case the corresponding Cobb-Douglas function was defined to depend not only on capital and labor, but also on energy.

2 The Method

In this section we briefly discuss the methods used in the paper. We begin by recalling that the Cobb-Douglas production function came into prominence after an economist Paul Douglas and a mathematician Charles Cobb with the aid of statistical analysis came up with an equation describing the relationship among the time series describing the US manufacturing output, labor input, and capital input for the period 1899–1922 Cobb and Douglas (1928). However, it must be mentioned that the function had already gained substential attention in the years prior to the 1928 paper by Cobb and Douglas (for more details, see Humpfrey Humphrey (1997)). In its most acceptable form, the function is defined by the formula

$$Y = AL^{\alpha}K^{\beta}, \tag{1}$$

where Y, L, and K are production, labor, and capital respectively, $A > 0$ denotes total productivity, while $\alpha > 0$ and $\beta > 0$ are elasticities of labor and capital. In most cases, the derivation of this function has been carried out by various authors either by employing analytical methods (see, for example, Sato Sato (1981)), or mostly through statistical treatment of existing data (see, for example, Cobb and Douglas Cobb and Douglas (1928), Rawski Jefferson et al. (1992), and Chow and Li Chow and Li (2002)). The same observation can be made about the criticisms by the authors who doubt the validity of the Cobb-Douglas function in conjunction with the study of economic data (see, for example, Felipe and Adams Felipe and Adams (2005)). Admittedly, different approaches to the derivation of the Cobb-Douglas function have led to misunderstanding of its true meaning and limitations. In this paper we continue the development of the combined approach proposed by two of the authors (RGS and KW) earlier Smirnov and Wang (2020, a, b) that aims to expoloit the natural synergy between the analytical and statistical methods employed in the past to study the Cobb-Douglas function and its properties. Recall that the analytical approach to the development and study of the Cobb-Douglas function is based on the assumption that production, capital, and labor of a given economy grow exponentially, namely, the dynamics is subject to the following simple system:

$$\dot{x}_i = b_i x_i, \quad b_i > 0, \quad i = 1, 2, 3, \tag{2}$$

where $x_1(t) = L(t)$, $x_2(t) = K(t)$, and $x_3(t) = Y(t)$ and the fixed parameters b_1, b_2, and b_3 characterize the corresponding exponential growth in capital, labor, and production as functions of time t. Then, integrating Eq. (2) to get the solutions

$$x_i = x_i^0 e^{b_i t}, \quad x_i^0, b_i > 0, \quad i = 1, 2, 3 \tag{3}$$

and eliminating t leads to the derivation of the Cobb-Douglas function as a time-independent invariant of the flow generated by (2) subject to an additional linearity condition:

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = 0, \tag{4}$$

for some parameters $a_1, a_2, a_3 \in \mathbb{R}$. Indeed, the product

$$x_1^{a_1} x_2^{a_2} x_2^{a_3} = (x_1^0)^{a_1} (x_2^0)^{a_2} (x_3^0)^{a_3} e^{(a_1 b_1 + a_2 b_2 + a_3 b_3)t}$$

is a time-independent invariant, provided the linearity condition (4) holds. Here x_i^0, $i = 1, 2, 3$ are the initial conditions. Solving the equation $x_1^{a_1} x_2^{a_2} x_3^{a_3} = C$ for x_3, we arrive at the Cobb-Douglas function of the form

$$x_3 = C^{\frac{1}{a_3}} x_1^{-\frac{a_1}{a_3}} x_2^{-\frac{a_2}{a_3}}, \tag{5}$$

where the constant $C = (x_1^0)^{a_1} (x_2^0)^{a_2} (x_3^0)^{a_3}$. Therefore, the Cobb-Douglas function in this context is consequence of exponential growth in production and the input factors (capital and labor), as well as the condition (4). Moreover, in view of (4) these conditions yield in fact a *family* of Cobb-Douglas functions, namely,

$$x_3 = C^{\frac{1}{a_3}} x_1^{\frac{b_3}{b_1} + \frac{a_2}{a_3} \frac{b_2}{b_1}} x_2^{-\frac{a_2}{a_3}}, \tag{6}$$

or, identifying $x_1 = L$, $x_2 = K$, $x_3 = Y$, $C^{\frac{1}{a_3}} = A$, and $\frac{a_2}{a_3} = \ell$, we have

$$Y = A L^{\frac{b_3}{b_1} + \frac{b_2}{b_1} \ell} K^{-\ell}. \tag{7}$$

We note that the exponential growth in the variables $x_1 = L$, $x_2 = K$, and $x_3 = Y$, as well as the linearity condition (4) make perfect sense from the economic standpoint. Indeed, they simply mean that on the one hand the economy is undergoing a robust growth represented by the growth in production, labor, and capital. On the other hand the variables are not independent. Specifically, $\ln x_1$, $\ln x_2$, and $\ln x_3$ are linearly connected via the condition (4) and the formula (6). This is also acceptable, because the three variables describe the same economy in which a change in one of the variables inevitably yields the corresponding change in the other two. One can assert that multicollinearity in this case is not a bag, it is a feature. For example, an increase in capital will affect both production and labor.

In order to specify the parameter ℓ in (7), we need an extra condition. One such a condition is the assumption of constant returns to scale (i.e., the condition $\alpha + \beta = 1$ in (1) and $\frac{b_3}{b_1} + \ell \frac{b_2}{b_1} - \ell = 1$ in (7), which yields the following Cobb-Douglas function

$$Y = A L^{\frac{b_3 - b_2}{b_1 - b_2}} K^{\frac{b_3 - b_1}{b_2 - b_1}}. \tag{8}$$

We note that the function (8) is economically sound, provided the elasticities of the inputs are positive, which implies that either $b_2 > b_3 > b_1$, or $b_1 > b_3 > b_2$. This is a further limitation on the economic growth determined by (2) with the meaning that the function (8) enjoys constant returns to scale iff production does not grow faster (slower) than both labor and capital.

Another well-known approach to the derivation of the Cobb-Douglas function (1) is statistical in nature and rooted in the study of the available data representing the growth in production, labor, and capital for a given economy. For example, that is exactly how Charles Cobb and Paul Douglas themselves employed the function (1) as the basis of a statistical procedure for estimating the relationship between production, labor, and capital. Specifically, they employed data from the US manufacturing sector for 1899–1922, assuming constant return to scale in (1), to fit the corresponding function $Y = AL^{1-\beta}K^{\beta}$ with the aid of statistical analysis to this data. The value for the elasticity of labor was found to be 0.75, while the National Bureau of Economic Research determined this value empirically to be 0.741 Cobb and Douglas (1928). In spite of the fact that this approach was later employed with much success to study other economic data sets (see Douglas Douglas (1976) and the relevant references therein), the question remained: *Can the data studied by Cobb and Douglas be fitted with another function of the type (1) that does not enjoy constant return to scale?* In view of the above observations, the answer to this question is *yes*, which was confirmed in Smirnov and Wang (2020a) by employing a modification of the statistical method used by Cobb and Douglas originally that incorporated the analytical approach briefly outlined above. More specifically, we proposed in Smirnov and Wang (2020a, b) the following approach. Given economic data representing growth in production, labor, and capital, we first verify whether the exponential model (3) can be fitted to the data, using statistical methods such as R programming and the method of least squares. If it is the case, we determine for each variable x_i, $i = 1, 2, 3$ the corresponding initial values x_i^0 and the parameters b_i, $i = 1, 2, 3$ representing exponential growth. Now we know for a fact that fitting a Cobb-Douglas function to the data is possible. Furthermore, if (the approximate values of) the parameters b_i, $i = 1, 2, 3$ satisfy either the inequality $b_2 > b_3 > b_1$, or $b_1 > b_3 > b_2$ the Cobb-Douglas function of the form (8) can be fitted to the given data. Otherwise, the elasticities of capital and labor in (1) satisfy either $\alpha + \beta < 1$, or $\alpha + \beta > 1$ for any element of the family (7). In the former case we have decreasing returns to scale determined by the inequalities $b_3 < b_2$ and $b_3 < b_1$, while in the latter—increasing returns to scale determined by the inequalities $b_3 > b_2$ and $b_3 > b_1$ for *all* elements of the family of the Cobb-Douglas functions (7) (see Smirnov and Wang (2020b) for more details).

It must be mentioned that the data originally studied by Cobb and Douglas in Cobb and Douglas (1928) has been further investigated using the algorithm outlined above Smirnov and Wang (2020a). Specifically, we have verified, using R, that indeed the time series representing the changes in production, labor, and capital approximately followed exponential growth with $b_1 = 0.025496$ (labor), $b_2 = 0.064725$ (capital), and $b_3 = 0.035926$ (production). Therefore, the family of the Cobb-Douglas functions compatible with the given data is determined from the formula (7) to be

$$Y = AL^{1.409084+2.538634\ell}K^{-\ell}. \tag{9}$$

Furthermore, we have $b_2 > b_3 > b_1$ and so constant return to scale is possible, that is the family of the Cobb-Douglas functions (9) contains the element of the form (8). Substituting these values for b_1, b_2, and b_3 into the formula (8), we found the elasticity of labor to be approximately 0.734125, which was very close to the value determined by Cobb and Douglas in Cobb and Douglas (1928) directly *under the assumption of constant return to scale*. However, this is not the only Cobb-Douglas function that affords a good fit to the data. For example, the function $Y = 0.471016LK^{0.161149}$, which is an element of the family (9) also affords a good fit to the data studied in Cobb and Douglas (1928). However, it no longer enjoys constant returns to scale. Nevertheless, the Cobb-Douglas function enjoying constant return to scale derived in Cobb and Douglas (1928) is the right choice because its value of the elasticity of labor was independently confirmed by the National Bureau of Economic Research. In what follows, we apply this method to study the Chinese economic data from the period 1978–2017.

3 The Data

This section focuses on China's economic data from the period 1978–2017. Our goal here is to collect, unify and tabulate the data representing China's growth in production, labor, and capital for this period. We first gather the data representing the nominal GDP for the period 1978–98 from Table 3-1 in China's Statistical Yearbook (CSY), Volume 2020. Next, we employ Chow and Li's method Chow and Li (2002) to obtain the production series data in 1978 prices through dividing nominal GDP by an adjusted deflator. The deflator for the period 1978–2017 employed here comes from the website Indexmundi (https://www.indexmundi.com/facts/china/gdp-deflator); it is given in terms of index values with the value at 1978 taken as 100%, which is consistent with implicit price deflator in Chow and Li (2002). The labor force figures from Table 2-10 in the CSY, Volume 2020 is used as the labor input. The capital time series data for 1978–1998 has been gathered and tabulated by Chow and Li Chow and Li (2002). We compute the capital series data after 1998 based on the capital stock values from 1978 to 1998 used by Chow and Li (see Table 1 in Chow and Li (2002)), employing their formula

$$K_t = K_{t-1} + RNI_t, \tag{10}$$

where K is capital and RNI represents real net investment (see Chow and Li (2002) for more details and references). The values of RNI are calculated using gross investment, net investment (gross investment less total provincial depreciation), and real gross investment (deflated gross investment in 1978 prices). We find gross investment and total provincial depreciation based on items in the tables of the GDP data by expenditure approach at provincial level published in the CSY, Volumes 1999–2012. It must be noted that the total provincial depreciation data for 2004, 2008, and 2013 are not presented in the CSY and so we estimate the data by averaging the total provincial depreciation values at their consecutive years. For example, depreciation for 2014 is obtained by finding the mean of values for the years 2013 and 2015.

We normalize the time series data by using dimensionless index values with values at 1978 taken as 100. We present the index values of capital, labor, and production from 1972 to 2017 in Table 3 on a logarithmic scale (see Appendix A). Next, we break the data from the period 1978–2017 into the following five data sets: 1978–1984, 1985–1991, 1992–2001, 2002–2009, and 2010–2017. As was already mentioned, each period is marked by specific events that significantly influenced China's economy.

4 The Results

Taking the logarithm of both sides in (3), we linearize the variables as follows:

$$\ln x_i = C_i + b_i t, \quad i = 1, 2, 3, \tag{11}$$

where $C_i = \ln x_i^0$, $x_1 = L$ (labor), $x_2 = K$ (capital), and $x_3 = Y$ (production).

Next, we recover the corresponding values of the coefficients C_i, b_i, $i = 1, 2, 3$ for Sets 1–5. Employing R and the method of least squares, we arrive at the following values.

Set 1 (1978–1984):

$$\begin{aligned} b_1 &= 0.030814,\ C_1 = 4.599695 \text{ (labor)},\\ b_2 &= 0.062199,\ C_2 = 4.593280 \text{ (capital)},\\ b_3 &= 0.084770,\ C_3 = 4.583829 \text{ (production)}. \end{aligned} \tag{12}$$

Set 2 (1985–1991):

$$\begin{aligned} b_1 &= 0.047535,\ C_1 = 4.794706 \text{ (labor)},\\ b_2 &= 0.096918,\ C_2 = 5.078597 \text{ (capital)},\\ b_3 &= 0.077071,\ C_3 = 5.283842 \text{ (production)}. \end{aligned} \tag{13}$$

Set 3 (1992–2001):

$$\begin{aligned} b_1 &= 0.010925,\ C_1 = 5.102944 \text{ (labor)},\\ b_2 &= 0.108844\ \ C_2 = 5.773019 \text{ (capital)},\\ b_3 &= 0.090424,\ C_3 = 5.911680 \text{ (production)}. \end{aligned} \tag{14}$$

Set 4 (2002–2009):

$$\begin{aligned} b_1 &= 0.004855,\ C_1 = 5.208945 \text{ (labor)},\\ b_2 &= 0.127195\ \ C_2 = 6.841267 \text{ (capital)},\\ b_3 &= 0.109907,\ C_3 = 6.781214 \text{ (production)}. \end{aligned} \tag{15}$$

Set 5 (2010–2017):

$$\begin{aligned} b_1 &= 0.002979,\ C_1 = 5.246106 \text{ (labor)},\\ b_2 &= 0.120791\ \ C_2 = 7.907700 \text{ (capital)},\\ b_3 &= 0.074385,\ C_3 = 7.644393 \text{ (production)}. \end{aligned} \tag{16}$$

We verify that the errors, represented by the $values in each estimation, are all less than 1, which suggests that the formulas (11) fit quite well to the data in Table 3 (all of the R programming codes used here are available upon request). We also obtain satisfactory values of the goodness of fit in each regression. Therefore, we arrive at the following families of Cobb-Douglas functions (7) associated with each set.

$$\begin{aligned} &\text{Set 1 (1978–1984): } Y_1 = AK^{2.751022+2.018531\ell}L^{-\ell},\\ &\text{Set 2 (1985–1991): } Y_2 = AK^{1.621353+2.038877\ell}L^{-\ell},\\ &\text{Set 3 (1992–2001): } Y_3 = AK^{8.276796+9.962838\ell}L^{-\ell},\\ &\text{Set 4 (2002–2009): } Y_4 = AK^{22.637899+26.214963\ell}L^{-\ell},\\ &\text{Set 5 (2010–2017): } Y_5 = AK^{24.969789+40.547499\ell}L^{-\ell}. \end{aligned} \tag{17}$$

We see that only the first family of Cobb-Douglas functions, corresponding to Set 1, does not contain the Cobb-Douglas function satisfying the condition of constant returns to scale, which is due to the inequality $b_3 > b_2 > b_1$. Next, in order to determine an appropriate element within each of the families (17), we will use additional empirical characterizations of each set. Recall that Cobb and Douglas in Cobb and Douglas (1928) verified the input elasticities for the function that they derived by comparing the value of α that they found its empirical value determined by the National Bureau of Economic Research. In what follows, we will employ a similar approach. Specifically, we make use of the fact that the labor elasticity α in (1) represents the (constant) value of the labor share, that is

$$\alpha = \frac{\partial Y}{\partial L}\frac{L}{Y},$$

which is compatible with the formula for the labor share derived in Smirnov and Wang (2020) bypassing the Cobb-Douglas function. To fix α and thus β and A within each set, we use this fact and the available empirical data to compute α directly, assuming that within each Set 1–5 the value of labor share is constant. It must be mentioned that

Table 1 China's labor share data by income approach from 2008–2017

Year	2008	2009	2010	2011	2012
Labor share	None	0.4662	0.4501	0.4494	0.4559
Year	2013	2014	2015	2016	2017
Labor share	None	0.4651	0.4789	0.4746	0.4751

Table 2 The values of parameters α, β and A that determine the corresponding Cobb-Douglas production functions for each of the five periods

Period	α	β	A	Error
1978–1984	0.594357	1.068429	0.046888	0.093819
1985–1991	0.598100	0.501866	0.875754	0.011126
1992–2001	0.579310	0.772623	0.222041	0.001630
2002–2009	0.494541	0.845208	0.206569	0.002338
2010–2017	0.464148	0.604368	1.537645	0.000108

although the recent empirical results Bentolila and Saint-Paul (2003), Bentolila and Saint-Paul (2003) show that the labor share is not constant at least in the medium run, in the short run this is a reasonable assumption. Indeed, the data studied in Chong-En and Zhenjie (2010), Qi (2020) have shown that China's labor share declined substantially between 1978 and the late 2000s/2010s, but roughly remained constant during the five aforementioned (short) periods of time. The labor share values for 1978–2007 are taken from the combined labor share in Table 1 presented in Chong-En and Zhenjie (2010). We compute China's labor share between 2008 and 2017 by employing the income approach. Thus, we have computed the provincial compensation of employees from the CSY (Vol.2008-Vol.2018) and obtained the values of annual labor share by dividing aggregate provincial compensation by nominal GDP (see Table 2). Then, we have determined the output elasticity α by calculating the mean values of labor share in each period. Note the provincial compensation data in 2008 and 2013 are not released, but this does not affect the mean values significantly.

As follows from our definition of the Cobb-Douglas function, constant returns to scale are not an intrinsic property of the Cobb-Douglas function that fits to given data. We substitute the above values of α into the formulas for each of the five families (17), thus determining the parameter ℓ leading to the the corresponding values of α and β. Next, we find the values of total factor productivity A employing the Brent regression method. We summarize the results in Table 2.

5 Concluding Remarks

Using the notion of the family of Cobb-Douglas functions given by (7) that is determined by the input and output variables exhibiting exponential growth (2), we were able to describe the growth of China's economy during each of the five consecutive periods from 1978 till 2017. In particular, we see that the growth in production was the strongest vs the growth in capital and labor from 2010 to 2017 (see Set 5 in (17)). During this period the highest was also the total productivity factor A. We also note that the growth in labor was the slowest during this last period. Overall, the growth in labor during the whole period appears to be logistic rather than exponential, which makes a perfect sense.

In summary, our model is based upon the following algorithm employed in this paper to study China's economy.

First, we check, using R, whether the output and input parameters can be accurately approximated by exponential functions. It is the case, we derive the corresponding parameters b_1, b_2, and b_3 representing exponential growth in each of the variables.

Next, we derive the corresponding family of the Cobb-Douglas functions (7). The problem is not solved yet, because we need some additional information needed to completely fix the parameters A, α, and β. To do this we make use of additional empirical data—in this case the labor share—to derive the Cobb-Douglas function that connects the output and inputs during a given period. The parameters A, α, and β are used to better understand and characterize the growth of production vs capital and labor for a given economy.

Our approach, which incorporate both statistical and mathematical methods, is a generalization of the approach employed by Cobb and Douglas in 1928. Indeed, employing our method, we can not only pick a Cobb-Douglas function that is a good fit for a data set representing economic growth, but we can also pick the right function and explain why there are other Cobb-Douglas functions that are compatible with a given data, which, nonetheless, do not relate the output and input variables for a given economy.

Appendices

A The Time Series Data from 1978–2017

See Table 3.

Table 3 The time series data from 1978–2017

Year	Production *Y*	Labor *L*	Capital *C*
1978	4.605170	4.605170	4.605170
1979	4.678628	4.626655	4.658306
1980	4.753533	4.658726	4.714056
1981	4.803614	4.690418	4.765696
1982	4.890866	4.725695	4.829077
1983	4.99653	4.750573	4.902155
1984	5.138630	4.787795	4.984775
1985	5.262161	4.821978	5.076008
1986	5.345325	4.849838	5.172084
1987	5.455029	4.878687	5.269186
1988	5.562040	4.907648	5.376683
1989	5.603906	4.925795	5.477815
1990	5.644398	5.083016	5.563794
1991	5.732475	5.094411	5.649897
1992	5.864016	5.104453	5.745329
1993	5.991187	5.114321	5.872274
1994	6.114023	5.123959	5.996493
1995	6.203785	5.132961	6.117242
1996	6.299963	5.145880	6.230647
1997	6.391264	5.158418	6.334437
1998	6.461915	5.170052	6.432802
1999	6.538906	5.180712	6.531172
2000	6.621475	5.190344	6.630719
2001	6.699336	5.200173	6.737104
2002	6.790904	5.206786	6.849440
2003	6.890539	5.212989	6.971409
2004	6.990493	5.220124	7.096924
2005	7.093971	5.225268	7.216096
2006	7.218866	5.229693	7.337764
2007	7.356048	5.234257	7.468736
2008	7.452116	5.237478	7.604464
2009	7.534165	5.240966	7.746788
2010	7.632617	5.244612	7.889437
2011	7.718726	5.248742	8.025847
2012	7.801369	5.252452	8.156867
2013	7.870421	5.256005	8.282785
2014	7.953807	5.259584	8.405889
2015	8.016168	5.262143	8.519031
2016	8.085520	5.264104	8.627565
2017	8.159216	5.264581	8.736350

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