





Strength and Deformability of Reinforced Wooden Beams of Variable Stiffness



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Abstract The method of designing reinforced wooden beams of variable stiffness with group reinforcement is described. The dependence of the breakage of reinforcement in the span on the reinforcement coefficient is given and the influence of changes in the stiffness of structures on the deformability is determined. Theoretical stress diagrams are analyzed, indicating the possibility of hazardous stress concentrations in reinforced beams of variable stiffness. Diagrams M and Q , as well as schemes of theoretical breakage of reinforcement in beams were constructed. The results of determining the influence of the reinforcement coefficient on the place of breakage of reinforcement in the span of beams are displayed graphically. The dependences of the anchorage length on the diameter and reinforcement coefficient, the relative length of the anchorage of the rods on the span of the reinforced beams have been constructed. Defined deflection of reinforced wooden beams of variable stiffness. A further direction of research is indicated, which consists in determining local stresses in zones of change in the stiffness of beams.

Keywords Timber · Building · Beams · Reinforcement of wooden beams

1 Introduction

To save materials of steel and epoxy compound, it became necessary to create the most efficient reinforced wooden structures [1–4]. Which can be obtained by rational placement of reinforcement in the span in accordance with the diagram of the greatest bending moments, i.e. create structures of variable stiffness by breaking the reinforcement in the span. But as shown by preliminary calculations performed by Boldyunum V.F. [5], in such structures it is possible that a dangerous stress

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concentration in the wood occurs at the ends of the reinforcing bars (Fig. 1). This is indicated by the dependence describing the distribution of shear stresses along the length of the reinforcement [6, 7]:

$$\tau_{\alpha} = -\frac{q_x}{S_e \psi} \left\{ 1 - \left[1 - \frac{\alpha}{2} (l^2 - b^2) \right] \frac{Sh \alpha x}{\alpha x Ch \alpha b} \right\} \quad (1)$$

The designations are taken from the statics condition.

Analysis of expression (1) shows that the magnitude of shear stresses is significantly influenced by the place of breakage of reinforcement in the span [8]. Based on this, it becomes necessary to theoretically determine the optimal place of breakage of reinforcement in the span of reinforced wooden structures [9–11], i.e. so that the resulting structures of variable stiffness would have the same or similar strength and stiffness indicators as structures of constant stiffness [12, 13].

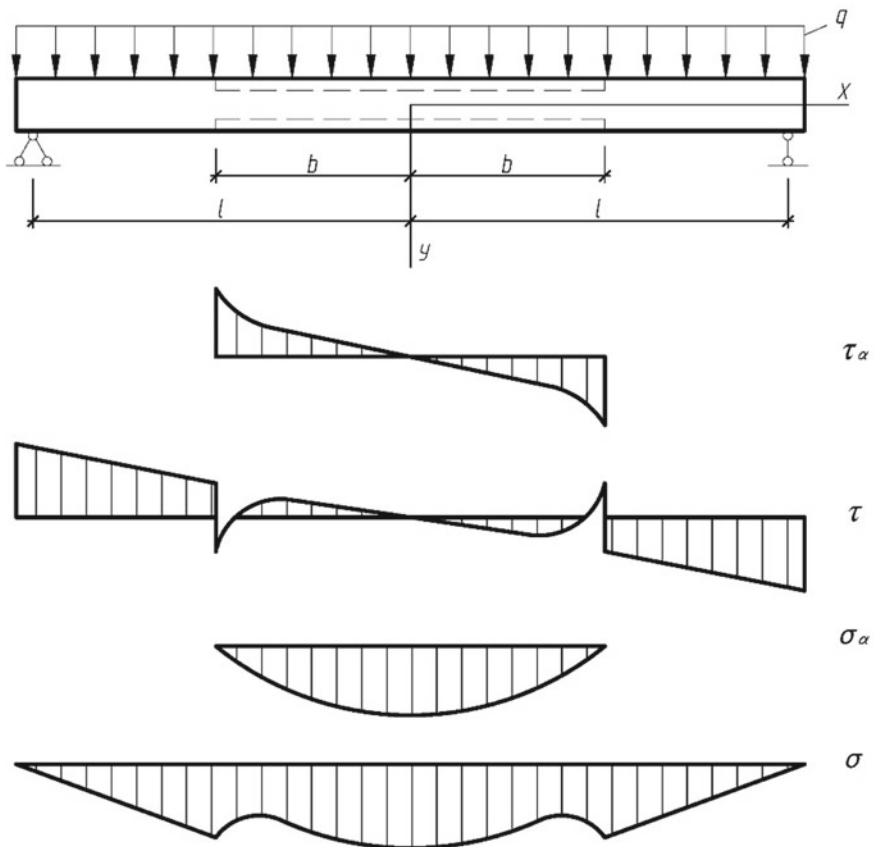


Fig. 1 Theoretical diagrams indicating the possibility of hazardous stress concentrations in reinforced beams of variable stiffness

2 Methods

It is more expedient to determine the place of breakage of reinforcement using a simpler method for calculating reinforced structures—according to the given sections [14–16].

The place of the theoretical breakage of reinforcement in bent structures is determined from the condition that the wood perceives a certain part of the bending moment M_{max} , acting on the complex structure (Fig. 2), i.e.:

$$M_x = W_w R_b \tag{2}$$

For a single-span element loaded with a uniformly distributed load— q , the value of the bending moment [17] at any point of the beam section is equal to:

$$M_x = \frac{q_x(l-x)}{2} \tag{3}$$

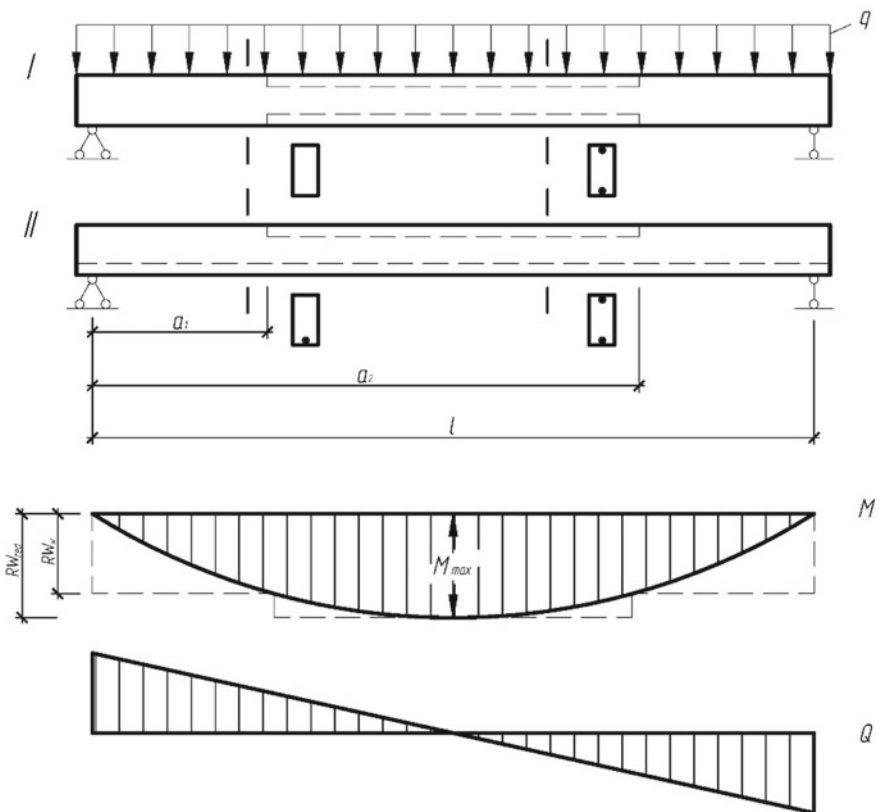


Fig. 2 Diagrams M, Q and circuit breakage theoretical reinforcement girders

Solving Eqs. (2) and (3) together, we determine the distance from the support to the place of theoretical breakage of the reinforcement:

$$a_{1,2} = \frac{l}{2} \pm \sqrt{\left(\frac{l}{2}\right)^2 - \frac{2R_b W_w}{q}} \quad (4)$$

The value of the calculated uniformly distributed load is found from the condition of the strength of the reinforced beams [18]:

$$\begin{aligned} M_{\max} &= W_{red} R_b = W_0(1 + 3\eta\mu) R_b \\ M_{\max} &= \frac{q l^2}{8} \end{aligned} \quad (5)$$

From identity (5) we determine the load:

$$q = \frac{8W_0(1 + 3\eta\mu)R_b}{l^2} \quad (6)$$

Substituting the load into Eq. (4), we get:

$$a_{1,2} = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{R_b W_w}{R_b W_0(1 + 3\eta\mu)}} \right) = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{M_w}{M_{\max}}} \right) \quad (7)$$

Let us assume that $W_w = W_0$ than:

$$a_{1,2} = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{1}{1 + 3\eta\mu}} \right) = \frac{l}{2} \left(1 \pm \sqrt{\frac{3\eta\mu}{1 + 3\eta\mu}} \right) \quad (8)$$

The break point of the reinforcement determined by the formula (8) takes into account the reinforcement termination, i.e. its establishment for the place of the theoretical break, determined by the formula (7), for a length from 0.0012 to 0.017 of the calculated span [19].

During the search experiment, the reinforcement anchorage zone for one series of beams was increased by introducing the safety factor $C = 2$ into the denominator of the radical expression of formula (8).

The place of breakage of reinforcement in this series of beams was determined by the formula:

$$a_{1,2} = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{1}{C(1 + 3\eta\mu)}} \right) \quad (9)$$

Analytical formulas for determining the theoretical break point of the compressed zone reinforcement in beams [20–22] with the tensile zone reinforcement along the entire span were derived using a similar method.

We solve jointly Eq. (3) with the equation describing the maximum bending moment perceived by an asymmetrically reinforced section of the beam, we get:

$$M_x = W_{red}^c R_b \quad (10)$$

Determine the location of the breakage of the reinforcement in the compressed zone:

$$a_{1,2}^c = \frac{l}{2} \pm \sqrt{\left(\frac{l}{2}\right)^2 - \frac{2R_b W_{red}^c}{q}} \quad (11)$$

Substituting the load determined from the strength condition of the symmetrically reinforced section [23, 24] by formula (6), we obtain:

$$a_{1,2}^c = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{W_{red}^c}{W_{red}}} \right), \quad (12)$$

where W_{red}^c —reduced moment of resistance of the compressed zone relative to the neutral axis (13) located at a distance h_c (14) from the upper edge of the beam [25]:

$$W_{red}^c = \frac{I_{red}}{h_c} \quad (13)$$

$$h_c = \frac{h_{n.a.}(1 + 2\eta\mu_{as})}{2(1 + \eta\mu_{as})} \quad (14)$$

The moment of inertia of an asymmetrically reinforced section [26] is determined by the formula:

$$I_{red} = \frac{bh_{n.a.}^3(1 + 4\eta\mu_{as})}{12(1 + \eta\mu_{as})} \quad (15)$$

Substituting expressions (14) and (15) into formula (13), we obtain the reduced moment of resistance of the compressed zone:

$$W_{red}^c = \frac{bh_{n.a.}^2(1 + 4\eta\mu_{as})}{6(1 + 2\eta\mu_{as})} \quad (16)$$

Let us assume that the design height of a symmetrically reinforced section is equal to the height of the section with an asymmetrical arrangement of reinforcement, i.e. $h_0 = h_{0as}$. In this case, the reinforcement coefficient for a section with an

asymmetrical arrangement of reinforcement will be half that of a symmetrical one, i.e. $\mu_{as} = \mu/2$. Then the place of breakage of the reinforcement in the compressed zone, taking into account the anchoring, should be determined by the formula:

$$a_{1,2}^c = \frac{l}{2} \left(1 \pm \sqrt{1 - \frac{1 + 2\eta\mu}{(1 + \eta\mu)(1 + 3\eta\mu)}} \right), \quad (17)$$

when substituting the reinforcement coefficient $\mu = F_a/bh_o$ —sections with symmetrically located reinforcement.

3 Results and Discussion

The place of breakage of reinforcement, determined by formula (17), takes into account the insertion of reinforcement beyond the place of theoretical break, determined by formula (11) for the anchorage length from 0.0018 to 0.025 of the calculated span of the beams. The anchorage length was obtained under the condition that during the operation of the beams in the zone of the maximum bending moment, the calculated strength characteristics of the materials: steel and wood are fully used. In this case, several factors affect the anchorage zone and the location of the breakage of the reinforcement: the reinforcement coefficient, the diameter of the reinforcement and the span of the beams.

The influence of the reinforcement coefficient on the place of breakage of reinforcement in the span of beams is determined by expressions (7)–(9), (17) and is shown in Fig. 3.

The dependence of the anchorage zone— l_{an} on the diameter of the reinforcement— d and the reinforcement coefficient is shown in Fig. 4. It is obtained on the basis of the assumptions made when deriving formulas (8) and (17). In the first case, due to the fact that the height of the unreinforced section of the beam is greater than the calculated one by the diameter of the bar $h = h_0 + d$, and in the second case, the calculated height of the asymmetrically reinforced section is greater than the calculated height of the symmetrically reinforced section $0.5d$ on the reinforcement.

The influence of the span of beams on the relative length of the embedment of rods $\lambda_{an} = l_{an}/d$ is shown in Fig. 5.

It should be noted that in practice the anchorage zone turns out to be much larger than that obtained by formulas (8) and (17). Since the main criterion in the design and operation of wooden beams, in most cases, is their deformability.

Consequently, the design load, as a rule, for such structures is taken to be significantly less than that determined by formula (6), from the condition of the strength of symmetrically reinforced beams. Therefore, the strength characteristics of materials, steel and wood, in the zone of action of the maximum bending moment, are not fully used.

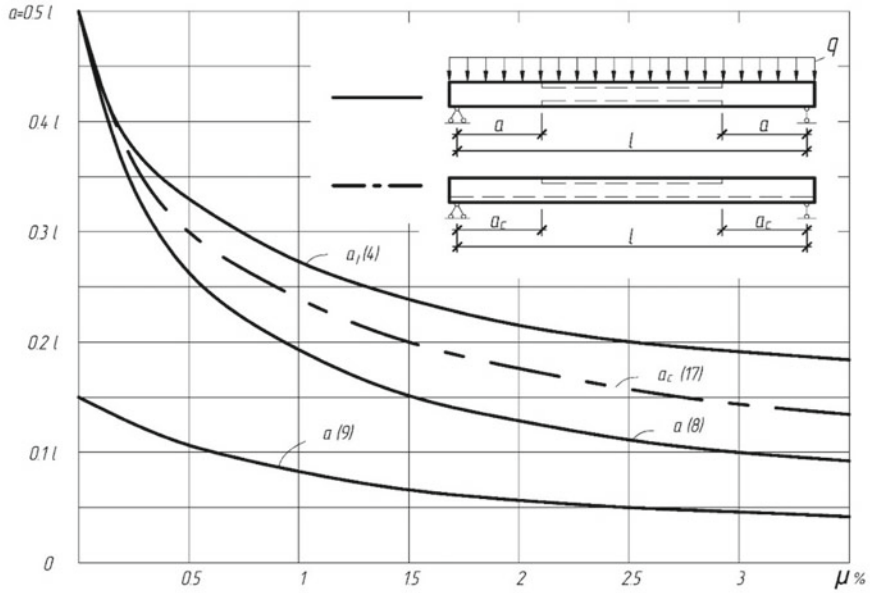


Fig. 3 Graph for determining the theoretical breakage location of reinforcement in beams of variable stiffness

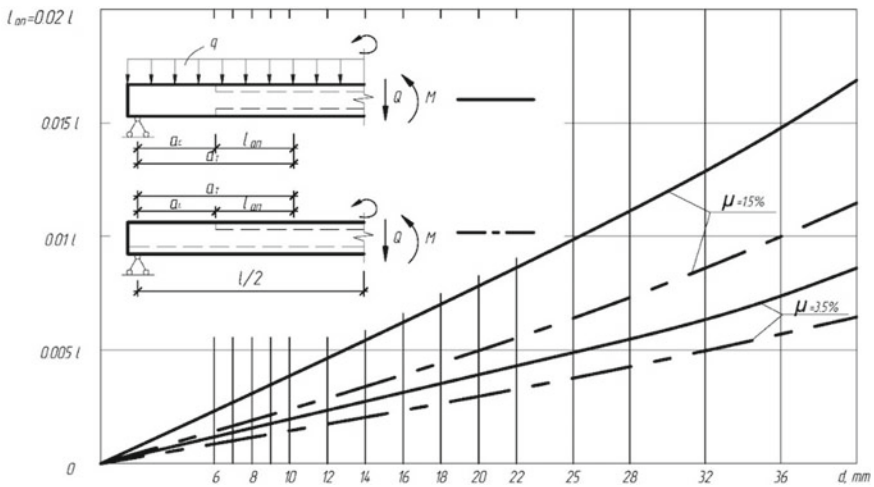


Fig. 4 Dependence of anchorage length on diameter and reinforcement coefficient

It is known that an increase in the anchorage zone leads to an increase in the strength and rigidity of the structure, and a change in the stiffness of beams, due to the breakage of reinforcement in the span, leads to an increase in deflection.

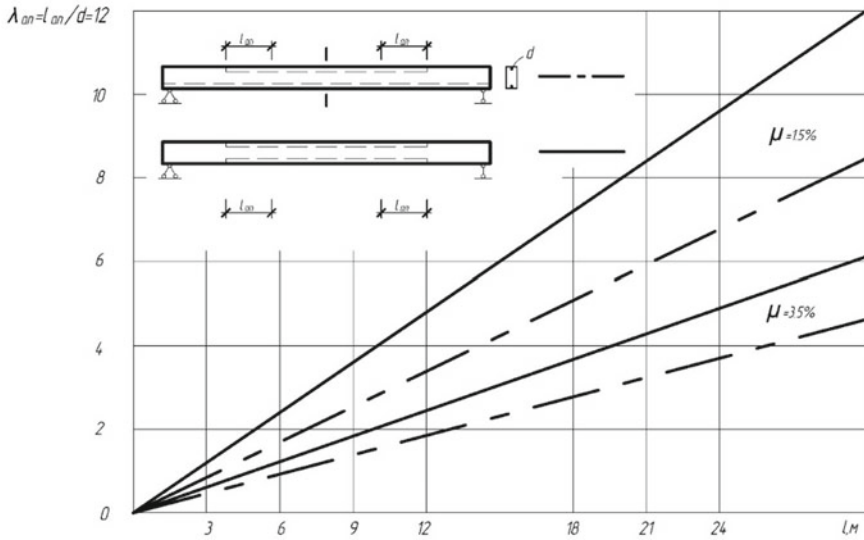


Fig. 5 Dependence of the relative length of anchoring of bars on the span of reinforced beams

Therefore, it became necessary to theoretically determine the deformability of beams of variable stiffness, loaded, as a rule, during operation with an evenly distributed load— q^n all over the span— l .

The dependence of the beam deflection on the accepted loading scheme is derived using the force method (Fig. 6):

$$f_0 = 2 \int_0^a \frac{M_x M_{x_1}}{EI_w} dz + \int_a^{l/2} \frac{M_x M_{x_1}}{EI_{red}} dz, \tag{18}$$

where $M_x = \frac{q^n z l}{2} - \frac{q^n z^2}{2}$ —bending moment in the current section of a given section of the beam from a uniformly distributed load acting on it;

$M_{x_1} = \frac{z}{2}$ —bending moment in the same section from a single load;

dz —beam axle length differential;

EI_w —stiffness of unreinforced area;

EI_{red} —stiffness of the reinforced section.

We transform expression (18):

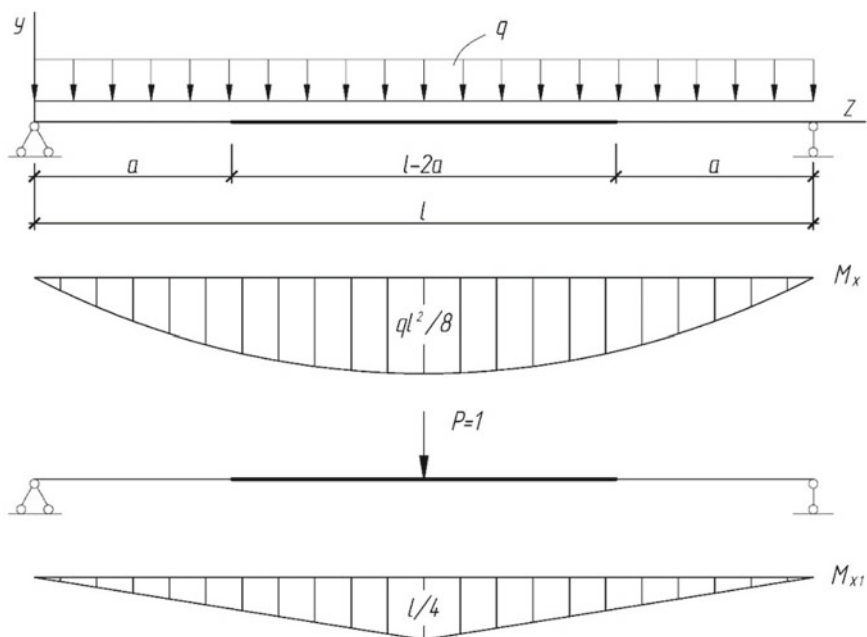


Fig. 6 Design scheme for determining displacements (deflections) of reinforced beams of variable stiffness

$$\begin{aligned}
 f_0 &= 2 \int_0^a \left(\frac{q^n l z^2}{4} - \frac{q^n z^3}{4} \right) \frac{dz}{EI_w} + 2 \int_a^{l/2} \left(\frac{q^n l z^2}{4} - \frac{q^n z^3}{4} \right) \frac{dz}{EI_{red}} = \frac{2}{EI_w} \left| \frac{q^n l z^3}{12} - \frac{q^n z^4}{16} \right|_0^a \\
 &+ \frac{2}{EI_{red}} \left| \frac{q^n l z^3}{12} - \frac{q^n z^4}{16} \right|_a^{l/2} = \frac{2}{EI_w} \left(\frac{q^n l a^3}{12} - \frac{q^n a^4}{16} \right) + \frac{2}{EI_{red}} \left(\frac{q^n l^4}{96} - \frac{q^n l^4}{256} - \frac{q^n l a^3}{12} + \frac{q^n a^4}{16} \right) \\
 &= \frac{5q^n l^4}{384EI_{red}} - \frac{4q^n l a^3 - 3q^n a^4}{24} \left(\frac{1}{EI_w} - \frac{1}{EI_{red}} \right)
 \end{aligned}$$

where I_w —the moment of inertia of an unreinforced section, in the case of reinforcement of a stretched zone along the entire span, and compressed on a part of the length is taken equal to:

$$I_{red.as} = \frac{bh_{n.a.}^3 (1 + 4\eta\mu_{as})}{12(1 + \eta\mu_{as})} \quad (19)$$

I_{red} —reduced moment of inertia of the section in the zone of action of the maximum bending moment:

$$I_{red} = \frac{bh_0^3}{12} (1 + 3\eta\mu) \quad (20)$$

It is known that the deflection of reinforced timber beams during bending generally depends on normal and shear stresses, i.e. consists of deformations from a bending moment, causing the rotation of cross-sections relative to the neutral axis, and deformations from shear forces, manifested in parallel shears relative to each other of the cross-sections of the base material—wood, as well as shear deformations of the reinforcement relative to wood.

As a rule, additional deflection due to lateral forces is not taken into account when calculating reinforced timber beams. This often leads to significant errors, since the shear modulus of wood G is about 20 times less than the modulus of longitudinal elasticity E . Especially in reinforced beams with height-to-span ratios $\frac{h}{l} > \frac{1}{17}$ and a reinforcement ratio $\mu = 2.5\%$, since the reinforcement of wooden beams leads to a decrease in lumber consumption, i.e. decrease in the height and width of structures in the plane of action of the maximum horizontal shear stresses.

It is known that the additional deflection in the middle of the span of beams due to the action of shear forces depends on the ratio $\frac{\tau_{max}}{\tau_{mid}}$, which takes into account the influence of the uneven distribution of shear stresses over the section and depends on the shape of the cross section. With plastic deformations of beams, the ratio $\frac{\tau_{max}}{\tau_{mid}}$ becomes a variable value, depending on the value of the elastic-stressed volume of the wood. Consequently, with an increase in the bending moment, the effect of shear forces on the total relative deflection of the beams in the elastic–plastic stage of work decreases. Therefore, the greatest influence of shear forces on the deflection of reinforced beams affects the conditionally elastic stage of work, when the presence of variable bending stiffness due to the flexibility of the steel–wood adhesive bond is only slightly manifested. Consequently, the additional deflection from the flexibility of the glue joint steel–wood, at the stage of conditionally elastic work of the reinforced wooden beams, can be neglected. However, when a constant or long-term load is applied in time, an increase in deflections of reinforced wooden structures is observed, which is caused by a change in the elastic modulus of wood and the effect of shear deformations of the adhesive bond of reinforcement with wood. Such an increase in deflections of complex structures must be taken into account when determining the total deflection by introducing a coefficient K_T , that takes into account the level of loading of reinforced structures.

Hence, the total deflection of reinforced wooden beams should be determined in accordance with the norms by the formula:

$$f = \frac{f_0}{k} \left[1 + C \left(\frac{h}{l} \right)^2 \right] K_T \quad (21)$$

Formula (21) takes into account the effect of shear deformation from shear forces by the coefficient— C , at $k = 1$ for beams with a constant section height.

Here, f_0 —is the deflection of reinforced wooden beams of constant or variable stiffness without taking into account shear deformations, is determined by the formula (18). The coefficient K_T in addition to the loading level, also takes into account the redistribution of stresses in time between the reinforcement and the wood, which leads to an increase in the normal stresses in the reinforcement and the glue seam and to a decrease in them in the wood. In beams with breakage of reinforcement in the span, the redistribution of stresses from wood to steel leads first to an increase in local stresses in the wood, and then to a decrease in them due to a change in the elastic modulus of wood over time.

Hence, it becomes necessary to determine local stresses in zones of change in the stiffness of beams, which is accepted as a priority task for further research.

4 Conclusions

As a result of the research carried out, the following conclusions can be drawn:

1. The theoretical definition of the optimal place of breakage of reinforcement in the span of reinforced wooden structures (beams of variable stiffness) has been carried out.
2. The influence of the reinforcement coefficient on the place of breakage of reinforcement in the span of beams has been determined.
3. Revealed the dependence of anchoring on the diameter of the reinforcement and the coefficient of reinforcement of complex structures.
4. The influence of the span of beams on the relative length of the embedment of the rods has been established.
5. The influence of the change in the stiffness of the reinforced structure on the deformability is considered.
6. A task for further research has been formulated.

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