# Deformation of 2D RC Beam-Column Joint

**Sergey Savin, Anastasia Obydina, and Ekaterina Mikhailik**

**Abstract** In the beam-column subassembly and adjacent bar sections of RC frames of frame-braced structural systems, a complex 2D stress–strain state arises. The computational models for such subassemblies presented in the scientific literature are mainly based on a simplified representation of the forces acting in them. These ones allow evaluate the implementation of the characteristic destruction mechanisms and further each of them is analysed separately. Therefore, in this article, a finite difference method is proposed for deformation analysis of a reinforced concrete beam-column monolithic subassembly. A distinctive feature of the proposed solution is the ability to take into account the discrete nature of the reinforcement, as well as the incomplete adhesion of reinforcement to concrete along the contact surface. Also, the article presents an example of calculating the deformed state of a beam-column monolithic subassembly of a RC frame scale model, for which a simulation was previously performed using bar FE. Comparison of the calculation results according to the proposed model and the traditional bar model shows the differences in the deformed state of the 2D RC beam-column joint. This can be explained by taking into account the 2D stress–strain state factors, which are not taken into account in the bar model.

**Keywords** Reinforced concrete · Beam-column joint · Finite difference · Numerical analysis · Deformation

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## 1 Introduction

In the reinforced concrete frames of buildings and structures at the beam-column junctions, a complex stress–strain state arises under external forces [1, 2]. In some cases, when the girders adjoin the column only from two opposite faces in the case of a frame-type assembly, such a stress–strain state can be conventionally considered as biaxial, assuming that the stresses are distributed relatively uniformly over the section width (from the plane of action of bending moments).

For a more accurate account of the deformed state of such 2D frame joints, as a rule, approaches are used that are similar to the applied element method [3, 4], which has gained wide recognition in recent years in modeling the deformation and destruction of reinforced concrete bearing systems of buildings and structures under special influences. Better convergence with experimental data when using the applied element method compared to the standard FEM procedure in a bar formulation is achieved by simulating the connection between short sections of structural elements using elastic-yielding springs.

Analysis of the calculated models of the girder-column interface nodes presented in the scientific literature shows that most of them are based on a simplified representation of a complexly stressed element by replacing stresses with generalized forces. Thus, highlighting two characteristic resistance mechanisms, truss and compressed inclined strip, Hwang and Lee [5] assess the possibility of their implementation separately. A similar approach to the analysis of the work of the girder and column interface units, which are somewhat different in their design, can be found in the works of other authors [6–8]. In works [9–11], the behavior of a plane joint element is modeled using elastic ties (springs). The introduction of such elastic-yielding bonds between elements into the computational model allowed Feng and Ning [9] to achieve better quantitative and qualitative convergence with experimental data than when using traditional rod models of the finite element method with rigid nodal joints. However, the use of the described approach is associated with the complexity of determining the compliance parameters of the elements-springs, taking into account the possibility of cracking, leading to the structural orthotropy of the plane-stressed joint [9, 12–14].

Summarizing the results of a brief analysis of the calculated models of the beam-column joints presented in the scientific literature, it can be concluded that the models of such nodes are mainly based on a simplified representation of the forces acting in them and an assessment of the implementation of one of the characteristic fracture mechanisms, the analysis of each of which performed separately. In this regard, the purpose of this work is to develop a generalized design model of a 2D reinforced concrete frame unit, which would take into account the presence and location of longitudinal and transverse reinforcement, as well as various loading options for beams and columns adjacent to the joint.

## 2 Method

To assess the nature of deformation and destruction of 2D nodes of reinforced concrete frame-tie frames of buildings and structures in this study, we will use a model of a bearing system consisting of universal physically nonlinear rod finite elements of girders and columns and special elements (by the type of super elements [15, 16]), modeling crossbar-column interface nodes. In this case, the stress–strain states of the sections at the ends of the bar elements will be the boundary conditions for calculating the nodal connection. The further solution of the problem of estimating the stress state of a node will be carried out in the "plane stress state" setting, neglecting the stresses from the plane of the load action, assuming that they are small in comparison with the components of the stress state in the plane. As a first approximation, the area bounding the junction of the column and the girder will be considered rectangular.

In contrast to the traditional approach to the calculation using super elements, in this study, the general solution to the problem of the plane stress state of a monolithic girder-column interface is based on the grid method in displacements. The use of the grid method makes it possible to construct an algorithm that is simpler than the FEM for assessing the convergence of the calculation results and adjusting the density of the region partitioning to achieve the required accuracy, as well as taking into account the contact interaction of reinforcement and concrete. The form of writing the problem in displacements, in turn, makes it possible to avoid solving additional problems of evaluating the values of the stress function at the nodes on the contour of the region that bounds the considered rectangular special element of the crossbar and column conjugation.

Following the accepted two-dimensional formulation of the problem of calculating the stress–strain state of the element of the joint of the girder and the column of a reinforced concrete frame, the equilibrium equations of an infinitesimal volume of an orthotropic body can be written in displacements in the following form:

$$\begin{aligned} \frac{E_x}{1-\nu^2}\frac{\partial^2 u}{\partial x^2} + \frac{E_y \nu_x}{1-\nu^2}\frac{\partial^2 v}{\partial x \partial y} + G_{xy}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = 0; \\ \frac{E_y}{1-\nu^2}\frac{\partial^2 v}{\partial y^2} + \frac{E_x \nu_y}{1-\nu^2}\frac{\partial^2 u}{\partial x \partial y} + G_{xy}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) = 0, \end{aligned} \tag{1}$$

where $u$, $v$ are displacements of grid nodes along orthogonal coordinate axes $OX$ and $OY$, respectively;

$E_x$, $E_y$ are reduced moduli of material deformations along the orthogonal coordinate axes OX and OY, respectively;

$G_{xy} = \sqrt{E_x E_y}[2(1+\nu)]^{-1}$ is a reduced shear modulus;

$\nu_x = \nu\sqrt{E_x/E_y}; \nu_y = \nu\sqrt{E_y/E_x}$,

$\nu$ is Poisson ratio, which takes value $\nu = 0.2$ for concrete.

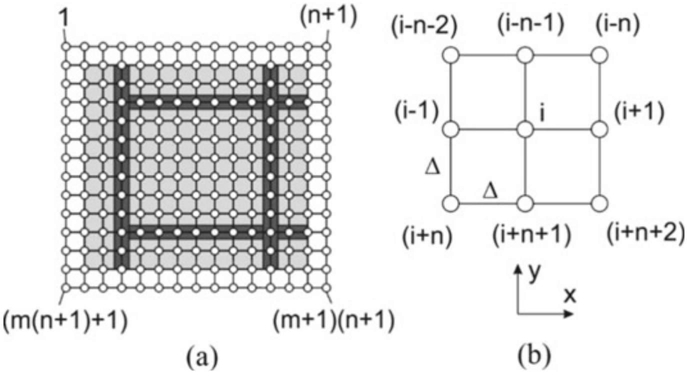


**Fig. 1** Scheme for the calculation of a joint element by the method of meshes: general view (**a**); accepted rule of numbering grid nodes (**b**)

In Eq. (1), we neglected the components of the volumetric forces from the self-weight of the material, since they do not make a significant contribution to the general stress state of the element at the loading stages considered in the study.

We divide the area bounding the joint element under consideration by a grid with the same horizontal and vertical spacing so that the centers of gravity of the reinforcing bars coincide with the grid lines (Fig. 1a).

# 3 Results and Discussion

## *3.1 Construction of a Computational Model and Finite-Difference Equations of the Grid Method*

Let us write down the derivatives in Eq. (1) in finite differences:

$$\begin{aligned}
\frac{\partial^2 u_i}{\partial x^2} &= \frac{u_{(i-1)} - 2u_i + u_{(i+1)}}{\Delta^2};\\
\frac{\partial^2 v_i}{\partial y^2} &= \frac{v_{(i-n-1)} - 2v_i + v_{(i+n+1)}}{\Delta^2};\\
\frac{\partial^2 u_i}{\partial y^2} &= \frac{u_{(i-n-1)} - 2u_i + u_{(i+n+1)}}{\Delta^2};\\
\frac{\partial^2 v_i}{\partial x^2} &= \frac{v_{(i-1)} - 2v_i + v_{(i+1)}}{\Delta^2};\\
\frac{\partial^2 u_i}{\partial x \partial y} &= \frac{u_{(i-n)} - u_{(i-n-2)} + u_{(i+n+2)} - u_{(i+n)}}{4\Delta^2};\\
\frac{\partial^2 v_i}{\partial x \partial y} &= \frac{v_{(i-n-2)} - v_{(i+n)} + v_{(i-n)} - v_{(i+n+2)}}{4\Delta^2},
\end{aligned} \qquad (2)$$

where $\Delta$ is a dimension of grid bay (Fig. 1b).

For the mixed derivative, we additionally write an expression in one-sided differences, which will allow us to relate the equilibrium equation (2) for the corner points of the contour of the element under consideration to the boundary conditions:

$$\begin{aligned}\frac{\partial^2 u_i}{\partial x \partial y} &= \frac{u_{(i-n)} - u_{(i-n-1)} + u_{(i+n+2)} - u_{(i+n+1)}}{\Delta^2} \\ &= \frac{u_{(i-n-1)} - u_{(i-n-2)} + u_{(i+n+1)} - u_{(i+n)}}{\Delta^2}; \\ \frac{\partial^2 v_i}{\partial x \partial y} &= \frac{v_{(i-n-2)} - v_{(i-1)} + v_{(i-n)} - v_{(i+1)}}{\Delta^2} \\ &= \frac{v_{(i-1)} - v_{(i+n)} + v_{(i+1)} - v_{(i+n+2)}}{\Delta^2}.\end{aligned} \tag{3}$$

Then Eq. (1) taking into account (2) for the $i$-th point of the considered plane stressed element (except for the corner points) can be rewritten as:

$$\begin{aligned}&4\left(a_{(i-1)}u_{(i-1)} - 2a_i u_i + a_{(i+1)}u_{(i+1)}\right) + \left(b_{(i-n-2)}v_{(i-n-2)} - b_{(i+n)}v_{(i+n)}\right. \\ &\quad \left. + b_{(i-n)}v_{(i-n)} - b_{(i+n+2)}v_{(i+n+2)}\right) \\ &\quad + \left(4d_{(i-n-1)}u_{(i-n-1)} - 8d_i u_i + 4d_{(i+n+1)}u_{(i+n+1)}\right. \\ &\quad \left. + d_{(i-n-2)}v_{(i-n-2)} - d_{(i+n)}v_{(i+n)} + d_{(i-n)}v_{(i-n)} - d_{(i+n+2)}v_{(i+n+2)}\right) = 0; \\ &4\left(c_{(i-n-1)}v_{(i-n-1)} - 2c_i v_i + c_{(i+n+1)}v_{(i+n+1)}\right) \\ &\quad + \left(b_{(i-n)}u_{(i-n)} - b_{(i-n-2)}u_{(i-n-2)} + b_{(i+n+2)}u_{(i+n+2)} - b_{(i+n)}u_{(i+n)}\right) \\ &\quad + \left(4d_{(i-1)}v_{(i-1)} - 8d_i v_i + 4d_{(i+1)}v_{(i+1)} + d_{(i-n)}u_{(i-n)}\right. \\ &\quad \left. - d_{(i-n-2)}u_{(i-n-2)} + d_{(i+n+2)}u_{(i+n+2)} - d_{(i+n)}u_{(i+n)}\right) = 0,\end{aligned} \tag{4}$$

For the corner points of the contour, instead of (2), expressions (4) should be substituted into Eq. (1).

When deriving Eq. (4), the following designations were adopted: ai, bi, ci, di are the coefficients of reduction of the deformability parameters of materials at the nodes intersected by the reinforcing bars, as well as at arbitrary nodes of the element in question in the deformed state and determined from the relations:

$$\begin{aligned}a_i &= \frac{E_{x,i}}{1-\nu^2} = \frac{E_b + kE_s \frac{A_{sx}}{b_{col}\Delta}}{1-\nu^2}; \\ b_i &= \frac{E_{y,i}\nu_{x,i}}{1-\nu^2} = \frac{E_{x,i}\nu_{y,i}}{1-\nu^2} = \frac{E_b + kE_s \frac{A_{sy}}{b_{col}\Delta}}{1-\nu^2}\nu_{x,i} = \frac{E_b + kE_s \frac{A_{sx}}{b_{col}\Delta}}{1-\nu^2}\nu_{y,i}; \\ c_i &= \frac{E_{y,i}}{1-\nu^2} = \frac{E_b + kE_s \frac{A_{sy}}{b_{col}\Delta}}{1-\nu^2}; \\ d_i &= G_{xy,i} = \frac{E_b\sqrt{\left(1 + k\frac{E_s}{E_b}\frac{A_{sy}}{b_{col}\Delta}\right)\left(1 + k\frac{E_s}{E_b}\frac{A_{sx}}{b_{col}\Delta}\right)}}{2(1+\nu)},\end{aligned}$$

where $E_b$, $E_s$ are deformation moduli for concrete and steel rebars respectively;

$A_{sx}$, $A_{sy}$ are—cross-sectional area of reinforcing bars along the OX and OY axes, respectively, whose axes pass through the $i$-th mesh node;

$b_{col}$ is a width of column cross section;

$\Delta$ is a dimension of grid bay for the beam-column joint under consideration;

$k$ is a coefficient taking into account the transfer of shear forces from concrete to the surface of reinforcing bars and taken in the range from 0 to 1, where 1 corresponds to the full transfer of forces, and 0 corresponds to the complete absence of adhesion between the reinforcement and concrete [17–19].

## 3.2 Calculation of the Beam-Column Joint Deformed State

Using the relations obtained in the previous section, we will calculate the deformed state of the T-shaped node along the "A" axis of the scale model of the reinforced concrete frame presented in [20]. The frame reinforcement scheme is shown in Fig. 2. The materials of the frames of the first series are following: concrete of compression strength class B25, for which the standard compression resistance is $R_{b,n}$ = 18.5 MPa, and the initial elastic modulus is $E_0$ = 30,000 MPa. The columns and crossbars are reinforced by spatial reinforcement cages with axial steel wire of class Bp500 ($R_{s,n}$ = 500 MPa) and transverse steel ties of class A300 ($R_{s,n}$ = 300 MPa).

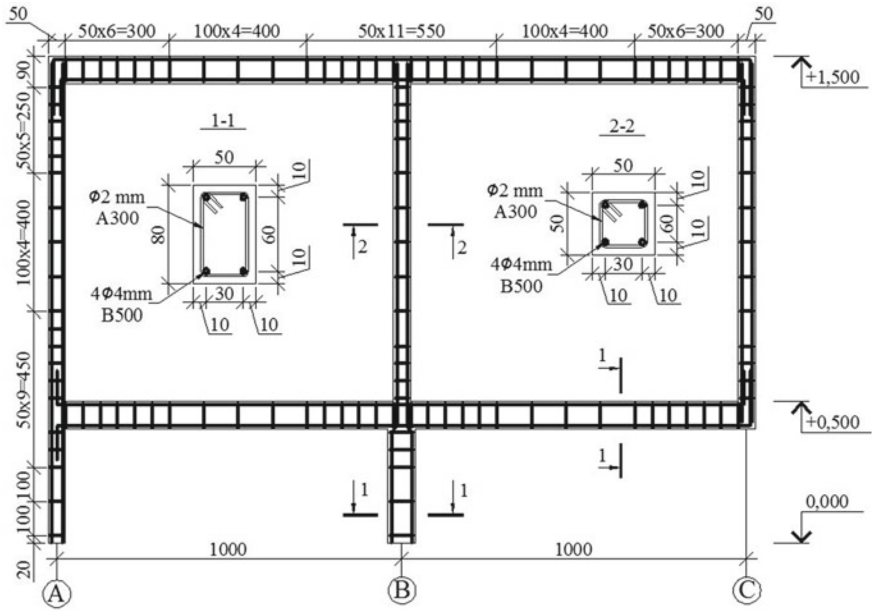


**Fig. 2** Reinforcement scheme of 2D scale model of RC moment frame

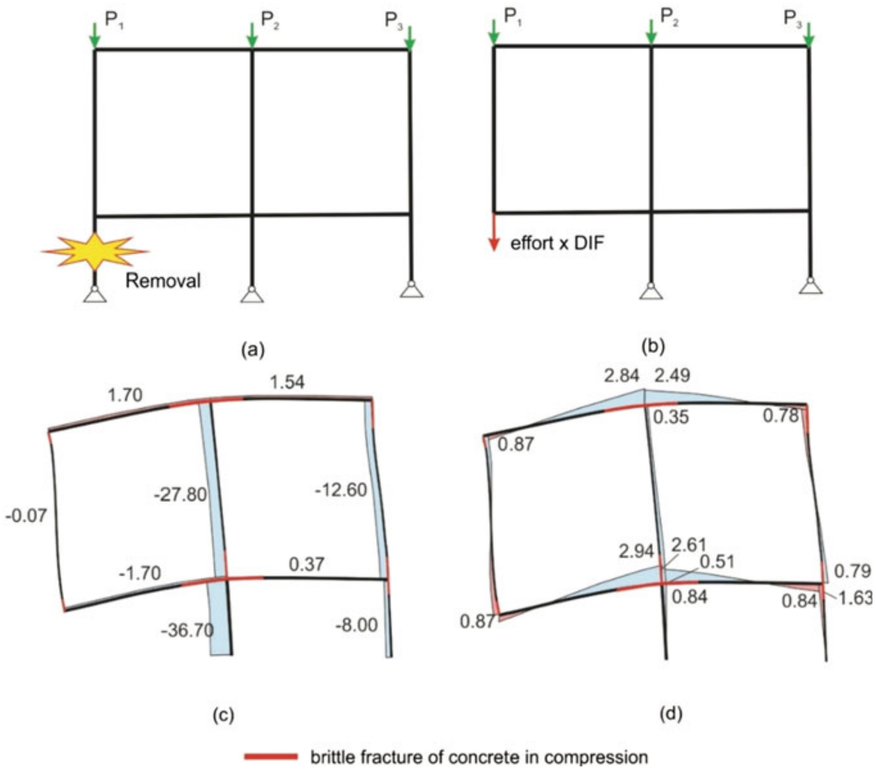


**Fig. 3** Calculation scheme of the experimental scale model of the RC frame: primary—“n” (**a**); secondary—“n − 1” (**b**); axial forces, kN (**c**) and bending moments, kNm (**d**) at t = T/4 after sudden corner column removal at the first floor

At the stage of normal operation, axial forces $P_1$ = 4 kN, $P_2$ = 20 kN, $P_3$ = 16 kN have been applied to the upper nodes of the frame as it is shown in Fig. 3. The calculation results for the 2D beam-column joint are presented in the form of displacement plats $U(x)$, m and $W(z)$, m in Figs. 4 and 5.

Therefore, we considered an example of calculating the deformed state of a girder-column node of a scale model of a reinforced concrete frame, for which a simulation was previously, performed using bar FE. Comparing results of calculation allows us to conclude that structural behavior of plane-stressed joint model and bar model differs. This can be explained by taking into account 2D stress–strain state within the 2D model of the finite difference method. That makes such solution more accurate.

In order to simulate the deformed state of a 2D reinforced concrete beam-column joint of the moment frame, the model based on a finite difference method is proposed. A distinctive feature of the proposed solution is the ability to take into account the discrete nature of the reinforcement, as well as incomplete adhesion of the reinforcement to concrete along the contact surface.

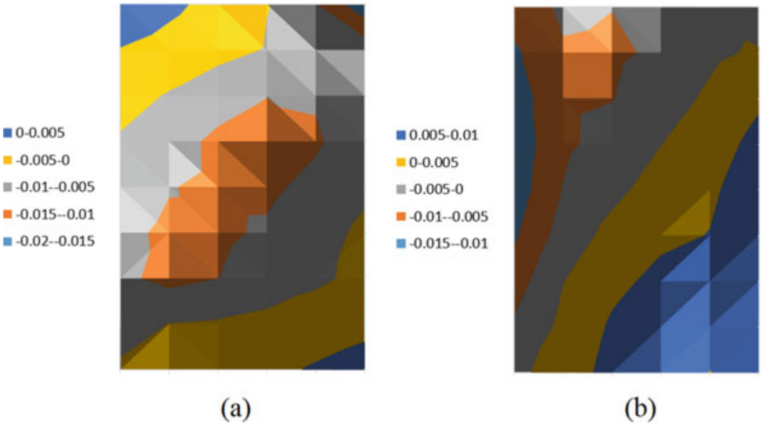


**Fig. 4** Displacement plats for the T-shaped RC frame beam-column joint along the "A" axis: U(x), m (**a**) and W(z), m (**b**)

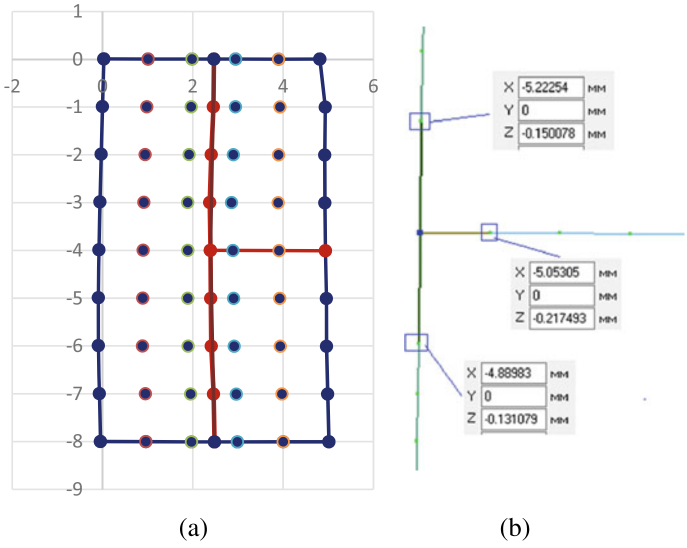


**Fig. 5** Deformed RC beam-column joint under 2D stress–strain state (**a**) and bar model (**b**)

## 4 Conclusions

Summarizing the results of the study, the following conclusions can be formulated:

1. A computational model of a 2D beam-column joint of a reinforced concrete frame in the form of a finite difference method has been built.

2. Comparison of the calculation results according to the proposed model and the traditional bar model shows the differences in the deformed state of the 2D RC beam-column joint. This can be explained by taking into account the 2D stress–strain state factors, which are not taken into account in the bar model.

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