Nonlinear Vibrations of an Orthotropic Viscoelastic Rectangular Plate Under Periodic Loads

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Abstract Modern methods and technologies for the manufacture of structures make it possible to obtain structures of various shapes and sizes. This, in turn, determines the possibility of using structures of variable thickness in modern technology and engineering. During operation, they are often subjected to various loads. Among the loads, periodic loads are of particular interest. On the basis of the Kirchhoff-Love theory, nonlinear parametric vibrations of an orthotropic viscoelastic rectangular plate of variable thickness are investigated without considering the elastic wave propagation. The mathematical model of the problem is described by a system of nonlinear integrodifferential equations, where the weakly singular Koltunov-Rzhanitsyn kernel is used as the relaxation kernel. The resolving equations of the problem are obtained by the Bubnov-Galerkin method and by a numerical method based on the use of quadrature formulas. The behavior of an orthotropic viscoelastic rectangular plate under the action of an external periodic load is investigated. The graphs obtained with the developed computer program show the effect on the amplitude-frequency response of the plate on various physical, mechanical, and geometrical parameters.

Keywords Rectangular plate · Viscoelasticity · Orthotropy · Variable thickness · Periodic load · Nonlinear parametric oscillations

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1 Introduction

Thin-walled shell structures of variable thickness made of composite materials are among the most widespread structures used in many fields of modern technology. This is due to the great functionality of such structures and the successful combination of their properties of lightness and strength. Modern technological progress makes it possible to manufacture structures of various shapes and sizes made of various materials.

At the same time, during operation, such structures are subjected to various dynamic loads. Among such loads, periodic loads are of particular interest for research. On the other hand, difficulties arise in the calculation and design of such structures, and the determination of their stress-strain state causes both computational and principal difficulties. Therefore, at present, the development of new mathematical models, improvement of calculation methods and algorithms is one of the urgent tasks.

Currently, there are many publications devoted to the construction of various theories, models and methods to assess the stress-strain state of thin-walled shell structures of constant and variable thickness under the action of various static and dynamic loads. A significant contribution to the study of such problems was made by Bolotin [1], Volmir [2], and over the past decades by Awrejcewicz and Krys'ko [3], Amabili [4], Grigorenko and Grigorenko [5].

There are a number of articles devoted to the study of vibrations and stability of plates and shells of variable thickness.

The study in [6] is devoted to the parametric vibrations of plates under the action of static and periodic loads. A numerical-analytical method for solving the problem using the Bolotin method was proposed. In that, the plates can have an arbitrary geometric shape.

In [7], the problem of parametric vibrations of an isotropic cylindrical shell of variable thickness under the action of a load along its generatrix is considered. An exact solution is obtained for different ratios of the parameters.

The study in [8] is devoted to the parametric vibrations of conical shells of variable thickness under static and periodic loads. Using the Galerkin method, the problem is reduced to solving an equation of the Mathieu type. The influence of various parameters on the region of dynamic instability was studied.

In [9], free vibrations of composite shells and plates of variable thickness are investigated. The solutions obtained are compared with analytical and numerical solutions known in the literature.

The development of a technique for solving the three-dimensional problem of bending of orthotropic plates of variable thickness is given in [10]. The problem is reduced to solving two independent problems, described by two independent systems of two-dimensional infinite equations.

Loja et al. [11] is devoted to the determination of the dynamic instability of composite plates of variable thickness. To solve the problem, the Bolotin method was used. The influence of various geometric parameters, as well as the properties of the material on the stability of the plate, was studied.

In [12], parametric vibrations of composite plates of variable thickness under periodic loads were investigated. The mathematical model of the problem is described by an equation of the Karman type. The finite element method is used to solve the problem.

The study of free vibrations of shells with a linear change in thickness under various boundary conditions is given in [13]. The finite element method is used to solve the problem. The influence of the variability of the thickness, dimensions of the shell and other parameters on the amplitude-frequency response of its vibrations was studied.

In [14], the finite element method is used to solve the problem of dynamic stability of rectangular panels of variable thickness under compressive loads. The problem is reduced to solving the system of Mathieu-Hill equations.

In [15], on the basis of the classical theory of shells in a nonlinear formulation, forced vibrations and dynamic stability of cylindrical shells of variable thickness subjected to mechanical stress are investigated. To derive the resolving equations, the methods of Galerkin and Runge-Kutta were used.

Analysis of published works shows that insufficient attention was paid to nonlinear parametric vibrations of nonhomogeneous viscoelastic plates and shells of variable thickness [16–18].

The article investigates nonlinear parametric vibrations of an orthotropic viscoelastic rectangular plate of variable thickness without considering elastic wave propagation.

2 Materials and Methods

Consider an orthotropic viscoelastic plate, rectangular in plan, with sides a and b, variable thickness $h = h(x, y)$, and with initial imperfections $w_0 = w_0(x, y)$; the plate is subjected to acting periodic load $P(t) = P_0 + P_1 \cos \Theta t$ ($P_0, P_1 = const$; Θ is the frequency of an external periodic load) along side a.

We will assume that there are no tangential inertial forces. Then the system of three equations [16] can be reduced to a system of two equations with two unknowns.

Following the results of [2], we introduce the stress function Φ in the middle surface in accordance with the following formulas:

$$\sigma_x = \frac{N_x}{h} = \frac{\partial^2 \Phi}{\partial y^2}, \quad \sigma_y = \frac{N_y}{h} = \frac{\partial^2 \Phi}{\partial x^2}, \quad \tau_{xy} = \frac{N_{xy}}{h} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{1}$$

The following equation of the Karman type is obtained:

$$
\begin{aligned}
&(1-\Gamma^*)\left\{\frac{h^3}{12}\left[B_{11}\frac{\partial^4(w-w_0)}{\partial x^4}+(8B+B_{12}+B_{21})\frac{\partial^4(w-w_0)}{\partial x^2\partial y^2}+B_{22}\frac{\partial^4(w-w_0)}{\partial y^4}\right]\right.\\
&\quad+\frac{1}{4}\left[2h\left(\frac{\partial h}{\partial x}\right)^2+h^2\frac{\partial^2 h}{\partial x^2}\right]\left(B_{11}\frac{\partial^2(w-w_0)}{\partial x^2}+B_{12}\frac{\partial^2(w-w_0)}{\partial y^2}\right)\\
&\quad+\frac{1}{2}h^2\frac{\partial h}{\partial x}\left[B_{11}\frac{\partial^3(w-w_0)}{\partial x^3}+(B_{12}+4B)\frac{\partial^3(w-w_0)}{\partial x\partial y^2}\right]\\
&\quad+\frac{1}{2}h^2\frac{\partial h}{\partial y}\left[B_{22}\frac{\partial^3(w-w_0)}{\partial y^3}+(B_{21}+4B)\frac{\partial^3(w-w_0)}{\partial x^2\partial y}\right]\\
&\quad+\frac{1}{4}\left[2h\left(\frac{\partial h}{\partial y}\right)^2+h^2\frac{\partial^2 h}{\partial y^2}\right]\left(B_{22}\frac{\partial^2(w-w_0)}{\partial y^2}+B_{21}\frac{\partial^2(w-w_0)}{\partial x^2}\right)\\
&\quad\left.+\left[2h\frac{\partial h}{\partial x}\frac{\partial h}{\partial y}+h^2\frac{\partial^2 h}{\partial x\partial y}\right]2B\frac{\partial^2(w-w_0)}{\partial x\partial y}\right\}\\
&\quad=\frac{12(1-\mu_1\mu_2)}{\sqrt{E_1E_2}}\left[h\frac{\partial^2 w}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}+h\frac{\partial^2 w}{\partial y^2}\frac{\partial^2\Phi}{\partial x^2}-2h\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\right]\\
&\quad+\frac{12(1-\mu_1\mu_2)}{\sqrt{E_1E_2}}q-P(t)\frac{\partial^2 w}{\partial x^2}-\frac{12(1-\mu_1\mu_2)}{\sqrt{E_1E_2}}\rho h\frac{\partial^2 w}{\partial t^2},\\
&\delta_2\frac{\partial^4\Phi}{\partial x^4}+2\delta_3\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\delta_1\frac{\partial^4\Phi}{\partial y^4}=(1-\Gamma^*)\left\{\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right\},
\end{aligned}
\tag{2}
$$

where $\delta_1=\frac{1}{E_1}$, $\delta_2=\frac{1}{E_2}$, $2\delta_3=\frac{1}{G}-\frac{\mu_1}{E_1}-\frac{\mu_2}{E_2}=\frac{1}{G}-\frac{2\mu_1}{E_1}$.

The solution of the system (2) with respect to the deflection w and the stress function Φ is found in the form

$$
w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\psi_{nm}(x,y),\quad \Phi(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\Phi_{nm}(t)\chi_{nm}(x,y) \tag{3}
$$

where $w_{nm}=w_{nm}(t)$ and $\Phi_{nm}=\Phi_{nm}(t)$—are the unknown time functions; $\psi_{nm}(x,y)$, $\chi_{nm}(x,y)$, $n=1,2,...,N$; $m=1,2,...,M$ are the coordinate functions that satisfy the boundary conditions of the problem.

Substituting (3) to (2) and introducing the following dimensionless quantities

$$
\begin{aligned}
&\overline{w}=\frac{w}{h_0};\ \overline{w}_0=\frac{w_0}{h_0};\ \overline{x}=\frac{x}{a};\ \overline{y}=\frac{y}{b};\ \overline{t}=\omega t;\ \overline{h}=\frac{h}{h_0};\ \lambda=\frac{a}{b};\ \delta=\frac{b}{h_0};\\
&\overline{q}=\frac{q}{\sqrt{E_1E_2}}\left(\frac{b}{h_0}\right)^4;\ \overline{\theta}=\frac{\theta}{\omega};\ \omega t;\ \frac{\Gamma(t)}{\omega};\ \delta_0=\frac{P_0}{P_{cr}};\ \delta_1=\frac{P_t}{P_{cr}};\ \Delta=\sqrt{E_1/E_2};\ g=\frac{G_{12}}{\sqrt{E_1E_2}}
\end{aligned}
$$

while retaining the previous notation with respect to the unknowns $w_{nm}=w_{nm}(t)$, $\Phi_{nm}=\Phi_{nm}(t)$, we obtain the following system

$$
\begin{aligned}
& 2\pi^4\lambda^4[1+\Delta\mu_2+2(1-\mu_1\mu_2)g]\sum_{n=1}^{N}\sum_{m=1}^{M}a_{k\ln m}\ddot{w}_{nm} \\
& \quad +\sum_{n=1}^{N}\sum_{m=1}^{M}p_{k\ln m}^2(1-2\mu_{k\ln m}\cos\Theta t)w_{nm}-(1-\Gamma^*)\sum_{n=1}^{N}\sum_{m=1}^{M}a_{1k\ln m}w_{nm} \\
& \quad =12(1-\mu_1\mu_2)\lambda^2\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}a_{2k\ln mij}w_{nm}\Phi_{ij}+12(1-\mu_1\mu_2)\lambda^4 q_{kl}, \\
& \quad \sum_{n=1}^{N}\sum_{m=1}^{M}b_{k\ln mij}\Phi_{nm}=\lambda^2\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}b_{1k\ln mij}w_{nm}w_{ij}, \\
& \quad w_{nm}(0)=w_{0nm},\dot{w}_{nm}(0)=\dot{w}_{0nm}
\end{aligned}
\tag{4}
$$

The solution of the system (3) is found by the numerical method proposed in [19]. This method is based on the use of quadrature formulas and eliminates the singularity in the relaxation kernel. In this case, a weakly singular Koltunov-Rzhanitsyn kernel of the following form is used as a relaxation kernel [20]:

$$\Gamma(t)=Ae^{-\beta t}\cdot t^{\alpha-1},\quad A>0,\quad \beta>0,\quad 0<\alpha<1 \tag{5}$$

An efficient computational algorithm was developed and a program in the Delphi algorithmic language was developed and implemented on a computer. The results of the study of parametric vibrations of orthotropic viscoelastic plates of variable thickness without taking into account the propagation of elastic waves at various physical and geometric parameters are given in the form of graphs.

The thickness variation law is set analytically and, in the general case, can be of any form. To obtain numerical results, the law of thickness variation is chosen in the form:

$$h=1-\alpha^*x, \tag{6}$$

where α^* is the parameter of the change in thickness.

3 Results and Discussion

The influence of the viscoelastic properties of the material on the behavior of the plate is shown in Fig. 1. The results of the study show that an account for the viscosity of the plate material leads to a decrease in the vibration amplitude.

Figure 2 shows the influence of the nonhomogeneity parameter Δ on the behavior of the viscoelastic plate. The results show that an account for the non-homogeneous properties of the plate material leads to a decrease in the vibration amplitude.

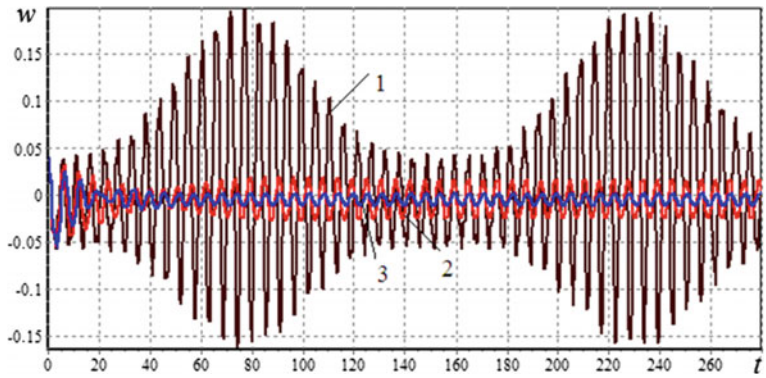

Fig. 1 The dependence of the deflection on time: $A_{ij} = 0$ (1); 2, $A_{ij} = 0.05$ (2); 3, $A_{ij} = 0.1$ (3)

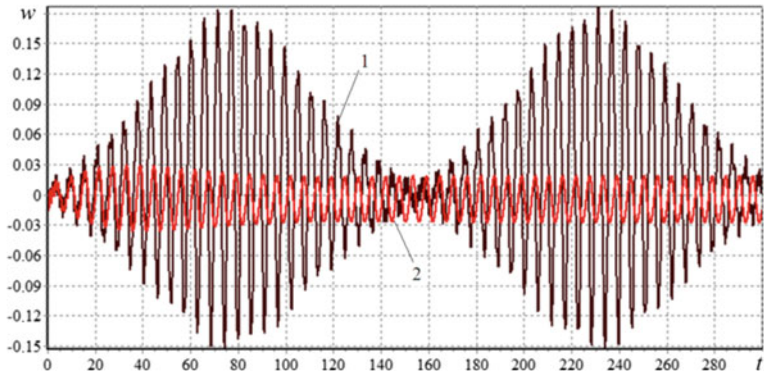

Fig. 2 The dependence of the deflection on time: $A_{ij} = 0.02$, $\Delta = 1.1$ (1); $A_{ij} = 0.02$, $\Delta = 1.3$ (2)

The change in the deflection of a viscoelastic plate as a function of time for different values of the thickness variability parameter α^* is shown in Fig. 3. It is seen that an increase in this parameter leads to a decrease in the vibration amplitude.

Figure 4 shows the results of studying the parametric vibrations of an orthotropic viscoelastic rectangular plate of variable thickness under various boundary conditions. The results obtained show that with an increase in the number of fixed sides of the plate, the vibration amplitude decreases, and the vibration frequency increases.

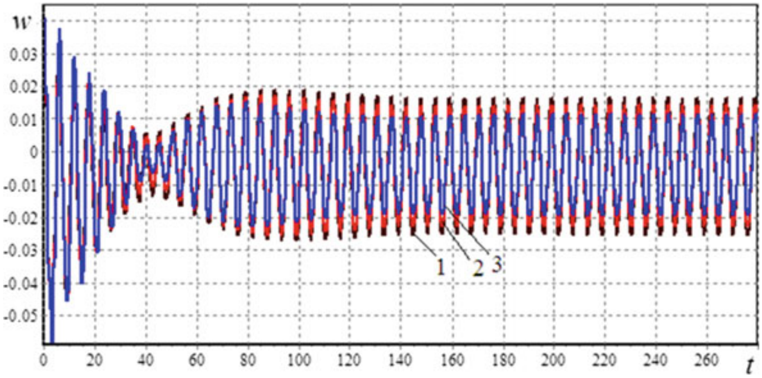

Fig. 3 The dependence of the deflection on time: α* = 0 (1); α* = 0.3 (2); α* = 0.5 (3)

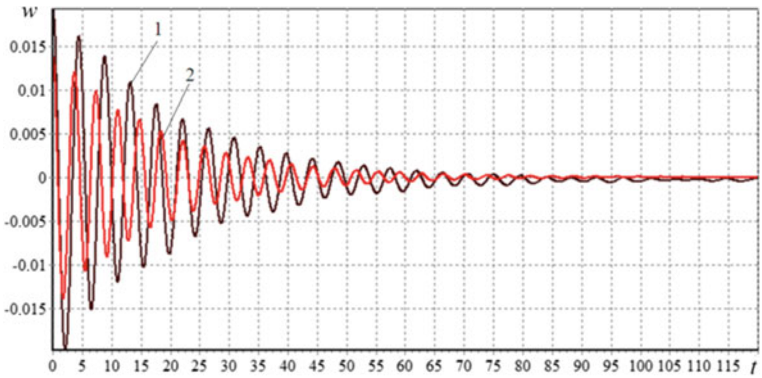

Fig. 4 The dependence of the deflection on time: two opposite sides are simply supported, the other two are clamped (1); all sides are clamped (2)

4 Conclusions

The study of parametric vibrations of orthotropic viscoelastic rectangular plates of variable thickness gave the following results:

1. A mathematical model, a method and an algorithm for solving the problem of parametric vibrations of orthotropic viscoelastic rectangular plates of variable thickness without considering the propagation of elastic waves were developed.
2. The proposed method can be used for various viscoelastic thin-walled structures such as plates, panels and shells of variable thickness.
3. The developed technique allows obtaining the results of the study of parametric vibrations of an orthotropic viscoelastic plate of variable thickness and for other laws of thickness variation.

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