



Development of a Mathematical Model of Electromechanical Dampers with a Regenerative Effect in Vehicle Wheel Suspensions

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Abstract. The article provides a brief classification of electromechanical dampers in vehicle suspensions. As investigated version of design of electromechanical shock absorber is suggested model with ball-screw drive of generator as investigated version of design of electromechanical shock absorber. The main components of the damping force are indicated: the force of inertia and the force of electromagnetic resistance. The given resistance forces are expressed taking into account the drive parameters. Presented mathematical model of electromechanical suspension damper of vehicle wheels considers parameters of motion as vehicle speed and height, and length of overcome irregularity is taken into account. Suggested model makes it possible to determine kinematic parameters of movable elements of electromechanical damper values of inertial and electromagnetic components of damping force and also generated electric energy, on conditions of vehicle movement and connected external load. Due to the presented dependencies, it was revealed that when determining the power of the generator for operation as part of an electromechanical damper, the most significant parameters are the rotation speed and efficiency of the generator, which in turn depend on the rotation speed of its rotor and the load connected to the stator outputs. The interrelationships presented in the work make it possible to investigate the influence of mass size and electromagnetic parameters of elements of electromechanical dampers on their operating characteristics, to use them as recommendations when designing electromechanical dampers.

Keywords: Electromechanical shock absorber · Moment of inertia · Damping · Recovery · Damping energy

1 Introduction

When moving the vehicle, the energy of mechanical vibrations in damping devices of wheel suspensions, which is significant, can be recovered into electric and then used in the onboard vehicle network, thereby reducing fuel consumption of cars with ICE operating on hydrocarbon fuel or increasing the range for electric vehicles. [1–3] Several embodiments of the electromechanical shock absorber are known [4–6]:

- Linear electromechanical shock absorber [7–9];
- Electromechanical shock absorber with hydraulic drive [10];
- Electromechanical shock absorber with mechanical drive [11, 12];
- Electromechanical shock absorber driven by ball-screw transmission, etc. [13, 14].

To study the effect of various design parameters of the electromechanical shock absorber on its performance, a physical model of the most promising damper equipped with a ball-screw drive and a rotary type generator is proposed (Fig. 1).

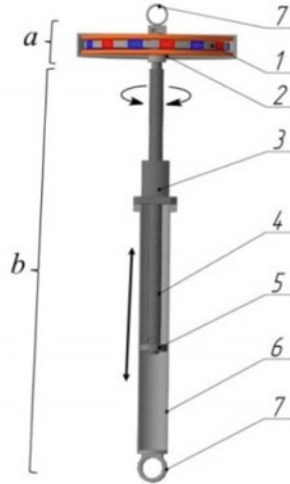


Fig. 1 Physical model of electromechanical shock absorber with a ball-screw drive. **a** Generator; **b** Ball-screw drive; (1) rotor; (2) stator; (3) drive nut; (4) shaft bushing; (5) movable support; (6) case; (7) eyelet

The principle of electromechanical shock absorber operation with a ball-screw drive consists the following: reciprocating movements of wheels relative to vehicle body perceived by eyelet, housing, and nut of ball-screw drive are converted into rotary movement of ball-screw drive shaft, which in turn drives generator rotor.

2 Methods and Equipment

When evaluating the damping and recuperative properties of the electromechanical shock absorber, two main components should be taken into account: mechanical, depending on the design parameters of the electromechanical shock absorber, and electromagnetic, depending on the operating characteristics of the generator and the external electrical load connected.

Inertial components include the force of inertia (N) in the case of reciprocating motion and the inertial moment ($N\ m$) of resistance in the case of rotational motion of

electromechanical shock absorber elements, including the generator rotor, determined by formulae (1) and (2), respectively.

$$F_i = m \cdot a \quad (1)$$

where m —weight, kg; a —acceleration of the body, m s^{-2} .

$$M_i = J \cdot \varepsilon \quad (2)$$

where J —moment of inertia of the rotating body, kg m^2 ; ε —angular acceleration of a body, radian/s .

Moment of inertia of the rotating body, kg m^2 :

$$J = \int r^2 dm = \int_v \rho r^2 dV \quad (3)$$

where r —distance from axis to elementary mass dm , m ; dV —elementary volume occupied by dm , m ; ρ —body density at the point where dm is, kg/m^3 .

Electromechanical component can be estimated by moment of resistance on generator rotor shaft, N m :

$$M_r = \frac{N_e \cdot \eta_g}{\omega} \quad (4)$$

where N_e —generator electric power at ω , W ; η_g —efficiency of the generator at ω and N_e ; ω —angular speed of generator rotor rotation, radian/s .

The reduced force from the moments of the inertial and electromagnetic resistance elements to the nut of the ball-screw drive (bsd) is expressed taking into account the parameters of the drive expressed by Eq. (5), H :

$$F_r = \frac{2\pi \cdot M \cdot \eta_{bsd}}{P} \quad (5)$$

where M —driven moment, (N m); P —ball-screw transmission pitch, (m); η_{bsd} —efficiency of ball-screw transmission.

The ratio of the rotation angle of the ball-screw gear shaft to the amount of movement of its nut is determined by the dependence:

$$\varphi = \frac{2\pi x}{Pk} \quad (6)$$

where x —nut movement, m ; k —number of ball-screw gear thread starts.

To assess the effect of electromechanical shock absorber on suspension operation as a whole, a mathematical model of vehicle suspension with electromechanical damper has been compiled (Fig. 2).

Figure 3 shows the model of the vehicle suspension with disturbing effect and without taking into account the stiffness of tires and ground $C_{(t+g)}$, so the ground and tire of the wheel are accepted as absolutely solid bodies.

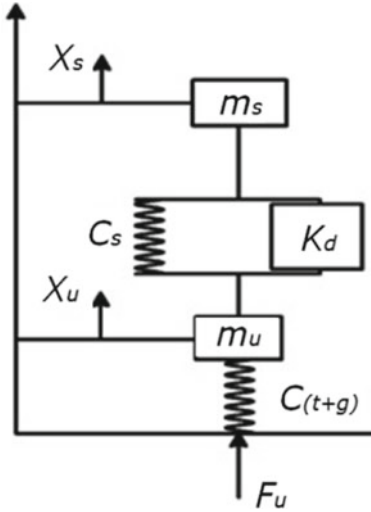


Fig. 2 Mathematical model of vehicle suspension. m_s —sprung weight, kg; x_s —movement of sprung weight, m; C_s —spring stiffness, N/m; K_d —damping coefficient, N•s/m; m_u —unsprung weight, kg; x_u —movement of unsprung weight, m; $C_{(t+g)}$ —tire and ground stiffness, N/m; F_u —the equivalent of all forces on the wheel transmitted from the unsprung part, N

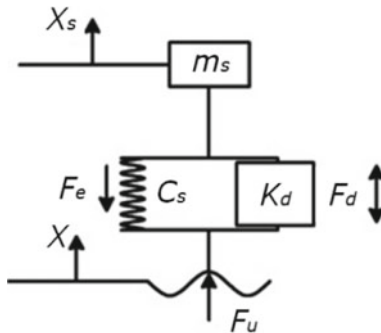


Fig. 3 Suspension model excluding tire and ground stiffness $C_{(t+g)}$

F_e —elastic force of springs, N; F_d —the strength of the damping, N; X —movement of unsprung mass (nut movement), m.

For the maximum comfort of the driver and passengers in the car, it will be ideal if the movement of the body, i.e., the sprung mass of the car, is zero ($x_u = 0$) under any disturbing effects. It is necessary to determine the resistance force at which this condition will be met.

The sprung part of the vehicle can start to move upward if the spring begins to expand or if the resistance force of the shock absorber is too high, as a result of which all the load from the unsprung part will be transferred to the sprung mass.

Equilibrium equation of this system:

$$m_s \cdot \ddot{x}_s = -m_s \cdot g - C_n \cdot (x_u - x_s) - K_d \cdot (\dot{x}_u - \dot{x}_s) + F_u \quad (7)$$

where \ddot{x}_s —acceleration of the compressed mass, m/s^2 ; \dot{x}_u —speed of unsprung mass movement, m/s ; \dot{x}_s —speed of the compressed mass movement, m/c. ; g —acceleration of free fall, m/s^2 ; m_s —sprung weight, kg ; K_d —damping coefficient, $\text{N}\cdot\text{s/m}$; C_n —spring stiffness, N/m ; F_u —equal to all forces on the wheel transmitted from the unsprung part, N .

To ensure the immobility of the pressed mass, Eq. 7 should take the form:

$$m_s \cdot g = -C_n \cdot x - K_d \cdot \dot{x} + F_u \quad (8)$$

In order for the spring part to be stationary, it is necessary that the gravity of the spring mass, together with the forces generated in the suspension itself, be compensated by the force transmitted from the spring part.

Force F_u —equal to all forces on the wheel transmitted from the unsprung part is:

$$m_u \cdot \ddot{x} = F_u \quad (9)$$

where m_u —unsprung mass, kg .

Consider the most interesting part of the presented model—the damping coefficient K_d , which is a characteristic of an electromechanical shock absorber, depending on its controlled and constant parameters. The damping coefficient characterizes the force of resistance to the movement of the ball-screw gear nut relative to the shaft depending on the driving conditions and is composed of the given forces and moments of inertia $F_{i,,}$, as well as the reduced electromechanical force of resistance F_{em} to the ball-screw gear nut.

The law of movement of the wheel from disturbing effects of the road is represented by Eq. (10):

$$x(t) = A \cdot \sin(\omega \cdot t) \quad (10)$$

where A —amplitude, m ; ω —frequency, Hz ; t —time, s .

Then, the law of movement of the ball-screw gear nut as part of the electromechanical shock absorber:

$$x(t) = i_s \cdot A \cdot \sin(\omega \cdot t) \quad (11)$$

where i_s —gear ratio of vehicle wheel suspension.

Converting Eq. 11 to describe the vehicle motion conditions taking into account the following parameters:

- vehicle speed, S_v (m/s);
- height of the unevenness to be overcome, A (m);
- length of irregularity, λ (m);

we will receive:

$$x(t) = i_s \cdot A \sin(\omega \cdot t) = i_s \cdot A \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (12)$$

$$\dot{x}(t) = i_s \cdot A \cos\left(\frac{2\pi \cdot S_v}{\lambda}\right) \cdot t \quad (13)$$

$$\ddot{x}(t) = -i_s \cdot A \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2 \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (14)$$

Figure 4 graphically shows the dependencies of movement (x), speed (\dot{x}) and acceleration (\ddot{x}) of unsprung mass on time.

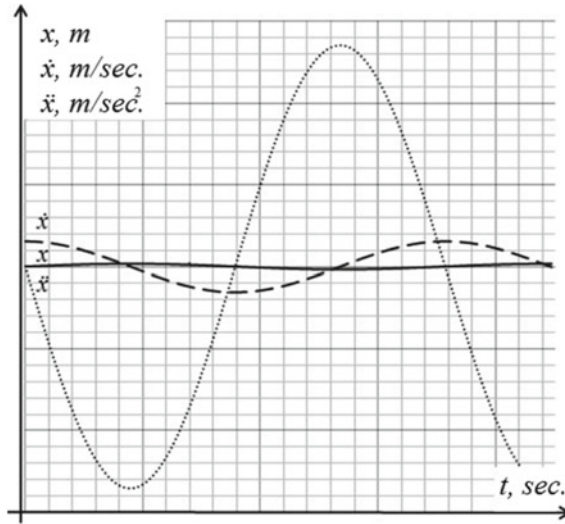


Fig. 4 Dependencies of movement, speed and acceleration of unsprung mass on time

The condition of the ball-screw drive shaft of the electromechanical shock absorber is described by the following equations:

$$\varphi(x) = \frac{2\pi \cdot x}{P \cdot k} = \frac{2\pi \cdot i_s \cdot A \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right)}{P \cdot k} \quad (15)$$

$$\dot{\varphi}(x_{np}) = \frac{2\pi \cdot \dot{x}}{P \cdot k} = \frac{2\pi \cdot i_s \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (16)$$

$$\ddot{\varphi}(x) = \frac{2\pi \cdot \ddot{x}}{P \cdot k} = -\frac{2\pi \cdot i_s \cdot A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (17)$$

These equations are also true for the generator rotor in the case of direct drive of the generator (Fig. 1). If a multiplier is used to increase the rotor speed of the generator, the equations of its motion will take the form:

$$\gamma(x) = \frac{2\pi \cdot i_m x}{P \cdot k} = \frac{2\pi \cdot i_s \cdot i_m \cdot A \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right)}{P \cdot k} \tag{18}$$

$$\dot{\lambda}(x) = \frac{2\pi \cdot i_m \cdot \dot{x}}{P \cdot k} = \frac{2\pi \cdot i_s \cdot i_m \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \tag{19}$$

$$\ddot{\gamma}(x) = -\frac{2\pi \cdot i_m \ddot{x}}{P \cdot k} = -\frac{2\pi \cdot i_s \cdot i_m \cdot A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \tag{20}$$

where i_m —multiplier gear ratio.

According to a given law of motion, it is possible to determine the state of the movable parts of the electromagnetic damper at any moment in time, therefore, to determine the effect of weight and size and design parameters on the operation of the damper as a whole.

So the force of inertia from the reciprocating parts, we define from the equation, N :

$$F_{IR}(x) = -i_s \cdot A \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2 \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \cdot \sum m_i \tag{21}$$

where $\sum m_i$ —sum of masses moving with one acceleration.

The damper under investigation is represented as a link consisting of inertial and electromagnetic elements connected in parallel to Fig. 5.

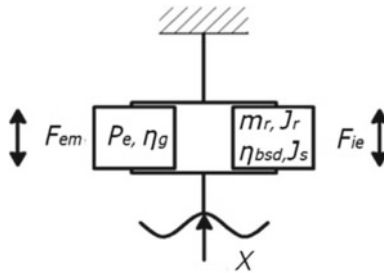


Fig. 5 Electromechanical damper model. F_{ie} —the reduced force to the upper eye of the damper from the inertial moment of resistance, N; F_{em} —the reduced force to the upper eye of the damper from the electromechanical moment of resistance, N; P_e —generator power, W; η_g —generator efficiency; m_r —weight of the rotor, kg; J_r —moment of inertia of the generator rotor, kg m²; η_{bsd} —ball-screw drive; J_s —moment of inertia of the shaft bushing, kg m²

Inertial moment of resistance from rotating elements of damper is equal to product of angular acceleration at moment of inertia, N m:

$$M_r(x) = -\frac{2\pi \cdot i_s A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \cdot \sum J_i \tag{22}$$

where $\sum J_i$ —sum of moments of inertia of elements moving with one angular acceleration.

For elements installed after the multiplier, N m:

$$M_{rm}(x) = -\frac{2\pi i_s \cdot i_m \cdot A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \cdot \sum J_i \quad (23)$$

The reduced force to the upper eye of the damper from the inertial moment of resistance is, N:

$$\begin{aligned} F_{ie}(x) &= \frac{2\pi \cdot M_r \cdot \eta_{bsd}}{P} \\ &= 2\pi \cdot \eta_{bsd} \left(-\frac{2\pi \cdot i_s A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \cdot \sum J_i \right) / P \quad (24) \end{aligned}$$

with multiplier:

$$\begin{aligned} F_{iem}(x) &= \frac{2\pi \cdot M_r \cdot \eta_{bsd} \cdot \eta_m}{P} \\ &= 2\pi \cdot \eta_{bsd} \cdot \eta_m \left(-\frac{2\pi \cdot i_s \cdot i_m \cdot A \cdot \left(\frac{2\pi \cdot S_v}{\lambda}\right)^2}{P \cdot k} \cdot \sin\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \cdot \sum J_i \right) / P \quad (25) \end{aligned}$$

Electromechanical moment of resistance of electromechanical shock absorber is expressed, N m:

$$\begin{aligned} M_e(x) &= \left(\frac{U^2}{R+r} \cdot \eta_g \right) / \frac{2\pi \cdot i_s \cdot \dot{x}}{P \cdot k} \\ &= \left(\frac{U^2}{R+r} \cdot \eta_g \right) / \frac{2\pi \cdot i_s \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (26) \end{aligned}$$

where U —voltage in the circuit, V; R —load resistance, Ohm; r —reactance, Ohm; η_g —generator efficiency.

or:

$$\begin{aligned} M_e(x) &= (I \cdot U \cdot \eta_g) / \frac{2\pi \cdot i_s \cdot \dot{x}_{np}}{P \cdot k} \\ &= (I \cdot U \cdot \eta_g) / \frac{2\pi \cdot i_s \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda} \cdot t\right) \quad (27) \end{aligned}$$

where I —current in the circuit, A.

The reduced force to the upper eye of the damper from the electromechanical moment of resistance is, N :

$$\begin{aligned}
 F_{em}(x) &= \frac{2\pi \cdot M_e \cdot \eta_{bsd}}{P} = 2\pi \cdot \eta_{bsd} \left(\left(\frac{U^2}{R+r} \cdot \eta_g \right) \Big/ \frac{2\pi \cdot i_s \cdot \dot{\gamma}_{np}}{P \cdot k} \right) \Big/ P \\
 &= 2\pi \cdot \eta_{bsd} \left(\left(\frac{U^2}{R+r} \cdot \eta_g \right) \Big/ \frac{2\pi \cdot i_s \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda}\right) \right) \Big/ P \quad (28)
 \end{aligned}$$

or:

$$\begin{aligned}
 F_{em}(x) &= 2\pi \cdot \eta_{bsd} \left((I \cdot U \cdot \eta_g) \Big/ \frac{2\pi \cdot i_s \cdot \dot{\gamma}_{np}}{P \cdot k} \right) \Big/ P \\
 &= 2\pi \cdot \eta_{bsd} \left((I \cdot U \cdot \eta_g) \Big/ \frac{2\pi \cdot i_s \cdot A \cdot \frac{2\pi \cdot S_v}{\lambda}}{P \cdot k} \cdot \cos\left(\frac{2\pi \cdot S_v}{\lambda}\right) \right) \Big/ P \quad (29)
 \end{aligned}$$

3 Research Results

It follows from the above that the sum of all forces brought to the nut of the electromechanical shock absorber ball-screw drive will be the resistance force of the damper.

$$F(x) = F_{iR}(x) + F_{ie}(x) + F_{iem}(x) + F_{em}(x) \quad (30)$$

Since the only controlled parameter in Eq. (30) is $F_{em}(x)$ —the electromechanical resistance force brought to the upper lug of the damper, it is advisable to consider it as the main component when calculating the characteristics of the elements of the electromechanical damper and express the electromechanical resistance force from Eq. (30).

$$F_{em}(x) = F_{iR}(X) + F_{ie}(x) + F_{iem}(x) - F(x) \quad (31)$$

From Eqs. (4)–(6), we express the generated electric power of the generator, W :

$$N_e = \frac{\omega \cdot F_{em} \cdot P}{2\pi \cdot \eta_g \cdot \eta_{bsd}} \quad (32)$$

where F_{em} —reduced damping force to the upper eye of the shock absorber from the moment of generator resistance, N .

It can be seen from Eq. (32) that when determining the power of a generator for operation as part of an electromechanical damper, the most significant parameters are the rotation speed and efficiency of the generator, which in turn depend on the rotation speed of its rotor and the load connected to the stator outputs.

4 Conclusions

The use of an electromechanical shock absorber in vehicle suspensions will allow changing the damping level in a wide range, as well as using recuperable energy to perform useful work. The presented dependencies can be used when studying the effect of mass and electromagnetic parameters of electromechanical shock absorber elements on its properties, as well as recommendations for designing an electromechanical shock absorber. To reduce the effect of inertia forces on the generator drive, for example, it is possible to use generators with a smaller rotor radius, with equal power characteristics, and also it is possible to install elastic and friction elements between the generator rotor and the ball-screw transmission shaft.

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