Plasticity Theory in Strength Calculations Concrete Elements Under Local Compression

Volodymyr Pohribnyi, Oksana Dovzhenko, Oleksiy Fenko, and Dmytro Usenko

Abstract To determine the strength of compressed elements, a variational method was proposed in the theory of concrete plasticity using the principle of virtual velocities. Plastic strain of concrete are considered to be localized in thin layers on the surface of failure (jump of velocities). The given basic dependences of the variational method. The problem of the strength of a concrete base when a rectangular stamp is pressed in by the method of characteristic lines under conditions of a plane stress state and plane strain has been solved and by the proposed method with a comparison of the results obtained. Two cases of destruction of a concrete base under local compression without splitting and during its implementation are considered. Dependencies provided for determining the ultimate load value under the stamp. The boundary between the cases of destruction is established depending on the ratio of the height of the base to the width of the stamp.

Keywords Principle of virtual velocities · Functional of the method · Stationary state · Strength · Concrete base · Indentation of a stamp

1 Introduction

In construction, widespread concrete elements with various shapes, geometric dimensions, the nature of the load application, the specifics of the stress state. As a result of the variety of proposals for assessing their strength [1–7], questions arise regarding the choice of design dependencies. Modern energy efficient structural solutions require refinement of calculations [8–10]. The empirical approach has a narrow field of application, limited by the experimental conditions, and the extension of the obtained formulas to other cases of operation of compressed elements can lead to errors in assessing their strength. Therefore, the creation of a fairly general methodology for calculating the strength of concrete elements in compression on a theoretical basis is an urgent task. As such a theoretical basis, the theory of plasticity can be used,

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the mathematical apparatus of which is widely tested for plastic materials. Scientific research is devoted to this direction [11–17]. At the National University «Yuri Kondratyuk Poltava Polytechnic» a variational method in the theory of plasticity of concrete using the principle of virtual velocities has been developed for calculating the strength of concrete and reinforced concrete elements under the shear [18–20]. The method considers discontinuous solutions and has experimental confirmation on keyed joints, models of a compressed zone above a dangerous inclined crack, samples that were recommended for determining the resistance of concrete with a "pure shear", etc. [21, 22]. There is a possibility of using it to solve problems of the strength of concrete elements under local compression [23]. Taking into account the specifics of their stress–strain state will make it possible to add additions to the basic dependencies of the variational method.

2 The Main Dependencies of the Principle of Virtual Speeds

The limited plastic properties of concretes cause localization of severe strain in the area of uneven compression in thin layers on the failure surface. Therefore, the functional of the principle of virtual velocities, at which only the strain rates vary, can be written in the form

$$J = \int_{S_l} (TH' + \sigma\xi')\Delta n dS - \int_{S_F} F_i \upsilon_i' dS, \tag{1}$$

where T – the shear stress intensity; H' – the intensity of shear strains; σ – the average stress; ξ' – the volumetric strain; S_l – the failure surface area; Δn – the thickness of the plastic layer; F_i – the surface force; υ_i' – the speed of the F_i application point; S_F – the area of action F_i.

The condition of concrete strength is accepted [24], which in the area of triaxial non-uniform compression is considered as a condition for the onset of yield of materials with different resistance to axial tension f_{ct} and compression f_c

$$T^2 + m\sigma - T_{sh}^2 = 0, \tag{2}$$

here $m = f_c - f_{ct}$, $T_{sh}^2 = f_c f_{ct}/3$.

Strength condition (2) at plane stress state and plane strain in coordinates $|\tau_n| - \sigma_n$, respectively, is written

$$|\tau_n| = \varphi(\sigma_n) = \sqrt{d^2 - \frac{1}{4}(\sigma_n - m)^2}, \tag{3}$$

$$|\tau_n| = \varphi(\sigma_n) = \sqrt{m\left(\sigma_n + \frac{1}{4}m + \frac{1}{12}n^2/m\right)}, \tag{4}$$

where $d = \sqrt{(f_c^2 - f_c f_{ct} + f_{ct}^2)/3}$, $n = f_c + f_{ct}$.

The functional of the principle of virtual velocities J [25] is investigated for a stationary state using the equation $\delta J = 0$.

In a plane stress state in the range of available slip planes at $\Delta n \to 0$, using (2) and (3), we have

$$\delta \int\limits_{S_l} \left[d\sqrt{4\Delta\upsilon_n'^2 + \Delta\upsilon_t'^2} - m\Delta\upsilon_n' \right] dS - \int\limits_{S_F} F_i \upsilon_l' dS = 0, \tag{5}$$

here $\Delta\upsilon_n'$ and $\Delta\upsilon_t'$ – jumps of normal and shear velocity components on the failure surface (velocity jump line).

Under conditions of plane strain, taking into account (2) and (4), the functional J is investigated using the equation

$$\delta \int\limits_{S_l} \left[\frac{d^2}{m} + \frac{m}{4}\left(\frac{\Delta\upsilon_t'}{\Delta\upsilon_n'} \right)^2 \right] \Delta\upsilon_n' dS - \int\limits_{S_F} F_i \upsilon_i' dS = 0. \tag{6}$$

Introducing the characteristic of strength $\chi = f_{ct}/f_c$ and parameters k' and $\tan\gamma'$, where υ_1' and υ_2' – the velocity components in the direction of stress σ_1 and σ_2, but γ'– the angle between the failure surface and the direction of action of the principal stresses σ_1, and solving Eq. (5) with respect to $\sigma_1 = F_1/S_F$, the basic dependence for the plane stress state is received

$$\frac{\sigma_1}{f_c} = \frac{2\sqrt{\left(1 - \chi + \chi^2\right)/3}\sqrt{\left(k' - \tan\gamma'\right)^2 + 0.25\left(1 + k'\tan\gamma'\right)^2} - (1 - \chi)\left(k' - \tan\gamma'\right) + k\sigma_2/f_c}{\tan\gamma}. \tag{7}$$

The value σ_1/f_c is set by varying the parameters k and γ and corresponds to the minimum power of plastic strain.

According to the obtained parameters k and $\tan\gamma$ on the failure surface, the level of shear stresses

$$\frac{\tau_n}{f_c} = \frac{\sqrt{(1 - \chi + \chi^2)/3}(1 + k\tan\gamma)}{\sqrt{4(k - \tan\gamma)^2 + (1 + k\tan\gamma)^2}}, \tag{8}$$

and normal stresses

$$\frac{\sigma_n}{f_c} = 1 - \chi - \frac{4\sqrt{\left(1 - \chi + \chi^2\right)/3}(k - \tan\gamma)}{\sqrt{4(k - \tan\gamma)^2 + (1 + k\tan\gamma)^2}}, \tag{9}$$

is established.

When used as a vary parameter of the angle $\psi = \pi/2 - 2\gamma$ between the tangent to the strength condition (3) and the direction of normal stresses σ_n, the following expressions were obtained to determine σ_1, τ_n and σ_n

$$\frac{\sigma_1}{f_c} = \frac{2\left[\sqrt{\left(1-\chi+\chi^2\right)/3}\sqrt{1+4\tan^2\psi'} - (1-\chi)\tan\psi'\right] + \left(\sqrt{1+\tan^2\psi'} + \tan\psi'\right)\sigma_2/f_c}{\sqrt{1+\tan^2\psi'} - \tan\psi'}, \tag{10}$$

$$\frac{\tau_n}{f_c} = \frac{\sqrt{\left(1-\chi+\chi^2\right)/3}}{\sqrt{1+4\tan^2\psi}}, \tag{11}$$

$$\frac{\sigma_n}{f_c} = 1-\chi-4\frac{\sqrt{(1-\chi+\chi^2)/3}\tan\psi}{\sqrt{1+4\tan^2\psi}}. \tag{12}$$

Under conditions of plane strain, the equations for determining the stresses σ_1, τ_n, σ_n have the form

$$\frac{\sigma_1}{f_c} = \frac{\left(\frac{1}{3}\ \frac{1-\chi+\chi^2}{1-\chi} + \frac{1-\chi}{4\tan^2\psi}\right)\psi + \frac{\sigma_2}{f_c}\left(\sqrt{1+\tan^2\psi} + \tan\psi\right)}{\sqrt{1+\tan^2\psi} - \tan\psi}, \tag{13}$$

$$\frac{\tau_n}{f_c} = \frac{1-\chi}{2\tan\psi}, \tag{14}$$

$$\frac{\sigma_n}{f_c} = \frac{1-\chi}{4}\left[\frac{1}{\tan^2\psi} - 1 - \frac{1}{3}\left(\frac{1+\chi}{1-\chi}\right)\right]. \tag{15}$$

3 The Problem of the Action of a Rectangular Stamp on a Concrete Base Under a Plane Stress State and Plane Strain

We consider the problem of determining the limit value of a uniformly distributed load q, which is transmitted to a concrete base through a rectangular stamp (Fig. 1).

To solve it in a plane stress state and plane strain, the well-known method of characteristic lines is used [24–26]. A thin and infinitely long base is adopted. The strains are considered small, so the change in the outline of the free surface can be neglected.

Two families of characteristic lines z and u and three characteristic areas of the stress state are considered: I (ABC) – simple stress state of uniaxial compression in the horizontal direction, which corresponds to a free rectilinear boundary; II (CBO) – a centered arear with an alternating stress state in the direction of the characteristics u with a family of radial straight lines $z = const$ centered at point B; III (BOB')

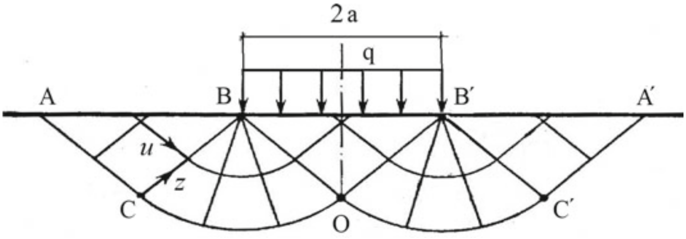

Fig. 1 Calculating the strength of a concrete base when exposed to a rectangular stamp

– biaxial compression with principal normal stresses $\sigma_1 = q$. The location of the OB'C' and C'B'A' arears relative to the central axis is symmetric to the OBC and CBA arears.

The parameters that determine the stress state are first set in the I arear with a sequential transition to the I arear, the value of the $C_{\mathrm{II}}(u)$ is set using the properties of the characteristic lines and the boundary conditions on the line BC (B'C').

The calculation is made for concrete with a ratio $\chi = f_{ct}/f_c = 0.1, d = 0.551 f_c$.

In a plane stress state, the angle of inclination of the characteristic lines in the I area to direction of action σ_1 is set by the formula

$$\gamma_{\mathrm{I}} = \frac{1}{2}\arccos\frac{m - 0.5 f_c}{1.5 f_c} = \frac{1}{2}\arccos\frac{0.5-\chi}{1.5} \tag{16}$$

and is equal to $\gamma_{\mathrm{I}} = 37.27^{\mathrm{o}}$.

For the convenience of consideration, the parameter is entered $t = (\sigma_1 - \sigma_2)/2$, which in the area I and when $\sigma_1 = f_c$ and $\sigma_2 = 0$ is equal $t_{\mathrm{I}} = 0.5 f_c$; $t_{\mathrm{I}}/d = 0.907$.

The equation on the characteristics $u = const$ is written as

$$\arcsin\left(\frac{5}{3} - \frac{8}{3}\frac{t_{\mathrm{III}}^2}{d^2}\right) + \frac{1}{2}\arcsin\left(\frac{5}{3} - \frac{2}{3}\frac{d^2}{t_{\mathrm{III}}^2}\right) = \arcsin\left(\frac{5}{3} - \frac{8}{3}\frac{t_{\mathrm{I}}^2}{d^2}\right) + \frac{1}{2}\arcsin\left(\frac{5}{3} - \frac{2}{3}\frac{d^2}{t_{\mathrm{I}}^2}\right) - \pi. \tag{17}$$

Substituting the values of the parameters t_{I} and d received: $t_{\mathrm{III}}/d = 0.82$, $t_{\mathrm{III}}/f_c = 0.452$.

The principal stresses in area III are determined from the equations

$$\sigma_1 = m + \sqrt{3\left(d^2 - t_{\mathrm{III}}^2\right)} + t_{\mathrm{III}} = \left(1 - \chi + \sqrt{1 - \chi + \chi^2 - 3t_{\mathrm{III}}^2} + t_{\mathrm{III}}\right) f_c, \tag{18}$$

$$\sigma_2 = m + \sqrt{3\left(d^2 - t_{\mathrm{III}}^2\right)} - t_{\mathrm{III}} = \left(1 - \chi + \sqrt{1 - \chi + \chi^2 - 3t_{\mathrm{III}}^2} - t_{\mathrm{III}}\right) f_c \tag{19}$$

and are equal to $\sigma_1 = 1.9 f_c$, $\sigma_2 = 0.995 f_c$.

Limit value of uniform load $q = 1.9 f_c$.

The angle between the characteristic u and the direction in area III is determined by the formula

$$\gamma_{\mathrm{III}} = \frac{1}{2}\arccos\frac{m - (\sigma_1 + \sigma_2)/2}{3t_{\mathrm{III}}} = \frac{1}{2}\arccos\frac{1 - \chi - (\sigma_1 + \sigma_2)/2}{3t_{\mathrm{III}}} \tag{20}$$

and is equal to $\gamma_{\mathrm{III}} = 57°$.

The sizes of areas I, II and III are: $BO = a\big/\sin\gamma_{\mathrm{III}} = 1.19a$; the radius of the u-characteristic at the boundary of arears I and II is defined as

$$r_{BC} = \sqrt[4]{\frac{4t_{\mathrm{III}}^2 - d^2}{4t_{\mathrm{I}}^2 - d^2}}\, r_{BO}, \tag{21}$$

$BC = r_{BC} = 1.1a$, $AB = 2r_{\max}\cos\gamma_1 = 1.75a$, $AA' = 2(AC + a) = 5.5a$.

Considering that $\chi = 0.15$: $\gamma_{\mathrm{I}} = 38.25°$, $\gamma_{\mathrm{III}} = 58.04°$, $t_{\mathrm{III}} = 0.428f_c$, $\sigma_1 = q = 1.85f_c$, $\sigma_2 = 0.989f_c$;$\chi = 0.05$: $\gamma_{\mathrm{I}} = 36.27°$, $t_{\mathrm{III}} = 0.475f_c$, $\sigma_1 = q = 1.95f_c$, $\sigma_2 = 0.999f_c$.

Under conditions of plane strain, the placement of arears I, II, and III is similar to that shown in Fig. 1.

In the arear I a simple stress state ($\sigma_2 = 0$) is realized, here the unknown angle of inclination of the characteristics u to the direction of the stress action σ_1, the value and boundary conditions in the plane of the BC, which is adjacent to the area II. The values of γ_{I} and σ_1 are set from the dependencies

$$\gamma_{\mathrm{I}} = \frac{1}{2}\arccos\frac{m}{m + 2d} = \frac{1}{2}\arccos\frac{1 - \chi}{1 - \chi + 2\sqrt{(1 - \chi + \chi^2)/3}}, \tag{22}$$

$$\sigma_{\mathrm{I}} = 2t_{\mathrm{I}} = m + 2d = \left(1 - \chi + 2\sqrt{(1 - \chi + \chi^2)/3}\right) f_c \tag{23}$$

and are equal to $\gamma_{\mathrm{I}} = 31.63°$, $\sigma_{\mathrm{I}} = 2f_c$, $t_{\mathrm{I}} = f_c$.

The condition on the characteristics $u = const$, which makes it possible to access the parameters of arear III through a centered arear, with plane strain is written in the form

$$\tan 2\gamma_{\mathrm{III}} - 2\gamma_{\mathrm{III}} = \tan 2\gamma_{\mathrm{I}} - 2\gamma_{\mathrm{I}} + \pi. \tag{24}$$

After substituting the angle values γ_{I} and d are installed: $2\gamma_{\mathrm{III}} = 79.53°$, $\gamma_{\mathrm{III}} = 39.77°$.

Parameter is equal to $t_{\mathrm{III}} = 0.5m/\cos 2\gamma_{\mathrm{III}} = 0.5(1 - \chi)/\cos 2\gamma_{\mathrm{III}} = 2.47f_c$.

The principal stresses in area III are determined from the equations

$$\sigma_1 = \frac{t_{\mathrm{III}}^2}{1 - \chi} + t_{\mathrm{III}} - \frac{(1 + \chi)^2}{3(1 - \chi)}, \tag{25}$$

$$\sigma_2 = \frac{t_{\mathrm{III}}^2}{1-\chi} - t_{\mathrm{III}} - \frac{(1+\chi)^2}{3(1-\chi)}, \tag{26}$$

$$\sigma_3 = \frac{\sigma_1+\sigma_2}{2} + \frac{m}{2} = \frac{\sigma_1+\sigma_2}{2} + \frac{1-\chi}{2} f_c \tag{27}$$

and are equal to $\sigma_1 = 9.18 f_c$, $\sigma_2 = 4.22 f_c$, $\sigma_3 = 7.15 f_c$.

The sizes of areas I, II and III are: $BO = a/\sin\gamma_{\mathrm{III}} = 1.56a$; the radius of the u-characteristics at the boundary areas I and II is defined as

$$r_{BC} = \sqrt{1 + \frac{2\gamma_{\mathrm{III}} + \pi - 2\gamma_{\mathrm{I}}}{\tan 2\gamma_{\mathrm{I}}}} r_{BO}, \tag{28}$$

$BC = r_{BC} = 2.58a$; $AB = 2r_{BC}\cos\gamma_{\mathrm{I}} = 4.39a$, $AA' = 2(AC + a) = 10.78a$.

Having that $\chi = 0.15$: $\gamma_{\mathrm{I}} = 31.92°$, $\gamma_{\mathrm{III}} = 39.80°$, $t_{\mathrm{III}} = 2.35 f_c$, $\sigma_1 = 8.75 f_c$, $\sigma_2 = 4.04 f_c$, $\sigma_3 = 6.82 f_c$; $\chi = 0.05$: $\gamma_{\mathrm{I}} = 31.39°$, $\gamma_{\mathrm{III}} = 39.75°$, $t_{\mathrm{III}} = 2.61 f_c$, $\sigma_1 = 9.66 f_c$, $\sigma_2 = 4.45 f_c$, $\sigma_3 = 7.53 f_c$.

According to [26], the distribution of velocities is as follows: the triangular area I moves downward with a speed V relative to areas II and III, which move away from the central axis and up. Velocity jumps take place along the boundaries of the arears on the CO and C'O lines. An analysis of the distribution of strains in this problem is given in [27].

In a simplified version for applying the dependences of the variational method, the influence of areas I and II on the strength of the concrete base is proposed to be taken into account by lateral compression.

Under the conditions of a plane stress state at $\sigma_2 = f_c$, the stress value σ_1 and the geometrical dimensions of the III area (angle γ_{III}) correspond to the characteristics obtained by the method above.

To determine the value of the ultimate load, the dependence is proposed

$$q = \sigma_1 = (2-\chi) f_c. \tag{29}$$

Under conditions of plane strain at $\sigma_2 = 4.22 f_c$ the stress value σ_1 and geometrical dimensions of the III area, the characteristics are similar to those obtained by the method. To determine the ultimate load and principal stresses when a rectangular stamp is applied to a concrete base under plane strain conditions, the following dependencies are proposed

$$q = \sigma_1 = (10 - 8\chi) f_c, \tag{30}$$

$$\sigma_2 = 4.7(1-\chi) f_c. \tag{31}$$

Consider the case of destruction of the concrete base when pressing the stamp with splitting in the tensile zone (Fig. 2).

The values of stresses in the tensile zone are penetrated by equal to the value of the resistance of concrete to axial tension f_{ct}.

Dependence for determining the value of the ultimate load in the kinematic scheme shown in Fig. 2, under conditions of plane stress has the form

$$q = \sigma_1 = \frac{f_c}{\tan\gamma'}\left[\sqrt{(1-\chi+\chi^2)/3}\sqrt{4(k'-\tan\gamma')^2+(1+k'\tan\gamma')^2}-(1-\chi)(k'-\tan\gamma')\right]+k' f_{ct}\left(\frac{h}{a}-\frac{1}{\tan\gamma'}\right). \tag{32}$$

The strength of the concrete base is influenced by the ratio of its height to the width of the stamp h/a.

For $\chi = 0.1$ a minimum power of plastic strain with varying parameters k and $\tan\gamma$ in relation to a ratio $\frac{h}{2a} = 6.32$ is equal to $q = \sigma_1 = 1.9 f_c$, the value of the load when concrete is destroyed only in the compressed zone.

The shear and normal stresses on the failure surface in the compressed zone are established from Eqs. (8), (9), (11) and (12) and are equal to $\tau_n = 0.55 f_c$,$\sigma_n = 0.9 f_c$.

The calculation results are given in Table 1.

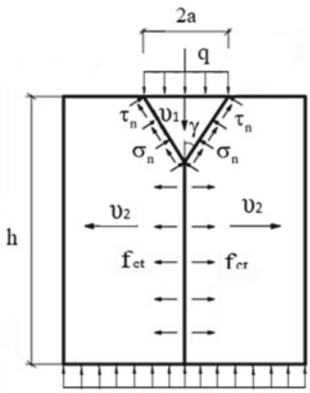

Fig. 2 Kinematic scheme of destruction of the concrete base when pressing a rectangular stamp with splitting

Table 1 The results of the calculation of the strength of the concrete base with simultaneous failure in compressed and tensile zones at $\chi = 0.1$

$\frac{h}{2a}$	$\frac{\sigma_1}{f_c}$	$\frac{\tau_n}{f_c}$	$\frac{\sigma_n}{f_c}$	k	$\gamma,^{\circ}$	$\psi,^{\circ}$
4	1.61	0.535	0.639	0.724	28.95	6.96
5	1.74	0.546	0.749	0.643	28.77	3.97
6	1.86	0.550	0.862	0,572	28.79	0.99
7	1.97	0.549	0.983	0.507	29.05	−2.16

For concretes with the ratio $\chi = 0.15$ and $\chi = 0.05$, the boundary between the cases of destruction only in the compressed zone and simultaneously in the compression and tension zones, respectively, is $\frac{h}{2a} = 4.38$ and $\frac{h}{2a} = 12.1$.

To establish the boundary between the cases of destruction, the dependence

$$\frac{h}{2a} = \frac{1}{3} + \frac{0.6}{\chi}. \tag{33}$$

For a larger value determined according to (33), the value $\frac{h}{2a}$ the I case of destruction is realized (Fig. 1), for a smaller value, the II case of destruction is realized (Fig. 2).

4 Conclusions

1. When solving the problem of the strength of a concrete base for indentation of a rectangular stamp by the method of characteristic lines and dependencies of the proposed variational method using the principle of virtual velocities and the criterion for the minimum power of plastic strain, the same values of the ultimate uniform load and geometric parameters of the area under the stamp were obtained.
2. The size of the principal stress σ_2 in the horizontal direction in the triangular arear of biaxial compression under the stamp at the plane stress state is equal to the principal stress $\sigma_1 = f_c$ in the triangular arear of uniaxial compression, which is adjacent to the free surface of the concrete base. Analysis of the stress state of the area under the stamp made it possible to propose for determining the values of large principal stresses σ_1 and the value of the ultimate load in it the dependence $q = 2f_c - f_{ct}$.
3. In conditions of plane strain, the value of the lower principal stresses in the area of biaxial compression under the stamp is equal to $\sigma_2 = 4.7(f_c - f_{ct})$ which correspond to the value of the principal stresses and the ultimate load on the concrete base $q = \sigma_1 = 10f_c - 8f_{ct}$.
4. The arear of realization of destruction only in the compressed zone near the stamp (case I) and simultaneous destruction in the zone of compression and tension (case II) at a plane stress state was determined. The boundary of cases of destruction corresponds to the ratio of the height of the concrete element to the width of the stamp $\frac{h}{2a} = \frac{1}{3} + 0.6\frac{f_c}{f_{ct}}$. If the ratio is greater than or equal to the specified value, the case I is realized, otherwise, the case of II.
5. The results obtained by the authors indicate that the theory of concrete plasticity is promising for solving problems of the strength of concrete elements under local compression.

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