Calculation of Composite Bending Elements

Taliat Azizov, Dmitry Kochkarev, Tatiana Galinska, and Oleksiy Melnyk

Abstract The article describes the method of calculation of stone beams strengthened with side reinforced concrete plates. The method comprises an imaginative dissection of a composite beam into stone and reinforced concrete parts and further inspection of the conditions of strain compatibility in the place where both parts of the combined structure are connected. The paper proves that it is not difficult to determine the conditions of strain compatibility. It is shown that the degree of a stone beam and reinforced concrete plates joint action depends on the quantity and diameter of bonds, their location, as well as on the strength and deformability characteristics of the stone beam and reinforced concrete plates. The article suggests that local deformations in bonds are better determined using empirical formulas. Using a stone beam strengthened with reinforced concrete plates as an example, the authors proved the efficiency of such strengthening and analyzed the factors of joint influence. The research suggests that in case of high bond stiffness, the composite structure can be calculated as monolithic, keeping in mind the usage of two different materials.

Keywords Composite beam · Bonds · Strain compatibility equations · Stone elements · Reinforced concrete plate

---

T. Azizov · O. Melnyk
Pavlo Tychyna Uman State Pedagogical University, Sadova St., 2, Uman 20300, Ukraine

D. Kochkarev
National University of Water and Environment Engineering, Soborna St., 11, Rivne 33028, Ukraine

T. Galinska (✉)
National University «Yuri Kondratyuk Poltava Polytechnic», 24, Pershotravnevij Ave, Poltava, Ukraine
e-mail: galinska@i.ua

V. Onyshchenko et al. (eds.), *Proceedings of the 3rd International Conference on Building Innovations*, Lecture Notes in Civil Engineering 181,
https://doi.org/10.1007/978-3-030-85043-2_3

1 Introduction

Problem statement and research analysis. Scientific research [1–7] suggests different ways to arrange clips for strengthening stone structures are proposed. Those are single or double-sided clips that are attached to the wall, usually through clamps or ruffs. All these structures are designed to strengthen stone structures against the effect of compressive forces. It is known that the calculation of stone structures is almost the same as the calculation of reinforced concrete structures. The calculation of bent structures is analyzed by [7, 8]. Research by [9] presents the calculation method which takes into account the nonlinear properties of concrete, Nevertheless all those studies do not consider cases when the element section contains materials with different characteristics, although it affects their stress-strain state.

Thus, research does not consider bent stone structures strengthened with side reinforced concrete plates, as well as strengthening structures made of light concrete blocks with reinforced concrete plates, including light concrete bent structures.

The possibility of using one-sided and two-sided reinforced concrete plates to strengthen bent stone structures, including structures made of light concrete blocks, is hindered by the lack of methods for calculating such structures. Both [10, 11] suggest the method and algorithm for calculating the above-described combined structures. However, those papers do not cover the issues of rigidity of anchors connecting the stone and reinforced concrete parts of the composite construction.

2 Method for Calculating

In this regard, the purpose of this article is to develop a method for calculating bent stone structures strengthened with side reinforced concrete plates.

Results and discussion. Consider a bendable element consisting of three layers (in a vertical plane) connected by bonds at individual points. Moreover, the side plates have the same thickness (Fig. 1). Due to the same thickness of the side plates, the action of the composite design will be subject to a flat bend. Such a structure can be calculated as one that is only affected by a bending load without torsion and oblique bending.

For the calculation, as in [11], we mentally divide the composite structure into two separate layers (two beams), layers 1 and 2 (Fig. 1). The first layer is the stone part of the composite beam. The second layer is a plate that has twice the thickness of each of the side plates.

Unknown vertical Si and horizontal Ti forces, where i is the number of the connection, will act in both the first and second cut-off layers (beams). These unknowns can be determined by composing the equations of strain (displacements) compatibility for layer 1 and layer 2 of the composite structure.

The components of the displacements will include:

Vertical displacements at the *i* point consist of the following components:

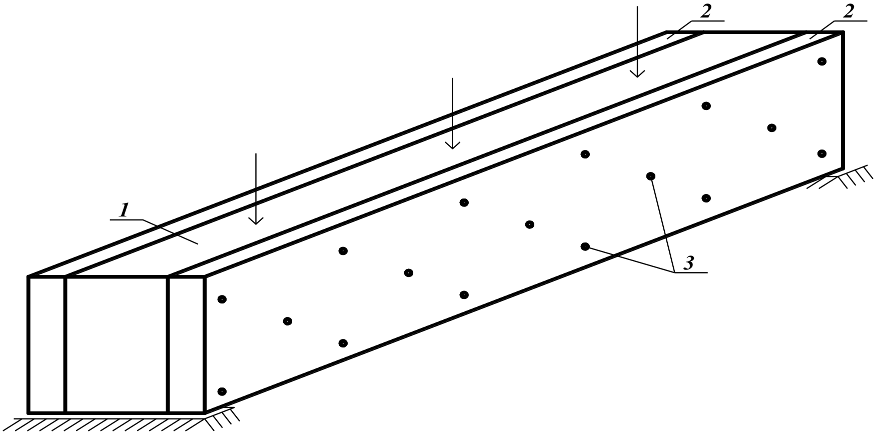
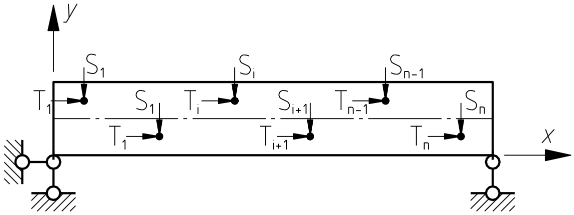


Fig. 1 Diagram of a two-layer bendable element. 1—stone beam; 2—concrete plates; 3—bonds

Fig. 2 Diagram of forces in bonds



1. from bending by external load;
2. from bending by vertical forces S_i (Fig. 2);
3. from bending moments created by horizontal forces T_i. In this case, if the force is above the neutral axis of the beam, the moment is positive. If it is lower, the moment is negative (Fig. 2);
4. from local deformation at the point of application of the vertical force S_i (bond deformation and concrete crumpling under the bond).
   Horizontal movements at the *i* point are made up of such components as (if the point is above or below the neutral axis, then the displacements can be either positive or negative):
5. from bending by external load;
6. from bending by vertical forces S_i;
7. from bending moments created by horizontal forces T_i in the XOY plane (Fig. 2);
8. from compression (stretching) by T_i forces;
9. from local deformation at the point of application of the horizontal force T_i.

All the components of displacements, except for displacements according to clauses 4 and 9, are determined by the known formulas for the resistance of materials [12].

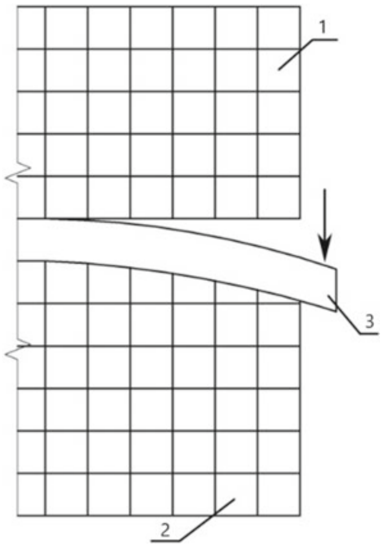


Fig. 3 Diagram of deformation of a reinforcing bar under the action of a transverse load 1—finite elements of concrete (stone); 2—finite elements of a reinforcing bar

Let us focus separately on displacements from local deformation. At first glance, the most accurate solution to this problem seems to be by modeling with three-dimensional finite elements using the well-known software *Ansys*, *Abacus*, *Lira*, etc. However, a detailed analysis reveals almost insurmountable obstacles. The most important of them is the correct modeling of the connection between the concrete and the anchor when it is acting on a load perpendicular to its axis. The fact is that when the anchor is working on a transverse load, one part of the concrete under the metal rod is crushed, and the opposite part "moves away" from the concrete almost without any resistance (Fig. 3).

In addition, it is known that the concrete in the contact zone with the reinforcement has mechanical characteristics that differ from the characteristics of the main part of the reinforced concrete element. According to M. M. Kholmyansky [13], microcracks are formed in the contact zone of reinforcing bars and concrete, which are difficult to account for by calculation. The deformability and strength of the contact layer may differ significantly from similar characteristics of the main concrete.

Modeling using one-way bonds (from the side of reinforcing bars separation from concrete) is also not acceptable due to the above-mentioned properties of the contact layer. The above facts are easily verified by experimental research.

In this regard, the displacements of the anchor from the transverse load should be determined experimentally. The recommendations [14] provide a formula obtained from processing experimental data, from which it is possible to determine the transverse displacement of a reinforcing bar loaded with a load perpendicular to its axis:

$$a_{loc} = 1000\frac{Q^2}{d_s^3 E_c^2} + \frac{Q}{d_s E_c} \quad (1)$$

where d_s and E_c are, respectively, the diameter of a reinforcing bar and the modulus of concrete deformations; Q is the force applied to the reinforcing bar in the direction perpendicular to its axis.

The empirical formula (1) integrally takes into account all the facts of the complex stress-strain state of the reinforcement-concrete system.

Equating the displacements $\boldsymbol{v_i}$ and $\boldsymbol{y_i}$ for the beam of the first layer (the stone part) to the same displacements for the beam of the second layer, we get the equation of strain compatibility at the *i* point of the structure. By composing such equations for all ***n*** points of the composite beam, it is not difficult to obtain a system of ***2*n*** equations with ***2*n*** unknowns $\boldsymbol{T_1 \ldots T_n}$, $\boldsymbol{S_1 \ldots S_n}$. The calculation algorithm based on a system of equations for the compatibility of deformations allows for a detailed analysis of the gain efficiency when varying different factors. At the same time, the software (the authors wrote it in *Pascal*) is quite simple, but the effectiveness of its use is high in terms of selecting the thickness of the reinforced concrete plate required by the designer, the number and diameter of anchors connecting the cage to the strengthened stone bending element.

After determining the unknowns, each of the two beams (stone and reinforced concrete plate) is calculated as a statically definable structure, which is affected by external forces and unknown forces $\boldsymbol{T_1 \ldots T_n}$, $\boldsymbol{S_1 \ldots S_n}$ in bonds determined from the solution of the system of compatibility equations.

The degree of compatibility of stone beams and side reinforced concrete plates depends on the number of anchors, connecting the parts of the composite beams, position of anchors, their diameter, and the mechanical characteristics of the stone part and reinforced concrete plates. Let us conditionally call this degree of compatibility "the gain coefficient". This coefficient ***k*** is equivalent to the ratio of the maximum bending moment in a non-strengthened stone beam from the action of an external load to the maximum bending moment in the stone part of the strengthened beam. In other words, if the calculation based on joint action shows that the maximum bending moment $\boldsymbol{M_{st}}$ acts in the stone part of the combined beam, and the maximum bending moment in the beam from the external load is ***M***, then the gain coefficient ***k*** will be determined in the following way:

$$k = M/M_{st} > 1 \tag{2}$$

It should be noted that when composing a system of equations for the strain compatibility, the malleability from local deformation of the first layer (the stone part) and the second layer (reinforced concrete plates) will be different.

The degree of influence of the bond stiffness on the operation of the composite structure will be shown by an example. Let's assume that there is a stone beam consisting of D500 aerated concrete blocks. Beam width $b_1 = 200$ mm, its height is 300 mm, the span is 3000 mm. Aerated concrete blocks are strengthened with side reinforced concrete plates arranged symmetrically on both sides. We will vary the thickness of reinforced concrete plates b2***/2*** (meaning that the total thickness of one plate is equal to $\boldsymbol{b_2}$). Let the bonds be arranged in two horizontal rows, located at a distance of 50 mm from the top and bottom of the axis of the composite beam. The

Table 1 Results of the composite beam calculation

Variant	b_2	n	Gain coefficient k when d_s equals			
			3	6.5	10	14
1	10	2	1,5	1,61	1,64	1,65
2	10	3	1,8	1,93	1,96	1,97
3	10	5	1,79	1,87	1,88	1,88
4	20	2	1,75	2,04	2,11	2,15
5	20	3	2,4	2,9	3,01	3,06
6	20	5	2,44	2,73	2,78	2,8
7	40	2	2,0	2,59	2,79	2,89
8	40	3	3,24	4,93	5,47	5,74
9	40	5	3,45	4,45	4,68	4,77

bond beams are distributed symmetrically along the length of the beam, their number $\boldsymbol{n}$ is variable. The diameter of bonds $\boldsymbol{d_s}$ also varies. Let a uniformly distributed load $\boldsymbol{q} = 10$ kN/m act on the stone part of the beam. Table 1 shows the calculation results of such a composite beam.

As can be seen from the table, the reinforcement efficiency is affected by the diameter of the anchors $\boldsymbol{d_s}$, the thickness of the concrete plate b2, and the quantity of anchors $\boldsymbol{n}$. A significant increase in the anchor diameter has little effect on increasing the gain coefficient. For example, with an anchor diameter of 14 and 10 mm, the gain coefficients differ slightly. The difference in the gain coefficients for different anchor diameters is greater the thicker the side reinforced concrete plate.

We can also conclude that for small spans, the most effective is a small number of anchors connecting a stone beam with a reinforced concrete plate, which differs from the requirements for the location of anchors when installing reinforced concrete clips [5]. These tables also show that the reinforced concrete plate can significantly strengthen the stone element (in the example given it is five or more times), which indicates a fairly high efficiency of using such structures.

It should be noted that the results shown in Table 1 were obtained without taking into account the crack formation in the composite beam and the nonlinear properties of materials. Research on the nonlinear properties of the material of a clip and a stone beam, as well as on their cracking can be performed iteratively, changing the equivalent stiffness of the stone part and the reinforced concrete plate at each iteration.

If the connections connecting the stone part and reinforced concrete plates are malleable, then the zero point of the stress plot in the stone part and in the reinforced concrete plates will be located at different distances in height (Fig. 4). In this case, it is possible that the zero stress point in the stone element is higher than the zero stress point in concrete and vice versa. The ratio depends on the degree of malleability of anchors, deformability of the stone and reinforced concrete parts, and their reinforcement. All these factors are easily determined by iterative calculation, when at each

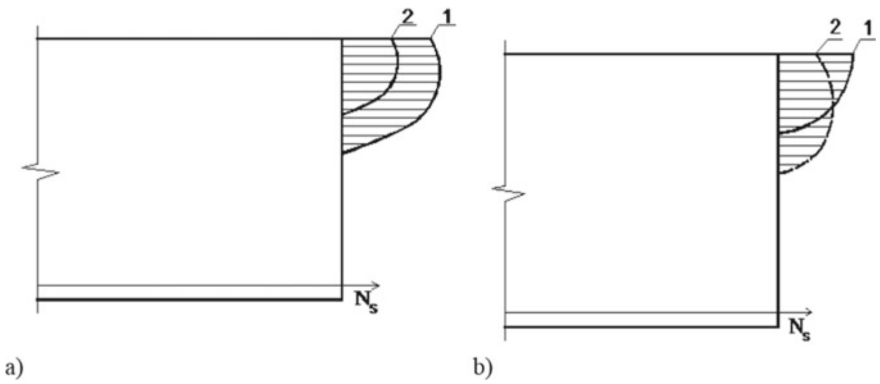


Fig. 4 Possible forms of stress plot in the compressed zone of the composite element. a)—The zero point of stress in the stone element is higher than the zero point of stress in concrete; b)—Vice versa. 1-stresses in the reinforced concrete plate; 2-stresses in the stone part

iteration, after determining the forces in the links, the stone and reinforced concrete beams are considered separately.

As a result of the flexibility of bonds, in addition to the various locations of the zero points of the stress plot, deflections of the stone part and reinforced concrete plates are different as well. There is a shift of the stone part relative to the reinforced concrete one. This factor is also easily determined using a computer-based calculation software developed by a method based on the compilation of a system of equations for the compatibility of deformations of two layers and the determination of unknown forces in the bonds. This theoretical fact of displacement of the stone part relative to the reinforced concrete plate has been empirically tested in [15–20].

If the stiffness of the anchors is high and local deformation can be ignored, then the composite beam can be considered as a monolithic consisting of two different materials in cross section. According to expression (1), the flexibility of the bonds depends on the diameter of the bond and the modulus of deformations of the concrete and stone part. The condition when the pliability of the bonds can be ignored is easily obtained by varying the stiffness of the bonds according to the software developed on the basis of the suggested methodology. The authors claim that the calculation of the composite structure, which can be considered monolithic, should be carried out using the method of calculated resistances of reinforced concrete [5], which is the subject of further research.

3 Conclusions

The paper suggests a method for determining the forces in the bonds connecting two parts of a composite structure, based on the compilation of conditions for the compatibility of deformations at the location of the bonds. The degree of joint action of stone beams and reinforced concrete plates is dependent on the number and diameter of bonds, their location and the strength and deformation characteristics of stone beams and reinforced concrete plates. With a high stiffness of the bonds, the composite structure can be calculated as a monolithic one, but taking into account the presence of two different materials in the cross section.

Further research is needed to develop a method for calculating the composite structure in the case when the malleability of anchors can be ignored, using the method of calculated resistances of reinforced concrete.

References

1. Azizov T, Melnik O, Myza O (2019) Strength and deformation of combined beams with side reinforced plates. https://doi.org/10.4028/www.scientific.net/MSF.968.234
2. Azizov T, Melnyk O, Orlova O, Kalenchuk-Porkhanova A, Vakal L (2017) Calculation of reinforced concrete ceilings with normal cracks accounting the chebyshev approximation. In: Paper Presented at the MATEC Web of Conferences, p 116. https://doi.org/10.1051/matecconf/201711602002
3. Azizov T, Jurkowska N, Kochkarev D (2019). Basis of calculation on torsion for reinforced concrete structures with normal cracks. In: Paper presented at the Proceedings of the Fib Symposium 2019: Concrete - Innovations in Materials, Design and Structures, pp 1718–1725
4. Kochkarev D, Azizov T, Galinska T (2020) Design of effective statically indeterminate reinforced concrete beams. https://doi.org/10.1007/978-3-030-42939-3_10
5. Kochkarev D, Azizov T, Azizova A, Galinska T (2021) Designing of standard cross sections of composite bending reinforced concrete elements by the method of design resistance of reinforced concrete. https://doi.org/10.1007/978-3-030-57340-9_25
6. Azizov T, Kochkarev D, Galinska T (2020) Reinforced concrete rod elements stiffness considering concrete nonlinear properties. Lect Notes Civil Eng 47:1–6. https://doi.org/10.1007/978-3-030-27011-7_1
7. Kochkarev D, Azizov T, Galinska T (2018) Bending deflection reinforced concrete elements determination. In: Paper Presented at the MATEC Web of Conferences, vol 230, p 02012. https://doi.org/10.1051/matecconf/201823002012
8. Kholmyanskij MM (1981) Kontakt armatury‘ s betonom. Strojizdat, p 184
9. Kamiani ta armokamiani konstruktsii (2011) Osnovni polozhennia: DBN V.2.6–162:2010. Minrehionbud Ukrainy, p 97 s
10. Piskunov VG, Gorik AV, Cherednikov VN (2000) Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads 2. Resolving equations and results. Mech Comp Mater. 36(6):445–452. https://doi.org/10.1023/A:1006798314569
11. Piskunov VG, Goryk AV, Cherednikov VN (2000). Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads. 1. Construction of a model. Mech Comp Mater 36(4):287–296. https://doi.org/10.1007/BF02262807
12. Pavlikov A, Kochkarov D, Harkava O (2019) Calculation of reinforced concrete members strength by new concept. In: CONCRETE. Innovations in Materials, Design and Structures: Proceedings of the fib Symposium 2019, Kraków, Poland, 27–29 May 2019, pp 820–827. http://reposit.pntu.edu.ua/handle/PoltNTU/6064
13. Pavlikov A, Kochkarev D, Harkava O (2020) Analysis of eccentrically loaded members of circular cross section by nonlinear deformation model. https://doi.org/10.1007/978-3-030-42939-3_19

14. Storozhenko L, Yermolenko D, Gasii G (2018). Investigation of the deformation state of a composite cable space frame structures with a photogrammetric method. Int J Eng Technol 7(3.2):442–446. https://doi.org/10.14419/ijet.v7i3.2.14568
15. Cherniha R, Pliukhin O (2013) New conditional symmetries and exact solutions of reaction-diffusion-convection equations with exponential nonlinearities. J Math Anal Appl 403(1):23–37. https://doi.org/10.1016/j.jmaa.2013.02.010
16. Kozak O, Zhuravskyi O, Delyavskyy MI (2019). Effect of the pre-stressed reinforcement curvature on the bearing capacity of inclined sections of monolithic beams. In: Paper presented at the AIP Conference Proceedings, p 2077. https://doi.org/10.1063/1.5091888
17. Panchenko OV, Ivanyts'kyi YL, Kun' PS, Zhuravs'kyi OD (2018) Determination of the durability of ferroconcrete bridge beams reinforced by composite strips. Mater Sci 53(5):660-665. https://doi.org/10.1007/s11003-018-0121-3
18. Semko O, Fenko O, Hasenko A, Harkava O, Kyrychenko V (2018) Influence of external and internal cooling at solidification on strength of brittle duralumin in compression. In: Paper Presented at the MATEC Web of Conferences, p 230. https://doi.org/10.1051/matecconf/201823002029
19. Hasenko A, Semko O, Drobotia O, Sirobaba V (2020) Experimental and numerical studies of nodes of light steel-reinforced concrete structures. In: Paper presented at the Proceedings of the 2020 Session of the 13th Fib International PhD Symposium in Civil Engineering, pp 173–178
20. Semko O, Hasenko A, Kyrychenko V, Sirobaba V (2020) The rational parameters of the civil building steel frame with struts. https://doi.org/10.1007/978-3-030-42939-3_25