Analytical Procedure for Design of Centrally Compressed Bars

Anton Makhinko, Nataliia Makhinko, and Oleg Vorontsov

Abstract The article is devoted to the study of stability problems of centrally compressed bars. The difficulties of the classical solution of this problem are highlighted. In analytical form, we propose a simplified dependence for calculating the stress reduction factor. Using the author's approach shows good agreement between the results received and the calculation results according to the normative methodology for a wide range of slenderness. An algorithm for determining the dimensions of the cross sections of steel elements loaded with a central force is also constructed. At the same time, solutions presented through the Lambert transcendental function are used to test rigidity. The convenience and advantages of using this algorithm are indicated. A practical example of the column sizing with the subsequent verification of the results according to current standards is given. The example of calculation shows the simplicity of the calculation according to the proposed algorithm and the full compliance of the result with the requirements of regulatory documents.

Keywords Stability · Buckling length · Centrally compressed bar · Buckling coefficient · Slenderness · Effective length · Lambert function

1 Introduction

The issues of stability theory are very important for displaying the stress–strain state and understanding the actual operation of compressed bars [1–5]. Stability testing is an integral part of real design [6–10]. However, it is important to have a simple engineering algorithm in order to solve practical problems and develop construction

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solutions. The analytical method presented in regulatory documents for calculating the stress reduction factor of compressed elements complicates this task [11]. First, it is connected with difficulties of a calculated nature. Thus, the aim of this study was to develop a methodology for calculating centrally compressed bars, which gives an accurate result, but is not complicated by unnecessary calculation procedures.

There is a centrally compressed bar with a length ℓ_k. The cross-section of the bar in general is arbitrary, but then the sections that are most popular in the field of metal structures are considered: equal angle, circular hollow section, channel section, composite I-beam. With the known strength characteristics of steel (yield strength R_y) it is necessary to propose an algorithm for determining one of the overall dimensions of the bar cross-section in a closed form from the condition of ensuring its stability.

2 Main Body

As you know, the calculation of centrally compressed elements in all world norms is performed according to the equation [11]

$$K_R = \frac{N}{\varphi R_y A_k} \leq 1.0, \tag{1}$$

where A_k is cross area; φ is stress reduction factor.

According to clause 8.1.3 of DBN «Steel Structures» [11], the stress reduction factor φ is a function of the element's slenderness λ, steel yield strength R_y and type of buckling curve. The main difficulty in selecting the cross sections of the elements is because the form of the function $\varphi(R_y, \lambda)$ is rather bulky and non-linear with respect to R_y

$$\varphi = \frac{E}{2\lambda^2 R_y}\left[\pi^2\left(1-\alpha+\beta\lambda\sqrt{\frac{R_y}{E}}\right)+\lambda^2\frac{R_y}{E}\right] - \frac{E}{2\lambda^2 R_y}\left[\sqrt{\left(\pi^2\left(1-\alpha+\beta\lambda\sqrt{\frac{R_y}{E}}\right)+\lambda^2\frac{R_y}{E}\right)^2+39.48\lambda^2\frac{R_y}{E}}\right], \tag{2}$$

$$\lambda = \frac{\ell_k}{i_{\min}\mu_k}, \tag{3}$$

where α and β are parameters of the buckling curve from Table 1 DBN [11]; ℓ_k is effective length; $i_{\min}$ is minimum section radius of inertia; μ_k is effective length factor, which takes into account the fastening of the ends of the rod.

To simplify the calculations, it was proposed to replace this «uncomfortable» expression with a more «convenient» one, which greatly simplifies the classical calculation of stability within the framework of norms. An exponential relationship of the form is proposed as such a «convenient» expression [12, 13, 15–19]

$$\varphi = \exp\left(-\delta \frac{\lambda^{\varepsilon}}{\pi^2} \frac{R_y}{E}\right), \tag{4}$$

where ε and δ are parameters of the stability curve similar to α and β.

The expression under the sign of the exponent resembles in its structure the well-known Euler formula for the buckling force and differs from it by the introduction of additional parameters ε and δ. In the general case, these parameters depend on the type of cross section of the element and the steel liquid limit. The parameter values can always be successfully chosen in such a way as to describe the curves of the stress reduction factor given in the design standards with a minimum error. However, we propose a simpler way, slightly affecting the accuracy of obtaining results. Its essence is that the degree of slenderness indicator ε is taken equal to 2, and the parameter δ is considered dependent only on the type of buckling curve: «a»: $\delta = 0.4$, «b»: $\delta = 0.5$, «c»: $\delta = 0.6$.

Thus, for the stress reduction factor, we will have

$$\varphi = \exp\left(-\delta \frac{\lambda^2}{\pi^2} \frac{R_y}{E}\right). \tag{5}$$

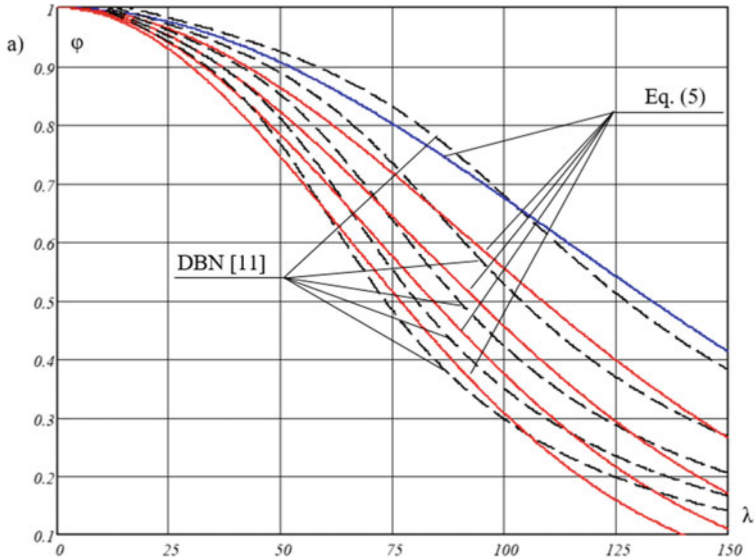
A comparison of the normative expression for the stress reduction factor (2) with the proposed relationship (5) is performed in Fig. 1 for two types of buckling curve («a» and «b»). The figure shows that in the range of element slenderness values from 0 to 100, the consistency of the results is quite acceptable, and for $\lambda > 100$, although the differences are large, firstly, Eq. (5) gives a lower estimate of the stress reduction factor φ, and secondly, by practice rarely has to deal with values $\varphi < 0.3$. Taking into account these considerations and the simplicity of the proposed relationship $\varphi(\lambda)$, we will further consider the application of Eq. (5) justified. Substituting Eq. (5) in Eq. (1), we obtain the following expression for checking the stability of centrally compressed elements

$$K_R = \frac{N}{\exp\left(-\delta \frac{\ell_k^2}{i_k^2 \pi^2} \frac{R_y}{E}\right) R_y A_k} \le 1.0, \tag{6}$$

where slenderness λ is represented through the effective length ℓ_k and radius of inertia i_k relative to the plane of interest.

Further, we will assume that the cross area A_k and the radius of inertia i_k of the element can always be represented as

$$A_k = b_k \cdot t_k \cdot f_1, \tag{7}$$



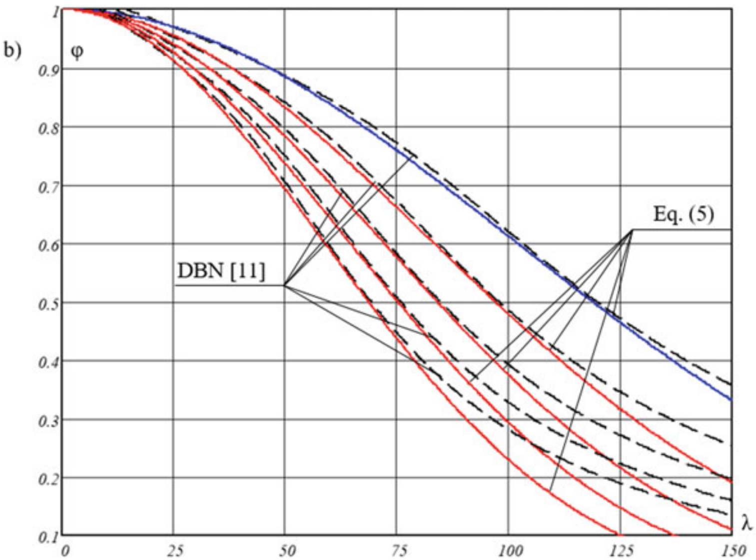


Fig. 1 Approximation buckling curve в DBN [11] according to Eq. (5): **a** - the type of buckling curve «a»; **b** - the type of buckling curve «b»

$$i_k = b_k \cdot f_2, \tag{8}$$

where b_k and t_k are some characteristic overall size and characteristic thickness of the cross section; f_1 and f_2 are some dimensionless coefficients, additionally characterizing other sizes of the section.

Table 1 represents the values for sections in the form of an equal angle, circular hollow section, channel section and I-beam.

Substituting expressions Eqs. (7) and (8) into Eq. (6), we obtain the following equation with respect to the desired cross-sectional characteristic b_k

$$N = K_R \exp\left(-\delta \frac{\ell_k^2}{b_k^2 f_2^2 \pi^2} \frac{R_y}{E}\right) R_y b_k t_k f_1. \tag{9}$$

It is convenient to represent the solution of this equation through the Lambert function. Omitting the intermediate transformations and simplifications, we give only the result

$$b_k = b_{ef} \cdot \Delta_R, \tag{10}$$

$$\Delta_R = \frac{\eta_R}{\sqrt{LambertW(\eta_R^2)}}, \tag{11}$$

Table 1 Characteristics of cross sections in the form of equal angle, circular hollow section, channel section and I-beam

<table>
<tr><th rowspan="2">Value</th><th colspan="4">Cross section</th></tr>
<tr><th>equal angle</th><th>circular hollow section</th><th>channel section</th><th>I-beam</th></tr>
<tr><td>b_k</td><td>width flange</td><td>diameter</td><td>width flange</td><td>width flange</td></tr>
<tr><td>t_k</td><td>flange thickness</td><td>thickness</td><td>flange thickness</td><td>flange thickness</td></tr>
<tr><td>h_k</td><td>–</td><td>–</td><td>depth</td><td>depth</td></tr>
<tr><td>t_w</td><td>–</td><td>–</td><td>web thickness</td><td>web thickness</td></tr>
<tr><td>f_1</td><td>2.0</td><td>π</td><td colspan="2">$2+\beta_k$</td></tr>
<tr><td rowspan="2">f_2</td><td rowspan="2">0.195</td><td rowspan="2">0.35</td><td colspan="2">plane of greatest rigidity
$0.5\beta_2\sqrt{(6+\beta_k)/(6+3\beta_k)}$</td></tr>
<tr><td colspan="2">plane of least rigidity
$\sqrt{(1+2\beta_k)/3}/(2+\beta_k)$ $1/\sqrt{6(2+\beta_k)}$</td></tr>
<tr><td colspan="5">Designations for sections in the form of a channel section and an I-beam
$\beta_k = \beta_1 \beta_2,\ \beta_1 = t_w/t_k,\ \beta_2 = h_k/b_k$</td></tr>
</table>

where b_{ef} is a value having a dimension of length, which we will call the effective dimension of the cross section. From the physical point of view, this is the dimension of the cross section calculated at bar in tension.

$$b_{ef} = \frac{N}{K_R\, R_y\, f_1\, t_k}, \tag{12}$$

where η_R is dimensionless characteristic showing how many times it is necessary to increase the cross-sectional dimension found from tensile analysis.

$$\eta_R = \frac{1}{\pi}\sqrt{2\delta \frac{R_y}{E}}\frac{\ell_k}{b_{ef}}\frac{1}{f_2}. \tag{13}$$

As for the Lambert function, it is defined as a solution of the functional equation

$$Lambert\ W(\eta_R)\exp[Lambert\ W(\eta_R)] = \eta_R. \tag{14}$$

This function is transcendental, i.e. the integer values of the argument correspond to the transcendental values of the function, and vice versa, the integer values of the function correspond to the transcendental values of the argument. The convenience of its use is explained by the fact that the dimensionless quantity Δ_R is enclosed in a very narrow range of values (from 1 to 10) and can be easily tabulated for all design schemes of compressed bars and cross-sectional forms of elements. The table of functional dependency values $\Delta_R(\eta_R)$ is given below.

Thus, in order to select the cross section of a compressed bar, it is necessary to select the required overall size from the conditions of its tensile operation and increase it by the value of Δ_R, found by the dimensionless parameter η_R. We give a numerical example.

3 Practical Calculation

According to the described approach, we perform the selection of a section of an I-beam profile for a 6 m long column with hinged fastening at the ends, with a known flange thickness $t_w = 8.0$ mm, $f_1 = 2.7$ and $f_2 = 0.25$ characteristics. We also set the compressive strength of $N_k = 1600$ kN and the value of the critical factor $K_R = 1.0$. The type of stability curve is taken as «b», which determines the value of the parameter $\delta = 0.5$.

In this case, the dimensionless argument of the Lambert function in the calculation by Eq. (13) is $\eta_R = 1.247$. According to Table 2, we find the value of the function $\Delta_R = 1.45$ and calculate the desired cross-sectional dimension (I-beam flange width) $b_k = 302.9$ mm.

Table 2 Table of values of function $\Delta_R(\eta_R)$

η_R	Δ_R	η_R	Δ_R
0.0	1.0	5.0	3.255
0.5	1.107	5.5	3.482
1.0	1.328	6.0	3.707
1.5	1.574	6.5	3.930
2.0	1.824	7.0	4.50
2.5	2.071	8.0	4.584
3.0	2.315	10.0	5.435
3.5	2.555	12.0	6.264
4.0	2.791	15.0	7.487
4.5	3.025	20.0	9.440

According to the assortment of rolled steel GOST 26,020-83 [14], we accept the 30K1 UC with geometric characteristics $b_k = 300$ mm, $h_k = 296$ mm, $t_w = 9.0$ mm, $A_k = 108$ cm^2, $i_{\min} = 75$ mm.

Let us check the cross section according to DBN [11]. The effective length factor for a bar with fastened ends is $\mu_k = 1$, and the slenderness of the element according to (3) will be $\lambda_k = 80$. Note that for the range of slenderness $\lambda < 100$ there is a high convergence of the results in terms of the stress reduction factor of the author's approach and the normative methodology. Using Eqs. (1) and (2), we calculate the stress reduction factor $\varphi = 0.697$ and the critical factor of the element, which is almost equal to the initial value $K_R^n = 0.996 \approx K_R = 1$.

4 Conclusions

1. To solve the stability problem of centrally compressed bars, an engineering algorithm is formulated. Using the author's approach makes it possible to simplify the procedure for determining the section dimensions. At the same time, the accuracy of the calculation remains quite high and complies with current standards.
2. The algorithm is based on the use of empirical dependence in determining the stress reduction factor. The study shows a fairly accurate correspondence of the results of the author's approach with the relationship given in current standards for a wide range of slenderness.
3. The original solution for the desired characteristics of the sections of centrally compressed elements is presented through the Lambert function. This allows you to choose the desired overall size of the compressed bar from the conditions of its operation in tension.

4. The given practical example of calculation shows the simplicity of the calculation according to the proposed algorithm and the full compliance of the result with the requirements of regulatory documents.

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