Approximation Model of the Method of Design Resistance of Reinforced Concrete for Bending Elements

Marta Kosior-Kazberuk, Dmytro Kochkarev, Taliat Azizov, and Tatiana Galinska

Abstract The paper considers approximation models of the method of design resistance of reinforced concrete for calculation of normal sections of bending reinforced concrete elements. It is proposed to use approximation dependences of two types—polynomial and linear. Both methods are based on the method of design resistance of reinforced concrete, which relies on universally accepted theory-based prerequisites and hypotheses. This method is based on the use of nonlinear deformation curves of concrete, Bernoulli hypothesis (the plane-sections hypothesis) is accepted as the correct one, and the extremum criterion for determining the bearing capacity (carrying force) based on a nonlinear deformation calculation model is used. The proposed techniques can massively simplify the calculation of bending reinforced concrete elements. They relieve from the necessity to use tables and perform complicated calculations with iteration methods, as it is intrinsic to the majority of existing methods. The possibility of determining on their basis both the bearing capacity (carrying force) and the relative height of the compressive zone of the concrete is shown in this article. The authors have conducted a check of the obtained approximation models of calculation on experimental samples of well-known researchers. The computational results indicate the computational accuracy, sufficient for practical calculations of the proposed methods. This paper presents the examples of determination of the carrying force and area of the effective reinforcement of normal sections of bending reinforced concrete elements by both offered methods. The proposed methods of calculating bending reinforced concrete elements can be widely used in design practice.

M. Kosior-Kazberuk
Bialystok University of Technology, Bialystok, Poland

D. Kochkarev
National University of Water and Environmental Engineering, Rivne, Ukraine

T. Azizov
Pavlo Tychyna Uman State Pedagogical University, Uman, Ukraine

T. Galinska (✉)
National University "Yuri Kondratyuk Poltava Polytechnic", Poltava, Ukraine
e-mail: galinska@i.ua

V. Onyshchenko et al. (eds.), *Proceedings of the 3rd International Conference on Building Innovations*, Lecture Notes in Civil Engineering 181,
https://doi.org/10.1007/978-3-030-85043-2_23

Keywords Nonlinear strain model · Bending moment · Deformation · Reinforced concrete beam · Design resistance

1 Introduction

The calculation of reinforced concrete elements at different types of force loads is usually performed taking into account nonlinear properties of materials [1–4, 9, 10]. The calculation of bending reinforced concrete elements involves the use of nonlinear or simplified diagrams of deformation of concrete [1–4]. The calculation of bending elements using nonlinear diagrams of deformation of concrete is quite complicated, and is usually performed using specially designed computer programs. Similar calculations on determination of the durability of normal sections of bending reinforced concrete elements by using simplified schemes are also quite complicated and may lead to certain imprecisions and inaccuracies. In most cases, such methods require the use of either pre-designed dependency graphs [1, 2] or the corresponding tables [4]. The simplest method, which involves the use of nonlinear diagrams of deformation of concrete in order to calculate reinforced concrete elements is the method of design resistance of reinforced concrete [4]. Instead, it also involves the use of certain tables. Let us consider an approximation model for calculating reinforced concrete elements based on this method.

The scientific developments of the authors of the article are associated with their preliminary studies, which are set out in the works [9–17], and are also a further development of the research of leading scientists Pavlikov [17, 18], Piskunov [19, 20], Pliukhin [21], Storozhenko [22], Zhuravskyi [23, 24] Semko [25–27].

2 Approximation Model of the Method of Design Resistance of Reinforced Concrete for Bending Elements

According to EN 1992-1-1, the calculation of bending reinforced concrete elements is performed using Bernoulli hypothesis (the plane-sections hypothesis), generally accepted deformation curves of concrete and reinforcement, as well as failure criteria. The main failure criterion is reaching the ultimate deformations of concrete and reinforcement. In the cases of calculation of beams with limited redistribution of forces, the relative height of the compressive zone of the concrete is also limited.

$$x/d \leq (\delta - 0.4)/(0.6 + 0.0014/\varepsilon_{cu}), \quad (1)$$

where x/d—the relative height of the compressive zone of the concrete, δ—the ratio of the distributed moment to the moment determined at the elastic stage, ε_{cu}—the ultimate strains of the compressive zone of the concrete. The remarkable thing is that according to [], a maximum redistribution of forces up to 20% is allowed for reinforcing steel of class A, and up to 30%—for reinforcing steel of class B.

Thus, regardless of the method of calculating bending reinforced concrete elements, it is necessary to check condition (1) in the case of redistribution of forces.

The calculation of bending elements according to the design codes [1] is as follows:

1. The mechanical reinforcement ratio is determined by the following expression:

$$\omega = \frac{\rho_f \cdot f_{yd}}{f_{cd}}, \tag{2}$$

   where ρ_f – the reinforcement ratio of the cross-section with longitudinal reinforcement, f_{yd} – design resistance of the longitudinal reinforcement, f_{cd} – design resistance of compressive strength of concrete.
2. Next, the parameter k_z $k_z = M/(f_{cd} \cdot b \cdot d^2)$ is determined according to the graph (Fig. 1).
3. The carrying force of the element is set by the expression:

$$M = k_z f_{cd} \cdot b \cdot d^2. \tag{3}$$

The procedure for calculating by the method of design resistance of reinforced concrete is similar to that described above:

1. The mechanical reinforcement ratio is determined by the formula (2).
2. The design resistance of reinforced concrete is set by the expression [1]

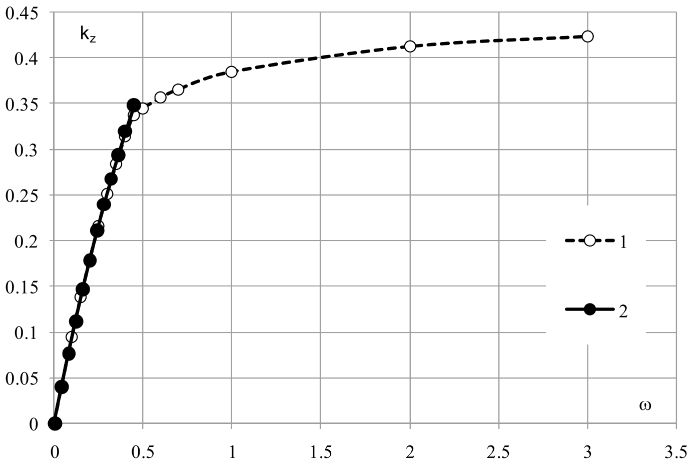

Fig. 1 Functional dependence $k_z = f(\omega)$ by the methods: 1–method of design resistance of reinforced concrete [4]; 2—EN 1992-1-1 [1]

$$f_z = 6 \cdot k_z f_{cd}. \quad (4)$$

3. The bearing capacity (carrying force) of the element is set by the formula:

$$M = f_z W_c, \quad (5)$$

where f_z – the design resistance of reinforced concrete in bending, MPa, W_c – moment resistance of working cross section of concrete, b·h^2/6, m^3.

The functional dependence of the method of design resistance of reinforced concrete, presented in Fig. 1, has a much larger number of values than the similar dependence of the method EN 1992-1-1 [1]. This is primarily due to the fact that the method of design resistance of reinforced concrete uses more accurate diagrams of deformation of concrete and theoretically substantiated failure criteria, in particular the extremum criterion. Therefore, the bearing capacity (carrying force) of the over reinforced elements should be determined more accurately by the method of design resistance of reinforced concrete. Methodology EN 1992-1-1 [1] limits ω to the value 0.45 (at C50 and below) and 0.36 (at C55 and above). This is due to the fact that such elements have brittle failure (fracture failure), which must be avoided in existent building structures. But when testing experimental samples with ω > 0.45, more precise values of the bearing capacity (carrying force) will correspond to the method of design resistance of reinforced concrete. The bearing capacity of the elements by the method EN 1992-1-1 [1] at ω > 0.45 will correspond to the bearing capacity of the elements at ω = 0.45. In the section at ω ≤ 0.45 bearing capacity of the bending elements will be the same for both methods.

The use of the dependency graph, shown in Fig. 1, in the calculation of bending elements somewhat complicates the calculation process itself, so we suggest to perform an approximation of the dependence $k_z = f(\omega)$.

The approximation will be performed in the range ω ≤ 0.45, avoiding the use of elements with brittle failure (fracture failure). We will perform the approximation by linear functions and a polynomial of the second degree (Fig. 1). Linear approximation involves somewhat approximate calculation of bending elements, so in order to simplify the calculation procedure, we will perform the approximation in the form of the following system:

$$k_z = \alpha_1 \cdot \omega, \ 0 < \varpi \leq 0.24; \quad (6)$$

$$k_z = \alpha_2 \cdot \omega, \ 0.24 < \omega \leq 0.45. \quad (7)$$

As a result of the approximation, the coefficients of the system of equations $\alpha_1 = 0.94, \ \alpha_2 = 0.82.$ were obtained. The coefficient of variation, determined by the fractional accuracies of this approximating dependence in comparison with the true function, is ν = 4.19%.

It ought to be noted that expressions, obtained by linear approximation, lead to the following expressions of the bearing capacity (carrying force):

$$M_{ED} = 0.94 \cdot A_s \cdot f_{yd} \cdot d,\ 0 < \varpi \leq 0.24; \quad (8)$$

$$M_{ED} = 0.82 \cdot A_s \cdot f_{yd} \cdot d, 0.24 < \omega \leq 0.45. \quad (9)$$

Approximating expressions in the form of a polynomial of the second degree have the following form:

$$k_z = \beta_1 \cdot \omega^2 + \beta_2 \cdot \omega,\ 0 < \varpi \leq 0.45; \quad (10)$$

The approximation coefficients for the expression (10) are $\beta_1 = -0.5,\ \beta_2 = 1.0$, whereby the coefficient of variation is $\nu = 0.82\%$ (Fig. 2).

1—dependence $k_z = f(\omega)$ of EN 1992-1-1 [1]; 2—linear approximation; 3—approximation by a polynomial of the second degree.

There is a relationship between the relative height of the compressive zone of the concrete and the mechanical reinforcement ratio [1]

$$x/d = 1.25 \cdot \omega,\ 0 < \omega \leq 0.45. \quad (11)$$

We shall confirm the validity of the obtained formulas on experimental samples.

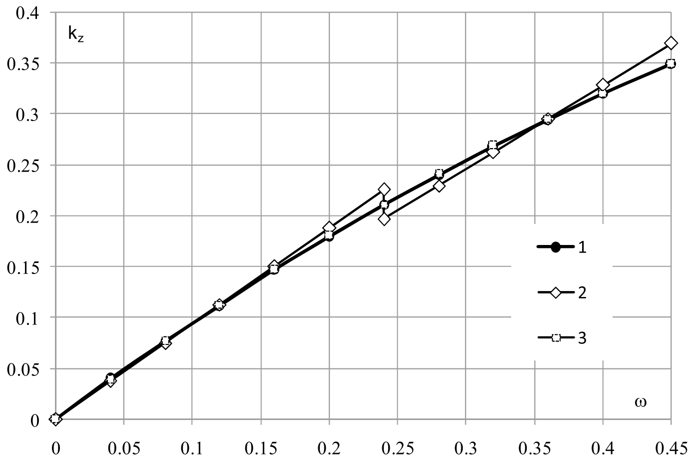

Fig. 2 Approximation of functional dependence $k_z = f(\omega)$:

Table 1 Verification of developed approximation models on experimental samples [5–8]

№.	Author of experiments, number of beams	Experimental data			Statistical value			
		f_c, *MPa*	f_y, *MPa*	ρ_s, %	x_l,	ν_l, %	x_p	ν_P, %
1	Pam et al. (2001) [5], 12 beams	36.4–102.5	520–579	0.76–3.14	1.00	13.17	1.00	13.17
2	Sarkar (1997) [6], 13 beams	77–107	442–470	1.03–4.04	0.95	15.66	0.95	12.66
3	Bernardo, Lopes (2004) [7], 19 beams	62.9–105.2	534–575	1.36–3.61	1.25	6.99	1.24	5.83
4	S.A. Ashour (2000) [8], 9 beams	48.61–102.4	530	1.18–2.37	0.99	4.07	0.99	3.17
	Total amount of samples 53				1.08	16.06	1.07	14.92

3 Verification of the Proposed Design Models on Experimental Samples

In order to check the proposed methods, we will determine the bearing capacity (carrying force) of experimental samples of well-known experiments [5–8]. Table 1 shows fractional accuracies of theoretically determined bearing capacity of the experimentally tested samples by the two abovementioned methods. The conducted calculations confirmed high accuracy of the developed approximating design models. The coefficient of variation of the ratio of theoretically determined and experimentally obtained bearing capacity for the linear dependence is $\nu_l = 16.06\%$, for polynomial dependence it is $\nu_P = 14.92\%$. When performing calculations, the parameter ω was limited to 0.45 (at C50 and below) and to 0.36 (at C55 and above).

4 Examples of Calculating Bending Reinforced Concrete Elements by the Offered Methods

Example 1. The task is to calculate the area of effective reinforcement, required for the reinforcement of the support section of a continuous beam made of concrete grade C20/25, $f_{cd} = 14.5$ MPa, $\varepsilon_{cu} = 350 \cdot 10^{-5}$, and of steel of class A500C, $f_{yd} = 435$ MPa, if the bending moment $M_{Ed} = 120$ кN·m works in the design section. Redistribution of forces in the amount of 15% ($\delta = 0.85$) should be carried out in the support section of the beam. It's advisable to take a rectangular cross-section of a beam with a single reinforcement with dimensions of b × d = 400 × 450 mm. The calculation must be performed by two approximation methods.

The solution. Let us determine the required design resistance of reinforced concrete.

$$f_{zM} = \frac{M_{ED}}{W_c} = \frac{6M_{ED}}{bd^2} = \frac{6 \times 120 \times 10^6}{400 \times 450^2} = 8.88\ MPa;$$

Then we will define the accessory parameter (fault-identifying variable):

$$k_z = \frac{f_{zM}}{6 \cdot f_{cd}} = \frac{8.88}{6 \cdot 14.5} = 0.102.$$

After that we shall determine the mechanical reinforcement ratio:

– by polynomial approximation

$$\omega = 1 - \sqrt{1 - 2 \cdot k_z} = 1 - \sqrt{1 - 2 \cdot 0.102} = 0.108;$$

– by linear approximation

$$\omega = k_z/0.94 = 0.102/0.94 = 0.109;$$

We will determine the limiting value of the height of the compressive zone of the concrete under the condition of redistribution of forces in the amount of 15% (δ = 0.85)

$$(\delta - 0.4)/(0.6 + 0.0014/\varepsilon_{cu}) = (0.85 - 0.4)/(0.6 + 0.0014/350 \cdot 10^{-5}) = 0.45.$$

Let us determine the height of the compressive zone of the concrete

– by polynomial approximation

$$x/d = 1.25 \cdot \omega = 1.25 \cdot 0.108 = 0.135 < 0.45.$$

– by linear approximation

$$x/d = 1.25 \cdot \omega = 1.25 \cdot 0.109 = 0.136 < 0.45.$$

Then we will determine the area of reinforcement and accept the reinforcement:

– by polynomial approximation

$$\rho_f = \frac{\omega \cdot f_{cd}}{f_{yd}} = \frac{0.108 \cdot 14.5}{435} = 0.0036 > \rho_{\min} = 0.0013.$$

$$A_s = \rho_f \cdot b \cdot d = 0.0036 \cdot 400 \cdot 450 = 648\ \text{mm}^2.$$

– by linear approximation

$$\rho_f = \frac{\omega \cdot f_{cd}}{f_{yd}} = \frac{0.109 \cdot 14.5}{435} = 0.00363 > 0.0013.$$

$$A_s = \rho_f \cdot b \cdot d = 0.00363 \cdot 400 \cdot 450 = 654 \text{ mm}^2.$$

As for the range of sizes, we will take 2Ø22 A500C, $A_s = 760\ mm^2$.

Example 2. The task is to determine the bearing capacity of a concrete beam, made of C20/25 strength grade of concrete, $f_{cd} = 14.5$ MPa, $\varepsilon_{cu} = 350 \cdot 10^{-5}$, and of steel of class A500C, $f_{yd} = 435$ MPa, if the beam is reinforced in the lower zone with reinforcement area $A_s = 760 \cdot mm^2$. The cross section of the beam is rectangular with single reinforcement with dimensions of b × d = 400 × 450 mm. The calculation must be performed by linear approximation methods.

The solution. Let us determine the mechanical reinforcement ratio:

$$\omega = \frac{A_S \cdot f_{yd}}{b \cdot d \cdot f_{cd}} = \frac{760 \cdot 435}{400 \cdot 450 \cdot 14.5} = 0.127,$$

Then we will determine the bearing capacity of the beam:

$$M_{ED} = 0.94 \cdot A_S \cdot f_{yd} \cdot d = 0.94 \cdot 760 \cdot 435 \cdot 450 \cdot 10^{-6} = 139, 84\ kN \cdot m.$$

It is quite interesting to determine the bearing capacity of the beam by area from the previous example at As = 648 mm²

$$M_{ED2} = 0.94 \cdot A_S \cdot f_{yd} \cdot d = 0.94 \cdot 648 \cdot 435 \cdot 450 \cdot 10^{-6} = 120, 34\ kN \cdot m.$$

The fractional accuracy is $\frac{120.34-120}{120} 100\% = 0.28\%$.

5 Conclusions

The considered approximation models of the method of design resistance of reinforced concrete enable to check quickly the durability of normal sections of bending reinforced concrete elements. The obtained dependences were confirmed by testing on a large number of experimental samples of well-known researchers, they fully correspond to the norms of design codes EN 1992-1-1 and can be widely used by engineers of different levels in design practice and in construction of reinforced concrete structures.

References

1. EN 1992–1 (1993) Eurokode- 2. Design of concrete structure. Part 1, General rules and rules for buildings, GEN
2. Bibi EV (2013) Designers' Guide to Eurocode 2: Design of reinforced concrete structures: designers' guide to EN 1992-1-1 and EN 1992-1-2. Eurocode 2: Design of reinforced concrete structures. General rules and regulations for buildings. Fire-fighting civil and structural engineering/Bibi EV, Narayanan RS; series editor Gulvanesyan X; Ministry of Education and Science of the Russian Federation, Federal State-Funded Educational Institution of Higher Professional Education "Moscow State University of Civil Engineering"; science editors Almazov VO, Plotnikov AI. Second edition, Moscow: MSUCE 2013, p 292 (The series "Published in MSUCE: Eurocodes". The science editor of the series –Almazov VO)
3. DBN V.2.6-98:2009. Konstruktsiyi budynkiv i sporud. Betonni ta zalizobetonni konstruktsiyi. Osnovni polozhennya., Minrehionbud, Kyiv
4. Kochkarev D, Azizov T, Galinska T (2018) Bending deflection reinforced concrete elements determination. In: Paper presented at the MATEC Web of Conferences, vol 230.https://doi.org/10.1051/matecconf/201823002012
5. Pam HJ, Kwan AKH, Islam MS (2001) Flexural strength and ductility of reinforced normal- and high-strength concrete beams. Struct Build 146(4):381–389
6. Sarkar S, Adwan O, Munday JGL (1997) High strength concrete: an investigation of the flexural behavior of high strength RC beams. Struct Eng 75(7):115–121
7. Bernardo LFA, Lopes SMR (2004) Neutral axis depth versus flexural ductility in high-strength concrete beams. ASCE J Struct Eng 130(3):425–459
8. Ashour SA (2000) Effect of compressive strength and tensile reinforcement ratio on flexural behavior of high-strength concrete beams. Eng Struct 22:413–423
9. Azizov T, Jurkowska N, Kochkarev D (2019) Basis of calculation on torsion for reinforced concrete structures with normal cracks. In: Proceedings of the fib Symposium 2019: Concrete - Innovations in Materials, Design and Structures 2019, pp 1718–1725
10. Kochkarev D, Azizov T, Galinska T (2020) Design of effective statically indeterminate reinforced concrete beams. https://doi.org/10.1007/978-3-030-42939-3_10
11. Kochkarev D, Azizov T, Azizova A, Galinska T (2021) Designing of standard cross sections of composite bending reinforced concrete elements by the method of design resistance of reinforced concrete. https://doi.org/10.1007/978-3-030-57340-9_25
12. Azizov T, Kochkarev D, Galinska T (2020) Reinforced concrete rod elements stiffness considering concrete nonlinear properties. Lecture Notes in Civil Engineering, vol 47, pp 1–6. https://doi.org/10.1007/978-3-030-27011-7_1
13. Azizov T, Derkowski W, Jurkowska N (2019) Consideration of the torsional stiffness in hollow-core slabs' design. https://doi.org/10.4028/www.scientific.net/MSF.968.330
14. Pavlikov A, Kosior-Kazberuk M, Harkava O (2018) Experimental testing results of reinforced concrete beams under biaxial bending. Int J Eng Technol (UAE) 7(3):299–305. https://doi.org/10.14419/ijet.v7i3.2.14423
15. Krassowska J, Kosior-Kazberuk M (2019) Experimental investigation of shear behavior of two-span fiber reinforced concrete beams. Arch Civil Eng 65(2):35–55. https://doi.org/10.2478/ace-2019-0017
16. Pavlikov A, Kochkarov D, Harkava O (2019) Calculation of reinforced concrete members strength by new concept. In: CONCRETE. Innovations in Materials, Design and Structures: Proceedings of the fib Symposium 2019, pp 820–827, Kraków, Poland, May 27–29 2019. http://reposit.pntu.edu.ua/handle/PoltNTU/6064
17. Pavlikov A, Kochkarev D, Harkava O (2020) Analysis of eccentrically loaded members of circular cross section by nonlinear deformation model. https://doi.org/10.1007/978-3-030-42939-3_19
18. Pavlikov A, Mykytenko S, Hasenko A (2018) Effective structural system for the affordable housing construction. Int J Eng Technol (UAE) 7(3):291–298. https://doi.org/10.14419/ijet.v7i3.2.14422

19. Piskunov VG, Gorik AV, Cherednikov VN (2000) Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads 2. resolving equations and results. Mech Composite Mater 36(6):445–452. https://doi.org/10.1023/A:1006798314569
20. Piskunov VG, Goryk AV, Cherednikov VN (2000) Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads. 1. construction of a model. Mech Composite Mater 36(4):287–296. https://doi.org/10.1007/BF02262807
21. Cherniha R, Pliukhin O (2013) New conditional symmetries and exact solutions of reaction-diffusion-convection equations with exponential nonlinearities. J Math Anal Appl 403(1):23–37. https://doi.org/10.1016/j.jmaa.2013.02.010
22. Storozhenko L, Yermolenko D, Gasii G (2018) Investigation of the deformation state of a composite cable space frame structures with a photogrammetric method. Int J Eng Technol 7(3.2):442–446. https://doi.org/10.14419/ijet.v7i3.2.14568
23. Panchenko OV, Ivanyts'kyi YL, Kun' PS, Zhuravs'kyi OD (2018) Determination of the durability of ferroconcrete bridge beams reinforced by composite strips. Mater Sci 53(5):660–665. https://doi.org/10.1007/s11003-018-0121-3
24. Kozak O, Zhuravskyi O, Delyavskyy M (2019) Effect of the pre-stressed reinforcement curvature on the bearing capacity of inclined sections of monolithic beams. In: Paper presented at the AIP Conference Proceedings, vol 2077. https://doi.org/10.1063/1.5091888
25. Semko O, Fenko O, Hasenko A, Harkava O, Kyrychenko V (2018) Influence of external and internal cooling at solidification on strength of brittle duralumin in compression. In: Paper presented at the MATEC Web of Conferences, vol 230. https://doi.org/10.1051/matecconf/201823002029
26. Hasenko A, Semko O, Drobotia O, Sirobaba V (2020) Experimental and numerical studies of nodes of light steel-reinforced concrete structures. In: Paper presented at the Proceedings of the 2020 Session of the 13th Fib International PhD Symposium in Civil Engineering, pp 173–178
27. Semko O, Hasenko A, Kyrychenko V, Sirobaba V (2020) The rational parameters of the civil building steel frame with struts. https://doi.org/10.1007/978-3-030-42939-3_25