Methods of Probabilistic Assessment of Building Enclosing Structures Thermal Reliability

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Abstract Probabilistic method for determining statistical characteristics of heat transfer resistance and enclosing structures inner surface temperature, which takes into account random nature of enclosing structures layers thickness, thermal characteristics of materials, indoor and ambient temperatures are developed. All random variables have normal distributions, and the air temperature is presented in a form of a sequence of 12 normally distributed random variables for each month of the year. Obtained statistical characteristics of the enclosing structure inner surface temperature for each of the heating period months enable estimating the duration of thermal failure (in minutes per year) by the criteria of comfort near the enclosure and formation of condensate on its inner surface. Relative durations of thermal failures can be roughly considered as annual failure probabilities. Developed techniques allow performing a comparative assessment of different enclosing structures thermal reliability level in the given climatic conditions of operation according to the criteria of sufficient heat transfer resistance, comfort in the room and possibility of condensation on the enclosures inner surface. Performed examples of calculations indicate a sufficient level of residential buildings walls thermal reliability with facade insulation, made in accordance with current design standards of Ukraine, and impossibility of normal operation of 510 mm thick brick walls without additional insulation, which were massively erected in the second half of last century.

Keywords Enclosing structures · Thermal reliability · Materials random properties · Random temperature effects

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1 Introduction

Enclosing structures should ensure not only buildings energy efficiency, but also the comfort of being indoors. Current design standards [1] establish four criteria for ensuring thermal reliability in enclosing structures design:

- specific annual energy consumption of the building should not exceed the maximum allowable value;
- heat transfer resistance must be not less than the minimum allowable value, depending on the type of enclosing structure and the temperature zone of its operation;
- temperature difference between the indoor air temperature and the enclosing structure inner surface temperature must not exceed the permissible value, which ensures the comfort of being near the enclosures;
- minimum value of the inner surface temperature in the areas of heat-conducting inclusions must be not less than the dew point temperature, which ensures impossibility of condensation on the enclosure inner surface.

Random nature of materials thermal characteristics, structural layers thickness, ambient air temperature and room temperature necessitates the use of probabilistic methods for assessing the level of enclosing structures thermal reliability of [2, 3].

Among a significant number of studies on atmospheric air temperature, we place emphasis on the works [4, 5], that substantiates possibility of atmospheric air temperature probabilistic representation in the form of a quasi-stationary random process or a sequence of 12 random variables with normal distribution and statistical characteristics of these models for almost 500 observation points of Ukraine. Similar studies of atmospheric air temperature made it possible to develop zoning according to climatic indicators [1, 6, 7].

Calculated values of indoor air temperature are set by norms [1] depending on the premises purpose. For civil buildings they are equal to 20–22 °C. In [3], the results of processing the systematic measurements of indoor air temperature, according to which the standard of indoor air temperature varies within 0.4…0.8 °C are presented.

The urgency of energy saving problem has led to a significant amount of research in the field of thermal engineering. Research and probabilistic representation of thermal insulation building materials characteristics was carried out in [8–11] and other works of the authors. It is established that coefficients of thermal conductivity can be represented in the form of normally distributed random variables with 0.03–0.14 variation coefficients. The average and calculated values of different thermal conductivity of building materials in different operating conditions are given in the standard [12]. Thermal characteristics of enclosing structures were analyzed in articles [2, 3, 11, 13, 14] and many other works. Considerable attention has also been given to the optimal values of thermal characteristics selection [15, 16] and the improvement of the buildings walls structures in order to increase their thermal reliability [17, 18].

One of the first approaches to assessing reliability of load-bearing structures by the criterion of strength is developed by O.R. Rzhanitsyn and methodically explicated in the monograph [19]. The essence of this method is to determine probability of structural element failure-free operation, taking into account bearing capacity random values and force from the current load. In the recent decades, methods for estimating the reliability indicators of load-bearing structures have been developed [20–23], which are based on potential input in form of random processes, sequences of maximum values and other probabilistic models.

The works [2, 3, 10, 11] are dedicated to probabilistic assessment of thermal reliability of enclosing structures. The disadvantage of these studies is incomplete consideration of calculated parameters statistical variability, in particular the outside and inside air temperature, thickness of enclosing structure layers and thermal conductivity of the materials used. In the article [3] it is offered to use not probability, but duration of thermal failure condition as numerical indicator of thermal reliability. This to some extent corresponds to the norms approach [24] to the calculation of structures according to the requirements of the serviceability limit state.

In general, the literature analysis showed that development of methods for assessing thermal reliability with a comprehensive account of the random nature of enclosing structures structural and thermal characteristics, temperature in the premises and ambient air temperature remains an urgent task.

The purpose of the study is to develop a method for assessing enclosing structures thermal reliability in the winter time, taking into account random nature of the influencing factors.

2 Prerequisites and Initial Data

The given below methods for assessing enclosing structures thermal reliability are based on the following prerequisites:

- the basis is made with calculation methods and formulas of thermal engineering given in the current design norms;
- enclosing structure layers thickness of the δ_i and coefficients of materials thermal conductivity λ_i are considered to be normally distributed random variables;
- outdoor air temperature, τ_{out} is a sequence of normally distributed random variables corresponding to individual months of the year;
- room temperature τ_{in} is a normally distributed random variable, the average value of which is set according to design standards, while the standard is set according to the results of field observations in residential buildings;
- dew point temperature τ_c is a random variable, statistical characteristics of which depend on the temperature and relative humidity in the premises;
- heat transfer coefficients of the enclosing structure inner α_{in} and outer α_{out} surfaces are considered to be deterministic values and are accepted according to the design norms;

Table 1 Wall layers construction and thermal conductivity characteristics

Wall construction	Wall layers thickness δ_i, m			S_δ m	Thermal conductivity characteristics		
	Type 1	Type 2	Type 3		R_i	M_λ	S_λ
Outer protective layer	0,01	0,01	–	0,002	0,011	0,860	0,043
Mineral wool plate insulation	0,12	0,08	–	0,002	2,449	0,047	0,004
Laying of hollow ceramic bricks	0,51	0,51	0,51	0,006	0,797	0,591	0,030
Internal plaster	0,02	0,02	0,02	0,006	0,025	0,749	0,037

– the minimum allowable value of heat transfer resistance of the enclosing structure R_{min} is a deterministic value adopted in accordance with the requirements of design standards.

Methods for thermal reliability assessing are illustrated by examples of calculation of residential buildings brick walls in the climatic conditions of Poltava and Kropyvnytskyi:

Type 1—brick wall, insulated in accordance with the requirements of the norms [1] regarding the minimum required heat transfer resistance $R_{min} = 3{,}3\ m^2 \cdot K/W$.

Type 2—brick wall, insulated in accordance with the requirements of the norms [1] regarding the minimum required heat transfer resistance $R_{min} = 2{,}475\ m^2 \cdot K/W$ reduced by 25% which is allowed provided ensuring overall energy efficiency of the building;

Type 3—brick wall of a residential building without additional insulation, which was built en masse in the second half of the twentieth century;

Wall constructions and statistical characteristics of thickness and thermal conductivity of each layer are given in Table 1. Mathematical expectations of layer thickness δ_i are accepted as equal to their design value, and S_δ standards are estimated by normative tolerances on product and layer dimensions taking into account 0.95 provision. Heat transfer supports of R_i wall layers are calculated according to calculated values of materials thermal conductivity from the norms [12], and their M_λ mathematical expectations and S_λ standards—from the results of experimental and statistical studies, performed by authors.

3 Enclosing Structure Heat Transfer Resistance

Calculated value the structure heat transfer resistance is determined by a well-known formula:

$$R = \alpha_{in} + \alpha_{out} + \sum_{i=1}^{N} \frac{\delta_i}{\lambda_i}, \quad (1)$$

where N –number of enclosing structure layers,

δ_i – thickness of i-th enclosing structure layer;

λ_i –calculated thermal material conductivity of the i-th layer of the structure;

α_{in}, α_{out} – heat transfer coefficients of enclosing structure inner and outer surfaces according to the data [12];

Random nature of δ_i enclosing structure layers thickness and λ_i materials thermal conductivity coefficients determines the randomness of heat transfer resistance resulting value (1). Considering the nonlinear operation of dividing random variables δ_i and λ_i in the formula (1), statistical characteristics of the resulting random variable R are obtained by the method of random variables functions linearization described in [25]. For this purpose, partial derivative functions (1) are determined from the random parameters δ_i and λ_i:

$$\frac{\partial R}{\partial \delta_i} = \frac{1}{\lambda_i}; \qquad \frac{\partial R}{\partial \lambda_i} = -\frac{\delta_i}{\lambda_i^2}. \quad (2)$$

Then, according to the method of linearization [25], mathematical expectation and standard of heat transfer resistance are equal to the following:

$$M_R = \alpha_{in} + \alpha_{out} + \sum_{i=1}^{N} \frac{M_{\delta i}}{M_{\lambda i}}, \quad S_R = \sqrt{\sum_{i=1}^{N} \left[\left(\frac{S_{\delta i}}{M_{\lambda i}} \right)^2 + \left(\frac{S_{\lambda i} M_{\delta i}}{M_{\lambda i}^2} \right)^2 \right]}, \quad (3)$$

where N – number of enclosing structure layers;

$M_{\delta i}$ and $S_{\delta i}$ – mathematical expectation and the standard of thickness of the i-th layer;

$M_{\lambda i}$ та $S_{\lambda i}$ – mathematical expectation and the standard of the i-th layer thermal conductivity.

As a result of calculations according to formulas (1)… (3) and the data from Table 1, the following calculated values of R, mathematical expectations M_R and standards S_R of heat transfer resistance of the wall of three types are obtained:

for wall type 1 – R = 3,440 $m^2 \times$ K/W, M_R = 3,612 $m^2 \times$ K/W, S_R = 0,226 $m^2 \times$ K/W;

for wall type 2 – R = 2,623 $m^2 \times$ K/W, M_R = 2,761 $m^2 \times$ K/W, S_R = 0,158 $m^2 \times$ K/W;

for wall type 3 – R = 0,819 $m^2 \times$ K/W, M_R = 0,873 $m^2 \times$ K/W, S_R = 0,036 $m^2 \times$ K/W.

Calculated values of heat transfer resistance of walls of types 1 and 2 correspond to the established norms [1] and the above-mentioned minimum permissible values for the walls of civil buildings in the first temperature zone of Ukraine. For wall type 3, the calculated value is 4 times lower than modern standards [1]. Mathematical expectations of heat transfer resistance of all walls are slightly higher than their calculated values.

4 Temperature of the Enclosing Structure Inner Surface

In the stationary mode of heat transfer, the temperature of enclosing structure inner surface can be determined from the known dependences of thermal engineering:

$$\tau_w = \frac{1}{R \cdot \alpha_{in}}[\tau_{in}(R \cdot \alpha_{in} - 1) + \tau_{out}], \tag{4}$$

where R – enclosing structure heat transfer resistance;

α_{in} – heat transfer coefficient of the enclosing structure inner surface;

τ_{in}, τ_{out} – indoor and outdoor air temperatures.

According to the accepted preconditions, all variables of formula (4) are normally distributed random variables. Therefore, the temperature of the enclosure structure inner surface is also a random variable, statistical characteristics of which can be determined by the method of linearization [25], as it was done for heat transfer resistance. Its mathematical expectation M_w and standard S_w are equal to the following:

$$M_w = \frac{1}{M_R \cdot \alpha_{in}}[M_{in}(M_R \cdot \alpha_{in} - 1) + M_{out}], \tag{5}$$

$$S_w = \frac{1}{M_R \cdot \alpha_{in}}\sqrt{S_{out}^2 + (M_{in} \cdot \alpha_{in} - 1)^2 S_{in}^2 + \frac{(1 - M_R \cdot \alpha_{in})^2}{M_R^2} \cdot S_R^2}, \tag{6}$$

where α_{in} – heat transfer coefficient of the enclosing structure inner surface;

M_R, S_R – mathematical expectation and standard of enclosure heat transfer resistance;

M_{in}, S_{in} – mathematical expectation and indoor air temperature standard;

M_{out}, S_{out} – mathematical expectation and outdoor air temperature standard.

According to formulas (5) and (6) it is possible to establish statistical characteristics of enclosing structure internal surface temperature for each of the heating period months. For this purpose it is necessary to substitute to formulas (5) and (6) mathematical expectations and standards of external air temperature in a certain month of the year, statistical characteristics of internal air temperature and enclosing structure heat transfer resistance (3). A simplified version of formula (6) without the third summand under the radical was obtained in [3], where the heat transfer resistance was considered a deterministic value.

5 Duration of the Thermal Failure State According to the Criterion of Comfort

This indicator is determined due to the probability that the difference Δ between the indoor air temperatures τ_{in} and the inner wall surface τ_w will exceed the permissible value Δ_{max} established by the norms [1]. According to the theorems on the numerical characteristics of random variables [25], the mathematical expectation and the standard of this difference are equal to the following:

$$M_\Delta = M_{in} - M_w, \quad S_\Delta = \sqrt{S_{in}^2 + S_w^2}, \tag{7}$$

where M_{in}, S_{in} – mathematical expectation and indoor air temperature standard;

M_w, S_w – mathematical expectation and temperature standard of the enclosing structure inner surface by (5), (6).

Absolute duration of the thermal failure state (in minutes per month) in the j-th month of the year is equal to the following:

$$Q_j = 43920\,[1 - F(\Delta_{max})], \tag{8}$$

where $F(\Delta_{max})$ – function of normal distribution of temperature difference Δ with numerical characteristics (8);

43,920 – number of minutes per month with an average duration of 30.5 days.

If changes in outdoor air temperature during the year are presented as a sequence of 12 normally distributed random variables, the annual duration of the Q_{year} thermal failure state is equal to the sum of the values (8) during the months relating to the heating period.

$$Q_{year} = \sum_j Q_j \tag{9}$$

and is expressed in minutes per year. For most of Ukraine, 7 months from October to April should be taken into account.

An alternative indicator is the relative duration of the thermal failure state Q_{rel}, calculated by dividing (9) by the length of the year in minutes

$$Q_{rel} = Q_{year}/525600. \tag{10}$$

Relative duration of thermal failure state (10) is a dimensionless quantity, which can be considered approximately the probability of thermal failure during the year.

Absolute (9) and relative (10) durations of the thermal failure state according to the comfort criterion are determined for three types walls described above in the climatic conditions of Poltava and Kropyvnytskyi. The following initial data are taken into account:

Table 2 The walls thermal failure state duration by the comfort criterion

City	Absolute duration of the state of thermal failure of wall types			Relative duration of the thermal failure state of wall types		
	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3
Poltava	16,3	53,5	41,100	$3,10 \times 10^{-5}$	$1,02 \times 10^{-4}$	$7,82 \times 10^{-2}$
Kropyvnytskyi	14,4	46,1	36,000	$2,75 \times 10^{-5}$	$8,77 \times 10^{-5}$	$6,86 \times 10^{-2}$

- statistical characteristics of outdoor air temperature for the cold months of the year from October to April M_{out} and S_{out} according to the data [4, 5];
- mathematical expectation of indoor air temperature for residential premises M_{in} = 20 °C, equal to the normative value according to [1] and the standard S_{in} = 0,6 °C according to the data [3];
- statistical characteristics of heat transfer resistance M_R and S_R, defined and given above for walls of three types;
- the maximum allowable value of the difference between the temperature of the indoor air and enclosing structure inner surface in residential buildings in accordance with the rules [1] is equal to $\Delta_{max} = 4$ °C.

The results of the calculations are shown in the Table 2.

The data in Table 2 indicate the close levels of thermal reliability of the walls being used in climatic conditions of Poltava and Kropyvnytskyi, located at a distance of about 200 km. Insignificant probable duration of the thermal failure state according to the comfort criterion indicates a sufficient level of thermal reliability of type 1 walls, the insulation of which fully meets the requirements of the norms [1]. The duration of thermal failures of type 2 walls is 3… 4 times longer, yet the discomfort per one hour a year can also be considered acceptable. Brick walls of type 3 without additional insulation, which were erected in the last century, can create uncomfortable conditions in the room for 25… 28 days per year. This duration of thermal failure state is clearly unacceptable, so the walls of type 3 are subject to mandatory thermal modernization by additional facade insulation.

6 Duration of the Thermal Failure State According to the Criterion of Condensate Formation

This figure is equal to probability that actual temperature of the enclosure structure inner surface τ_w will fall below the dew point τ_d. Statistical characteristics of the temperature of the enclosing structure inner surface are determined by formulas (5), (6). In [3] it was shown that the dew point temperature for living premises conditions can be considered a random variable with a mathematical expectation and a standard $M_d = 10,6$ °C, $S_d = 1,7$ °C.

Table 3 Duration of the thermal failure state of the walls by the criterion of condensate formation

City	Absolute duration of thermal failure state of wall types			Relative duration of thermal failure state of wall types		
	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3
Poltava	0,20	0,38	126	$3{,}77 \times 10^{-7}$	$7{,}21 \times 10^{-7}$	$2{,}40 \times 10^{-4}$
Kropyvnytskyi	0,19	0,35	102	$3{,}54 \times 10^{-7}$	$6{,}62 \times 10^{-7}$	$1{,}95 \times 10^{-4}$

According to the theorems on the numerical characteristics of random variables [25], mathematical expectation and the standard of the temperature difference $\Delta = \tau_w - \tau_d$ in the j-th month of the year are equal to:

$$M_\Delta = M_w - M_d; \quad S_\Delta = \sqrt{S_w^2 + S_d^2}. \tag{11}$$

Absolute duration of the thermal failure state (in minutes per month) in the j-th month of the year is equal to probability that the temperature difference $\Delta = \tau_w - \tau_d$ will be less than zero:

$$Q_j = 43920\ F(0). \tag{12}$$

Annual duration of thermal failure state by the criterion of condensate formation Q_{year} (in minutes per year) is determined by formula (9), and the relative duration—by (10). The results of calculations of thermal failure absolute and relative duration according to the criterion of condensate formation of three types of walls in the climatic conditions of Poltava and Kropyvnytskyi are given in Table 3.

The data in Table 3 indicate practical impossibility of condensation on the first and second types walls inner surface. The total duration of conditions for condensate formation on the type 3walls inner surface is 2 h per year. It is possible that condensate formation can damage the finishing layers, thus level of thermal reliability of the 3 type wall should be considered insufficient.

Tables 2 and 3 show relative durations of thermal failures of the 1 and 2 type walls are close and even lower than failure probabilities of load-bearing structures recommended by current design standards of Ukraine [24]. Relative durations of thermal failures of type 3 walls exceed the values recommended by these standards.

7 Conclusions from Research Results

1. The technique is developed for defining statistical resistance characteristics of heat transfer and temperature of an internal surface of enclosing structures taking into account random character of the structures sizes and thermal characteristics of materials, as well as casual temperatures of internal and atmospheric air.

2. As indicators of thermal reliability it is expedient to use absolute and relative duration of thermal failures state on criteria of comfort and formation of condensate on enclosing structures internal surfaces. The proposed probabilistic methods for determining these indicators enables performing a comparative assessment of thermal reliability level of different enclosing structures in the given climatic conditions of operation.
3. The performed examples of calculations indicate a sufficient level of thermal reliability of the residential buildings walls with facade insulation, made in accordance with current design standards of Ukraine, and the impossibility of normal operation of 510 mm thick brick walls without additional insulation, which were built en masse in the second half of the last century
4. To more accurately determine the indicators of thermal reliability of enclosing structures, it is necessary to perform experimental and statistical studies in order to obtain and probabilistically represent the thermal conductivity of structural and insulating building materials.

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