Generalized Intensity-Dependent Multiphoton Jaynes–Cummings Model

V. Bartzis, N. Merlemis, M. Serris, and G. Ninos

Abstract In this chapter, we study the Jaynes–Cummings model under multiphoton excitation and in the general case of intensity-dependent coupling strength given by an arbitrary function f. The Jaynes–Cummings theoretical model is of great interest to atomic physics, quantum optics, solid-state physics, and quantum information theory with several applications in coherent control and quantum information processing. As the initial state of the radiation mode, we consider a squeezed state, which is the most general Gaussian pure state. The time evolution of the mean photon number and the dispersions of the two quadrature components of the electromagnetic field are calculated for an arbitrary function f. The mean value of the inversion operator of the atom is also calculated for some simple forms of the function f.

1 Introduction

The Jaynes–Cummings model [1–3] is a theoretical model that describes the system of a two-level atom interacting with a single mode of the quantum electromagnetic field. The model is considered to be of great importance in quantum optics because it

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N. J. Daras, Th. M. Rassias (eds.), *Approximation and Computation in Science and Engineering*, Springer Optimization and Its Applications 180,
https://doi.org/10.1007/978-3-030-84122-5_6

is the simplest solvable model that describes the interaction of radiation with matter. The model allows for a fully quantum mechanical treatment of atoms interacting with an electromagnetic field, thus revealing a number of novel features, in contrast to the semi-classical approximation, in which only the atom is treated quantum mechanically and the electromagnetic field is assumed to behave according to the classical electromagnetic theory.

The mathematical formulation of the model is based on the Hamiltonian formalism of the full system, which after the rotating wave approximation [3] it can be expressed in terms of the inversion, raising, and lowering operators of the atom, denoted by $\sigma_3, \sigma_+, \sigma_-$ and annihilation and creation operators a, a^+ of the radiation mode. The full system's Hamiltonian consists of the atomic excitation Hamiltonian, the free field Hamiltonian, and the Jaynes–Cummings interaction Hamiltonian:

$$H = \frac{1}{2}\hbar\omega_0\sigma_3 + \hbar\omega a^+ a + \hbar\lambda(\sigma_+ a + \sigma_- a^+) \tag{1}$$

Here ω_0 is the transition frequency of the atom, and ω is the single mode angular frequency. The parameter λ is the coupling constant for the radiation–atom interaction. The operators $\sigma_3, \sigma_+, \sigma_-$ are 2×2 Pauli matrices

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ \sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{2}$$

The σ and a obey the following algebra:

$$[\sigma_3, \sigma_\pm] = \pm 2\sigma_\pm,\ \ [\sigma_+, \sigma_-] = \sigma_3,\ \ \left[a, a^+\right] = 1 \tag{3}$$

$$\sigma_+\sigma_- = \frac{1}{2}(1 + \sigma_3),\ \sigma_-\sigma_+ = \frac{1}{2}(1 - \sigma_3),\ \sigma_3^2 = 1 \tag{4}$$

In a series of articles [4–7], Sukumar, Buck, and Singh considered two generalized Jaynes–Cummings models with the following interaction Hamiltonians:

$$H_{\text{int}} = \hbar\lambda(\sigma_+ a\sqrt{a^+ a} + \sigma_-\sqrt{a^+ a}a^+) \tag{5}$$

$$H_{\text{int}} = \hbar\lambda(\sigma_+ a^m + \sigma_- a^{+m}) \tag{6}$$

We note that in the first model (5), the coupling strength depends on the number operator $n = a^+ a$ (or otherwise on the radiation intensity), whereas in the second model (6), the transmission of the atom from one level to the other is accompanied by the absorption or emission of m photons. The model described by Eq. (6) has been studied by Nayak and Mohanty [8] with $m = 2$ in order to obtain the steady-state photon statistics in a two-photon laser in which the decay of the lasing levels was taken into account. In addition, Haroche et al. [9] have observed the two-photon laser emission in Rydberg atoms of Rb, and Eq. (6) has been also widely applied to study the dynamics of the field and atomic variables in Rydberg atoms [10, 11].

Bartzis [12] has already studied the intensity-dependent two photon Jaynes–Cummings model with interaction Hamiltonian

$$H_{\text{int}} = \hbar\lambda(\sigma_+ a^2\sqrt{a^+ a} + \sigma_- \sqrt{a^+ a}\ \mathrm{a}^{+2}) \tag{7}$$

and the Jaynes–Cummings model with atomic motion [13].

N. Nayak and V. Bartzis [14, 15] have also used the three-level and the two-level Rydberg atom interacting with two nondegenerate modes, thus showing the differences in the dynamics. In another work, Bartzis, Patargias, and Jannussis have presented results in the case of one or two cavity modes interacting with both a three-level atom and Kerr-like medium [16, 17].

The atomic spin squeezing of N two-level and three-level atoms has been observed by Nayak et al. [18–20]. In recent years, more generalized Jaynes–Cummings models have been proposed [21–29], and the intensity dependence has also been considered in the work of Saha et al. [23]. In the case of multilevel atomic systems and multiphoton processes, the theoretical description is easier using the semi-classical approximation, for example, in potassium atoms in order to study two-photon excitation, multiphoton emissions, and other nonlinear processes [30–33].

In this work, we continue the generalization of the Jaynes–Cummings model by considering the interaction Hamiltonian that has the form

$$H_{\text{int}} = \hbar\lambda(\sigma_+ a^m f(a^+ a) + \sigma_- f(a^+ a)\ \mathrm{a}^{+m}) \tag{8}$$

This Hamiltonian describes a multiphoton process, since the transmission of the atom from one level to the other is accompanied by absorption or emission of m photons. In addition, the coupling strength in Eq. (8) is intensity dependent with the dependency described by an arbitrary function $f(a^+ a)$. In the standard Jaynes–Cummings model, the coupling strength is considered to have a constant value. However, it is reasonable to assume that the coupling strength depends on the intensity since radiation intensity is observed to depend on time. As initial state of the radiation mode, we consider a squeezed state [34–40], the most general Gaussian pure state, which is defined as

$$|\alpha, z\rangle = S(z)D(\alpha)|0\rangle \tag{9}$$

where $D(\alpha) = \exp(\alpha a^+ - \alpha^* a)$ is the Weyl displacement operator and

$$S(z) = \exp\left[\frac{1}{2}(\mathrm{za}^2 - z^* a^{+2})\right], \mathrm{z} = \mathrm{re}^{-i\theta} \tag{10}$$

represents the squeeze operator.

In the n-representation, the squeeze state takes the form [35]

$$|\alpha, z\rangle = \sum_n C_n |n\rangle,$$

$$\mathrm{C}_n = \frac{1}{\sqrt{n!\mu}}\left(\frac{\nu}{2\mu}\right)^{n/2} H_n\left[\alpha(2\mu\nu)^{-1/2}\right]\exp\left[-\frac{1}{2}|\alpha|^2 + \frac{\nu^*}{2\mu}\alpha^2\right] \tag{11}$$

where $\mu = \cosh \mathrm{r}$, $\mathrm{v} = e^{i\theta}\sinh \mathrm{r}$

The mean photon number for a squeezed state has the form

$$\bar{n} = |\hat{\alpha}|^2 + |\nu|^2, \quad where \quad \hat{\alpha} = \mu^*\alpha - \nu\alpha^* \tag{12}$$

The two quadrature components are defined as

$$X_1 = \frac{1}{2}(a + a^+) \tag{13}$$

$$X_2 = \frac{1}{2i}(a - a^+) \tag{14}$$

Consequently, the electric field of the radiation mode has the form

$$E(t) = X_1 \cos\omega \mathrm{t} \ + X_2 \sin\omega t \tag{15}$$

The dispersions of X_1 and X_2 for a squeezed state with $\theta = 0$ are

$$\langle(\Delta X_1)^2\rangle = \frac{1}{4}e^{-2r} \tag{16}$$

$$\langle(\Delta X_2)^2\rangle = \frac{1}{4}e^{2r} \tag{17}$$

The squeezing phenomenon is observed in Eqs. (16) and (17), since the quantum noise is lower in one quadrature component than that of the coherent state ($\langle(\Delta X_i)^2\rangle = 1/4, i = 1,2$) and higher in the other.

2 Time Evolution of the Atom Inversion Operator

In order to compute the time evolution of the system, we use Eq. (8) for the interaction, and the Hamiltonian of our model takes the form

$$H = \hbar\omega\left(a^+a + \frac{m}{2}\sigma_3\right) + \frac{\hbar\Delta}{2}\sigma_3 + \hbar\lambda(\sigma_+ a^m f(a^+a) + \sigma_- f(a^+a)a^{+m}) \tag{18}$$

where $\Delta = \omega_0 - m\omega$.

We can define the operators

$$C = a^{+}a + \frac{m}{2}\sigma_3 \tag{19}$$

$$B = \hbar\lambda(\sigma_{+}a^{m} f(a^{+}a) - \sigma_{-} f(a^{+}a)a^{+m}) \tag{20}$$

$$D = \hbar\lambda(\sigma_{+}a^{m} f(a^{+}a) + \sigma_{-} f(a^{+}a)a^{+m}) \tag{21}$$

so finally the Hamiltonian (18) takes the form

$$H = \hbar\omega C + \frac{\hbar\Delta}{2}\sigma_3 + D \tag{22}$$

It is easy to prove that $[C, H] = [C, B] = 0$, $[\sigma_3, B] = 2D$, $[\sigma_3, D] = 2B$

For the calculation of the time evolution of the operator σ_3 (represents the atom population inversion), we will work in the Heisenberg picture. The Heisenberg equations of motion for the operators σ_3 and B are

$$i\hbar\dot{\sigma}_3 = [\sigma_3, H] = [\sigma_3, D] = 2B \tag{23}$$

$$i\hbar\dot{B} = [B, H] = -\hbar\Delta D + [B, D] \tag{24}$$

The commutator of B and D is calculated to be

$$[B, D] = \hbar^2\lambda^2 \left\{\sigma_3 \left[a^{m} f(a^{+}a),\, f(a^{+}a)a^{+m}\right]_{+} + \left[a^{m} f(a^{+}a),\, f(a^{+}a)a^{+m}\right]\right\} \tag{25}$$

where the symbol $[,]_{+}$ represents the anticommutator.

So solving the above system of Eqs. (23) and (24), we obtain the following differential equation for σ_3:

$$\begin{aligned}\ddot{\sigma}_3 =& \frac{2\Delta}{\hbar}(H - \hbar\omega C - \frac{\hbar\Delta}{2}\sigma_3) \\ &- 2\lambda^2 \left\{(\sigma_3 + 1)a^{m} f^2(a^{+}a)a^{+m} + (\sigma_3 - 1) f(a^{+}a)a^{+m}a^{m} f(a^{+}a)\right\}\end{aligned} \tag{26}$$

The differential equation (26) cannot be solved in general for any arbitrary function $f(a^{+}a)$ so in the following discussion, we present the solution considering the two simple cases of $f(a^{+}a)$=1 and $f(a^{+}a)$=$\sqrt{a^{+}a}$. For these cases, Eq. (26) takes the form

$$\ddot{\sigma}_3 + \omega'^2\sigma_3 = \frac{2\Delta}{\hbar}(H - \hbar\omega C) \tag{27}$$

where $\omega'^2 = \left\{\begin{array}{c} 4\lambda^2\kappa(\kappa-1)\cdots(\kappa+1-m) + \Delta^2 \text{ for } f(a^{+}a) = 1 \\ 4\lambda^2\kappa^2(\kappa-1)\cdots(\kappa+1-m) + \Delta^2 \text{ for } f(a^{+}a) = \sqrt{a^{+}a} \end{array}\right\}$
and $\kappa = C + \frac{1}{2}m$.

The operator C is a constant of motion, so the solution of the above equation is

$$\sigma_3(t) = \sigma_3(0)\cos\omega' t + \frac{2B(0)}{i\hbar\omega'}\sin\omega' t + \frac{2\Delta}{\hbar\omega'^2}(H - \hbar\omega C)(1 - \cos\omega' t) \tag{28}$$

We suppose that the atom is initially at the excited state and the field in a squeezed state; thus, the solution takes the form

$$\langle\sigma_3(t)\rangle = \sum_n \left\{\frac{\Delta^2}{\omega_n'^2} + \left(1 - \frac{\Delta^2}{\omega_n'^2}\right)\cos\omega_n' t\right\}|C_n|^2 \tag{29}$$

where $\omega_n'^2 = \begin{Bmatrix} 4\lambda^2\kappa_n(\kappa_n - 1)\cdots(\kappa_n + 1 - m) + \Delta^2 \text{ for } f(a^+a) = 1 \\ 4\lambda^2{\kappa_n}^2(\kappa_n - 1)\cdots(\kappa_n + 1 - m) + \Delta^2 \text{ for } f(a^+a) = \sqrt{a^+a} \end{Bmatrix}$
and $\kappa_n = n + m$.

3 Field Statistics of the Generalized Intensity-Dependent Multiphoton Jaynes–Cummings Model

The Hamiltonian of the system is given by Eq. (18). We define the operators

$$C = a^+a + \frac{m}{2}\sigma_3 \tag{30}$$

$$N = \frac{\Delta}{2}\sigma_3 + \lambda(\sigma_+ a^m f(a^+a) + \sigma_- f(a^+a)a^{+m}) \tag{31}$$

We easily can prove that

$$[C, N] = [H, N] = [H, C] = 0 \tag{32}$$

Consequently, the time evolution operator can be written in the form

$$U(t, 0) = e^{(-i/\hbar)\mathrm{Ht}} = e^{-i\omega\mathrm{Ct}}e^{-\mathrm{iNt}} \equiv U_1(t, 0)U_2(t, 0) \tag{33}$$

In the two-dimensional atomic subspace, the matrix representation of the operators U_1 and U_2 has the form

$$U_1(t, 0) = e^{-i\omega\alpha^+\alpha t}\sum_{n=0}^{\infty}\frac{(-\frac{\mathrm{im}\omega t}{2})^n}{n!}\sigma_3^{(n)} \tag{34}$$

or

$$U_1(t, 0) = e^{-i\omega\alpha^+\alpha t}\begin{pmatrix} e^{\frac{-\mathrm{im}\omega t}{2}} & 0 \\ 0 & e^{\frac{\mathrm{im}\omega t}{2}} \end{pmatrix} \tag{35}$$

Similarly, we can prove that operator

$$U_2(t,0) = e^{-\mathrm{iNt}} = \sum_{n=0}^{\infty} \frac{(-\mathrm{it})^n}{n!} N^{(n)} \tag{36}$$

has the following form:

$$U_2(t,0) = \begin{pmatrix} K & L \\ M & Q \end{pmatrix} \tag{37}$$

where *K, L, M,* and *Q* are calculated as

$$\begin{aligned} K =& \cos\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^m f^2(a^+a)a^{+m}}\right\} \\ & - i\frac{\Delta}{2}\frac{\sin\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^m f^2(a^+a)a^{+m}}\right\}}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^m f^2(a^+a)a^{+m}}} \end{aligned} \tag{38}$$

$$L = -i\lambda \frac{\sin\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^m f^2(a^+a)a^{+m}}\right\}}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^m f^2(a^+a)a^{+m}}} a^m f(a^+a) \tag{39}$$

$$M = -i\lambda \frac{\sin\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^{+m} f^2(a^+a+m)a^{m}}\right\}}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^{+m} f^2(a^+a+m)a^{m}}} f(a^+a)a^{+m} \tag{40}$$

$$\begin{aligned} Q =& \cos\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^{+m} f^2(a^+a+m)a^{m}}\right\} \\ & + i\frac{\Delta}{2}\frac{\sin\left\{t\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^{+m} f^2(a^+a+m)a^{m}}\right\}}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 a^{+m} f^2(a^+a+m)a^{m}}} \end{aligned} \tag{41}$$

In addition, from Eq. (33), the operator $U(t,0)$ is written as

$$U(t,0) = e^{-i\omega a^+ a} \begin{pmatrix} \Psi & Z \\ Y & W \end{pmatrix} \tag{42}$$

where $\Psi = e^{\frac{-\mathrm{i}m\omega t}{2}} K \quad Z = e^{\frac{-\mathrm{i}m\omega t}{2}} L \quad Y = e^{\frac{\mathrm{i}m\omega t}{2}} M \quad W = e^{\frac{\mathrm{i}m\omega t}{2}} Q$

We can easily show that

$$UU^{+} = U^{+}U = 1$$

Assuming that the atom is initially the excited state, we have the density operator of the field as

$$\begin{aligned}
\rho_f(t) &= \mathrm{Tr}_{\mathrm{atom}} \left\{ U(t,0) \begin{pmatrix} \rho_f(0) & 0 \\ 0 & 0 \end{pmatrix} U^{+}(t,0) \right\} \\
&= e^{-i\omega a^{+}at} \Bigg[\Bigg(\cos\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)} \right] \\
&\quad - i\frac{\Delta}{2} \frac{\sin\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)} \right]}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)}} \Bigg) \rho_f(0) \\
&\quad \Bigg(\cos\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)} \right] \\
&\quad + i\frac{\Delta}{2} \frac{\sin\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)} \right]}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a+m)!}{(a^{+}a)!} f^2(a^{+}a+m)}} \Bigg) \\
&\quad + \lambda^2 \frac{\sin\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a)!}{(a^{+}a-m)!} f^2(a^{+}a)} \right]}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a)!}{(a^{+}a-m)!} f^2(a^{+}a)}} f(a^{+}a) a^{+m} \rho_f(0) a^m f(a^{+}a) \\
&\quad \frac{\sin\left[t\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a)!}{(a^{+}a-m)!} f^2(a^{+}a)} \right]}{\sqrt{\frac{\Delta^2}{4} + \lambda^2 \frac{(a^{+}a)!}{(a^{+}a-m)!} f^2(a^{+}a)}} \Bigg] e^{i\omega\alpha^{+}\alpha t}
\end{aligned} \tag{43}$$

We consider as initial state of the system a squeezed state (9–11). So we can calculate the matrix elements of $\rho_f(t)$ in the $|n\rangle$-basis

$$
\begin{aligned}
\langle n|\rho_f(t)|n'\rangle = C_n C^*_{n'} e^{-i\omega(n-n')t} &\left(\cos\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n+m)!}{(n)!}f^2(n+m)}\right]\right.\\
&\left. -i\frac{\Delta}{2}\frac{\sin\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n+m)!}{(n)!}f^2(n+m)}\right]}{\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n+m)!}{(n)!}f^2(n+m)}}\right)\times\\
&\times\left(\cos\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n'+m)!}{(n')!}f^2(n'+m)}\right]\right.\\
&\left. +i\frac{\Delta}{2}\frac{\sin\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n'+m)!}{(n')!}f^2(n'+m)}\right]}{\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n'+m)!}{(n')!}f^2(n'+m)}}\right)+\\
&+\lambda^2 C_{n-m}C^*_{n'-m}e^{-i\omega(n-n')t}f(n)f(n')\\
&\times\sqrt{\frac{n!n'!}{(n-m)!(n'-m)!}}\frac{\sin\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n)!}{(n-m)!}f^2(n)}\right]}{\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n)!}{(n-m)!}f^2(n')}}\\
&\times\frac{\sin\left[t\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n')!}{(n'-m)!}f^2(n')}\right]}{\sqrt{\frac{\Delta^2}{4}+\lambda^2\frac{(n')!}{(n'-m)!}f^2(n')}}
\end{aligned}
\tag{44}
$$

Finally, the time evolution of the mean photon number is calculated as

$$
\overline{n} = \mathrm{Tr}_{\text{field}}\left[a^+a\rho_f(t)\right] = \sum_n n\langle n|\rho_f(t)|n\rangle \tag{45}
$$

We next consider the time of the dispersions of the quadrature operators

$$
X_1 = \frac{1}{2}(a+a^+) \tag{46}
$$

$$
X_2 = \frac{1}{2i}(a-a^+) \tag{47}
$$

which are finally calculated to have the form

$$
\begin{aligned}
\langle(\Delta X_1)^2\rangle = \tfrac{1}{4}&\left\{1+\right.\\
\sum_n\left[2n\langle n|\rho_f(t)|n\rangle+\sqrt{(n+1)(n+2)}(\langle n+2|\rho_f(t)|n\rangle+\langle n|\rho_f(t)|n+2\rangle)\right]&\\
\left.-(\sum_n[\sqrt{n+1}(\langle n+1|\rho_f(t)|n\rangle+\langle n|\rho_f(t)|n+1\rangle)])^2\right\}&
\end{aligned}
\tag{48}
$$

$$\langle(\Delta X_2)^2\rangle = \frac{1}{4}\Bigg\{1 + \sum_n \big[2n\langle n|\rho_f(t)|n\rangle - \sqrt{(n+1)(n+2)}(\langle n+2|\rho_f(t)|n\rangle + \langle n|\rho_f(t)|n+2\rangle)\big] + (\sum_n[\sqrt{n+1}(\langle n+1|\rho_f(t)|n\rangle - \langle n|\rho_f(t)|n+1\rangle)])^2\Bigg\} \quad (49)$$

4 Conclusions

The mathematical formalism for the generalized intensity-dependent multiphoton Jaynes–Cummings model is presented for an arbitrary mathematical function f describing the dependency on the intensity. The time evolution of the mean value of the atom inversion operator is calculated for two simple cases of the function f. The mean photon number and the dispersions of the two quadrature components are also calculated for an arbitrary function f in the case of a squeezed state as initial state of the electromagnetic field.

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