Chapter 1
Cosmophysics of Modified Gravity

Ruth Lazkoz

1.1 Breaking Scientific and Geographic Boundaries Through the Physics of Gravitation During a Societal Plight

One of the biggest achievements of Physics has been tailoring a standard cosmological model to describe the Universe quite successfully through a vast multitude of observations. In this architectural miracle the role of buildings bricks and joists is played by exquisite observational data, informing us about different epochs, regions and regimes. Evidence ranges from cosmic microwave background fluctuations to supernovae luminosity distances, along with baryon acoustic oscillations, cluster mass measurements and several other probes. Note that our access to an exquisite profusion of data is a fortunate sign of our times, while that was not certainly the case when the architects of modern cosmology set the foundations of our discipline. They certainly did not work in such gilded stage, as Nobel Laureate James Peebles recalls about his graduate years:

> I was very uneasy about going into cosmology because the experimental observations were so modest.

However, our standard cosmological paradigm is a little patchy picture: in general, transitions between different epochs and scales are somewhat poorly depicted. Traditionally, cosmologists have aspired to fitting as much physics as possible into a full-fledged single model, but this goal faces challenges which have recently become quite an issue. In this respect we can mention two that have gathered some attention for different reasons.

Firstly, we witness the discrepancy between reputed teams of cosmologists as to what is the speed at which astronomical bodies are hurtling away from us, unprivileged observers. This results in the so called tension on the current value of the H_0

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parameter (see Chap. 32 by Di Valentino), a concern that has been around us for some years now and that does not cease to be a lively source of controversy, quite the opposite.

Secondly, I wish to add to this very short editor's choice of baffling problems another bold departure from (cosmological) orthodoxy that has been put forward according to reliable evidence coming from *Planck* data: the spatial curvature of the Universe might be non zero [1]. The debate is alive and kicking, although it might be closed by a thoughtful identification of degeneracy-breaking data set, cosmic chronometers, which, perhaps not so anecdotally rely heavily on our understanding on galactic evolution [2].

We may be tempted to regard these two quandaries as astrophysical rather gravitational, but gravity is by all means the dominant interaction on the scales of astrophysical interest, which are revealing to us every day more exquisitely. For this reason, it is clear that these are times when a fluid dialogue between theory and experiment is very much needed. As Albert Einstein had it:

> A theory is something nobody believes, except the person who made it. An experiment is something everybody believes, except the person who made it.

It may seem from the previous paragraphs that the routes to test the gravitational interaction are restricted to macroscopic realms where concentrations of mass and/or energy are very significant, but even though full characterizations of effects in some modified gravity scenarios are still lacking, our improved understanding (and in particular beyond Riemannian frameworks) tells us that microscopic experiments might bring surprises (see Chap. 21 by A. Delhom). Just keep in mind that:

> We often fail to notice things that we are not expecting. —Lisa Randall

Another possible summary from the preceding paragraphs is that the array of fundamental problems the community is involved is certainly quite vast, and hence I could have easily arranged a different set of questions to tackle rather than the pair up above (even without venturing into the quantum gravity dominion). Yet, in my personal view, the current wide spectrum of enigmas is nothing but a manifestation of our need to improve and extend the standard body of our knowledge in gravity. Fortunately, even though history tells us that early motivations to go beyond General Relativity were more intellectually motivated by Mathematics, it is Physics itself where such explorations find their roots at present (see Chap. 3 by C. Boehmer and references therein). Needless to say that a portion of our readership may benefit from the review Chap. 2 by J. P. Mimoso on basic aspects of General Relativity that acts as a kick-off to this volume.

One of the most tempting routes of modification of General Relativity (GR) has been the possibility that our spacetime has more than four dimensions, with the extra ones inaccessible to low-energy exploration methods (as explained in a much broader context in Chap. 8 J. A. R. Cembranos). But it is well known that if we want to describe our macroscopic experience of the Universe in those scenarios, we have to roll up these extra dimensions. Such operation typically results in specific modifications of the gravitational Lagrangian, which can be found among the many covered in the huge assortment of possibilities discussed in the present manuscript (endorsed with their physical motivation, definitely).

But extra-dimensional gravity has not been the major operation ground of the European network CANTATA, whose work is celebrated by this and all following chapters. In general, our large team has rather set the focus on other enticing possibilities, those in which the gravitational Lagrangian is modified in a “why not” intellectually legit spirit, thus allowing other degrees of freedom to play a role in the construction of our theoretical settings. Put in a bit rough way: most modifications to be found in the following pages rely on modifying the action by resorting to scalars that can be built with either the metric tensor or different pieces of the affine connection.

Note that I have inconspicuously introduced in my dissertation the two main building blocks of our gravitational theory: differential geometry as its mathematical formulation on one hand, and the Lagrangian as the encoder of its physical content on the other hand. The necessary association among them emerges when we eventually find out how a manifold, furnished with a metric and a connection, dictates how all the particles that live in it move, and conversely, how all the physical manifestations of those particles cipher the metric and the connection. For a less over-scrupulous redemption of this powerful idea the reader can resort to the *ad nauseam* quoted pronouncement by John Archibald Wheeler:

> Space-time tells matter how to move; matter tells space-time how to curve.

Let me elaborate further on this by working for a moment on a setting in which Newton’s mechanics (and theory of gravity) is valid, say, Special Relativity (SR). In a covariant fashion, the (vanishing) acceleration we require to describe the motion of massive particles in free fall takes the following form for any observer other than the local inertial one:

$$a^{\lambda} \equiv \frac{d^2x^{\lambda}}{d\tau^2} + \Gamma^{\lambda}_{\mu\nu}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}, \tag{1.1}$$

where $\Gamma^{\lambda}_{\mu\nu}$, according to usual notation, represents the Christoffel symbol of the second kind, also known as the Levi–Civita connection, which is defined through the usual operation of combining partial derivatives of the metric and contractions (with the metric itself). Note that all the rest of notation used is so standard that explanations can hopefully be waived.

Now, in order to preserve our ability to switch consistently between the physical conclusions of different observers, we need to pinpoint rigorously some key aspects within the broad concept of “variation”. For instance, in this specific problem I have resorted to a definition of acceleration that works the same in all coordinate systems. It turns out that the (mere) partial derivative of the velocity with respect to the proper time does not render a covariant vector which can be interpreted as the acceleration.

This is where the covariant derivative enters the picture and lets us define the acceleration rigorously and (in principle) exactly as above. But this difficulty with derivatives is not just a particular problem of the velocity: partial derivatives in general do not play as good tensor operators, whereas the covariant derivative does indeed allow for parallel transport of tangent vectors along curves, i.e. it produces derivatives with tensorial nature. In this fashion, the acceleration is nothing but the covariant derivative of the velocity along the curves which the velocity field itself constructs, that is, $a^\mu \equiv u^\mu{}_{;\nu} u^\nu$. Nevertheless, there is a subtle point here, as this geometric route brings us back formally to the geodesics equation, that is, Eq. 1.1, but with the caveat that in this setting $\Gamma^\lambda_{\mu\nu}$ can represent any connection whatsoever, and not necessarily the Levi–Civita one, which is metric compatible and torsion free (and therefore symmetric). Roadmaps to geometric concepts such as the last two, which will be revisited often in this volume, can be found in Chap. 10 by D. Iosifidis and E. N. Saridakis and in Chap. 11 by T. Koivisto).

Before proceeding any further, an apology is in order, though, because I am taking advantage of the many more details other contributions are to offer in this collection to simplify my dissertation, and therefore I am giving myself permission to follow the Nobel Laureate Roger Penrose's recommendation:

> Do not be afraid to skip equations (I do this frequently myself).

The ambitious and broad next step would then to perform experiments to inform us of the motion of test particles, which would then reveal the specific form of the connection through their geometric properties. In this respect, this volume offers deeper insights into the boundless question of how curvature affects the motion of particles (see Chap. 26 by L. Á. Gergely for an overview on gravitational lensing). But in order to paint a master work of art and not a mere sketch, us physicists have to associate the equations of motion governing the pertinent trajectories with a physical framework, that is, we need to match particles and fields.[1]

The, perhaps, most elegant way to accomplish this task is the application of the variational principle to an action. The advantage of this method is how (relatively) undemanding it becomes to compare GR and its modifications with other classical field theories (such as Maxwell's theory). Nevertheless, for the sake of fluidity of this chapter, I feel compelled to write here the so very well-known action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_m \right], \tag{1.2}$$

where, again, standard textbook notation is being used, and therefore $\kappa = 8\pi G/c^4$, R denotes the Ricci scalar, and g is the determinant of the $g_{\mu\nu}$ metric. In the usual manner, I also let $\mathcal{L}_m$ account for any matter fields. Once the degrees of freedom of the action have been established, variation upon them will yield the (relevant) field equations, which will, in turn, be the route to the equations of motion. In this way, the geometric and matter pieces in the above Lagrangian yield the left and right hand side of the Einstein equations:

[1] Not that I pretend by any means to be an unpaired artist of gravitation myself. I rather mean the work of art, that the edifice of gravitational theories represent, will be or has been the result of collective efforts such as our network's.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}\,. \tag{1.3}$$

The mathematical identity represented by the vanishing covariant derivative of the left hand side, implies the vanishing of the right hand side, and four constraint equations follow. In the particular case that the energy-momentum tensor is that of incoherent matter, a straightforward[2] calculation can be followed to get exactly the same geodesics equation as above.

In case the reader finds this justification for the use of the variational principle not strong, I can rough out a two-fold additional justification. On one hand, the action provides a most fundamental starting point, in the sense that it offers a link with the underlying high energy physics. On the other hand, it provides precisely the classical equations of motion, and this takes root on Wheeler's First Moral Principle:

> Never make a calculation until you know the answer. Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle.

In fact, this can be used as a *mantra* from now on, as any modification of GR we may fancy, must be reducible to it under some restrictions, or in the correct limit, as meekly as GR becomes SR in the absence of gravity, or Newton's gravity in the weak-field and low-velocity limit. But this would be a reduction pursued along the route of physical properties, whereas we can follow a more mathematical track (or as least one which is not so physically grounded at its onset, which however converges with Physics as it progresses). This route to modify Einstein's gravity is that offered by considering a non-trivial connection, as it allows to produce two extra fundamental objects with valuable relevant geometric information (see Chap. 11). The first one is the non-metricity tensor:

$$Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}. \tag{1.4}$$

The second one is the torsion, which stems from the antisymmetric part of the connection:

$$T^{\alpha}_{\mu\nu} \equiv \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}. \tag{1.5}$$

As explained in more detail in Chap. 10, the respective effects of torsion under parallel transport is to crack parallelograms into pentagons whereas what non-metricity produces are changes in dot products and vector lengths.

If the torsion and non-metricity tensors vanish (that is, if connection is symmetric and metricity holds), the Levi–Civita connection is recovered, and gravity "à la Einstein" is (in which the metric is the only degree of freedom). Conversely, the metric and the connection could be considered as independent objects, whose relations would be given by the field equations. This is the so-called Palatini formalism (see here below).

[2] Needless to say that in this framework straightforward actually means "inexperienced lecturer mind blowing".

In addition, and in view of the previous discernment, a general connection $\Gamma^{\alpha}_{\mu\nu}$ can be decomposed as [3, 4]:

$$\hat{\Gamma}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}, \tag{1.6}$$

where

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2} T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)} \tag{1.7}$$

is the contortion, and

$$L^{\alpha}{}_{\mu\nu} = \frac{1}{2} Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)} \tag{1.8}$$

is the disformation.

The Riemann tensor for this general connection turns out to have a too phenomenal look for me to avoiding to portrait it here, and I will restrict myself to just mentioning that, by resorting to some clever tricks, it can be made vanish, thus leading to two equivalent formulations of gravity. These two rely upon the definition of the invariants $\mathbb{T}$ or $\mathbb{Q}$, obviously related to the torsion and non-metricity. Specifically the corresponding two new settings stem from the Einstein–Hilbert Lagrangian upon the replacements $R \to -\mathbb{T}$ or $R \to -\mathbb{Q}$, which lead respectively to the metric teleparallel or symmetric metric teleparallel versions of GR. Full fledged chapters of this volume, namely Chaps. 11 and 14 by S. Bahamonde, K. F. Dialektopoulos, M. Hohmann, J. L. Said, are devoted to these modifications of gravity and the reader is invited to dwell into them.

However, it is not convenient to lose sight of the physical perspective. Let us go back to the need of celebrating the ability of Einstein's relativity to describe most of the physical behaviours we have access too, and acknowledge at the same time that many puzzles are still standing, and therefore modifications attempting at solving them must have a sensible GR limit. This is, of course, one of the key aspects of one of the most popular ways to modify Einstein's gospel: the so called scalar-tensor theories. Historically, they originated as an arguably better way to incorporate Mach's principle into gravitational theory, a trick to make it impossible to gauge away the gravitational field completely, leaving behind a remnant which depends on an universal quantity such as the mass of the whole Universe. Under these rules, the Lagrangian will then typically become dependent on a new quantity, the scalar field ϕ.

In the pioneering presentation of this seductive idea, Brans and Dicke considered

$$S = \int d^4x \sqrt{-g} \left[\frac{\phi}{16\pi} R + \mathcal{L}_{\phi}(\phi, g_{\mu\nu}, \phi^{;\alpha}) + \mathcal{L}_m \right], \tag{1.9}$$

with a carefully tailored $\mathcal{L}_{\phi}$ so that the field equations do not display derivatives of order higher than second. This is important so as to guarantee the theory remains

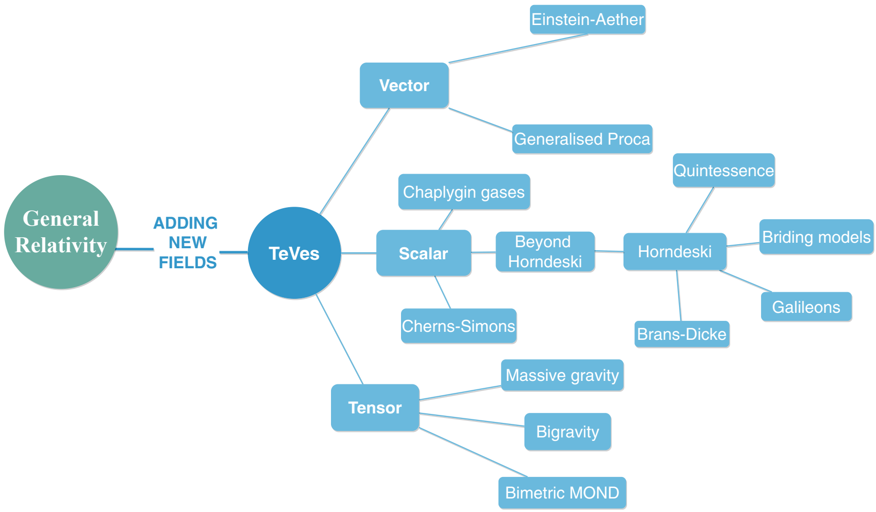

Fig. 1.1 Schematic categorisation of the Tensor-Vector-Scalar (TeVeS) class of theories, arising by adding new fields to General Relativity

ghost free (that is, no Ostrogradski instabilities appear). One can of course consider to replace the ϕR with $f(\phi)R$ by means of an arbitrary function, which again requires a wise choice of the term $\mathcal{L}_\phi$ to avoid divergences.

More generally, by relaxing some requirements, it is possible to engineer the family of all models with a Lagrangian containing second order derivatives of the field leading to second order equations of motion. These are the so called Horndeski scenarios, which were rediscovered some decades after their first appearance, and were then dubbed Galileon models. In a compact way we can rewrite them as

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_H(\phi, g_{\mu\nu}, \phi^{;\alpha}, \phi^{;\alpha}{}_{;\beta}, R^{\eta\sigma}{}_{\lambda\tau}) + \mathcal{L}_m\right] . \qquad (1.10)$$

This is the right place to invite the reader to have a look at the contribution by P. Martín Moruno in Chap. 6, and references therein, where a more detailed account is offered on scalar-tensor theories such as those defined above and others beyond. In Fig. 1.1 we present a schematic categorisation of the Tensor-Vector-Scalar class of theories, arising by adding new fields to General Relativity.

But unstoppable as curiosity and imagination are, the continuously growing set of generalisations of GR has one subset that stands above all others because of its popularity and (if we may) promises, the $f(R)$ proposal, where the Ricci scalar R is replaced in the original Einstein–Hilbert (plus additions) action:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_m\right] . \qquad (1.11)$$

With their generality, tractability and flexibility, these scenarios offer chances to reproduce a wide assortment of cosmological kinematics (see the staggering overview in Chap. 5 by Á. de la Cruz-Dombriz). They are constestably able to provide mechanisms to explain either early or late-time acceleration (fitting *customer's* needs) and, in a sense, they represent the simplest general modification of Einstein's gravity than can be thought of. The understanding of the capability of this broad context to reproduce a succession of necessary stages (radiation-like, matter-like, and dark energy-like) has improved over the year along with the restrictions they are subject to. But then one should not be fooled by the potentialities offered by $f(R)$ theories: hardly ever has there been in the history of Physics an one-size fit to all theoretical framework, which is actually a feature that has made our discipline precisely to play its unmatched role in science. From a strictly very personal perspective, it is somewhat frustrating how often we hear/read the claim that these scenarios can always be tweaked so they fit the data, which is quite a too loose statement if one does not provide an accompanying report of the quality of fit and other demanded statistical criteria. At the end of the day, if $f(R)$ theories (and other modified gravity routes) were the "holy grail" there would be no justification to write a volume such as ours. So, brushing aside this perhaps hypercritical tone, we should celebrate modified gravity models as laboratories to tests ideas, predict difficulties and venture solutions. Remember, we get challenged and, as put in the words of Neil de Grasse Tyson,

> The Universe is under no obligation to make sense to you.

Let me now, however, briefly sketch other relevant works shaping the section of the volume devoted to research in the theoretical front (that of Working Group (WG)1), so as to offer a tidbit of the high tea menu it represents. Clearly, a simple gaze at the table of contents reveals that the role played by the metric-affine or Palatini formalism is major in modified gravity, in general. It allows to relax the "standard" convention between the metric and the connection, so that the latter can be worked out from first principles [5]. I will make no distinction between the Palatini and the metric-affine formalism, as it is customary in many works, and in some of our chapters, although formally a bit of extra rigour (and a higlight of differences) is in order if fermions are present in the matter Lagrangian [2, 6], which is not really necessary for most cosmological applications.

The metric-affine scheme progresses initially from the construction of a curvature scalar using the connection and the metric tensor as independent quantities, but then it can be enlarged considerably by constructing Lagrangians with other scalar terms built from the symmetric part of the Ricci tensor and its contractions with the metric tensor. These settings offer many new attractive possibilities, such as the removal or smoothing out of cosmological singularities which are generic in GR scenarios, as discussed in Chap. 12 by A. Delhom and D. Rubiera-Garcia). Another remarkable point of the Palatini formulation is that, unlike in the metric approach, second-order field equation are always obtained, even for Lagrangians with terms which are non linear in the scalar curvature or include the above mentioned less standard scalars. A related approach is hybrid-metric Palatini gravity, where the standard R term coming

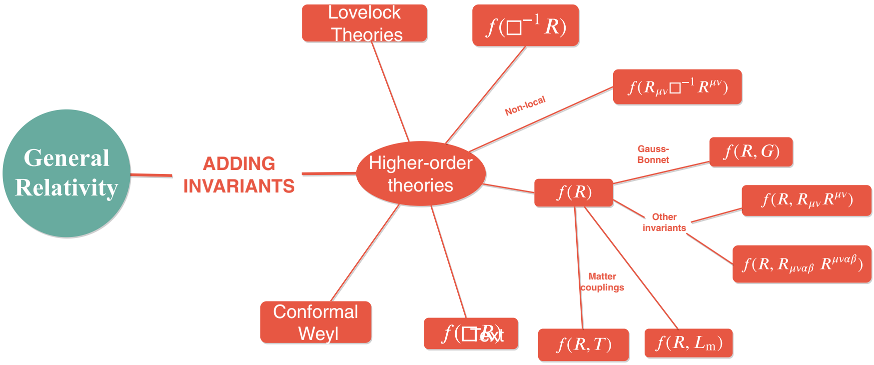

Fig. 1.2 Schematic categorisation of the higher-order class of theories, arising by adding higher-order invariants in the Lagrangian of General Relativity

from the metric connection is added to extra terms depending on an alternative curvature scalar, $\mathcal{R}$, derived from an independent connection, such terms offering a richer phenomenology as to the evolution of matter inhomogeneities. These and many other aspects are covered in the contribution by F. S. N. Lobo in Chap. 13. In Fig. 1.2 we present a schematic categorisation of the theories, arising by adding higher-order invariants in the Lagrangian of General Relativity.

Alternatively, a (wilder and) completely separate route to describe gravity as a manifestation of geometry can be pursued too: teleparallel gravity and its extensions. These replace curvature with torsion and are build upon a curvature-free connection (therefore flat). Following the recipe I anticipated above, one possible Lagrangian starting point for teleparallel descriptions of gravity is:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,[f(\mathbb{T}) + \mathcal{L}_m]\,. \tag{1.12}$$

Among the attractive features of teleparallel gravity we see again the second-order nature of the equations and the broadness of the whole setting, which allows to rewrite many theories built from the metric and the Levi–Civita connection. It is also certainly very appealing that the theory allows to waive the equivalence principle, thus re-framing gravitation as an interaction more similar to other fundamental ones. Many aspects of teleparallel gravity are covered into the sizeable contribution in Chap. 14, where the authors do not forget to acknowledge the absolute need to continue to examine these extensions in the light of observations. Actually, the collection of new features offered by these framework is tremendous, as the possibility of interpreting it as a gauge theory of translations contributes to filling the gap between gravity and other interactions in Nature, or as the intellectually not less challenging question of the definition and characterization of singularities in this alternative formulation of gravitational physics. In Fig. 1.3 we present a schematic categorisation of the theories arising by modifying the geometry of General Relativity.

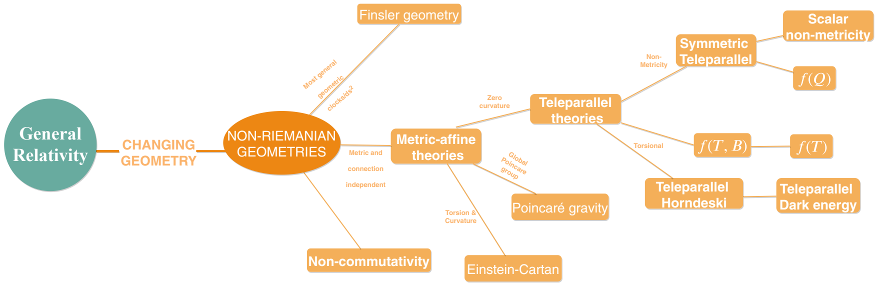

Fig. 1.3 Schematic categorisation of the theories arising by modifying the geometry of General Relativity

Recapitulating, there seem to be two main and disparate ways to manufacture modifications to GR: the addition of new fields on one hand, and geometry based restylings on the other hand. This splitting is largely a matter of convention, as prescriptions to go from one to another can be typically found with manageable techniques (see again Chaps. 5 and 13). But this dual possibility has proved so far more enriching than a source of confusion, and it continues to provide motivation and inspiration.

Nevertheless, this whole collection of possible alternatives to Einstein gravity may seem as if the community was meandering rather than following a straight course. But remembering Roger Penrose's words again, is just opportune:

> Sometimes it's the detours which turn out to be the fruitful ideas.

In this sense, I also invite specialists to explore the contribution by N. Voicu and C. Pfeifer in Chap. 15 and learn about the possibilites of relativistic extensions of Finsler geometry, with their intrinsic unusual features such as multiple covariant derivatives and matter dynamics.

Perhaps, some readers have noticed the absence of references to the colossal problem of our lack of substantial understanding of gravity at the quantum level. The reason is mainly that it falls beyond the major objectives of the Action. This choice is really not a gesture of contempt, but rather a need to concentrate efforts on an otherwise major attempt, which is to understand gravity at the mainly classical level. Our team however is happy to boast about intrepid representatives which explore the quantum side of gravity through (again) extensions of Einstein's framework. In this respect see Chap. 9 by G. Calcagni, Chap. 16 by R. Garattini, Chap. 17 by M. Bouhmadi-López and P. Martín-Moruno, and Chap. 27 by R. Casadio and A. Giusti. In Fig. 1.4 we present a schematic categorisation of the theories arising by the use of quantum arguments.

Let me again stress that the two main routes to knowledge about modifications of our understanding of gravity are theory and data analysis, which must act synergistically, as it has been the case of the activities of our Action. This volume is a veracious reflection of that fact, with a carefully tailored choice of contributions at the forefront of the field in its broadest manifestation.

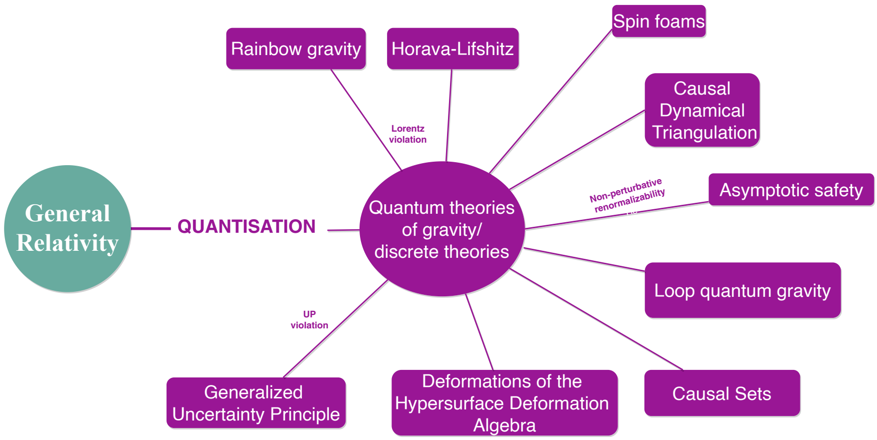

Fig. 1.4 Schematic categorisation of the theories arising by the use of quantum arguments

CANTATA researchers whose theoretical work makes strong contact with observations have contributed to WG2 (testing relativistic effects) an WG3 (observational discriminators) mainly. Their daily research encourages them to reflect pointedly about the questions that these new observational findings pose. This allows these scientists to perform masterly forecasts of constraints to be placed on theoretical frameworks by the coming generation of cosmological observations. Remember in this context the wise words by Vera Rubin:

> Science progresses best when observations force us to alter our preconceptions.

One important area of operation where scientists in WG2 of our team have completed proficient works are tests of gravity at scales well below 1 Mpc, that is, non-cosmological ones (clusters are not covered in Part II, but rather in Part III), ranging from earth laboratory tests to orbits around compact objects. Within this domain, screening mechanisms in scalar-tensors theories with a scalar field coupled to matter have gathered a lot of interest. The reason is that the screening treats differently traditional gravity tests and laboratory tests in the sense that it allows effects to evade detection by the former, while revealing to the latter, as if some kind of reward for clever and inspired novel experiments were offered. Several screening mechanisms have been proposed so far [7], and among the most popular we find those with canonical kinetic terms on one hand, such as the chameleon and the symmetron mechanisms, and then a third one on the other, the Vainshtein mechanism, which emerges from derivative non-linearities. In the contribution by Anne-Christine Davies and Benjamin Elder in Chap. 19 the focus is mainly set on tests on chameleon scalar fields through different approaches such as vacuum chambers, atom interferometry, torsion balances, Casimir force searchers and others. The symmetron field is covered as well, but with less extension, as research on this area is more recent. In a wide sense the dynamics of the scalar field is governed by an effective potential:

$$V_{\text{eff}}(\phi) = V(\phi) + (A(\phi) - 1)\rho_m. \tag{1.13}$$

The (square of) the auxiliary function $A(\phi)$ encapsulates the coupling between matter and the scalar field in the form of a conformal rescaling between the metric used to express the matter sector of the action (Jordan metric/frame) and the metric used to render the Einstein equations in the "orthodox" fashion (Einstein metric/frame). Further details can be found in Chap. 20 by P. Brax. But the zoo of screening mechanisms contains more species, and when one lets curiosity (or imagination) run and non canonical kinetic terms enter the picture, we can find other mechanisms such as K-mouflage (see Chap. 20) described in this volume, a screening possibility which has so far not been tightly constrained by gravitational wave observations.

Now, if Earth-based laboratories are at the bottom of the scale ladder of experiments to test modifications of gravity, the solar system is the obvious next: a weak-field limit realm which is typically treated under the parametrized post-Newtonian formalism, an in-between stratagem to wrap-up conjectures about both observations and theories being considered, which connects the former with the latter through a set of parameters. This is explained in detail in the contribution by M. Hohmann in Chap. 24, where both the standard version and some extensions are discussed. In fact, a large class of extensions of GR can be accommodated into the customary formulation, but the community has not stopped there, and broader versions have been devised for cases demanding so. The (well-known) first two parameters of the expansion (γ and β) estimate how mass creates curvature on one hand and the degree of nonlinearity in the superposition law for the gravitational potentials on the other hand. Stringent bounds have been obtained through refined experiments and can offer light on the viability of modifications of gravity with significant impact on those parameters and subsequent ones.

But the prevailing weak-field/strong-field battle in physics explorations demands we also turn our attention (as a team) to representatives of largest curvature and highest densities regimes: compact stars and black holes. Modelling stellar structure is a demanding task in the default GR setting, and modifications of gravity introduce additional difficulties, both from the mathematical and physical perspectives. Hardly any features can be studied from a general formulation and individual investigations are typically the way to go. New knowledge will, for instance, allow to spot (if any) changes in the mass-radius relation of neutron stars, as it will depend on new parameters characterizing the modifications (see Chap. 22 by G. Olmo, D. Rubiera-Garcia, and A. Wojnar). The analogous operates on black holes, in the sense that the famous absence of hair theorem may need a tweak in the presence of additional gravitational degrees of freedom.

These are just some of the surprises that compact objects (whether extreme or not) may have in store. Another is spontaneous scalarization, which consists in the emergence of a non-trivial configuration of a scalar field without sources and which vanishes asymptotically. For this to occur a scalar field must exist which is non-minimally coupled to gravity. This highly nonlinear effect may offer tight limits to the modified scenarios through observations of, for instance, binary systems as discussed in Chap. 23 by J. L. Blázquez-Salcedo, B. Kleihaus, and J. Kunz.

When we move to (much) larger scales we must publicize that efforts of CANTATA researchers have served the international effort of the community on the design of future accurate tests of gravity (modified or standard) and the improvement of current ones with their expertise on numerics and computation. Some of our colleagues have made distinguished contributions to large teams expected to lead the design and operation of terrestrial and space-based surveys which will be operating in the near future. A glimpse of the breathtaking future is offered by observations of gravitational waves, which have brought strong implications for modified theories of gravity. At the cosmological background level [8], an additional friction that adds to the Hubble one appears, and this modifies the amplitude of the waves, whereas an effective (anomalous) mass and consequent atypical speed alter the phase, as discussed in Chap. 34 by J. M. Ezquiaga. Notoriously, gravitational waves provides a new observational channel for testing gravity theories which predict a non-null mass for the graviton. In this context we can highlight the closely related massive gravity and bigravity settings, which belong to the field theory framework of gravity and of which a glimpse is offered in Chap. 7 by L. Heisenberg.

But more sophisticated and challenging effects could also arise in the physics of gravitational waves when modified gravity scenarios are considered, as not only additional polarisations emerge (up to four extra ones), but also they could get mixed up and frequency mutations might be produced too; see Chap. 25 by M. Sakellariadou. Actually, the detection of these additional polarisation modes represents a significant technical challenge.

Into the bargain, the absolutely greatest promise of gravitational waves is the cosmological realm. Events that can be regarded as standard sirens [9] (may be able to) probe the redshift evolution of the luminosity distance of the gravitational wave source, which is proportional to the inverse of the amplitude. In principle, it can be different from the electromagnetic luminosity distance of a companion event, and therefore their ratio will offer a test for parameters associated with physics beyond GR and also with dark energy evolution (which is also referred to, sometimes, as "new physics", as differing from the ΛCDM setting).

Now, along those lines, the scheme which makes currently most sense is to combine those measurements with observations of the large scale structure coming from surveys which will probe the Universe in different scale regimes as compared to the size of the horizon. Further details on this classification are given in the contribution by Y. Akrami and M. Martinelli in Chap. 29, where readers can find a summary of one of the most popular formulations of perturbations valid for the linear regime (that is, for horizon size scales or intermediate scales).

The starting point for this approach is to rewrite the Friedmann-Lemaître-Robertson-Walker metric in the Newtonian gauge as

$$g_{\mu\nu} = \mathrm{diag}\left[-(1+2\Phi), a^2(1-2\Psi)\delta_{ij}\right], \tag{1.14}$$

where a is the scale factor and Φ and Ψ are the gauge-invariant Bardeen potentials.

One route to progress is making use of the quasi-static approximation, which is valid for the subhorizon linear regime and significant breadth of gravitational theories. This approach produces a sizeable simplification of the pertinent equations, as the extra degrees of freedom associated with departures from GR can be jumbled up in the (generally) time and space dependent functions μ and η, respectively the effective Newton's constant and the gravitational slip. The second function (that is, η) makes the two Bardeen potentials differ from each other, unlike the GR case, and leads to observable weak lensing effects through the combination of μ and ν; whereas ν (on its own) affects the growth of structure; see Chap. 33 by L. Kazantzidis and L. Perivolaropoulos.

Related to the previous discussion, one analytic method based on the perturbation theory framework I particularly like to highlight is the Effective Field Theory (EFT) approach, which can capture interesting effects on scales smaller than the intermediate one, and in particular the onset of the transition from GR to a modified gravity regime, which is crucial for the research objectives of our Action. In particular, it depicts physical effects germane to macroscopic scales by integrating out short-distance features, so they appear on long-distance characteristics as extra/perturbative parameters. This chassis supports any dark energy or modified gravity model possessing one additional scalar degree of freedom in the way described in detail in Chap. 31 by N. Frusciante and S. Peirone.

These theoretical developments are, of course, a concoction to make the community ready to take advantage of future large scale structure surveys when they become available. In this respect even though the current constraints are of $\mathcal{O}(1)$, they are expected to get as low as $\mathcal{O}(10^{-1})$, and there is hope that violations of the current observational inference of $\mu(z = 0) = \nu(z = 0)$ could be detected (see again Chap. 29).

Among the abundance of outputs of future large scale structure surveys we must mention the sensitivity of the clustering of galaxies to the (specific) theory of gravity under play. Effects less tangible than density perturbations and redshift-space distorsions will place unprecedented constrains which are obtained by confronting pertinent gauge-invariant quantities (the two Bardeen potentials being among them again) with the information provided by the power spectrum and its multipole expansion, exploring effects which are currently neglected in current surveys; see the contribution of C. Bonvin in Chap. 30. Note that the importance of these studies is twofold. On one hand they vindicate the role of galaxies as concentrations of baryonic (and therefore electromagnetically accessible) concentrations of matter for the study of the Universe, and on the other they offer a fundamental channel to explore the vital role of dark matter. In addition, it is relevant that the interplay between baryons and dark matter [10], which takes place in regions with high matter density, also allows to test the equivalence principle [11]

The teamwork between dark and baryonic matter and its proportions in the Universe can be literarily cast into the dainty mould of these words by one of the clearest minds in gravitation, Arthur S. Eddington:

> An ocean traveler has even more vividly the impression that the ocean is made of waves than that it is made of water.

But having stated the paramount importance of galaxies to understand modified theories of gravity the Universe, we cannot forget that the Universe offers us even better laboratories, galaxy clusters. Their ambivalence as both astrophysical and cosmological objects can help us discriminate between gravitational effects and cumbersome astrophysical phenomena as thoroughly covered in Chap. 36 by I. D. Saltas and L. Pizzuti, where prospect of kinematic, thermal and lensing explorations are reviewed yet again resorting to perturbative quantities considered in other contributions, such as the gravitational slip η. Nevertheless, for a flavour on the sort of discriminating criteria clusters can offer when time evolution of scalar fields is addressed, it is worth check the contribution by D. Mota in Chap. 37, which closes those related to WG3.

But then again, going back to cosmological scales, I must stress that studies with perturbative ingredients will necessarily be flawed until some pending disquietudes regarding the cosmological background and its linear perturbations are relieved. The most talked about one being the tension between high and low redshift data estimates of the Hubble constant, H_0 (see Chap. 32 for a state-of-the-art review and a sketch of what lies road ahead). The second actor in this stage play of tensions is the apparent discrepancy in the amplitude of the matter power spectrum as set by σ_8, the root-mean-square fluctuations in the matter mass density in a comoving sphere of diameter 8 Mpc. Large compilations of redhift-space distorsions and other dynamical probes show a statistically significant discrepancy with *Planck* data and would favor an evolving and weaker effective Newton's constant (through μ), and readers can head to the contribution of Chap. 33 for a detailed discussion and review of the topic. Clearly, all complementary routes offered by tests of dynamical features are destined to play a most relevant role, for instance weak lensing. Its very-hard-to-spot effects are again encoded in the Bardeen potentials, and forecasts have been carried out for surveys such as *Euclid* in the theoretical context of Horndeski theories; see Chap. 35 by V. Pettorino and A. S. Mancini.

It should be clear by now to the reader that has come so far, that the route to erudition in this limitless field is grievous, and will unavoidably require all the literature anybody can digest so as to get an exquisite insight on caveats of these scenarios. Such an exercise would undoubtedly lead the candidate to a multifaceted and more thorough understanding of (her/his favourite flavour of) gravity through questions such as whether the correct weak field limit is attainable, whether instabilities occur (a typical pathology of higher order theories), and whether the initial value problem is well posed. Nevertheless, and here comes the crux of the matter, no proficient understanding of a modified gravity framework can be reached without an analysis of the formation (and sustenance) of structures. In this context I cannot but highlight again the key question of whether its cosmological perturbations of the cosmological background are capable of leaving a blueprint agreeable with the currently observed patterns in the cosmic microwave background and the large-scale structure itself. The intimate connection between theory and observations is therefore an unbreakable bond, and the seed sown by our team's work will surely thrive and feed our knowledge hungry community, and, again, as a team, our feeling is that:

> This is the way —The Mandalorian

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