Unsteady Resonant Oscillations of a Gyroscopic Rigid Rotor with Non-linear Damping and Non-linear Rigidity of the Elastic Support

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Abstract. The article is concerned with the effect of linear and cubic non-linear damping of an elastic bearing on forced resonant vibrations of a gyroscopic vertical rigid rotor taking into account non-linear stiffness of the cubic nature of the bearing material. It is confirmed that non-linear cubic damping of the support can suppress not only the maximum amplitude, but also the amplitudes of forced unsteady oscillations behind the rotation speed corresponding to the maximum amplitude and the variation of its values in time along the main curve, around its mean values. It shifts the speed of rotation of the amplitude maximum, with rigid and soft non-linear elastic characteristics of the support material downwards and upwards, respectively. It is shown that with a "slow" increase in the shaft rotation speed, an increase in the absolute value of the angular acceleration is accompanied by a shift of the amplitude peak towards high speeds, with a "slow" decrease in the shaft rotation speed – towards low speeds with a decrease in the amplitude of oscillations. It is shown that during the rotor takeoff run, the maximum amplitude for the case with a rigid non-linear elasticity characteristic of the support material is greater than the same value for the case with a soft non-linear elasticity characteristic of the support material, and conversely, during the rotor run-down for similar cases.

Keywords: Gyroscopic rotor · Non-linear rigidity · Non-linear damping · Unsteady oscillation

1 Introduction

The operating speeds of rotary machines can be above or between critical speeds. In the practice of rotary machines operation there were cases when the machines had unacceptably high vibrations during the transition through the critical speed(s). It is known that one of the main causes of shaft vibration is the inertial forces of unbalanced masses.

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A simplified model with lumped parameters of the rotor system, as a rule, is used to study the dynamics of the shaft of one rotor on the bearing supports. It is very important to use properties and characteristics of the material of the supports for attenuation and damping of vibration in order to stabilize movement of an unbalanced rotor and vibration systems. Supports are the means of connecting the device between the rotor and the supporting structure, which have various shapes and designs, depending on specific assumptions.

A convenient way to introduce attenuation to support bearings in a rotor system on viscoelastic flexible rubber supports [1]. In parallel with the development of viscoelastic material modeling, which helps to describe the complexity of material properties, the use of viscoelastic components in the dynamics of the rotor and vibration systems also, increased as a whole, in particular with non-linear elastic characteristics and damping. So, for example, in works [2, 3] the influence of quadratic non-linear damping on resonant oscillations and stability of a gyroscopic rotor with quadratic or cubic non-linear stiffness of an elastic support was considered. Studies [4–8] show that linear and non-linear cubic damping can significantly suppress the resonant peak of the fundamental harmonic, eliminate the jump-like phenomena of the non-linear system. In non-resonance regions, where the vibration frequency is higher than its resonance value, non-linear cubic damping, unlike linear damping, can reduce the amplitude of the rotor vibration. Therefore, in all regions of oscillation frequency (rotation speed), only non-linear cubic damping can support the performance characteristics of the vibration isolator. The work [4] provides an excellent overview of research on linear and non-linear vibration-isolating systems.

Non-linear damping suspension can affect the stability of the flexible rotor in short journal bearings. In the work [9] a numerical method is used to solve the equations of motion, and bifurcation diagrams, orbits, Poincaré maps, maps and amplitude spectra are used to display motions. The results of works [2–8] are confirmed.

Under unsteady oscillations, the amplitude and frequency of disturbances change, they differ significantly from the oscillations observed under constant frequency and amplitude of disturbances.

Recently, unsteady oscillations have begun to be studied as transient processes in systems in connection with the spread of methods of direct and analytical-numerical modeling of the equations of the oscillatory process. Therefore, there are ample opportunities for new research in this direction.

This article examines the unsteady vibrations of a gyroscopic rotor with a vertical rigid shaft mounted on the lower hinge and upper elastic bearings. An ideal system is modeled, non-linear differential equations of the rotor motion are solved analytically by the method of varying amplitude, which allows obtaining a system of shortened equations and equations of unsteady oscillations of the rotor. Assuming that the speed of rotation of the shaft is a function of "slow" time, these equations are solved and their results are compared. The influence of non-linear cubic damping of elastic support, non-linear characteristics of support elasticity, rate of "slow" change in shaft rotation speed on the amplitude-frequency dependence of rotor oscillations is investigated. The numerical results of solving the rotor motion equations are compared with the analytical results.

2 Equations of Motion

The rotor is considered, the structural diagram of which is shown in Fig. 1. Rotor consists of a shaft with a length L, mounted vertically by means of a lower hinge and an upper elastic support spaced from it at a distance l_0 and a disk fixed at the free end of the shaft, having a mass m, a polar moment of inertia I_P and a transverse moment of inertia I_T the same for any direction. The elastic support has linear stiffness k_1, non-linear stiffness k_3, linear damping μ_{d1}, non-linear cubic damping μ_{d3}. The speed of the shaft rotation $\dot{\varphi} = \omega$ is such that the rotor can be viewed as a gyroscope, the fixed point of which is the lower shaft support. The position of the geometric center of the disk S is determined by coordinates x, y in a fixed coordinate system $Oxyz$, and the position of the shaft and the rotor as a whole in space by the Euler angles α, β and the angle of rotation φ. The angles α, β are small, the movement of the rotor in the direction of the coordinate axis z is neglected. Next, denote the coordinates of the center of mass m of the disk through x_m and y_m. Assume also that the linear eccentricity e lies in the direction of the N axis of the $ONKZ$ coordinate system rotating with the rotor. Restrict to small deviations of the rotor axis.

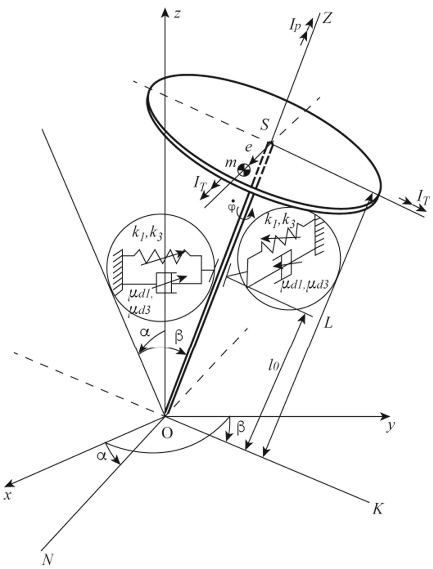

Fig. 1. Rotor geometry

Expressing the projections of the angular velocity of the rotor in the coordinate axes of the $ONKZ$ system, the coordinates of the center of mass of the disk and the coordinates of the upper support through the angular coordinates α, β and φ, finding expressions for the kinetic energy, potential energy of the rotor, the Rayleigh function and the projections of the moments of forces acting on the system, substituting them into the Lagrange equations of the second kind using the following dimensionless parameters

$$\begin{gathered} l = l_0/L;\ \bar{t} = t\omega_0;\ \overline{\tau} = \tau\omega_0;\ \Omega(\overline{\tau}) = \omega(\tau)/\omega_0;\ \overline{I}_T = I_T/(mL^2); \\ \overline{I}_P = I_P/(mL^2);\ \overline{K}_1 = k_1/\left(m\omega_0^2\right);\ e_r = e/\left[L\left(1+\overline{I}_T\right)\right]; \\ I_{P1} = \overline{I}_P/\left(1+\overline{I}_T\right);\ \overline{G} = g/\left(L\omega_0^2\right);\ K_3 = k_3 l_0^4/\left[mL^2\omega_0^2\left(1+\overline{I}_T\right)\right]; \\ \mu_1 = \mu_{d1}/\left[mL^2\omega_0\left(1+\overline{I}_T\right)\right];\ \mu_3 = \mu_{d3}\omega_0/\left[mL^2\left(1+\overline{I}_T\right)\right], \end{gathered} \tag{1}$$

where is $\omega_0 = \sqrt{\left(k_1 l_0^2 - mgL\right)/\left[mL^2 - \left(I_p - I_T\right)\right]}$ the natural frequency of the damped rotor system, obtain the equations of motion of the rotor in the form

$$\begin{aligned} \alpha'' + I_{P1}\Omega(\bar{\tau})\beta' + \mu_1\alpha' + \mu_3\alpha'^3 + \omega_n^2\alpha + K_3\alpha^3 = e_r\left(\Omega^2(\bar{\tau}) + \bar{G}\right)\cos\varphi, \\ \beta'' - I_{P1}\Omega(\bar{\tau})\alpha' + \mu_1\beta' + \mu_3\beta'^3 + \omega_n^2\beta + K_3\beta^3 = e_r\left(\Omega^2(\bar{\tau}) + \bar{G}\right)\sin\varphi, \end{aligned} \tag{2}$$

where $\Omega(\overline{\tau})$ is the dimensionless rate of the shaft rotation, depending on the "slow" dimensionless time $\overline{\tau} = \varepsilon\ \overline{t}$, $\varepsilon \ll 1$, is a small parameter [10].

On the right-hand part of the system of Eqs. (8) perturbations containing φ'', were discarded, since in the region close to the resonance velocity $\varphi'' \ll \Omega^2$, and perturbations having a parameter $\overline{I}_P$ (in what follows, assuming that $\overline{I}_P \ll \overline{I}_T$) and values of the second and higher orders of smallness with respect to α, β, their derivatives, and their combinations. The indicated disturbances are small in comparison with disturbances, the amplitudes of which are proportional to the angular velocity squared.

Consider a rotor system close to a linear system. Therefore, choose one of the asymptotic methods, for example, the method of slowly varying amplitudes [11]. For the direct use of this method, the following restrictions are taken to solve Eqs. (2). The projections of the moments of the damping forces $\mu_1\alpha'$, $\mu_1\beta'$ and $\mu_3\alpha'^3$, $\mu_3\beta'^3$, as well as the moment of the cubic component of the restoring force $K_3\alpha^3$, $K_3\beta^3$, the moments of the centrifugal force of the imbalance of mass and gravity $e_r\left(\Omega^2(\overline{\tau}) + \overline{G}\right)\cos\varphi$, $e_r\left(\Omega^2(\overline{\tau}) + \overline{G}\right)\sin\varphi$ are considered small in comparison with the projections of the moments of the vibration inertia force and the linear restoring force acting in the system. Assuming that $\overline{I_P} \ll \overline{I_T}$ the projections of the moment of the passive gyroscopic force can also be considered small, $I_{P1}\Omega(\overline{\tau})\alpha'$ $I_{P1}\Omega(\overline{\tau})\beta'$. we will also limit ourselves to considering a spinning rotor: $\Omega^2(\overline{\tau}) \gg \overline{G}$ and motion in the resonance range, where the frequency of free oscillations ω_n is close to the frequency of forced oscillations Ω, i.e.$\xi = \varepsilon\xi_1 = \Omega(\overline{\tau}) - \omega_n \ll \omega_n$.

Equations (2), at small values of the quantity ξ and restrictions accepted above will take the following form:

$$\begin{gathered}\alpha^{''} + \Omega^2(\overline{\tau})\alpha = \\ e_r\Omega^2(\overline{\tau})\cos\varphi - I_{P1}\Omega(\overline{\tau})\beta^{'} - \mu_1\alpha^{'} - \mu_3\alpha^{'3} - \omega_n^2\alpha - K_3\alpha^3 + 2\xi\alpha, \\ \beta^{''} + \Omega^2(\overline{\tau})\beta = \\ e_r\Omega^2(\overline{\tau})\sin\varphi + I_{P1}\Omega(\overline{\tau})\alpha^{'} - \mu_1\beta^{'} - \mu_3\beta^{'3} - \omega_n^2\beta - K_3\beta^3 + 2\xi\beta,\end{gathered} \tag{3}$$

where is $\omega_n = \sqrt{\left(\overline{K}_1 l^2 - \overline{G}\right)/\left(1 + \overline{I}_T\right)}$ the dimensionless natural frequency of the linear rotor system (3) at $\overline{I_T} \gg \overline{I_P}$.

Equations (3) are a system of second order nonlinear ordinary differential equations with respect to α, β.

3 Solutions of Motion Equations

In an oscillatory system, under the influence of damping forces, which cause attenuation of higher harmonics, single-frequency oscillations of the fundamental tone are established with a frequency close to the frequency of the disturbing force. The single-frequency method allows us to consider both stationary oscillations and the process of the rotor transition through critical speeds under very general conditions - causing the variability of the coefficients of the differential equations, in the presence of elastic supports with a non-linear characteristic of elasticity and non-linear damping. Although the law of variation of angular speed of the rotor can be obtained only on the basis of processing the results of experimental studies of acceleration and running down of the machine, but to determine the general nature of the transient process, the single-frequency method allows solving the problem with the arbitrary law of variation of angular speed of the rotor. The only limitation that determines the applicability of this method is the requirement for a slow change in the angular velocity with respect to the value of the natural frequency of the system under study.

Therefore, search for solutions (3) in the form:

$$\alpha = A\left(\overline{t}\right)\cos\left[\varphi + \theta\left(\overline{t}\right)\right], \beta = A\left(\overline{t}\right)\sin\left[\varphi + \theta\left(\overline{t}\right)\right]. \tag{4}$$

Here is $A\left(\overline{t}\right)$ the slowly varying amplitude, $\theta\left(\overline{t}\right)$ is the phase shift of the oscillations relative to the forced harmonic moment.

Further, using the method of varying amplitudes [11], obtain the equations of the transient process in the form

$$A^{'} = [e_r\Omega^2(\overline{\tau})\cos\varphi + \Big(2\xi\Omega(\overline{\tau}) - I_{P1}\Omega^2(\overline{\tau})\Big)Acos(\varphi+\theta) + \mu_1\Omega(\overline{\tau})Asin(\varphi+\theta) +$$

$$\mu_3\Omega^3(\overline{\tau})A^3sin^3\mu_3\Omega^3(\overline{\tau})A^3sin^3(\varphi+\theta) - K_3A^3cos^3(\varphi+\theta)]sin(\varphi+\theta), \quad (5)$$

$$A\theta^{'} = -[e_r\Omega^2(\overline{\tau})\cos\varphi + \Big(2\xi\Omega(\overline{\tau}) - I_{P1}\Omega^2(\overline{\tau})\Big)Acos(\varphi+\theta) + \mu_1\Omega(\overline{\tau})Asin(\varphi+\theta)$$
$$+ \ \mu_3\Omega^3(\overline{\tau})A^3sin^3(\varphi+\theta) - K_3A^3cos^3(\varphi+\theta)]cos(\varphi+\theta). \quad (6)$$

After performing averaging of Eqs. (5) and (6), the system of equations for the transient process of the rotor is obtained in the following form

$$A^{'} = -e_r\Omega^2(\overline{\tau})sin\theta/2 - \mu_1\Omega(\overline{\tau})A/2 - 3\mu_3\Omega^3(\overline{\tau})A^3/8 \quad (7)$$

$$\theta^{'} = -e_r\Omega^2(\overline{\tau})cos\theta/(2A) - \left[\Omega(\overline{\tau}) - \omega_n - \frac{1}{2}I_{P1}\Omega(\overline{\tau})\right]\Omega(\overline{\tau}) + 3K_3A^2/8 \quad (8)$$

4 Unsteady Oscillations

To illustrate the influence of the value of non-linear cubic damping of the support on the development of the oscillatory process when passing through the resonant region, consider calculation of the unsteady mode of motion of the rotor system under the assumption that the speed of the shaft rotation Ω is also a "slowly" changing parameter according to the law $\Omega = \Omega_0 + \nu\bar{t}$. The equations of the unsteady process (7) and (8), (5) and (6) were modeled in the Mathlab-Simulink package. The angular speed of the shaft rotation Ω increased "slowly" uniformly ($\nu > 0$) or decreased uniformly over ($\nu < 0$) time.

The system parameters have the following values: $e_r = 0.0346$, $\omega_n \approx 1$, $I_{P1} = 0.021$, $\mu_1 = 0.01$.

For K_3=0.1, choose the initial conditions for the case with $\nu > 0 : \bar{t} = 0 :$ 1) $\Omega_0 = 0.81$, $A_0 = 0.067$, $\theta_0 =$ -0.02521 with $\mu_3 = 0.01$; 2) $\Omega_0 = 0.79$, $A_0 = 0.06254$, $\theta_0 =$ -0.02297 with $\mu_3 = 0.02$; 3) $\Omega_0 = 0.79$, $A_0 = 0.0625381$, $\theta_0 =$ -0.0230614 with $\mu_3 = 0.043$, for the case with $\nu < 0 : \bar{t} = 0 :$ 1) $\Omega_0 = 1.39$, $A_0 = 0.0640693$, $\theta_0 = 0.0134013$ with $\mu_3 = 0.01$; 2) $\Omega_0 = 1.39$, $A_0 = 0.0640692$, $\theta_0 = 0.0134806$ with $\mu_3 = 0.02$; 3) $\Omega_0 = 1.39$, $A_0 = 0.0640691$, $\theta_0 = 0.0136628$ with $\mu_3 = 0.043$.

For $K_3 =$ -0.1, accept the initial conditions for the case with $\nu > 0 : \bar{t} = 0 :$ 1) $\Omega_0 = 0.80$, $A_0 = 0.0664575$, $\theta_0 =$ -0.0240624 with $\mu_3 = 0.01$; 2) $\Omega_0 = 0.79$, $A_0 = 0.0626448$, $\theta_0 =$ -0.0230045 with $\mu_3 = 0.02$; 3) $\Omega_0 = 0.79$, $A_0 = 0.0626447$, $\theta_0 =$ -0.0231013 with $\mu_3 = 0.043$, for the case with $\nu < 0 : \bar{t} = 0 :$ 1) $\Omega_0 = 1.39$, $A_0 = 0.0640315$, $\theta_0 = 0.0133933$ with $\mu_3 = 0.01$; 2) $\Omega_0 = 1.39$, $A_0 = 0.0640315$, $\theta_0 = 0.0134724$ with $\mu_3 = 0.02$; 3) $\Omega_0 = 1.39$, $A_0 = 0.0640313$, $\theta_0 = 0.0136543$ with $\mu_3 = 0.043$.

The abscissa axis has two scales: the scale Ω and the corresponding time scale $\bar{t}$ Resonance curves of non-stationary oscillations of the rotor, constructed on the results of modeling Eqs. (7) and (8), (5) and (6), are shown in Figs. 2 - 6. All the plots clearly show that increase in the value of the non-linear cubic damping of the elastic support μ_3 from 0.01 to 0.043 suppresses not only the maximum amplitude and its variation around the mean value, but also the oscillation amplitude and its variation below the resonance and over the resonance rotation speed. It shifts the shaft rotation speed corresponding to the maximum amplitude with a rigid non-linear elastic characteristic ($K_3 > 0$) of the support material downward, and with a soft non-linear elastic characteristic ($K_3 < 0$) of the support material toward an increase. Comparison of the plots in Fig. 2a and Fig. 2b shows that with an increase in the value of ν from 0.00025 to 0.0005 with a rigid non-linear characteristic ($K_3 > 0$) of the support elasticity, the resonance peak of the amplitude shifts towards high speeds of rotation and its value decreases [10].

Comparison of Fig. 3 and Fig. 2, shows the identity of the results of solving the equations of the non-stationary process before averaging (5) and (6) with the results of solving the equations of the non-stationary process after averaging (7) and (8) over time, although in Fig. 3, there is a variation in the values of the amplitude of oscillations in time along the main curve, around its mean values.

From Figs. 2 and 4, it is clearly seen that at $K_3 > 0$ and $\nu > 0$ (run-up) the resonance amplitude is greater than at $\nu < 0$ (run-down).

Changes in the nonlinear stiffness characteristics of an elastic support significantly affect the description of the resonance curves. The amplitude-frequency characteristics of the rotor during the transient process and the soft characteristic of the non-linear elasticity of the support ($K_3 < 0$) are shown for the take-off run ($\nu > 0$) of the machine in Fig. 5, for run-down ($\nu < 0$) of the machine – in Fig. 6. From these graphs, it is noticed that ($\nu > 0$) the resonance peak of the amplitude is less during the run-up than during the run-down ($\nu < 0$), i.e. on the contrary, than in the case with a rigid non-linear characteristic of the support elasticity ($K_3 > 0$) [12]. When the absolute value of the angular acceleration ν changes from 0.00025 to 0.0005 in the case of starting ($\nu > 0$) the machine, the resonance peak of the amplitude shifts towards an increase in the rotation speed [10], then in the case of braking ($\nu < 0$) towards a decrease in the shaft rotation speed.

Comparison of the amplitude-frequency characteristics of the rotor for the case with a rigid non-linear elastic characteristic (Fig. 2, 3, 4) and for the case with a soft non-linear elastic characteristic (Fig. 5, 6) of the support material shows that during the run-up, ($\nu > 0$) the maximum amplitude of the resonance curves for $K_3 > 0$ is greater than the analogous value for $K_3 < 0$; at the run-down ($\nu < 0$), the maximum amplitude for $K_3 > 0$ is less than the similar parameter for $K_3 < 0$.

In order to check whether the considered transient process is really "resonant", the equation of the reference line of the resonance curve is derived from the equations of motion (3)

$$\Omega = \omega_n/(2 - I_{P1}) + \sqrt{[\omega_n/(2 - I_{P1})]^2 + 3K_3A^2/[4(2 - I_{P1})]}. \tag{9}$$

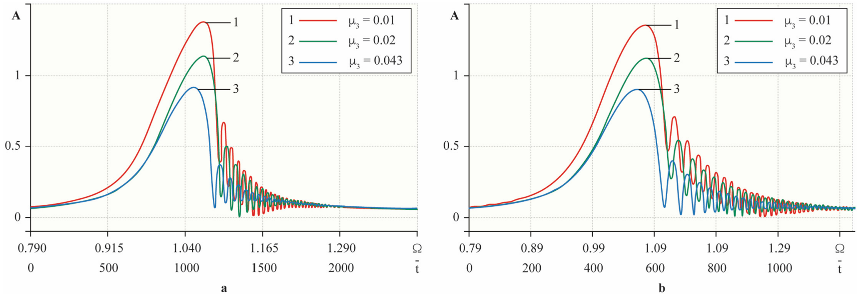

Fig. 2. Transition through resonance at $K_3 > 0$, according to the results of modeling Eqs. (7) and (8) with *a* - ν = 0.00025, *b* - ν = 0.0005

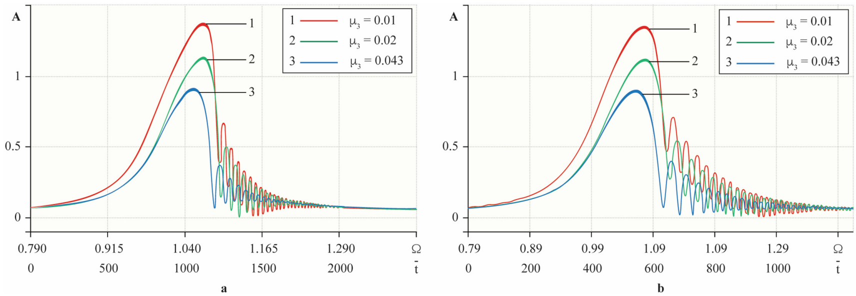

Fig. 3. Transition through resonance at $K_3 > 0$, according to the results of modeling Eqs. (5) and (6) with *a* - ν = 0.00025, *b* - ν = 0.0005.

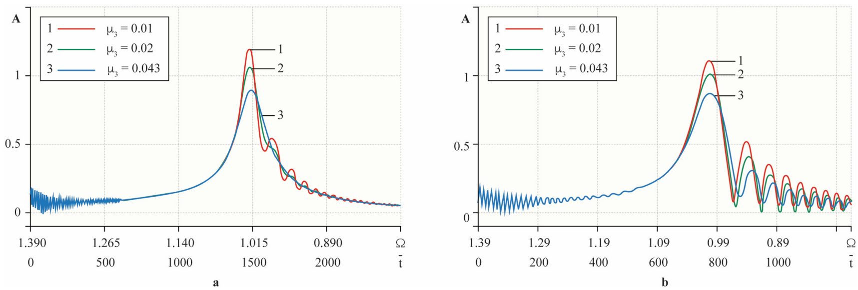

Fig. 4. Transition through resonance at $K_3 > 0$, according to the results of modeling Eqs. (7) and (8) with *a* - ν = -0.00025, *b* - ν = -0.0005

Assuming that, $\nu \ll \Omega^2$ the peak amplitudes and the corresponding rotational speeds of the resonance curves approximately satisfy Eq. (9). So, for example, for $K_3 = 0.1$, $\mu_1 = 0.01$, $\mu_3 = 0.01$, ν = 0.00025 the maximum amplitude $A = 1.360$ corresponds to the rotation speed $\Omega = 1.075$ (1 resonance curve in Fig. 2), for $K_3 =$ -0.1, $\mu_1 = 0.01$, $\mu_3 = 0.01$, ν = 0.00025 the maximum amplitude $A = 1.163$ – the rotation speed $\Omega =$ 0.9570 (1 resonance curve in Fig. 5).

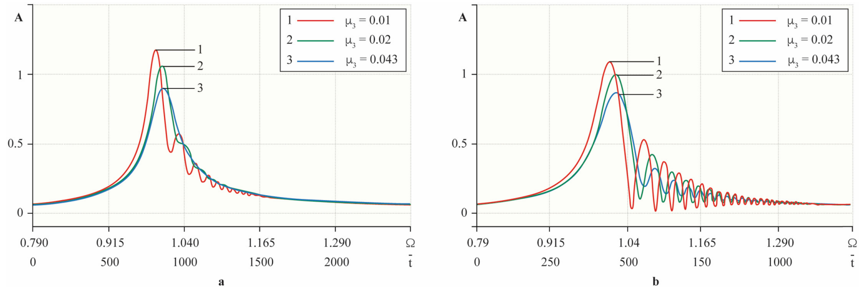

Fig. 5. Transition through resonance at $K_3 < 0$, according to the results of modeling Eqs. (7) and (8) with a - ν = 0.00025, b - ν = 0.0005

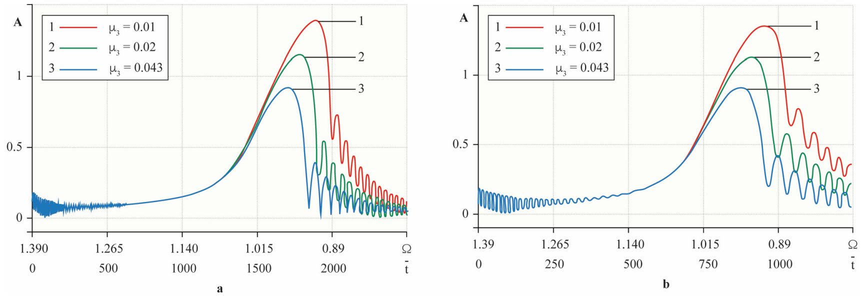

Fig. 6. Transition through resonance at $K_3 < 0$, according to the results of modeling Eqs. (7) and (8) with a - ν = -0.00025, b - ν = -0.0005

To confirm the analytical study, Eqs. (3) were solved directly numerically. Figure 7 shows the numerical results for passing through the resonance with a rigid non-linear elastic characteristic of the support of the support material and a "slowly" varying value of the angular velocity of rotation Ω. In this figure, the effects of damping of the oscillations amplitude having the value of μ_3 and beating of similar oscillations are observed. These results are consistent with previous analytical results shown in Fig. 2 and Fig. 4. The differences lie in the width of the region of clearly visible vibrations, the magnitude

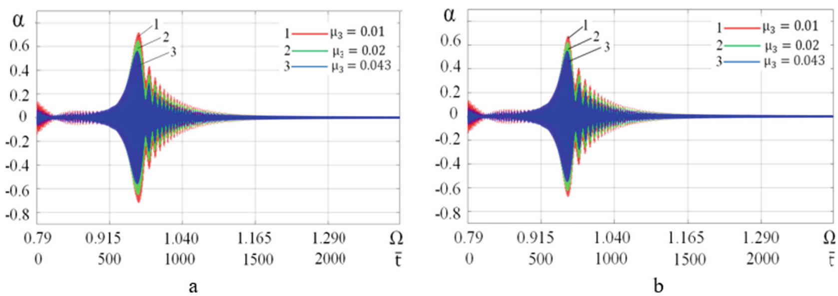

Fig. 7. Transition via resonance with ν = 0.00025 according to the results of the numerical solution of Eqs. (3) for a - $K_3 > 0$, b - $K_3 < 0$

of the maximum vibration amplitude and the displacement of the corresponding shaft rotation speed. Despite this, the basic behavior of the transient process persists. Jumping effects are not detected.

5 Conclusions

Differential equations of motion of a gyroscopic rigid unbalanced rotor with non-linear cubic stiffness and non-linear cubic damping are constructed and solved by the method of varying amplitude. Differential equations of unsteady oscillations of the rotor are obtained, which were solved numerically for the transient process through the resonance region.

It is shown that non-linear cubic damping significantly suppresses not only the maximum amplitude and its variation, but also the oscillation amplitude and its variation below the resonance and over the resonance rotation speed. It shifts the resonant rotation speed of the shaft downwards with the rigid non-linear elastic characteristic ($K_3 > 0$) of the support material and upwards with the soft non-linear elastic characteristic ($K_3 < 0$) of the support material.

It was confirmed that with a "slow" increase in the shaft rotation speed ($\nu > 0$), an increase in the absolute value of the angular acceleration is accompanied by a shift of the amplitude maximum towards high rotation speeds, with a "slow" decrease in the shaft rotation speed ($\nu < 0$) - towards low rotation speeds with a decrease in the amplitude of oscillations.

The results from the analysis of studies of influence of non-linear characteristics of elasticity ($K_3 > 0$ and $K_3 < 0$) of the support material during the run-up ($\nu > 0$) and run-down ($\nu < 0$) of the rotor on the peak amplitude of oscillations are presented.

There is an agreement between the results of analytical solutions and numerical solutions of the equations of rotor motion.

The research results can be used in the manufacture of a vibration isolator, which significantly suppresses the peak amplitude, and the amplitude of oscillations below the resonance and over the resonance rotation speed, for a vibrating system, incl. rotary one.

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