Minimization of Shaking Moment in Fully Force Balanced Planar Four-Bar Linkages

Vigen Arakelian[1,2](✉)

[1] INSA Rennes / Mecaproce, 20 Avenue des Buttes de Coesmes, CS 70839, 35708 Rennes, France
vigen.arakelyan@insa-rennes.fr
[2] LS2N-ECN UMR 6004, 1 rue de la Noë, BP 92101, 44321 Nantes, France

Abstract. This paper deals with a solution of shaking force and shaking moment balancing of planar four-bar linkages. The shaking moment balancing is realized by displacement of the axis of rotation of the counterweight connected with the input link. The conditions for balancing are formulated by the minimization of the root-mean-square value of the shaking moment. This approach is well known. However, the paper describes another of its properties. It is about the choice of the shaking force balancing solution, which significantly affects the minimization of the shaking moment. It is well known that the shaking force in four-bar linkages can be balanced in various ways. The aim of this paper is to show that the choice of the balancing scheme of shaking forces can influence the minimization of shaking moment. To show this difference, two balancing schemes are compared: by two and three counterweights. It is shown that the application of the mentioned balancing technique for minimization of the shaking moment is more efficient for shaking forces balancing by three counterweights. Numerical simulations carried out via ADAMS software illustrate the mentioned observations.

Keywords: Shaking force · Shaking moment · Dynamic balancing · Planar four-bar linkage · Inertia forces · Minimization · Root-mean-square value

1 Introduction

The balancing of mechanisms is a well-known problem in the field of high-speed machinery because the variable dynamic loads cause vibration and noise of the machines. The resolution of this problem consists in the balancing of the shaking force and shaking moment, fully or partially, by internal mass redistribution or by adding auxiliary links [1].

A reliable and simple way to balance shaking forces is to redistribute the mass of the moving links of the mechanism by adding counterweights. It is widespread and quite attractive for industrial applications.

However, balancing of the shaking moment is more challenging and can only be reached by a considerably complicated design of the initial mechanism or by unavoidable increase of the total mass.

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R. Berkof [2], Ye and Smith [3], Arakelian and Smith [4], Feng [5] have proposed methods for complete shaking moment balancing by planetary gear trains. Esat and Bahai [6] used a toothed-belt transmission to cancel the shaking moment in four-bar linkages. Kochev [7] proposed to balance shaking moment by a prescribed input speed fluctuation achieved by non-circular gears or by a microprocessor speed-controlled motor.

Moore, Schicho and Gosselin have proposed all possible sets of design parameters for which a planar four-bar linkage is balanced: both shaking force and shaking moment [8]. Briot and Arakelian [9] used this approach for complete shaking force and shaking moment balancing of four-bar linkages.

The complete shaking force and shaking moment balancing of four-bar linkages via copying properties of pantograph systems formed by gears was also considered [10].

A comparison of various shaking moment balancing principles has been carried out by van der Wijk, Herder and Demeulenaere [11]. This overview summarizes, compares and evaluates the existing principles of complete shaking force and shaking moment balancing regarding the addition of mass and the addition of inertia.

As was mentioned above, the complete shaking moment balancing can often be achieved by a considerably complicated design of the initial mechanism and by unavoidable increase of the total mass. This is the reason why methods of partial dynamic balancing of mechanisms have also been developed.

Freudenstein, J.P. Macey, E.R. Maki [12] derive the equations for minimizing any order of combined pitching and yawing moments by counterweighting the driveshaft or a shaft geared to the driveshaft. The equations are given directly as a function of the harmonic coefficients of pitch and yaw and apply to any plane machine configuration. J.L. Wiederrich and B. Roth [13] proposed simple and general conditions for determination of the inertial properties of a four-bar linkage that allow partial momentum balancing. Dresig and Schönfeld [14, 15] examined the optimum balancing conditions for various structural forms of planar six and eight-bar linkages. A last-square theory for the optimization of the shaking moment of fully force-balanced inline four-bar linkages, running at constant input angular velocity, is developed in the studies of J.L. Elliot and D. Tesar [16] and R.S. Haines [17].

V.A. Shchepetilnikov [18] suggested the minimization of the unbalance of shaking moment by transferring the rotation axis of the counterweight mounted on the input crank. In his works the first harmonic of the shaking moment is eliminated by attaching the required input link counterweight, not to the input shaft itself, but to a suitable offset one which rotates with the same angular velocity. This approach is original in that, while maintaining the shaking force balance of the mechanism, it is possible to create an additional moment, reducing thereby the shaking moment. The similar studies have been developed in [19, 20].

This paper represents the further development of shaking moment balancing technique based on the last mentioned principle, i.e. by parallel displacement of the rotation axis of the counterweight mounted on the input crank. The improvement of the known approach resides in the fact that the choice of the scheme of the shaking force balancing essentially influences at the level of the shaking moment minimization.

2 Shaking Moment Minimization

Let us consider an in-line four-bar linkage with constant input angular velocity: $\dot{\varphi}_1 = d\varphi_1/dt$. Two schemes of the shaking force balancing of the linkage will be considered: by two and three counterweights (Fig. 1a and Fig. 1b).

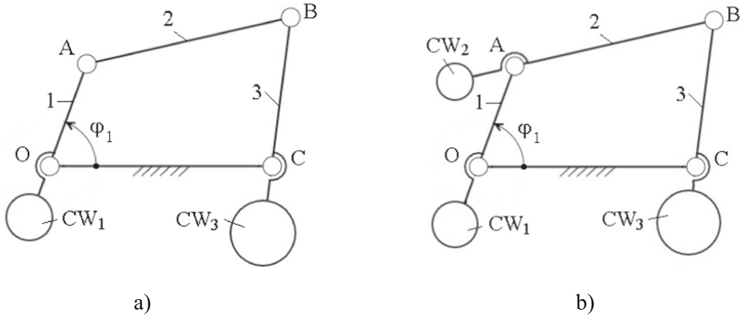

Fig. 1. Force-balanced in-line four-bar linkage: a) by two counterweights and b) by three counterweights.

After shaking force balancing of the in-line four-bar linkage by two counterweights connected to links 1 and 3 (Fig. 1a), the shaking moment can be expressed as [21]:

$$M^{sh} = K_2\ddot{\varphi}_2 + K_3\ddot{\varphi}_3 \tag{1}$$

with $K_2 = -m_2\left(k_2^2 + r_2^2 - l_2r_2\right)$ and $K_3 = -\left(m_3 + m_{CW_3}\right)\left(k_3^2 + r_3^2 + l_3r_3\right)$, where, m_2 is the mass of link 2, m_3 is the mass of link 3, m_{CW_3} is the mass of the counterweight mounter on the link 3, k_2 is the radius of gyration of link 2, k_3 is the radius of gyration of link 3, $l_2 = l_{AB}$ is the length of link 2, $l_3 = l_{BC}$ is the length of link 3, $r_2 = l_{AS_2}$ is the distance of the joint center A from the center of mass S_2 of link 2, $r_3 = l_{CS_3}$ is the distance of the joint center C from the center of mass S_3 of link 3.

In the case of the shaking force balancing by three counterweights (Fig. 1b), considering that the center of mass of the rocker 3 is on the axis of the joint C and the center of mass of the connecting rod 2 is on the axis of the joint B, the shaking moment may be expressed as: $K_2 = -\left(m_2 + m_{CW_2}\right)k_2^2$ and $K_3 = -\left(m_3 + m_{CW_3}\right)k_3^2$, where, m_{CW_2} is the mass of the counterweight mounter on the link 2.

By parallel displacement of the axis of rotation of the counterweight CW_1 (Fig. 2) from center O to the center $O_1(x_1, y_1)$, the balancing of the shaking force of the mechanisms can be maintained, but, in addition to the unbalanced shaking moment, a supplementary moment M_1^{bal} will be created:

$$M_1^{bal} = F_1(x_1 \sin\varphi_1 - y_1 cos\varphi_1) \tag{2}$$

with $F_1 = m_{CW_1} r_{CW_1} \dot{\varphi}_1^2$, where, φ_1 is the angle of rotation of the input link, m_{CW_1} is the mass of the counterweight mounted on the input link, $r_{CW_1} = l_{O_1S_{CW1}}$ is the rotation radius of the center of mass of the counterweight with respect to center O_1.

The counterweight with mass m_{CW_1} moved in parallel is driven (by gears or toothed belts for example) at the same rotational speed as the input link, i.e. $\dot{\varphi}_1$. For clarity, the driving mechanisms are not shown here.

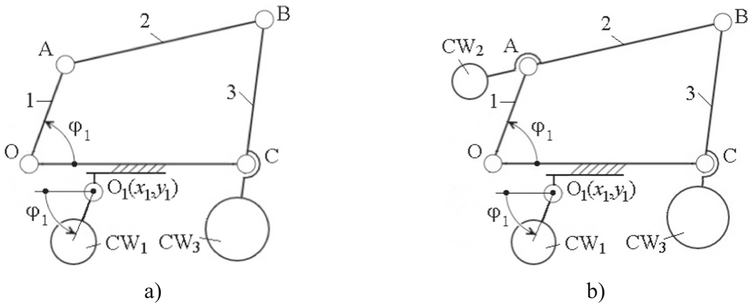

Fig. 2. Shaking moment balancing of a force-balanced four-bar linkage.

For minimization of the root-mean-square value (rms) of the shaking moment of the modified mechanism, it is necessary to minimize the sum:

$$\Delta_{rms} = \sum_{i=1}^{N} \left(M_1^{bal} + M^{sh} \right)^2 \rightarrow \min_{x_1, y_1} \tag{3}$$

where, N is the number of calculated positions of the linkage.

For this purpose, the following conditions must be fulfilled:

$$\partial \Delta_{rms} / \partial x_1 = 0 \text{ and } \partial \Delta_{rms} / \partial y_1 = 0 \tag{4}$$

Conditions (4), taking into account that $\sum_{i=1}^{N} \sin \varphi_1 \cos \varphi_1 = 0$ for $\varphi_1 \in [0; 2\pi]$, lead to a system of linear equations, from which the following expressions are obtained:

$$x_1 = \sum_{i=1}^{N} M^{sh} \sin \varphi_{1i} / F_1 \sum_{i=1}^{N} \sin^2 \varphi_{1i} \text{ and } y_1 = -\sum_{i=1}^{N} M^{sh} \cos \varphi_{1i} / F_1 \sum_{i=1}^{N} \cos^2 \varphi_{1i} \tag{5}$$

Observations showed that the choice of the shaking force balancing scheme influences the minimization of the shaking moment. In order to demonstrate this for an arbitrarily four-bar linkage, a numerical comparison has been carried out.

3 Illustrative Example with Numerical Simulations

The in-line four-bar linkage used for numerical simulations has the following parameters: the lengths of links: $l_{OA} = 0.2\ m$; $l_{AB} = 0.45\,m$; $l_{BC} = 0.45\,m$; $l_{OC} = 0.6\,m$, the location of the centers of mass: $l_{OS_1} = 0.1\,m$; $l_{AS_2} = 0.225\,m$; $l_{CS_3} = 0.225\,m$, the masses:$m_1 = 2\,kg$; $m_2 = m_3 = 4\,kg$, the axial inertia moments: $I_{S_2} = I_{S_3} = 0.08\,kg\,m^2$.

The shaking force of the four-bar linkage has been balanced via two mentioned methods.

a) By two counterweights (Fig. 1a) with following parameters: the location of the counterweights' centers of mass: $r_{CW_1} = r_{OS_{CW1}} = 0.1\,m$; $r_{CW_3} = r_{OS_{CW3}} = 0.225\,m$, the masses of counterweights: $m_{CW_1} = 6\,kg$; $m_{CW_3} = 8\,kg$, the axial inertia moments after shaking force balancing: $I_{S_2} = 0.08\,kg\,m^2$; $I_{S_3} = 0.5\,kg\,m^2$.

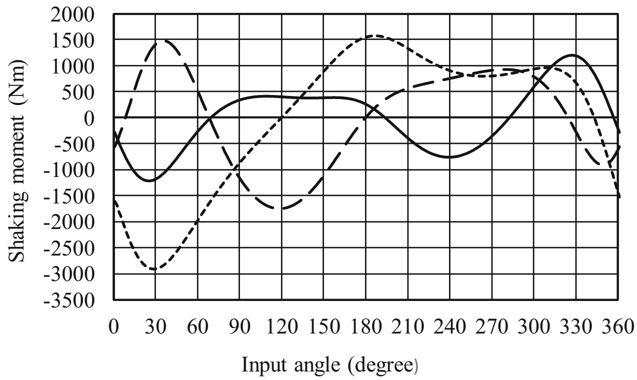

Fig. 3. Shaking moments of the four-bar linkage balanced by two counterweights: unbalanced (dash line), force-balanced (dot line) and with minimized shaking moment (solid line).

b) By three counterweights (Fig. 1b) with following parameters: the location of the counterweights' centers of mass: $r_{CW_1} = r_{OS_{CW1}} = 0.15\,m$; $r_{CW_2} = r_{BS_{CW2}} = 0.225\,m$, $r_{CW_3} = r_{OS_{CW3}} = 0.225\,m$, the masses of counterweights: $m_{CW_1} = 12\,kg$; $m_{CW_2} = 4\,kg$; $m_{CW_3} = 4\,kg$, the axial inertia moments after shaking force balancing: $I_{S_2} = 0.3\,kg\,m^2$; $I_{S_3} = 0.3\,kg\,m^2$.

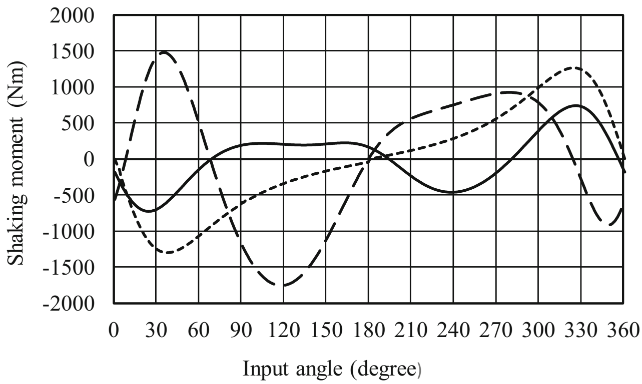

Fig. 4. Shaking moments of the four-bar linkage balanced by three counterweights: unbalanced (dash line), force-balanced (dot line) and with minimized shaking moment (solid line).

According to expressions (5) the following values of the coordinates of the axis O_1 have been obtained: a) for the linkage balanced by two counterweights (Fig. 1a): $x_1 = -0.48\,m$ and $y_1 = 0.56\,m$; b) for the linkage balanced by three counterweights (Fig. 1b): $x_1 = -0.11\ m$ and $y_1 = 0$.

The obtained results (Fig. 3) show that for the force-balanced mechanism given in Fig. 2a, a 31% reduction in the shaking moment is achieved. With regard to the force-balanced mechanism given in Fig. 2b, a 50% reduction in the shaking moment is achieved (Fig. 4). This comparison was made according to the maximum values of the shaking moments of the unbalanced and the moment-balanced mechanisms.

4 Conclusions

In the paper, it is shown that when applying the method of shaking moment minimization of four-bar linkages by transferring the rotation axis of the counterweight mounted on the input crank, the choice of the shaking force balancing approach influence on the moment minimization. To evaluate the efficiency of the shaking moment balancing of four-bar linkages, two force-balanced linkages are numerically compared: by two and three counterweights. At first sight, the balancing approach carried out by two counterweights seems more attractive as it leads to a smaller increase in the total mass of the mechanism. Moreover, from the point of view of the design, the shaking force balancing of the four-bar linkage by two counterweights mounted on the crank and the rocker is easier to implement. However, as shown in the paper, the application of the mentioned balancing technique for minimization of the shaking moment is more efficient for shaking forces balancing by three counterweights.

One should not get the impression that a solution with three counterweights is always more optimal from the point of view of minimizing the shaking moment according to the described method. The conclusion that should be retained is that different shaking force balancing schemes lead to different shaking moment minimization results. Therefore, when applying the described method to minimize the shaking moment, it is important to choose an optimal shaking force balancing scheme, as it affects the results of minimization.

References

1. Arakelian, V., Briot, S.: Balancing of linkages and robot manipulators. Advanced Methods with Illustrative Examples, Springer, Switzerland (2015)
2. Berkof, R.S.: Complete force and moment balancing inline four-bar linkages. Mech. Mach. Theory **8**, 397–410 (1973)
3. Ye, Z., Smith, M.R.: Complete balancing of planar linkages by an equivalence method. Mech. Mach. Theory **29**(5), 701–712 (1994)
4. Arakelian, V., Smith, M.R.: Complete shaking force and shaking moment balancing of linkages. Mech. Mach. Theory **34**(8), 1141–1153 (1999)
5. Feng, G.: Complete shaking force and shaking moment balancing of four types of six-bar linkages. Mech. Mach. Theory **24**(4), 275–287 (1989)
6. Esat, I., Bahai, H.: A theory of complete force and moment balancing of planar linkage mechanisms. Mech. Mach. Theory **34**, 903–922 (1999)

7. Kochev, I.S.: Active balancing of the frame shaking moment in high-speed planar machines. Mech. Mach. Theory **27**(1), 53–58 (1992)
8. Moore, B., Schicho, J., Gosselin, C.M.: Determination of the complete set of shaking force and shaking moment balanced planar four-bar linkages. Mech. Mach. Theory **44**, 1338–1347 (2009)
9. Briot, S., Arakelian, V.: Complete shaking force and shaking moment balancing of in-line four-bar linkages by adding a class-two RRR or RRP Assur group. Mech. Mach. Theory **57**, 13–26 (2012)
10. Arakelian, V.H., Smith, M.R.: Shaking force and shaking moment balancing of mechanisms: a historical review with new examples. J. Mech. Des. **127**(2), 334–339 (2005)
11. van der Wijk, V., Herder, J.L., Demeulenaere, B.: Comparison of Various Dynamic Balancing Principles Regarding Additional Mass and Additional Inertia. J. Mech. Robot. **1**(4), 041006 (2009)
12. Freudenstein, F., Macey, J.P., Make, E.R.: Optimum balancing of combined pitching and yawing moments in high-speed machinery. J. Mech. Des. **103**(3), 571–577 (1981)
13. Wiederrich, J.L., Roth, B.: Momentum balancing of four-bar linkages. J. Eng. Ind. **98B**(4), 1289–1295 (1976)
14. Dresig, H., Schönfeld, S.: Rechnergestützte Optimierung der Antribs-und Gestellkraftgrössen ebene. Koppelgetriebe-Teil 1. Mech. Mach. Theory **11**, 363–370 (1976)
15. Dresig, H., Schönfeld, S.: Rechnergestützte Optimierung der Antribs-und Gestellkraftgrössen ebene, Koppelgetriebe-Teil 2. Mech. Mach. Theory **11**, 371–379 (1976)
16. Elliot, J.L., Tesar, D.: The theory of torque, shaking force and shaking moment balancing of four link mechanisms. J. Eng. Ind. **99B**(3), 715–722 (1977)
17. Haines, R.S.: Minimum RMS shaking moment or driving torque of a force-balanced 4-bar linkage using feasible counterweights. Mech. Mach. Theory **16**, 185–190 (1981)
18. Shchepetilnikov, V.A.: Balancing of mechanisms. Mashinostroenie, Moscow (1982)
19. Arakelian, V., Dahan, M.: Partial shaking moment balancing of fully shaking force balanced linkages. J. Mech. Mach. Theory **36**(11–12), 1241–1252 (2001)
20. Arakelian, V., Smith, M.: Shaking moment minimization of fully force-balanced linkages. In: Proceedings of the 11th World Congress in Mechanism and Machine Science, vol. 2, pp. 633–636. Tianjin, China, 1–4 April 2004
21. Berkof, R., Lowen, G.: Theory of shaking moment optimization of force-balanced four-bar linkages. ASME J. Eng. Ind. **93**(1), 53–60 (1971)