Analysis of the Stress-Strain State of Rotating Drill Strings with a Drilling Mud

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Abstract. This paper studies the stress-strain state of a rotating drill string complicated by the effect of a drilling mud flow and external compressive and twisting forces. The drill string is considered in the form of a homogeneous isotropic elastic rod with constant cross-section. A nonlinear mathematical model of the drill string spatial lateral vibrations based on Novozhilov's nonlinear theory of elasticity is utilized. The Bubnov-Galerkin approach that allows reducing the given PDEs to ODEs and the numerical stiffness-switching method are applied to obtain a solution of the model. To analyze the stress-strain state of the drill string, we use the maximum stress intensity criterion and construct the graphs demonstrating changes of strain and stress in the chosen drill string cross-section with time, and stress distribution over the drill string length with time for nonlinear and linear cases. The research results indicate the significance of accounting for geometric nonlinearity to examine the drill string stress-strain state under the influence of the drilling mud.

Keywords: Drill string · Nonlinearity · Lateral vibrations · Stress-Strain state · Drilling mud

1 Introduction

The problem of strength of structures, parts of machines and mechanisms is of great importance in engineering practice. It is inextricably related to the determination of the magnitudes of stress and strain arising in deformable solids. Neglecting the requirements of physical and mathematical consistency, also associated with the use of basic stress-strain relationships, may result in large errors in calculations and serious ensuing consequences [1], especially when modelling the drill string dynamics complicated by factors of various nature.

The main reason of premature failure of drill string components, deterioration of the well trajectory and other undesirable phenomena emerging during the drilling process is the drill string vibrations, amongst which the coupled transverse mode is considered to be the most dangerous one [2]. Many drill string failures are preceded by fatigue fractures of metal, which are formed at multiple changes of magnitude and direction of loading at the sites of stress concentrations; however, they do not have a significant effect on the deformation of the drill string elements. At the same time, bending is a primary cause of residual stresses occurring during the drill string rotation.

J. Beran et al. (Eds.): TMM 2020, MMS 85, pp. 114–122, 2022.
https://doi.org/10.1007/978-3-030-83594-1_12

The effect of the drilling mud (fluid) on the drill string motion was investigated in a number of works. In [3], the authors studied the impact of drilling hydraulics on drill stem vibrations using the Euler-Bernoulli beam theory and a finite element formulation for the drill stem discretization. The Herschel-Bulkley and power law fluid models were utilized to characterize the rheology of the drilling fluid. The results showed that the fluid dynamic pressure had a significant influence on the drill stem lateral frequencies. The effect of the drilling mud on the drill string dynamics based on the Timoshenko beam theory with the use of a simplified fluid-structure interaction model was studied in [4]. The impact of internal and external fluid flows on vibrations of the flexible cantilever pipe, which is similar in nature to the drill string, was investigated in [5]. It was obtained that the external fluid flow might cause a loss of the system stability by divergence. Analysis of coupled nonlinear lateral vibrations of a drill string under the effect of external supersonic gas and internal fluid flows was also carried out [6]. The research results indicated the presence of nonlinear effects and need for further study of the drill string dynamics in the gas and fluid flows in nonlinear formulation. For these reasons, this paper aims at studying the stress-strain state of the drill string taking into account geometric nonlinearity, the internal drilling mud flow and external loadings.

2 Governing Equations

Consider spatial lateral vibrations of a rotating drill string modelled in the form of a homogeneous isotropic elastic rod of length l with constant cross-section (Fig. 1).

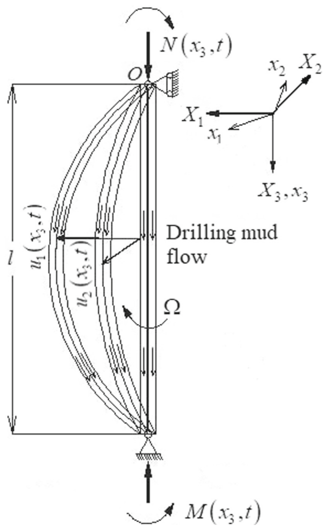

Fig. 1. Scheme of a drill string with a drilling mud.

The drill string is subject to the effect of an axial compressive force $N(x_3, t)$ and a torque $M(x_3, t)$. A drilling mud flow moves along the inner tube of the drill string in a positive direction of the vertical drill string axis x_3. Two Cartesian coordinate systems $OX_1X_2X_3$ and $Ox_1x_2x_3$ associated with lateral displacements of the drill string and accounting for its rotation with angular speed Ω, respectively, are utilized.

Since the drill string spatial lateral vibrations are studied, the displacement components of any point of the rod are given by

$$\begin{aligned}
&U_1(x_1, x_2, x_3, t) = u_1(x_3, t),\\
&U_2(x_1, x_2, x_3, t) = u_2(x_3, t),\\
&U_3(x_1, x_2, x_3, t) = -\frac{\partial u_1(x_3,t)}{\partial x_3}x_1 - \frac{\partial u_2(x_3,t)}{\partial x_3}x_2,
\end{aligned} \tag{1}$$

where the lateral displacements $u_1(x_3, t)$ and $u_2(x_3, t)$ are determined by solving the following nonlinear mathematical model including the effect of external compressive and twisting loadings and the drilling fluid flow pressure (see [7] for details):

$$\begin{aligned}
&EI_{x_2}\frac{\partial^4 u_1}{\partial x_3^4} - \rho I_{x_2}\frac{\partial^4 u_1}{\partial x_3^2\partial t^2} + \frac{\partial^2}{\partial x_3^2}\left(M(x_3, t)\frac{\partial u_2}{\partial x_3}\right) + \frac{\partial}{\partial x_3}\left(N(x_3, t)\frac{\partial u_1}{\partial x_3}\right)\\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^3 - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right)\\
&+\left(\rho A + \rho_f A_f\right)\left(\frac{\partial^2 u_1}{\partial t^2} - 2\Omega\frac{\partial u_2}{\partial t} - \Omega^2 u_1\right) - \rho_f I_{x_2}\left(\frac{\partial^4 u_1}{\partial x_3^4} + 2\frac{\partial^4 u_1}{\partial x_3^3\partial t} + \frac{\partial^4 u_1}{\partial x_3^2\partial t^2}\right)\\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_1}{\partial x_3^2} + 2V_f\frac{\partial^2 u_1}{\partial x_3\partial t} - 2V_f\Omega\frac{\partial u_2}{\partial x_3}\right)\\
&+\left(\rho A + \rho_f A_f\right)g\left(\frac{\partial u_1}{\partial x_3} - (l - x_3)\frac{\partial^2 u_1}{\partial x_3^2}\right) = 0,
\end{aligned} \tag{2.1}$$

$$\begin{aligned}
&EI_{x_1}\frac{\partial^4 u_2}{\partial x_3^4} - \rho I_{x_1}\frac{\partial^4 u_2}{\partial x_3^2\partial t^2} - \frac{\partial^2}{\partial x_3^2}\left(M(x_3, t)\frac{\partial u_1}{\partial x_3}\right) + \frac{\partial}{\partial x_3}\left(N(x_3, t)\frac{\partial u_2}{\partial x_3}\right)\\
&-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^3 - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^2\right)\\
&+\left(\rho A + \rho_f A_f\right)\left(\frac{\partial^2 u_2}{\partial t^2} + 2\Omega\frac{\partial u_1}{\partial t} - \Omega^2 u_2\right) - \rho_f I_{x_1}\left(\frac{\partial^4 u_2}{\partial x_3^4} + 2\frac{\partial^4 u_2}{\partial x_3^3\partial t} + \frac{\partial^4 u_2}{\partial x_3^2\partial t^2}\right)\\
&+\rho_f A_f\left(V_f^2\frac{\partial^2 u_2}{\partial x_3^2} + 2V_f\frac{\partial^2 u_2}{\partial x_3\partial t} + 2V_f\Omega\frac{\partial u_1}{\partial x_3}\right)\\
&+\left(\rho A + \rho_f A_f\right)g\left(\frac{\partial u_2}{\partial x_3} - (l - x_3)\frac{\partial^2 u_2}{\partial x_3^2}\right) = 0.
\end{aligned} \tag{2.2}$$

Here E is Young's modulus, $I_{x_1},\ I_{x_2}$ axial inertia moments with respect to the x_2- and x_1-axes, ρ the drill string density, ν Poisson's ratio, A the cross-sectional area of the drill string, ρ_f the drilling mud density, A_f the internal cross-sectional area of the drill string, V_f the drilling mud flow speed.

Governing Eqs. (2.1), (2.2) meet the boundary conditions corresponding to the case of a simply supported rod:

$$\begin{aligned}
&u_1(x_3, t) = u_2(x_3, t) = 0 \qquad (x_3 = 0,\ x_3 = l),\\
&EI_{x_2}\frac{\partial^2 u_1(x_3,t)}{\partial x_3^2} = EI_{x_1}\frac{\partial^2 u_2(x_3,t)}{\partial x_3^2} = 0 \qquad (x_3 = 0,\ x_3 = l).
\end{aligned} \tag{3}$$

The initial conditions are written as

$$\begin{aligned} &u_1(x_3,t) = u_2(x_3,t) = 0 \qquad (t=0), \\ &\tfrac{\partial u_1(x_3,t)}{\partial t} = C_1, \quad \tfrac{\partial u_2(x_3,t)}{\partial t} = C_2 \qquad (t=0), \end{aligned} \tag{4}$$

where $C_1,\ C_2$ are constants defining the displacement rates of the rod cross-section from initial position in the Ox_1x_3- and Ox_2x_3-planes, respectively, at the initial time moment.

3 Determination of the Stress-Strain State

According to Novozhilov's nonlinear theory of elasticity [8], the strain tensor components ε_{ij} are determined through the displacement projections given by Eq. (1) as follows:

$$\begin{aligned} &\varepsilon_{ii} = \tfrac{\partial U_i}{\partial x_i} + \tfrac{1}{2}\left(\tfrac{\partial U_j}{\partial x_j}\right)^2, \quad i,j=\overline{1,3}, \\ &\varepsilon_{ij} = \tfrac{\partial U_i}{\partial x_j} + \tfrac{\partial U_j}{\partial x_i} + \tfrac{\partial U_k}{\partial x_i}\tfrac{\partial U_k}{\partial x_j}, \quad i,j,k=\overline{1,3} \quad (i \neq j). \end{aligned} \tag{5}$$

To find the stress tensor components σ_{ij}, the equations of generalized Hooke's law for a linear elastic homogeneous isotropic solid body in terms of Lame parameters $G,\ \lambda$ are applied:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \quad i,j,k=\overline{1,3}, \tag{6}$$

where $G = \frac{E}{2(1+\nu)},\ \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$.

The maximum stress intensity criterion describing the transition to plastic deformation in metals with sufficient accuracy and proved by experiments [9] is used for determining the stress-strain state of the rotating drill string:

$$\sigma_* = \sqrt{\frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)}, \tag{7}$$

where σ_* is the critical stress, $\sigma_1,\ \sigma_2, \sigma_3$ principal stresses.

4 Numerical Results

To obtain a solution of the nonlinear mathematical model of drill string vibrations (2.1), (2.2) with boundary and initial conditions (3), (4), we utilize the Bubnov-Galerkin method that allows reducing the given PDEs to ODEs with further application of the numerical stiffness-switching method, as in [6]. The axial compressive force and torque are assumed to be distributed over the drill string length, i.e. $N(x_3,t) = N$, $M(x_3,t) = M$. All the computations are conducted in the Wolfram Mathematica package.

The initial values of the drilling system parameters used for numerical analysis are presented in Table 1.

Table 1. Drilling system parameters.

System parameter	Value
Drill string length, l	$500\,\text{m}$
Angular speed of rotation, Ω	$2\,\text{rad}/\text{s}$
Young's modulus, E	2.1×10^{11} Pa
Drill string material density, ρ	$7800\,\text{kg}/\text{m}^3$
Poisson's ratio, ν	0.28
Outer diameter of the drill string, D	63.5×10^{-3} m
Wall thickness, h	4.5×10^{-3} m
Drill string cross-sectional area, A	$0.834 \times 10^{-3}\,\text{m}^2$
Axial compressive load, N	3.5×10^3 N
Torque, M	$10^4\,\text{N} \cdot \text{m}$
Drilling mud density, ρ_f	$1120\,\text{kg}/\text{m}^3$
Internal cross-sectional area, A_f	$2.33 \times 10^{-3}\,\text{m}^2$
Drilling mud flow speed,V_f	$3.57\,\text{m}/\text{s}$

Figure 2 illustrates the strain change in the drill string cross-section $x_3 = 0.8l$, Fig. 3 – the stress change in the considered cross-section with time (in seconds) at the initial values of the drilling system parameters. The cross-section point with coordinates $(x_1, x_2) = (0.03,\ 0.03)$ is chosen while drawing the graphs for conducting the comparative analysis of the application of nonlinear and linear mathematical models.

It is worth noting that the strain graphs are constructed for mean strain ε_0, which is calculated by the formula:

$$\varepsilon_0 = \frac{1}{3}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}). \tag{8}$$

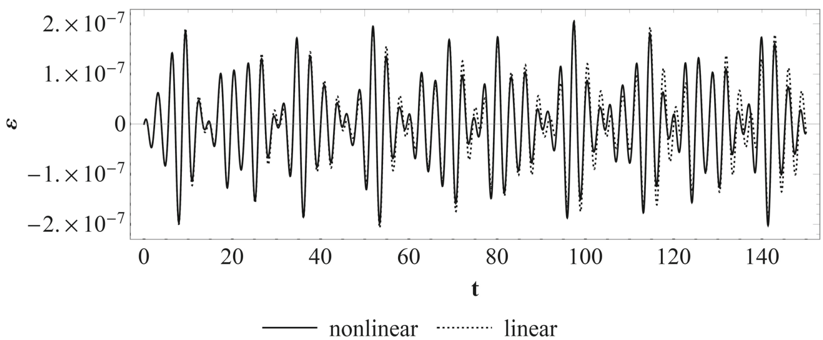

Fig. 2. Drill string strains at the initial parameter values.

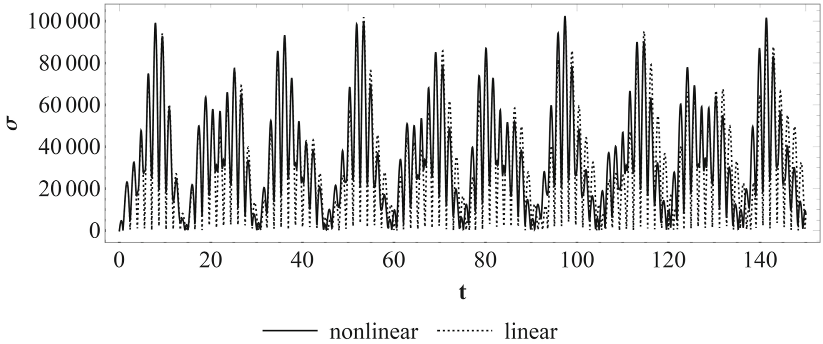

Fig. 3. Drill string stresses at the initial parameter values.

The linear model used for the comparative analysis is obtained on the basis of classical linear elasticity theory, according to which the strain components ε_{ij} linearly depend on the displacement projections U_i, and represents the linearized version of the governing Eqs. (2.1), (2.2). Hence, it does not involve the terms bringing the influence of geometric nonlinearity, namely the fifth and sixth terms in both Eqs. (2.1) and (2.2).

It follows from Figs. 2 and 3 that there is no significant difference in maximum values of strain and stress when applying the nonlinear model and its linear analogue at the initial system parameters.

Figures 4, 5 demonstrate the strain and stress changes in the selected cross-section of the rod $x_3 = 0.8l$, respectively, and Figs. 6, 7 give the stress distribution over the drill string length (in metres) with time when increasing the axial compressive load up to $N = 8 \times 10^3$N.

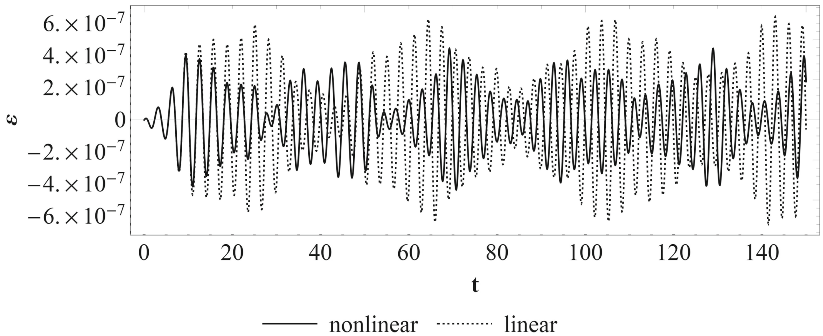

Fig. 4. Drill string strains under the axial force $N = 8 \times 10^3$ N.

As can be seen from Figs. 4 and 5, the increase of the axial force and accounting for geometric nonlinearity in the model results in considerable decrease of the mean strain and stress magnitudes in the given cross-section of the rotating drill string under the effect of the drilling mud flow. The stress distribution over the drill string length shows the sharp rise of stress in the lower part of the drill string (near 500 m) for nonlinear

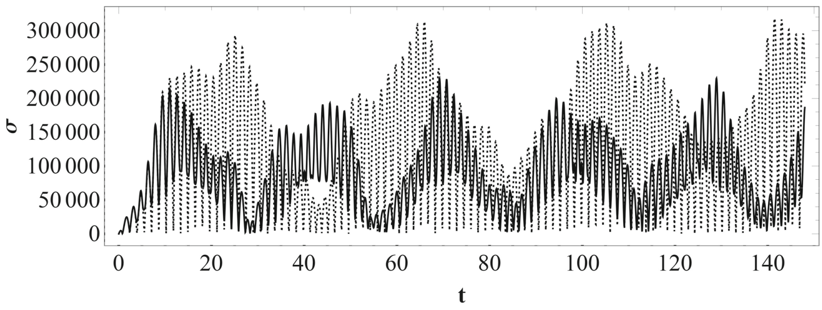

Fig. 5. Drill string stresses under the axial force $N = 8 \times 10^3$ N.

model (Fig. 6), which brings a correction to the results obtained in linear case when the most critical stresses are observed at a depth of 350–450 m (Fig. 7).

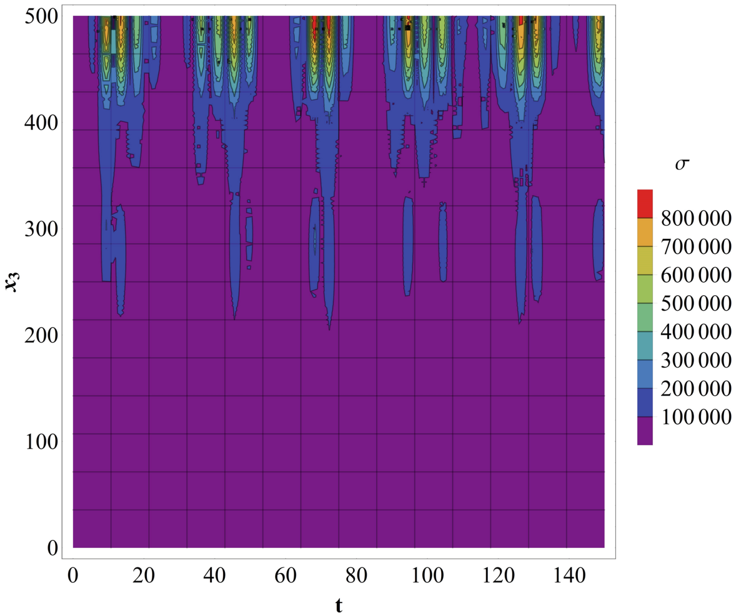

Fig. 6. Stress distribution over the drill string length at $N = 8 \times 10^3$ N for the nonlinear model.

The obtained results indicate the importance of accounting for geometric nonlinearity in mathematical models when increasing the values of the drilling system parameters to analyze the drill string stress-strain state under the influence of the drilling mud and complicated by external loadings.

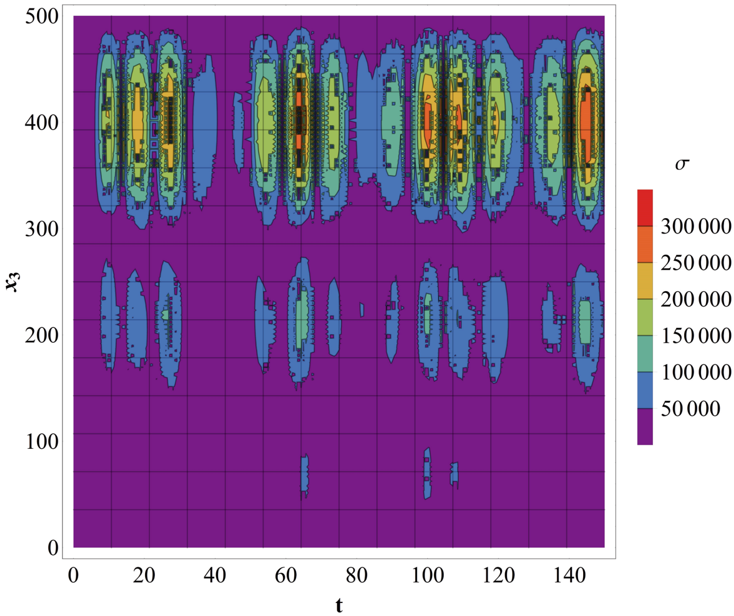

Fig. 7. Stress distribution over the drill string length at $N = 8 \times 10^3$ N for the linear model.

5 Conclusion

In this work, the analysis of the stress-strain state of a rotating drill string taking into account a drilling mud flow, an axial compressive load and a torque was conducted. A nonlinear mathematical model of the rod lateral vibrations, based on Novozhilov's nonlinear theory of elasticity, was utilized to describe the complex oscillatory process of the drill string and bring it closer to a real drilling process.

The graphs of the strain and stress change with time in a particular drill string cross-section, constructed on the basis of the nonlinear model, showed the significant decrease of the strain and stress magnitudes compared to its linear analogue when the compressive force was increased. The important result was obtained for stress distribution over the drill string length with time in nonlinear case. It revealed a shift of critical stresses to the bottom of the drill string compared to the results of linear model.

To further our research, the obtained results can be extended to a more complex problem taking into account the intermittent contact of the drill string with the borehole wall and damping effects.

Acknowledgements. This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP09261135).

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