Modeling of Nonlinear Dynamics of Planar Mechanisms with Elastic and Flexible Pre-stressed Elements

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Abstract. The motion of planar hinge-lever mechanisms with flexible and elastic links in a closed pre-stressed contour is considered. Modeling of the mechanism motion is carried out on the basis of their kinetic-elastodynamic analysis, which takes into account the inertial relationship between the large-scale motion of mechanisms as a rigid body and nonlinear vibrations of the links as a result of their elastic deformation. This work pays attention to both longitudinal and lateral vibrations of elastic links. The equations of motion of the mechanisms are obtained by the use of Novozhilov's nonlinear theory of elasticity, according to which the link deformations are assumed to be finite. Based on Biot's theory of incremental deformations, the field of initial stresses in flexible elements is taken into account due to their preliminary tension, which determines the geometric nonlinearity of dynamic models. As an example, the dynamics of a planar five-link hinge-lever mechanism with closed pre-stressed contour is studied.

Keywords: Dynamics · Mechanisms · Links · Elasticity · Nonlinearity · Pre-stress

1 Introduction

The idealization of the links of mechanisms and machines as rigid ones significantly narrows the range of problems under consideration for most of the dynamic problems of modern mechanical engineering. Basically, they are limited to quasi-static and pre-critical operating modes [1, 2]. Deformations of flexible and elastic elements of mechanisms and machines cause, when unaccounted for, complex dynamic processes in them. They affect the strength and operating characteristics of the system as a whole, arousing scientific and practical interest amongst researchers.

In contrast to rigid elements when the main problems of the machine dynamics are

1) studying the forces acting on the linkages under conditions of a given law of motion of the machine;

2) studying the true law of motion of a machine under the influence of given forces; which are solved separately and quite easily, in case of elastic elements, a separate

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consideration of the problems cannot take place since the kinematics of the mechanisms is complicated by deformations of the linkages [1].

These questions are reflected in the method of kinetic-elastodynamic analysis, widely used nowadays for the dynamic analysis of mechanisms and machines, considering the deformability of links and elements. It takes into account the nonlinear inertial relationship between the motion of a mechanism as a rigid body and the vibrational process resulting from the elastic deformation of its linkages. Amongst the first works in this area, one can note the works of Sadler, Sandor [3, 4], Chu, Pan [5] and others, where the elastic motion of planar mechanisms is modeled with limiting the number of elastic elements and their representation by the Euler-Bernoulli beam. In case of mechanisms of spatial topology, the models are presented discretely with limited degrees of freedom of elastic elements by Dubowsky et al. [6, 7], Winfrey [8, 9], Shabana [10]. It is worth mentioning the work of Erdman, Sandor, Oakberg [11], where the method of kinetic-elastodynamic analysis and synthesis of mechanisms using a flexibility matrix was developed. Beams with different loading options depending on the boundary conditions are taken as a model for elastic analysis of the mechanism links. One of the first works on the dynamic analysis of planar mechanisms accounting for finite deformations of elastic links is the work of Viscomi, Ayre [12]. Despite the fact that only one link was assumed elastic, it was the most complete, meaningful work and underlie modern research on modeling the nonlinear dynamics of elastic elements of structures, mechanisms and machines when removing restrictions on the magnitude of their deformations.

Modern works aim at studying the spatial motions of rotational-vibrational mechanical elastic systems, which are widely used in mechanical and instrument engineering, power engineering, transport, and many other fields of modern technology as driving elements, elements of percussion mechanisms, drilling equipment, etc. These systems are based on finite segments of elastic rectilinear rod elements. Modern works on the nonlinear dynamics of rod elements with no restrictions on the magnitude of their deformations are of particular interest. Amongst them are the works of Erofeev [13, 14], Asghari [15], Gulyaev [16] and others. Flexible elements by their nature are also nonlinear; they are presented in sufficient detail in the works of Svetlitskii. As a rule, flexible elements are tensioned to eliminate their sagging. There is little research on the influence of initial stress caused by the constructive necessity of mechanisms on their dynamics. At the same time, as shown by the research results of Guz' and others [17, 18], the wave speed depends significantly on the initial stress tensor. This fact was noted in Ogden's works studying the influence of initial stresses of pre-stressed media subject to finite deformations. Ogden's works regarding the study of incremental motion superimposed on an initially stressed configuration subject to finite deformation are known. They investigated the effect of initial stress on infinitesimal wave propagation [19, 20]. Chadwick and Ogden obtained formulas for a pre-stressed material in the absence of residual stress.

Therefore, study of the mutual influence nature of the static fields of initial stress and the disturbed state of the elastic links of mechanisms during their operation is of practical interest.

The authors of this paper investigate the planar hinge-lever mechanisms with elastic links and pre-tensioned flexible elements in a closed elastic contour. The considered approach is based on the kinetic-elastodynamic analysis of mechanisms with flexible pre-stressed elements and elastic links taking into account their finite deformations. The nonlinear dynamics of such mechanisms is modeled using Hamilton's variation principle, as well as the widely known Biot's theory of incremental deformations [21] and Novozhilov's theory of finite deformations [22].

2 Kineto-Elastodynamic Analysis of Planar Hinge-Lever Mechanisms

In most works on kineto-elastodynamic analysis of planar hinge-lever mechanisms, the number of elastic elements is restricted to one link. Moreover, its elastic displacements are assumed to be small. The nature of the link deformation is defined by the elastic deflection, and axial forces are assumed to be constant along the length of the elastic elements. It restricts the model since the variability of the axial forces along the link length significantly affects the vibrational process, causing the effect of time-varying stiffness or "frequency modulation". This phenomenon is of particular importance for mechanisms with a sufficiently large ratio of the lengths of connecting rods to the length of a crank.

This paper proposes an extension to the model by assuming all links to be elastic, increasing the degrees of freedom of the link deformation and considering the finiteness of their magnitudes.

Based on the generalized model of spatial deformation of the rod element [23] and the transition to its particular topologies, a dynamic model of elastic motion of the entire mechanism is constructed. The field of longitudinal and lateral displacements $u_i(x,t)$ and $v_i(x,t)$ of the links is given by

$$\begin{aligned} U(x,y,z,t) &= u(x,t) - \frac{\partial v(x,t)}{\partial x} y \ , \\ V(x,y,z,t) &= v(x,t) \ , \\ W(x,y,z,t) &= \ 0 \ , \end{aligned} \tag{1}$$

where $u_i(x,t)$ is the translational displacement of the section of the i-th link along the x-axis, $v_i(x,t)$ the displacement of the flexural center of the i-th link cross-section along the y-axis owing to bending.

The generalized model of elastic motion of links is shown in Fig. 1. Figure 1a corresponds to the simply supported driven links, whereas Fig. 1b shows elastic deformation of the driving link as a cantilever beam.

The position of the points of the i-th link of the deformed mechanism in the inertial coordinate system OXY is determined by the radius vector R_i;$X_i(t)$, $Y_i(t)$, $\theta_i(t)$ determine the plane motion of the undeformed i-th element relative to the inertial (global) coordinate system OXY; $O_i x_i y_i$ is a moving coordinate system associated with the i-th link and describing its deformed configuration (local coordinate system); u_i, v_i longitudinal and lateral displacements of the i-th link in the section x as a result of the link deformation, respectively. The kinematics of the mechanisms is defined by methods known in the literature.

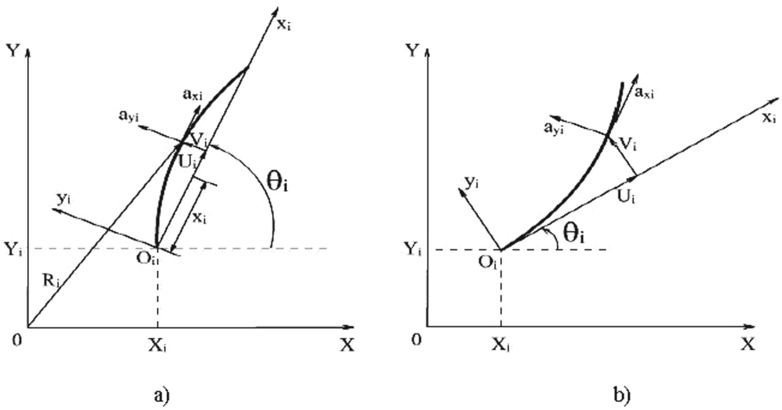

Fig. 1. Generalized model of deformation of elastic links for hinge-lever mechanisms.

When deriving the equations of motion of the mechanism as a system of rod elements, it is necessary to ensure the connection between the elements through kinematic pairs in view of its multi-link structure. For that the force analysis of the mechanisms is carried out. Figure 2a shows the diagram of the loading forces of the linkages; the force diagram in a kinematic pair connecting two adjacent links is given in Fig. 2b.

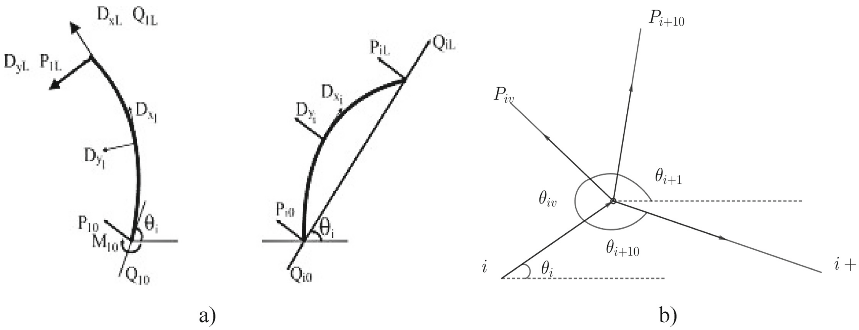

Fig. 2. Diagrams of the loading forces of the driving link and i-th driven links (a) and ones in a kinematic pair connecting two adjacent links (b).

The unknown components of reactions in the *i*-th link hinges are determined from the equilibrium condition of forces and moments acting on this link:

$$\begin{aligned} & Q_{i0} + Q_{iL} - \int_0^{l_i} m_i a_{xi} dx = 0, \\ & P_{i0} + P_{iL} - \int_0^{l_i} m_i a_{yi} dx = 0, \\ & P_{i0} l_i + \int_0^{l_i} m_i a_{xi} v dx - \int_0^{l_i} m_i a_{yi} x dx = 0. \end{aligned} \quad (2)$$

A system of 3*N* equations was obtained to govern the reactions in the hinges (*N* is the number of moving links of the mechanism). The missing conditions for the solvability of the system of Eqs. (2) are obtained from the conditions of equilibrium of forces in the hinges connecting adjacent links. According to Fig. 2, the equations of equilibrium of forces in the hinges in case of two adjacent driven links (simply supported beams) have the form:

$$\begin{aligned} & Q_{iL} \cos\theta_i + Q_{i+10} \cos\theta_{i+1} - P_{iL} \sin\theta_i + P_{i+10} \sin\theta_{i+1} = 0, \\ & Q_{iL} \sin\theta_i + Q_{i+10} \sin\theta_{i+1} + P_{iL} \cos\theta_i + P_{i+10} \cos\theta_{i+1} = 0. \end{aligned} \quad (3)$$

If a kinematic pair connects more than two links, then the number of terms in (3) increases.

Figure 3 shows a diagram of the force loading of a flexible element.

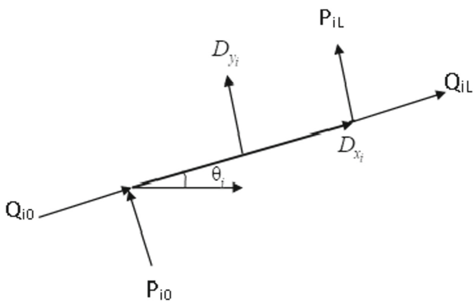

Fig.3. Diagram of loading forces for flexible elements.

Flexible elements have a number of advantages and benefits: reduced metal capacity, the absence of threat of longitudinal bending of the link, unpretentiousness in assembly and accuracy of manufacturing kinematic pairs, etc. Therefore, they are widely used in transmission mechanisms of machines. However, flexible elements in the form of a cable, belt, chain and other transmissions, carrying out a one-way connection, sag during operation. There are two main approaches to address this deficiency:

1) tension of flexible elements, creating an initial tension;

2) power closure of the kinematic chain for the implementation of alternating work of two flexible elements. Both these approaches are applied in mechanisms with closed elastic pre-stressed contour.

The mutual influence of elastic motion of the linkages is taken into account by transferring forces through their hinged joints. The force analysis of flexible elements is carried out similarly to the force analysis of elastic elements, i.e. reactions in hinges are determined from the condition of equilibrium of forces and moments of each link, and the missing equations for closing the system are taken from the condition of equality of forces in the hinges connecting flexible and elastic linkages.

3 Equations of Motion of Flexible and Elastic Linkages

The motion of a rotating rod element in cases of its plane and spatial topology of deformation is modeled in [24, 25]. The equations of motion are derived on the basis of Ostrogradsky-Hamilton's variation principle. Also, the finiteness of the link deformations is assumed according to Novozhilov's theory of finite deformations. Here, in contrast to an unlinked element, when its motion can be specified locally, elastic linkages between the system of elements lead to the complication of the model.

In this case, the motion of the entire system must be determined in the global coordinate system, specifying the nominal motion of elements and elastic displacements resulting from their deformation.

In accordance with the deformation scheme (1), the kinetic and potential deformation energies of any i-th link are defined as

$$\begin{aligned} \mathrm{T}_i = \frac{m_i}{2}\int_0^{l_i}\left(\dot{X}_i\cos\theta_i + \dot{Y}_i\sin\theta_i + \dot{u}_i - v_i\dot{\theta}_i\right)^2 dx \\ + \frac{m_i}{2}\int_0^{l_i}\left(\dot{Y}_i\cos\theta_i - \dot{X}_i\sin\theta_i + \dot{v}_i + x_i\dot{\theta}_i + u_i\dot{\theta}_i\right)^2 dx. \end{aligned} \tag{4}$$

$$\mathrm{U_i} = \frac{E_iJ_{iy}}{2}\int_0^{l_i}\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx + \frac{E_iF_i}{2}\int_0^{l_i}\left[\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2\right)^2 + \frac{1}{4}\left(\frac{\partial v}{\partial x}\right)^4\right] dx. \tag{5}$$

As a result, a system of coupled nonlinear equations of motion of the i-th type is obtained:

$$
\begin{aligned}
&E_iJ_iv_{xxxx} - E_iF_i\left(v_{xx}u_x + v_xu_{xx} + 3v_x^2v_{xx}\right) + \rho_iF_iv_{tt} \\
&= \rho_iF_i\left[\sum_j\left[l_j\dot{\theta}_j\sin(\theta_i-\theta_j)(2\dot{\theta}_i-\dot{\theta}_j) - l_j\ddot{\theta}_j\cos(\theta_i-\theta_j)\right] + (x+u)\ddot{\theta}_i + v\dot{\theta}_i^2 + 2u_t\dot{\theta}_i\right], \\
&E_iF_i(u_{xx}+v_xv_{xx}) - \rho_iF_iu_{tt} = \rho_iF_i\left[\sum_j\left[l\ddot{\theta}\sin(\theta_i-\theta_4) + l_j\dot{\theta}_j\cos(\theta_i-\theta_j)(\dot{\theta}_i-\dot{\theta}_j)\right]\right. \\
&\left.+\sum_j l_j\dot{\theta}_j\dot{\theta}_i\cos(\theta_i-\theta_j) + \ddot{\theta}_iv - (x+u)\dot{\theta}_i^2 + 2\dot{\theta}_i\dot{v}\right], \qquad j = \overline{1 \div i-1}.
\end{aligned}
\tag{6}
$$

It specifies the axial and transverse displacements of the links of the mechanism relative to their undeformed position.

The summation sign by j on the right-hand side of Eqs. (6) is associated with the determination of the position functions of the i-th deformable link through the previous $i-1$ links of the mechanism. An "overdots" on the right-hand side of Eqs. (6) indicate derivatives with respect to time. Depending on the type of kinematic pairs of the mechanism links, the corresponding boundary conditions that include both external and internal forces and moments are chosen. For driven links (simply supported beams), they are specified as

$$
v_i(0,t) = u_i(0,t) = \frac{\partial^2 v_i}{\partial x^2}(0,t) = 0, \quad v_i(l,t) = u_i(l,t) = \frac{\partial^2 v_i}{\partial x^2}(l,t) = 0. \tag{7}
$$

The boundary conditions for driving links (cantilever beam) are given by

$$
\begin{aligned}
&v_1(0,t) = u_1(0,t) = \frac{\partial^2 v_1}{\partial x^2}(0,t) = 0, \quad E_1F_1\left.\frac{\partial^3 v}{\partial x^3}\right|_{x=0} = P_{10}, \\
&E_1F_1\left.\frac{\partial u_i}{\partial x}\right|_{x=0} = Q_{10}, \quad E_1I_1\left.\frac{\partial^2 v}{\partial x^2}\right|_{x=0} = M_{10}.
\end{aligned}
\tag{8}
$$

In the case of the action of external forces and moments on the i-th element of the mechanism, the influence of the latter on the mechanism dynamics can be accounted for in the boundary conditions.

The equations of motion of flexible elements are based on the fundamental relations of Biot's theory of initial stresses [21]. The latter are obtained from the equations of equilibrium of a volume element of a deformed medium during the transition to the initial state of the medium:

$$
\frac{\partial A_{xx}}{\partial x} + \frac{\partial A_{xy}}{\partial y} + \frac{\partial A_{xz}}{\partial z} + X(\xi,\eta,\varsigma) = 0, \tag{9}
$$

where

$$A_{xx} = \bar{\sigma}_{\xi\xi}\frac{d(\eta,\varsigma)}{d(y,z)} + \bar{\sigma}_{\xi\eta}\frac{d(\varsigma,\xi)}{d(y,z)} + \bar{\sigma}_{\xi\varsigma}\frac{d(\xi,\eta)}{d(y,z)} \quad (10)$$

$\frac{d(\eta,\varsigma)}{d(y,z)}$ are Jacobians of transformation of pairs of $x,\ y,\ z$ variables into pairs ξ, η, ς;

$$\bar{\sigma}_{\xi\xi} = \sigma_{11}^0 + \bar{s}_{\xi\xi}, \quad \bar{\sigma}_{\xi\eta} = \sigma_{12}^0 + \bar{s}_{\xi\eta}, \quad \bar{\sigma}_{\xi\varsigma} = \sigma_{13}^0 + \bar{s}_{\xi\varsigma} \quad (11)$$

σ_{11}^0 initial stresses in a Cartesian coordinate system;
$\bar{s}_{\xi\xi}$ increment of stresses as a result of deformation.
The rest equations are obtained by cyclic substitution.
By introducing simplifications in (10), we can obtain various equations of motion with nonlinear effects and different topology of deformation.

4 Modeling of Elastic Motion of a Planar Five-Link Mechanism with Closed Pre-stressed Contour

As an example, the motion of a planar five-link hinge-lever mechanism with closed elastic pre-stressed contour is considered (Fig. 4). Such mechanisms are used as a drive for machines performing reciprocating motion. The motion of mechanisms is modeled taking into account the elastic properties of the links and the initial stresses in flexible elements.

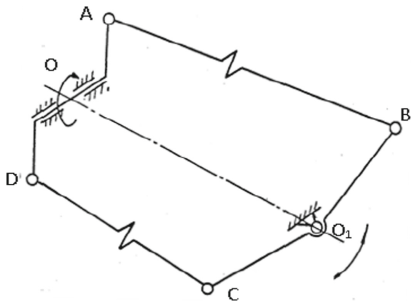

Fig. 4. Planar five-link hinge-lever mechanism with closed pre-stressed contour.

When the mechanism operates, the field of initial stresses of flexible elements transits from a static state to the dynamic one. Moreover, it interacts with the disturbed state of other elastic links of the mechanism. The equations of motion of elastic elements, taking into account the finiteness of deformations, will be determined by a nonlinear system of the form (6).

Passing to the one-dimensional case in Eqs. (9), (10) and generalizing it for the case of finite deformations, the equation of motion of flexible elements is obtained:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial \sigma^{\circ}_{xx}/\partial x}{E + \sigma^{\circ}_{xx}(1 + e_{xx})} \frac{\partial u}{\partial x}\left(1 + \frac{1}{2}\frac{\partial u}{\partial x}\right) + \frac{\partial Q/\partial x}{F(E + \sigma^{\circ}_{xx}(1 + e_{xx}))} = \frac{\rho}{E + \sigma^{\circ}_{xx}(1 + e_{xx})}\frac{\partial^2 u}{\partial t^2}. \tag{12}$$

Depending on the type of supports and mountings of flexible elements with adjacent links, the appropriate boundary conditions are specified for the equations of motion (12). For hinged joints, they are written as

$$E_i F_i \left(\frac{\partial u}{\partial x}\right)\bigg|_{x=0} = -Q_{i0}, \quad E_i I_i \left(\frac{\partial u}{\partial x}\right)\bigg|_{x=L} = -Q_{iL}. \tag{13}$$

Thus, the mathematical model of elastic motion of the considered mechanism is given by the following system of equations:

$$\begin{aligned} & E_i F_i \frac{\partial^2 u_i}{\partial x^2} - m_i \frac{\partial^2 u_i}{\partial t^2} =_i (\varphi_i, \varphi_j, \dot{\varphi}_j, \ddot{\varphi}_j, u_i, v_i, \dot{v}_i), \\ & E_i J_i \frac{\partial^4 v_i}{\partial x^4} + m_i \frac{\partial^2 v_i}{\partial t^2} = G_i(\varphi_i, \varphi_j, \dot{\varphi}_j, \ddot{\varphi}_j, u_i, \dot{u}_i, v_i)\ ,\ j = 1,\ \overrightarrow{i} - 1, \\ & \frac{\partial^2 u}{\partial x^2} + \alpha(E, \sigma^{\circ}_{xx}, e_{xx}) \frac{\partial u}{\partial x}\left(1 + \frac{1}{2}\frac{\partial u}{\partial x}\right) + q = \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} \end{aligned} \tag{14}$$

with boundary conditions (7), (8), and (13).

5 Conclusion

Based on the kineto-elastodynamic analysis, the generalization of the model of motion of the planar hinge-lever mechanisms with pre-stressed flexible and elastic elements was carried out. The nonlinear inertial relationship between the nominal motion of the links as rigid bodies and the vibrational process as a result of deformation of all the mechanism links was taken into account. The geometric nonlinearity of the model due to the finiteness of deformations of elastic links and accounting for the field of initial stresses in flexible elements was observed.

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