Chapter 10
Computational Models for Affect Dynamics

Niels Vanhasbroeck , Sigert Ariens , Francis Tuerlinckx, and Tim Loossens

Abstract Computational models of affect dynamics are ubiquitous. These models are appropriate for either exploring intensive longitudinal data or testing theories about affect dynamics. In this chapter, we give a brief overview of some of the computational models that have been applied in the field of affect dynamics, focusing on both discrete-time and continuous-time models. The emphasis of this chapter lies on describing the core ideas of the models and how they can be interpreted. At the end, we provide references to other important topics for the interested reader.

10.1 Introduction

Studying emotion dynamics, or how emotions change over time, implies that one has to collect intensive longitudinal data (ILD; i.e., longitudinal data with many measurements over time; Y. Chen & Zhang, 2020). Advances in technology have greatly facilitated the collection of such data (Hamaker et al., 2015), both in daily life (Bolger et al., 2003; Larson & Csikszentmihalyi, 1983; Myin-Germeys et al., 2018) and the laboratory (Seeley et al., 2015). When ILD have been collected, the challenge remains to analyze them and interpret the results. For this, we make use of *computational models*, that is, statistical or mathematical models that formalize how we believe a system works. In the case of emotion dynamics, these models formalize properties of ever-evolving emotions, so that they may change in their presence, intensity, and frequency over time.

However, formalizing the operation of the affective system is easier said than done. Luckily, the increased use of ILD has led to an increased interest in models

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that quantify time-dependent changes in a whole range of psychological processes. Knowing what models exist and when it is appropriate to use them is thus tantamount for any researcher who wants to understand how emotions change over time.

In this chapter, some of the available computational models for temporal processes will be considered, with a focus on those models that have been used to study affect dynamics. The chapter will focus on computational models for the flow of emotional experiences, rather than models that zoom in on one specific emotional experience and how it unfolds over time (Kuppens & Verduyn, 2015; Verduyn et al., 2009).

10.1.1 Why Computational Models?

Why should we bother with these often complicated models of affect dynamics? To appreciate the role models may play in advancing research, we discern two general goals of their application and refer the interested reader to Breiman (2001) and Koopmans (2011) for a more elaborate discussion.

First, computational models can be used as data analysis tools. In this sense, they enable the researcher to study complex patterns of change in empirical data and validate hypotheses concerning these patterns (Hamaker et al., 2015). The way such computational models are being used is no different than ANOVA: It is a generic model that can be applied to a wide range of data without presuming that the model provides a complete account of the data generating process. The usefulness of such models is determined by whether they can account for some of the features observed in empirical data. For example, it is widely believed that the intensity of an emotional or mood state is in some way related to its past intensity (i.e., when one feels bad now, this will likely continue for some time; Kuppens & Verduyn, 2017). Models that do not take such relation into account, may be less appropriate for analyzing affective time series.

Secondly, computational models can be formulated as theories of affect dynamics (Farrell & Lewandowsky, 2010; Hamaker et al., 2015; Luce, 1995). Like verbal theories, they may describe how emotions behave over time and what underlies their fluctuations. Each model has its own focus or emphasis, as well as its own set of assumptions. Furthermore, it is possible to derive hypotheses and test them to provide either evidence for or against the model (Jekel, 2019).

There are some advantages to using these theoretical computational models compared to verbal theories (see Farrell & Lewandowsky, 2010 and Smaldino, 2017 for a more elaborate discussion). Firstly, defining a computational model forces us to specify all aspects of a theory in an explicit and detailed manner. Secondly, when multiple computational models describe the same process, comparison of model performance (i.e., how well a model describes empirical data) may guide us towards better theories of affect dynamics. Lastly, discussion about the theory does not rest upon the interpretation of others, but rather on the set of mathematical properties of the model. This limits misinterpretation of the theory and moves the literature away

from communication of theories to the testing of them. However, here also lies a disadvantage of these models: They may be too technical to understand straightaway, which may still lead to misunderstanding of what these models can, and perhaps more importantly, cannot explain.

The boundary between a computational model that is used as an analysis tool and a computational model that is considered a theory is often fuzzy—the same model may be used as a data analysis tool by some and as a theoretical model by others. Therefore, we will use the term "computational model" in a general sense, referring to both instances.

10.1.2 Characteristics of Affective Time Series

As mentioned before, a minimal requirement for computational models of affect dynamics is that they take into account at least some features of emotional change. Here, we shortly discuss the most general characteristics of an affective time series that are accounted for by most computational models of affect, namely the *baseline*, *variability*, and *regulation* of the process.

Figure 10.1 provides an illustration of an affective time series in which the happiness of a single individual has been measured at 25 different time points. A first thing one may notice is that measured happiness fluctuates around the dotted line in the figure. This dotted line is called the baseline and represents the affective state that comes naturally to the individual, or the affective state in which the individual is most likely to find him-/herself. If happiness increases or decreases relative to this baseline, one would feel more or less happy than usual (Brickman & Campbell, 1971; Kuppens et al., 2010). The fact that happiness does not stick to the baseline, but shows some variability over time, is a second characteristic of the time series. This variability indicates that happiness is not a stable construct, but that it changes over time. Finally, when a grave change in happiness has occurred, it tends to move

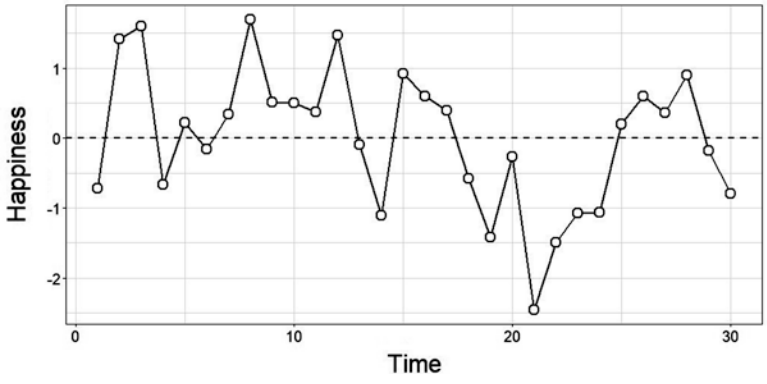

Fig. 10.1 Illustration of an affective time series

back towards the baseline (e.g., timepoint 21–25 in Fig. 10.1). In other words, affect is regulated back to the baseline: a third characteristic of affective time series (Gross, 2015; Kuppens & Verduyn, 2017).

In the subsequent parts of this chapter, we will introduce the reader to some computational models of affect dynamics. We make a distinction between *discrete-time* and *continuous-time* models—two classes of models that differ in how they treat time, which we will discuss later. In our discussion, we will not shy away from showing the mathematical equations of these models, as we deem it important to expose the interested reader to them before they might delve into the, often technical, literature around them. We follow some conventions for mathematical notation. Lowercase letters are used for scalars or single values. When they are in bold, however, they represent a collection of such scalars called *vectors*. Uppercase letters are used for matrices. Greek letters are used for parameters that should be estimated, while Roman letters describe observed values.

10.2 Discrete-Time Models

The class of *discrete-time* models enjoys a wide popularity in psychological research. These models relate measurements at a given time point to measurements at previous time points, usually by means of *difference equations* or *maps* (Strogatz, 2018). As such, discrete-time models describe changes in the affective time series in discrete steps, from one measurement to another.

First, we will introduce the *autoregressive models*, which are among the most prominent in the affect dynamics literature (Hamaker et al., 2015) and often serve as a building block for more complicated models. Afterwards, we will focus on *reinforcement learning models* as a more recently proposed class of models for affect dynamics.

10.2.1 Autoregressive Models

10.2.1.1 The Autoregressive Model

Imagine that we track someone's happiness across time, using a continuous slider ranging from 0 (*not happy*) to 100 (*extremely happy*). Let y_j (with $j \in \{0,1,\ldots,N\}$) denote the ratings obtained at time points $t_0 < \cdots < t_j < \cdots < t_N$. The lag-1 *autoregressive* or *AR(1)* model relates the rating y_j at time t_j to the rating y_{j-1} at time t_{j-1} (i.e., the immediate predecessor) by means of a linear regression:

$$y_j = \delta + \varphi y_{j-1} + \varepsilon_j$$

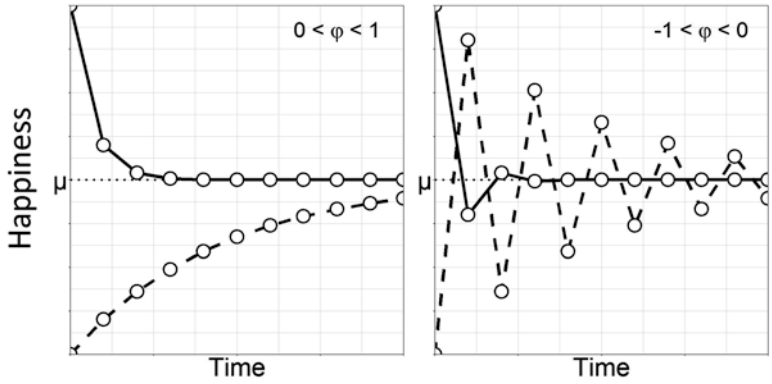

Fig. 10.2 Emotional fluctuations as captured by a standard AR model (without random noise). In the left plot, the autoregressive coefficient $\boldsymbol{\varphi}$ is positive, lying in the interval]0, 1[. This leads to an exponential decay, which becomes more gradual as $\boldsymbol{\varphi}$ approaches 1. In the right plot, $\boldsymbol{\varphi}$ is negative, lying in the interval $]-1, 0[$. This leads to emotional switching, in which the emotional state crosses the baseline at each future time point. The strength of this overshoot dies out over time, and is related to the size of $\boldsymbol{\varphi}$, so that values of $\boldsymbol{\varphi}$ closer to -1 lead to longer dying-out time. In the plots, the solid line is always generated by a small autoregressive effect ($\boldsymbol{\varphi} = -0.2$ or $\boldsymbol{\varphi} = 0.2$), while the dashed line is generated by a large autoregressive effect ($\boldsymbol{\varphi} = -0.8$ or $\boldsymbol{\varphi} = 0.8$)

The parameter δ is a constant referred to as the *intercept*. This intercept can take any value and is related (but not equal) to the baseline (see Appendix 1).

The parameter φ ($\in [-1,1]$) is called the *autoregressive coefficient*; it describes the temporal dependence of the variable y with itself at lag 1. In other words, it summarizes how strongly happiness scores depend on previous happiness scores. Values closer to -1 or 1 imply a strong temporal dependence, whereas values closer to zero imply that there is little to no carry-over effect. When this effect is positive, an initial happiness score is expected to be regulated towards the baseline exponentially fast. When it is negative, however, we expect happiness to overshoot the baseline with each additional measurement. This overshoot is damped, so that it dies out over time. These autoregressive effects are visualized in Fig. 10.2.

The stochastic variables ε_j are often referred to as the *innovations*. These are used to describe unpredictable effects due to internal and external processes, capturing the variability in the dependent variable. They are typically assumed to be uncorrelated over time, independent of past values of the variable y, and normally distributed with mean 0 and variance σ^2_ε:

$$\varepsilon_j \sim N\left(0, \sigma^2_\varepsilon\right)$$

Order The AR(1) model is a model of *order* 1 or lag 1. This means that the ratings y_j at times t_j are regressed on the ratings y_{j-1} at times t_{j-1}. However, the AR(1)

model is generalizable to higher orders, so that emotional states further in the past may also contribute to an emotional state at present.

The AR(p) model of order p is defined as:

$$y_j = \delta + \sum_{p}^{k=1} \varphi_k y_{j-k} + \varepsilon_j,$$

where an autoregressive effect is assigned to each lagged variable y_{j-k}. In this chapter, we will confine ourselves to the discussion of models of order 1, although results may be generalized to models of order p. Because of this restriction, we will also simplify our notation from AR(1) to AR.

10.2.1.2 The Vector Autoregressive Model

In the context of affect dynamics researchers are usually interested in the change and interactions of multiple emotions or affective components over time. To accommodate this need, the AR model can be extended to take multiple variables into account—an extension also known as the vector autoregressive (VAR) model.

A VAR model with d variables is defined as:

$$\begin{aligned} \boldsymbol{y}_j &= \boldsymbol{\delta} + \Phi \boldsymbol{y}_{j-1} + \boldsymbol{\varepsilon}_j \\ \boldsymbol{\varepsilon}_j &\sim N\left(\mathbf{0}, \Sigma_\varepsilon\right) \end{aligned} \tag{10.1}$$

with

$$\boldsymbol{y}_j = \begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_d \\ \\ \\ \end{bmatrix}_j, \boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_d \\ \\ \\ \end{bmatrix}, \boldsymbol{\varepsilon}_j = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_d \\ \\ \\ \end{bmatrix}_j$$

$$\Phi = \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1i'} & \cdots & \varphi_{1d} \\ \vdots & \ddots & \vdots & & \vdots \\ \varphi_{i1} & \cdots & \varphi_{ii'} & \cdots & \varphi_{id} \\ \vdots & & \vdots & \ddots & \vdots \\ \varphi_{d1} & \cdots & \varphi_{di'} & \cdots & \varphi_{dd} \end{bmatrix}, \Sigma_\varepsilon = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1i'} & \cdots & \sigma_{1d} \\ \vdots & \ddots & \vdots & & \vdots \\ \sigma_{i1} & \cdots & \sigma_{ii'} & \cdots & \sigma_{id} \\ \vdots & & \vdots & \ddots & \vdots \\ \sigma_{d1} & \cdots & \sigma_{di'} & \cdots & \sigma_{dd} \end{bmatrix}$$

The values of the variables at time t_j are contained in the $d \times 1$ vector $\boldsymbol{y}_j$. The intercepts and innovations of each variable are contained in the $d \times 1$ vectors $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}_j$. The autoregressive effects reside in the $d \times d$ transition *matrix* Φ—more specifically on its diagonal (i.e., φ_{ii} represents the autoregressive effect of variable y_i). The off-diagonal elements are called *crossregressive effects* and represent the temporal relations between the different variables. This means that a variable at time t_{j-1} may be related to another variable at time t_j. As an example, fatigue early in the day may be associated to irritability later in the day. Or being relaxed in the morning may be associated to feeling happier in the afternoon (if you did not miss any deadlines because you were a bit too relaxed, that is).

The (co)variances of the innovations are captured in the $d \times d$ innovation covariance matrix Σ_ε. The diagonal contains the variances of the variables and the off-diagonal elements coincide with the covariances between the variables. Allowing innovations to covary means that, on average, random perturbations to the first process at time t_j are not independent from perturbations to a different process at time t_j. Innovation covariance is therefore usually assumed to reflect common responses to external stimuli. Note that such innovation covariances are different from the direct lagged effects in that they do not make specific which of the two processes drives the other.

The VAR model has several interesting properties, such as regulation to a baseline, growing uncertainty of predictions further away in the future, and the possibility to derive the autocorrelation from the transition matrix (see Appendices A and B). Furthermore, it allows researchers to study complex patterns of affect dynamics without the requirement to make a priori decisions on parameter structure (see Fig. 10.3), so it can be applied to a wide range of problems. Because of its simplicity, its versatility, and its usefulness in describing basic properties of affect dynamics, it is no surprise that VAR models have received much attention in the field, both in the empirical (e.g., Congard et al., 2011; Kuppens et al., 2012a; Kuranova et al., 2020; Wichers et al., 2020) and statistical literature (e.g., Adolf et al., 2017; Bringmann et al., 2018; Bulteel et al., 2016). However, these advantages come at a price, as the large number of parameters requires a large amount of data to be estimated adequately (Loossens, Dejonckheere, Tuerlinckx, & Verdonck, 2021).

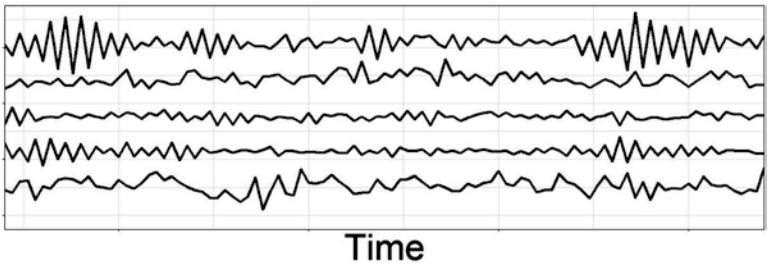

Fig. 10.3 Simulated trajectories for five variables of a VAR model. The VAR can capture many complicated, and often realistic patterns of emotional fluctuation that can be found in data

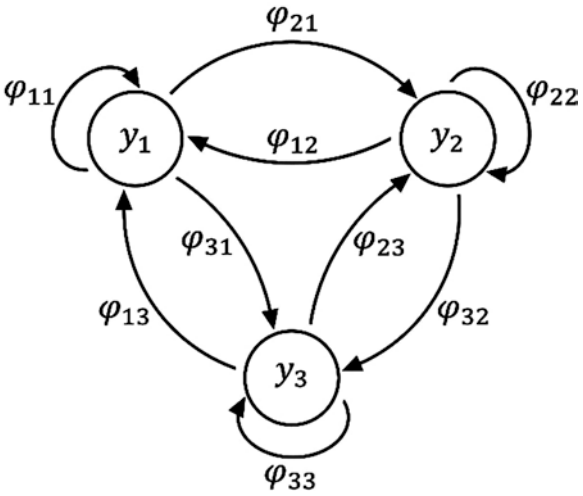

Fig. 10.4 Example of a directed network with three nodes and six unidirectional edges. The nodes of the network consist of three variables y_1, y_2, and y_3. The strength of the lagged associations between these variables is estimated by using the auto- and cross-regressive effects that reside in the transition matrix Φ of a VAR model

10.2.1.3 Network Models

A network model is a representation of how different observed variables are related to each other. In Fig. 10.4, an example of such as network is given, where different variables or *nodes* are associated to each other through their *edges*. These edges may be weighted and unweighted, such that in the former case, you estimate the strength of the association, while in the latter case you remain agnostic of the association strength. Edges can furthermore be directed or undirected, such that directed edges describe the unidirectional influence of node i on node i', while undirected edges describe the bidirectional influence between these two nodes (i.e., both nodes exert an equal influence on each other; Smith et al., 2018).

There are many ways to build network models, depending on the association measures used as the edges and the values of the variables. In affect dynamics, a VAR model is often used as a basis of network,[1] presenting results in a more approachable way, easing interpretation (Borsboom & Cramer, 2013; Bringmann et al., 2013; Bulteel et al., 2016; Epskamp, 2020a). Building a network model from an individual's affective time series can then be accomplished with a few basic steps. First, you define the nodes to be equal to the variables y. Then, you estimate the parameters of the VAR model and equate the (directed) edges to the auto- and crossregressive effect in the transition matrix Φ (see Fig. 10.4). Finally, the network is visualized and interpreted. These steps can be accomplished by using several R packages, such as *graphicalVAR* (Epskamp, 2020b; see also Epskamp et al., 2018) and *qgraph* (Epskamp et al., 2012). A network built on the transition matrix of a VAR model is temporal in nature, as the lagged association between the variables is

[1] Because of this, we present network models in the section on autoregressive models. This was a practical choice, and we do not mean to imply that network models are always autoregressive in nature. In fact, most networks are not (e.g., Ising models; Kruis & Maris, 2016).

used as an association measure. Such a network therefore shows how variables are expected to change over time.

The network approach has several advantages. First, it allows researchers to make graphs of the estimated network, which may aid interpretation of the results. Furthermore, network models allow the use of several measures that define characteristics of the network, such as *centrality* of nodes (i.e., how interconnected a specific node is) and *clustering* between different nodes (Borsboom & Cramer, 2013; Watts & Strogatz, 1998). Such measures have been used to define several characteristics of emotional networks. For example, it has been found that in patients, several symptoms cluster together and that these clusters are only connected to each other through *bridge symptoms* (e.g., Borsboom et al., 2011; Fried et al., 2017; Greene et al., 2020; but see Groen et al., 2020). Similarly, it has been found that positive and negative emotions cluster together (Bringmann et al., 2013).

Despite these advantages, there are also several interpretational issues with network models. First, network models are often conceptualized as complex systems consisting of causal agents (Epskamp, 2020a). In other words, the relation of one node to another is usually interpreted as a causal one, meaning that changes in one node cause changes in another node. However, research in affect dynamics is generally correlational in nature and therefore not amenable to causal analysis in the first place. As succinctly put by Box (1966), "To find out what happens to a system when you interfere with it you have to interfere with it (not just passively observe it)" (p. 629).

Another hazard is specific to networks built on VAR models. The use of a network approach may obscure some known sources of bias in the parameters of VAR models. Two sources of bias have been identified by Bulteel et al. (2016). Firstly, differences in the variability of the variables may lead to inflated or deflated association estimates. Secondly, auto- and crossregressive coefficients reflect unique direct effects, which means that when the variables are interrelated, a portion of the temporal relations between variables is ignored. This can be seen by calculating the autocorrelation $\rho_{t\ t-1}$ for variable y_1 in VAR model with two variables and with $\boldsymbol{\delta} = [0,0]^{'}$, which is equal to (see Appendix 2):

$$\rho_{t,t-1} = \frac{\varphi_{11}\sigma_1^2 + \varphi_{12}\sigma_{12}}{\sigma_1^2}$$
$$= \varphi_{11} + \frac{\varphi_{12}\sigma_{12}}{\sigma_1^2}$$

The autocorrelation calculated here represents the temporal relation of the variable to itself, and does not only depend on the autoregressive effect, but also on the crossregressive effect of y_2 on y_1, scaled (in part) by how much the two variables relate contemporaneously (captured by σ_{12}). Using only φ_{11} as a measure of autorelation in a network model may thus not capture the full relation, which can lead to erroneous conclusions. Only in a few situations can the autoregressive effect

capture the full relation, namely when there is no covariance between the process variables (Bulteel et al., 2016).

10.2.1.4 Extensions

Many extensions to the AR and VAR models exist, and they are often introduced for one of two reasons. Firstly, they can be introduced to deal with some of the limitations of classical autoregressive models, such as the issue of *nonstationarity*. Secondly, extensions have been introduced to take the influence of contextual factors into account. Both reasons and their related extensions are discussed in detail below.

Nonstationarity. Stationary processes are processes that display *constant* statistical properties over time (Scargle, 1981). In other words, while emotions change over time, their means and (co)variances remain the same over time (see Appendix 1). For example, one's happiness will regulate back to the same baseline over and over again (e.g., Brickman & Campbell, 1971).

The AR-based models discussed in this section assume that the processes they model are stationary. However, that assumption may be overly restrictive. There may be situations in which we expect meaningful changes in the dynamical properties of a time series, for instance due to interventions, experimental manipulations, or major life events. Indeed, nonstationary time series have been observed in affect dynamics in clinical populations (Bonsall et al., 2012; Nelson et al., 2017; van de Leemput et al., 2014) and in situations where the environment has a meaningful (and possibly enduring) influence on an individuals' affect dynamics (Dunn et al., 2018). Studying nonstationarity and its origins may therefore be relevant, since it could, for instance, signal an individual's transition from a healthy state to mood disorders (van de Leemput et al., 2014).

Nonstationarity is a broad concept: Changes in mean, covariance, and higher order statistical moments can occur suddenly or gradually, or even display recurring patterns over time (e.g., Chow et al., 2005; Larsen et al., 2009). Sometimes researchers may have explicit hypotheses about likely sources of nonstationarity. Other times researchers may instead wish to treat nonstationarity as a nuisance that should be taken care of prior to the actual analysis. Each of these distinctions has implications for how one should deal with nonstationarity (see also Hamaker & Wichers, 2017). The topic is thus vast, and many approaches to modeling nonstationary series have been suggested. Here, we will only consider model-based solutions. For data-driven ways of handling nonstationarity prior to analysis (such as differencing and detrending), we refer the reader to Box and Jenkins (1970), Dickey and Fuller (1979), Hamilton (1994b), Lütkepohl and Xu (2012), and Velicer and Molenaar (2012).

Time-Varying VAR An intuitive extension of the typical VAR model is to allow some of its parameters to change over time, which is exactly what the time-varying

VAR (tvVAR) does (Bringmann et al., 2018). The general model for order 1 would be:

$$\boldsymbol{y}_j = \boldsymbol{\delta}_j + \Phi_j \boldsymbol{y}_{j-1} + \boldsymbol{\varepsilon}_j$$
$$\boldsymbol{\varepsilon}_j \sim N\left(\mathbf{0}, \Sigma_\varepsilon\right)$$

with

$$\boldsymbol{\delta}_j = \boldsymbol{f}\left(t_j\right)$$
$$\Phi_j = g\left(t_j\right)$$

where the index j in $\boldsymbol{\delta}_j$ and Φ_j indicates that these components change over time. The functions $\boldsymbol{f}(t_j)$ and $g(t_j)$ are smooth functions of time and are estimated using regression splines (for more details and an empirical application, see Bringmann et al., 2018).

Despite its intuitive appeal and applicability for exploratory analyses, this model typically needs a lot of data in order to be estimated accurately. Furthermore, the multivariate version of this model fixes the innovation covariances to zero, indicating that it cannot—at present—capture contemporaneous relations between variables.

Regime-Switching Models An alternative method is applicable to situations in which dynamical features change abruptly. One type of models that incorporates these abrupt changes are the regime-switching models (Cabrieto et al., 2018; Hamilton, 2010). These regimes consist of separately estimated VAR models, thus allowing regimes to be different on all kinds of dynamical features. The way in which one switches regimes depends on assumptions made by the researcher. For example, Markov regime-switching models determine these shifts by a hidden Markov model (Hamilton, 2010). Another possibility is to formalize an hypothesized relationship between the probability of switching and an observed covariate. These models are typically called threshold autoregressive models (Tong, 2011).

In general, regime-switching models can be formalized as:

$$\boldsymbol{y}_j = \boldsymbol{\delta}_{r_j} + \Phi_{r_j} \boldsymbol{y}_{j-1} + \Psi_{r_j} \boldsymbol{\varepsilon}_j$$
$$\boldsymbol{\varepsilon}_j \sim N\left(\mathbf{0}, \Sigma_\varepsilon\right)$$

where the model coefficients depend on the value of r_j, denoting the regime r at time t_j. The matrix Ψ allows for regime-dependent innovations without explicitly changing the innovation matrix in a specific regime. To determine switching, a latent estimate of the probability p_{kl} of switching from regime k to regime l is made:

$$p_{kl} = P\left(r_j = l \mid r_{j-1} = k\right)$$

While these models have been successfully used in the context of affect dynamics (e.g., switching between depressive and manic states in bipolar disorder; Bonsall et al., 2012; transitioning from healthy to depressive state; Albers & Bringmann, 2020), they suffer from the limitation that they require more than one model to capture the complete time series (Hamilton, 2010). As a consequence, the number of parameters to be estimated substantially increases with each additional regime. To be able to estimate such a model reliably, sufficiently long time series are required.

Context. Until now, the models have assumed that the emotional system evolves in a contextual vacuum. We have often referred to the dependent variables as reflecting affect, without including any external contextual information in the model. Nonetheless, the inclusion of contextual information in computational models may greatly enhance our understanding of how events may elicit emotions, which aspects of the emotional process are influenced, what the emotional system does when it anticipates them, and how long these emotions last (Daros et al., 2019; Voelkle et al., 2013).

Fixed Moderated VAR One way in which such information can be included is fixed moderated VAR (fmVAR, Adolf et al., 2017), an extension of the VAR framework which allows parameters to change depending on the value of a lagged external variable, by including it as a moderator variable. The general model can be formulated as:

$$\begin{aligned} \boldsymbol{y}_j &= \boldsymbol{\delta}_j^* + \Phi_j^* \boldsymbol{y}_{j-1} + \boldsymbol{\varepsilon}_j \\ \boldsymbol{\varepsilon}_j &\sim N\left(\mathbf{0}, \Sigma_{\varepsilon j}^*\right) \end{aligned}$$

with

$$\begin{aligned} \boldsymbol{\delta}_j^* &= \boldsymbol{f}\left(\boldsymbol{x}_{j-1}\right) \\ \Phi_j^* &= g\left(\boldsymbol{x}_{j-1}\right) \\ \Sigma_j^* &= h\left(\boldsymbol{x}_{j-1}\right) \end{aligned}$$

where $\boldsymbol{x}$ is a vector of values of the moderator at each time point.

The functions $\boldsymbol{f}$, g, and h are *link functions* which define the functional form of how the moderator influences the process. For example, a linear link function for δ_j^* would assume a linear relationship between the moderator and the intercept of the process $\boldsymbol{\delta}$. This is a relatively straightforward way of accommodating changes in the level of the series, for instance by tying a dummy-coded moderator signaling a major life event to the process intercept.

Note that the moderator is assumed to be observed and measured without error. As such, fmVAR models are in particular applicable to situations in which contextual factors are controlled by the researcher like, most notably, in lab studies. Missing moderator values cannot be handled within the model, which need to be imputed a priori (Adolf et al., 2017).

10.2.2 Reinforcement Learning

Where autoregressive models (and most other models in this chapter) mainly describe relations between variables, *reinforcement learning models* go a step further and link environment with emotions in a more explicit way.[2] Reinforcement learning is a computational framework that describes how an agent or individual uses its experience to update its behavior, often with the goal of maximizing future rewards (Bennett et al., 2020; Sutton & Barto, 2018). Central to this framework is the notion that we get feedback on our actions, which is then used to change our behavior. To make this more concrete: imagine a child that gets caught taking candy from the cupboard without asking first. The parents may be very angry with the child, an outcome he/she probably does not like. In light of this feedback, the child may then change his/her future behavior and ask the parents for candy first (or perhaps steal the candy more stealthily). Reinforcement learning thus provides a way to integrate emotional states with environmental cues, goals, and behavior, bringing action tendencies to computational modeling (Frijda, 2007).

We discuss two reinforcement learning-based models, one that is only concerned with explaining emotional states (Rutledge et al., 2014; Villano et al., 2020) and another that links these emotional states to behavior and learning biases (Bennett et al., 2020). These are, however, not the only models in this rich field (see e.g., Doll et al., 2012; Eldar & Niv, 2015; Eldar et al., 2015; Sutton & Barto, 2018).

10.2.2.1 Computational Model of Happiness

This unnamed model was originally proposed by Rutledge et al. (2014) and has been used by several other authors (e.g., Villano et al., 2020; Vinckier et al., 2018). It was proposed within the context of a study on the influence of gambling outcomes on fluctuations in happiness. For this, Rutledge et al. (2014) used a gambling experiment in which participants had to choose between a certain reward (value c) or a gamble. When participants choose a gamble, they had a 50/50 probability of receiving a higher (denoted as h) or lower outcome (denoted as l) than the certain reward. The reward given at the end of the trial can be denoted as o. Based on this experiment, the model was formulated as[3]:

[2] Note that we specifically talk about reinforcement learning in the context of emotion dynamics: This class of model is applicable to many more subjects, like decision-making, conditioning, and learned behavior (see Sutton & Barto, 2018).

[3] To remain in line with the mathematical notations of this chapter, we changed the notation of this model (see Rutledge et al., 2014).

$$y_j = \delta + \omega_1 \sum_{j}^{i=1} \gamma^{j-i} c_i + \omega_2 \sum_{j}^{i=1} \gamma^{j-i} v_i + \omega_3 \sum_{j}^{i=1} \gamma^{j-i} p_i + \varepsilon_j$$
$$\varepsilon_j \sim N\left(0, \sigma^2\right)$$

where δ is the intercept, and the variables c, v $\left(= \frac{(h+l)}{2}\right)$, and p $(= o - v)$ represent the value of the certain reward, the expected value of the gamble, and the reward prediction error (i.e., the difference between the obtained reward and the expected value). Note that for trials in which the gamble is chosen, c is zero, and for trials in which the certain reward is chosen, v and p are zero. Importantly, past outcomes also play a role, although their influence decays over time. This decay is captured by the value of γ ($\in [0,1[$), which is called the forgetting factor (see Rutledge et al., 2014). The greater the value of γ, the longer the rewards linger and the greater their influence on current happiness. The influence of all rewards (both at the current time point as well as the previous ones) is scaled by the ω's. With this model, Rutledge et al. (2014) found that prediction errors are an important driver of fluctuations in happiness, a result that has subsequently been confirmed by Rutledge et al. (2017) and Vanhasbroeck et al. (2021).

A limitation of the model is that it can only be applied to situations in which there is a certain and uncertain outcome. However, in reality, rewards may always be uncertain. To accommodate this limitation, Villano et al. (2020) made a small adjustment to the model:

$$y_j = \delta + \omega_1 \sum_{j}^{i=1} \gamma^{j-i} o_i + \omega_2 \sum_{j}^{i=1} \gamma^{j-i} p_i + \varepsilon_j$$
$$\varepsilon_j \sim N\left(0, \sigma^2\right)$$

where p represents the prediction error and o the uncertain outcome. With this adjusted model, Villano et al. (2020) also found that prediction error is an important driver of fluctuations in affect in daily life.

10.2.2.2 Integrated Advantage Model of Mood

While the previous models were able to establish the role of prediction error in affect dynamics, they do not specify how happiness or affect may guide behaviors. The Integrated Advantage Model of Mood (IAMM) goes a step further and explicitly links behavior to mood, and mood biases to behavior (see Bennett et al., 2020). In this chapter, we limit our discussion to the basics of the IAMM, leaving out some of the details of the model. We refer the interested reader to the preprint of Bennett et al. (2020) for a more thorough discussion of the model.

Central to the IAMM is the notion of *advantage*, which can be defined as the difference between the outcome of a chosen action and the value of the state within

which this action has been taken (Bennett et al., 2020). More concretely, advantage tells you something about how well you are doing now compared to before. Let's return to the example of the child: the child craved some candy (a low-valued state), so he/she decided to look in the cupboard and take some candy (an action), after which the craving is satiated (a high-valued state). The advantage of the child's action is thus positive, as he/she moves from a low- to a high-valued state. This advantage may lead to a strengthening of this behavior, i.e., the child may choose to act this way again in the future. If, however, at some point the parents catch him/her, the child suddenly find him/herself in a low-valued state again, decreasing the advantage of this same action. As such, the overall advantage of the action "taking candy" may depend on how many times the child was able to take candy without being caught, and on how the child values angering the parents (e.g., if a child does not really care that the parents are angry, then he/she will probably continue acting the same way).

Based on this notion of advantage, the IAMM defines mood that results from an action as:

$$y_j = y_{j-1} + \eta\left(\hat{\alpha}^{\pi}\left(s_{j-1}, a_{j-1}\right) - y_{j-1}\right) \tag{10.2}$$

where η plays a similar role as the parameter γ in the model of Rutledge et al. (2014), and where $\hat{\alpha}^{\pi}\left(s,a\right)$ is the estimated advantage of doing action a (taking candy) in situation s (craving candy) under a certain behavioral policy π. The behavioral policy can be seen as a rulebook that links actions to situations. For example, in the situation "craving candy", the action "taking candy" may be more likely taken than the action "asking parents first". However, based on the new experience of the child, and the associated negative advantage of performing this action, its probability may decrease. As such, $\hat{\alpha}^{\pi}\left(s,a\right)$ plays a crucial role in updating the policy π. The IAMM thus goes further than just describing mood states: It also describes how advantage can change behavioral tendencies over time (Bennett et al., 2020).

This is, however, only part of the story. Bennett et al. (2020) suggest that behavioral updating—based on advantage of an action—may be influenced by mood in the form of *momentum*. Momentum is a term from machine learning that describes an optimization algorithm for estimating parameters in which information of past updates is integrated with information on newly proposed ones (Rojas, 1996).

In terms of behavior, this comes down to the following: Suppose we act in a way that leads to positive advantage, then this advantage will lead to elevated mood and behavioral updating. Because of our elevated mood, we will also be more likely to update our behavior in the future (in combination with advantage, that is). In mathematical terms, this comes down to the updating of the parameters θ of the behavioral policy π, the details of which can be found in the preprint of Bennett et al. (2020).

$$\varepsilon_j = \lambda \varepsilon_{j-1} + \nabla_\theta \log\left(\pi\left(a_j \mid s_j\right)\right)$$

$$u_j = \zeta \varepsilon_j \left(\widehat{\alpha}^\pi\left(s_j, a_j\right) + \frac{1-\eta}{\eta} y_j\right)$$

$$\theta \leftarrow \theta + u_j$$

where υ represents the update in the parameters $\boldsymbol{\theta}$ of the behavioral policy, thus updating behavioral tendencies. Without going into detail, we will note that ε is the *eligibility trace* of the model, which determines the sensitivity with which certain action tendencies will update, depending on whether they were important in generating the taken action.

The question remains how one estimates the advantage of an action. Bennett et al. (2020) propose several estimators that may be used, one of which is the prediction error. A full description of these estimators is, however, beyond the scope of this chapter. If one is interested in knowing more about the model, we encourage the reader to read the preprint of Bennett et al. (2020).

10.2.2.3 Limitations

Reinforcement learning models have the clear advantage that they combine individual and environment in a more explicit way than is typically done by using autoregressive models. They can take into account the learning history of an individual (e.g., learned regulation strategies; Gross, 2015; learned action tendencies; Frijda, 2007) and biased reward perception (Mason et al., 2017). As such, it is no surprise that reinforcement learning-based models have been suggested as theoretical models of mood disorders (Bennett & Niv, 2018, June 7; Eldar et al., 2015).

A disadvantage of these models, however, is their reliance on known rewards: In order for the models to be estimated accurately, you should have reliable information about rewards received after performing given actions and the value that is attached to them by the individual. This information may not always be available (e.g., rewards may be intrinsic), which may influence the estimation of the parameters of the models. This makes reinforcement learning models ideal to analyze experimental data, but more difficult to apply to data from more naturalistic studies (e.g., daily life studies).

10.3 Continuous-Time Models

Why do we refer to discrete-time models as *discrete-time* models? A discrete-time model relates (or maps) observations at the discrete time points t_{j-1} to observations at the discrete time points t_j. In other words, discrete-time models are only

concerned with how the process changes from one measurement to the next. With such mapping comes the assumption that all observations are separated by precisely the same time interval. A failure to meet this assumption has been found to bias parameter estimates (de Haan-Rietdijk et al., 2017; Hamaker & Wichers, 2017). The problem is that this requirement is nigh impossible to achieve in practice, so we may assume that most (if not all) applications of these models will lead to biased estimates. While some methods have been proposed to limit this bias, no perfect solutions exist within the discrete-time approach (de Haan-Rietdijk et al., 2017; Rindskopf, 1984).

Continuous-time models constitute another solution to this problem. These models attempt to estimate the function of the continuously-evolving variables with the use of *differential equations*, which have the advantage of incorporating change at any time interval by explicitly accounting for time (Deboeck, 2013; Strogatz, 2018). This mindset makes for an arguably more natural way of thinking about affective processes, which do not stop and wait for a next observation to express themselves, but evolve continuously over time (Cunningham et al., 2013; Ekman, 1992; Feldman Barrett, 2009; Frijda, 2007; Moors & Fischer, 2019; Scherer, 2005). Because of this continuity-assumption, emotions may inherently be better off being studied in continuous time (Boker, 2002). Furthermore, continuous-time models can deal with unequal time intervals between measurements, which may be better for ecological validity (Hektner et al., 2006) and for capturing relevant information about the underlying process (Voelkle & Oud, 2013).

10.3.1 Differential Equations

Continuous-time models are often differential equations, which relate the current value of a variable to the speed with which this same variable is regulated towards its baseline. In the context of affect dynamics, this implies that the strength with which a certain affective state is regulated, depends on the severity of the disruption of affect.

Because we provide mathematical formula for the models in this section, it is important to have a notion of what differential equations look like and how they can be interpreted. Our introduction is not exhaustive. We refer the interested reader to Deboeck (2013) for a low-level treatment of differential equations, and to Strogatz (2018) for a more thorough introduction to differential equations.

Differential equations can be written in a few, analogous ways, namely:

$$\frac{dy(t)}{dt} = f\left(y(t)\right)$$
$$dy(t) = f\left(y(t)\right)dt$$
$$\dot{y}(t) = f\left(y(t)\right)$$

where *f* is a function of the variable, which may be linear or nonlinear (see later), and deterministic or stochastic (i.e., with or without a random component). In these equations, we can think of $dy(t)$ as being a change in value of the variable $y(t)$, and dt as a change in time. Hence, in the first notation, the left side represents the speed with which the variable $y(t)$ changes (change in the variable divided by a change in time), which is then related to the current value of the variable on the right side. In the second notation, the change in time dt is moved to the right side of the equation, so that you now relate the change in the variable $y(t)$ directly to the current state of the variable and the amount of time that has passed. The third notation is equivalent to the first one, so that $\dot{y}(t) = {}^{dy(t)}\!/_{dt}$. Personally, we prefer the second notation, and will try to use it where possible. In case the differential equations would become too complicated, however, we will use the first, standard notation.

10.3.1.1 Interpretation

The interpretation of differential equations may seem more alien than that of discrete-time models. However, there are several methods that make interpretation of continuous-time models easier. Besides directly interpreting the parameters (when they have clear-cut meanings), we will focus on one other method, namely considering the *vector fields*. For additional methods, we refer the reader to Ryan et al. (2018) and Strogatz (2018).

Plotting the vector field of a differential equation probably constitutes the most straightforward way for their interpretation (Strogatz, 2018). Vector fields represent the expected trajectories of emotional change, given specific initial conditions.[4] Plotting them thus provides us with some interesting characteristics of emotional change, such as the location of the baseline, the expected evolution of emotions, and the strength of regulation. Consider Fig. 10.5 in which different one- (top) and two-dimensional (bottom) vector fields are plotted. In panels (A) and (B), the derivative $\dot{y}(t)$ is plotted against the value of $y(t)$. In panel (A), it can be seen that when $y(t)$ is greater than 0, the derivative is lower than 0 (and vice-versa when $y(t)$ is lower than 0). This means that if $y(t)$ is greater than 0, it will move towards lower values of $y(t)$ with a speed that is determined by $\dot{y}(t)$. This movement stops when $\dot{y}(t)$ is equal to 0. The value of $y(t)$ in which this occurs is called a *fixed point*, and in this case the attractor, as values of $y(t)$ are regulated towards it (see the arrows). In panel (B), we see an opposite pattern, such $y(t)$ moves away from the fixed point, making it a *repellor* (note that if $y(t) = 0$, it stays put and does not move away from this fixed point). In panels (C) and (D), two equivalent vector fields for two-dimensional systems are shown with either arrows (size of the arrow indicates velocity of regulation) or trajectories.

[4] Importantly, this implies that vector fields are deterministic—they show what the model would expect if there were no perturbations to the system.

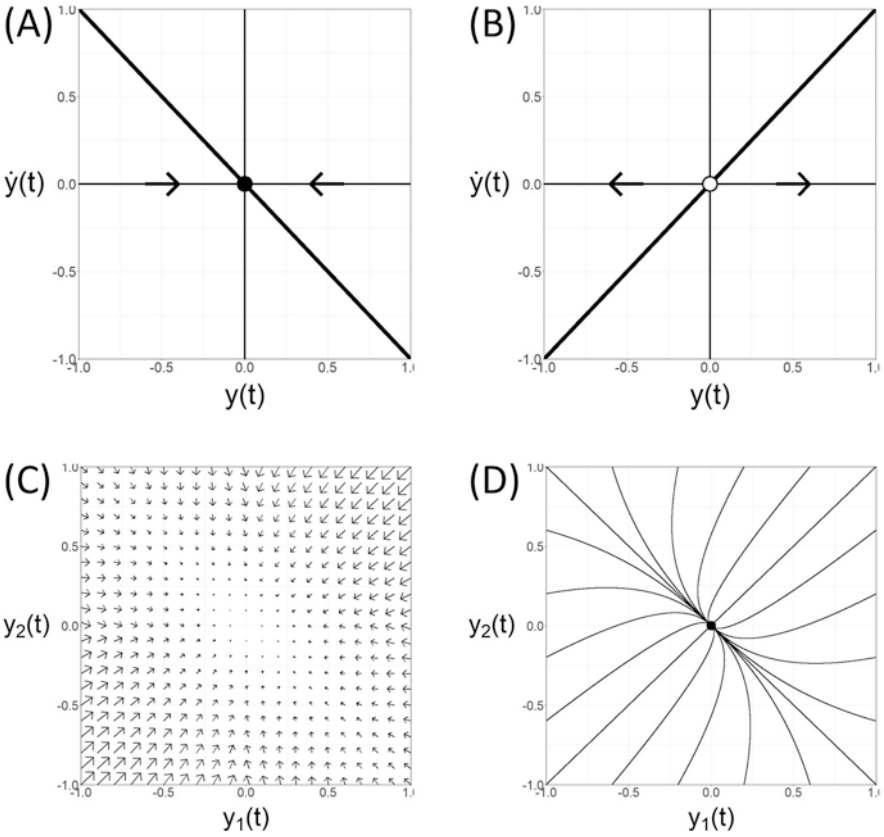

Fig. 10.5 Examples of vector fields for one- and two-dimensional systems. In plot (**A**), the derivative $\dot{y}(t)$ is plotted against the variable $y(t)$, resulting in a one-dimensional vector field. This vector field is specific to the simple system $\dot{y}(t) = -y(t)$, in which speed of regulation is linearly related to the emotional state. Arrows indicate the value to which the emotional state is regulated (here, the origin). This value is known as an *attractor*. In plot (**B**), another one-dimensional vector field is shown, this time for the system $\dot{y}(t) = y(t)$. Now, there is no regulation, but rather explosion of the emotional system, as indicated by the arrows. The origin takes on the role of *repellor*, which repels, rather than attracts values of $y(t)$. Plots (**C**) and (**D**) show two-dimensional vector fields with an attractor. In plot (**C**), direction and strength of regulation is shown through the use of arrows that differ in size. In plot (**D**), the deterministic trajectories towards the attractor are shown as solid lines

Constructing one- to two-dimensional vector fields can be achieved by following the following steps: (a) specify the parameters of the model to plot, (b) specify the initial condition of $y(t)$, (c) compute a trajectory from this initial condition for a specific amount of time, and (d) compute other trajectories and plotting them together. To approximate a trajectory, one can make use of the Euler method (Strogatz, 2018):

$$\Delta y_t = y_t \Delta_t$$
$$y_{t+1} = y_t - \Delta y_t$$

where Δy_t represents the discretized change in the variable $y(t)$, and Δt represents the discretized change in time. In practice, you should choose Δt to be small enough so that the system does not fully return to baseline from the first go, but also not too small so that you can barely see any regulation. Other approximation methods exist as well, but will not be reviewed here (see Strogatz, 2018). Some software allows you to create such vector fields through the use of a function (e.g., *dynr*; Ou et al., 2019).

We will begin our discussion with linear continuous-time models of affect dynamics. These models are of the form (Strogatz, 2018):

$$d\boldsymbol{y}(t) = A\boldsymbol{y}(t)dt + c \tag{10.3}$$

where c denotes a collection of terms that does not depend on the state vector $\boldsymbol{y}(t)$ and A is a $d \times d$ matrix.

10.3.2 Linear models

10.3.2.1 Continuous-Time VAR

In the psychological literature, the term *continuous-time VAR* model is sometimes used as a synonym for the *Ornstein-Uhlenbeck* (OU) model, named after Ornstein and Uhlenbeck who formalized the properties of this continuous-time model (Uhlenbeck & Ornstein, 1930). This is because the OU model and the discrete-time VAR(1) model are closely related: if y(t) is a continuous-time OU process (e.g., happiness) and you take equally-spaced measurements of it, then the resulting observations behave according to a discrete-time VAR(1) model (for more details on the relation between the OU and discrete-time VAR(1), we refer the interested reader to Bergstrom, 1984; Oud, 2007; Oud & Jansen, 2000; see also Fig. 10.6). Because of this simple relationship, the OU model has been proposed as an alternative analysis tool for emotion data (Driver & Voelkle, 2018a; Voelkle & Oud, 2013), and is slowly being used by researchers (e.g., Booij et al., 2020; Guthier et al., 2020; Kuppens et al., 2010; Steele et al., 2018). For brevity, we will use the term OU model to denote the continuous-time VAR model and we will keep on using the term VAR model to denote the discrete-time VAR model.

The OU model is defined as (following Oravecz et al., 2011; related formulations by Deboeck & Preacher, 2016; Driver & Voelkle, 2018b; Oud & Jansen, 2000; Voelkle & Oud, 2013):

$$d\boldsymbol{y}(t) = \Theta\left(\boldsymbol{\mu} - \boldsymbol{y}(t)\right)dt + \Gamma d\boldsymbol{w}(t) \tag{10.4}$$

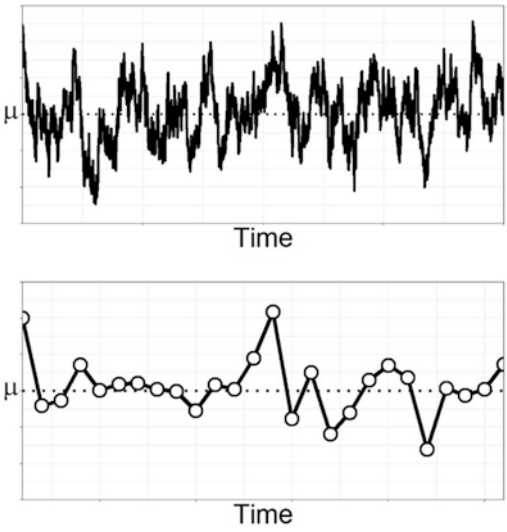

Fig. 10.6 Simulated data of an OU model (top) and associated equidistant measurements (bottom). These measurements can be described by a VAR model

with

$$\boldsymbol{y}(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_i(t) \\ \vdots \\ y_d(t) \end{bmatrix}, \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_i \\ \vdots \\ \mu_d \end{bmatrix}, \boldsymbol{w}(t) = \begin{bmatrix} w_1(t) \\ \vdots \\ w_i(t) \\ \vdots \\ w_d(t) \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \theta_{11} & \dots & \theta_{1i'} & \dots & \theta_{1d} \\ \vdots & \ddots & \vdots & & \vdots \\ \theta_{i1} & \dots & \theta_{ii'} & \dots & \theta_{id} \\ \vdots & & \vdots & \ddots & \vdots \\ \theta_{d1} & \dots & \theta_{di'} & \dots & \theta_{dd} \end{bmatrix}, \Gamma = \begin{bmatrix} \gamma_{11} & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ \gamma_{i1} & \dots & \gamma_{ii'} & \dots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ \gamma_{d1} & \dots & \gamma_{di'} & \dots & \gamma_{dd} \end{bmatrix}$$

The vector $\boldsymbol{\mu}$ is a constant vector that represents the baseline or attractor. Regulation of $\boldsymbol{y}(t)$ towards $\boldsymbol{\mu}$ is determined by the $d \times d$ matrix Θ, also known as the *drift matrix*. It fulfills a similar role as the matrix Φ for the VAR model (see Eq. (10.1)), but they are not the same: Θ is a nonlinear transformation of Φ that describes regulation on an infinitesimal time scale (see e.g., Oud, 2007).

Equation (10.4) is a *stochastic differential equation*. It does not only include a deterministic part (also known as *drift term*), but also a stochastic part (also known as *diffusion term*). This stochasticity comes about through the time-dependent fluctuations generated by d independent Wiener processes contained within the vector $\boldsymbol{w}(t)$. These fluctuations are scaled by the $d \times d$ lower-triangular matrix Γ. This

matrix is not the continuous-time extension of the innovation matrix of the VAR model, but the latter can be computed from the former as $\Sigma = \Gamma\Gamma^T$.

The OU model, like the VAR model, is very general, which means that it can be a good exploratory tool, but in itself it says little about what the emotional system looks like. Nevertheless, we know of two theoretical applications of the OU model, one of which we turn to next.[5]

DynAffect. DynAffect is a theoretical OU model that assumes that fluctuations in affect occur within a two-dimensional valence-arousal affective structure (Kuppens et al., 2010; Oravecz et al., 2011). As a theoretical model for affect dynamics, it assumes that (a) there is a baseline to which individuals regulate their emotions, (b) the strength of regulation depends on the distance from the current affective state to this baseline, and (c) there are individual differences to be found in the baseline and regulation strength.

Evidence from a number of studies supports each of these assumptions (Kuppens et al., 2010; Oravecz et al., 2018; Oravecz & Brick, 2019; Wood et al., 2018). But, these assumptions are also very general. As such, their a priori probability of being true may be great, and the lack of falsification is thus less informative than one may have initially anticipated (Popper, 1959). Even so, DynAffect is a useful model of affect dynamics, as it provides a structural framework that can be used by other researchers (e.g., Pellert et al., 2020; Schweitzer & Garcia, 2010) and allows the investigation of individual differences in affect dynamics (e.g., Santangelo et al., 2016; Wood et al., 2018).

10.3.2.2 Damped Linear Oscillator

The *damped linear oscillator* is a linear model that has been specifically proposed to capture regulatory processes (Boker & Nesselroade, 2002; Chow et al., 2005; Hu et al., 2014; Steele & Ferrer, 2011). In physics, the damped linear oscillator is a well-known model for a pendulum that slows down due to friction (and of the movements of a spring in a viscous fluid, but we believe the pendulum speaks more to the imagination). When we instigate the movement of a pendulum, we can see its mass swing down to a central position, and then back up to the other side, only to swing down on a following turn. Each oscillation, i.e. each time the pendulum swings back, the amplitude of the swing will decrease, until at some point, the mass reaches a resting state (Boker & Graham, 1998). Another analogy is that of a thermostat: If we increase the temperature of the room, the room will heat up and slightly overshoot the specified temperature. It will then cool down until it overshoots the same temperature, after which it will heat up again (and vice-versa for decreasing the temperature of the room; Boker & Nesselroade, 2002; Chow et al., 2005). These movements are visualized in Fig. 10.7.

[5] In the application that we do not discuss, the OU model was only part of a series of equations (see Pellert et al., 2020; Schweitzer & Garcia, 2010).

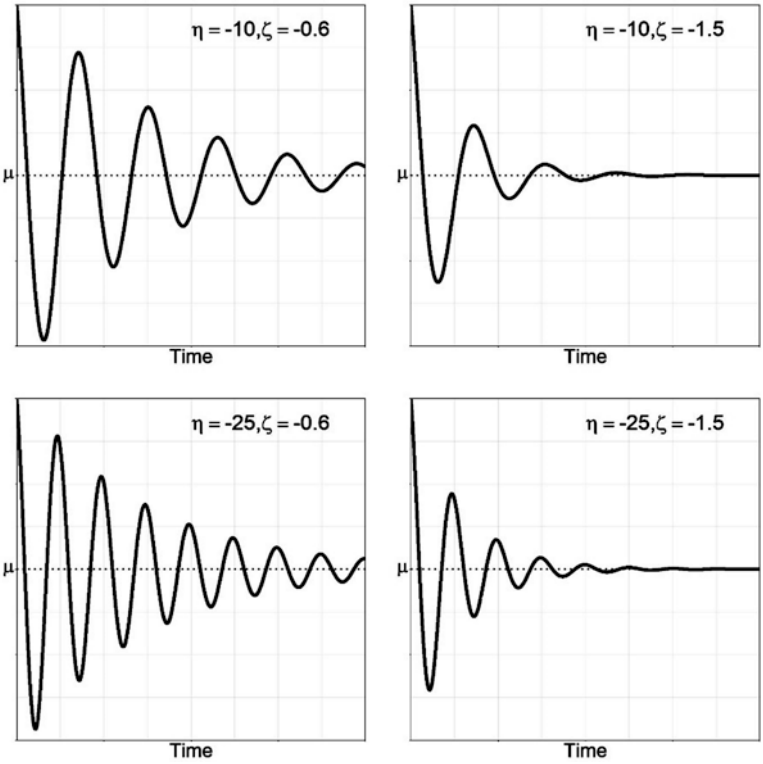

Fig. 10.7 Examples of regulation patterns to be expected by the damped linear oscillator (for one-dimensional systems). The trajectories were generated for a same initial condition and a same amount of time. When we decrease the value of η, we create a greater frequency of oscillations, visible in the number of times $y(t)$ overshoots the baseline (compare this for values $\eta = -10$ and $\eta = -25$). When we decrease the value of ζ, we increase damping, meaning that the overshoot will die out sooner (compare this for values $\zeta = -0.6$ and $\zeta = -1.5$). Note that these expected trajectories resemble the ones for negative autoregressive effects in the AR model (see Fig. 10.2)

Based on these examples, one may start to see why this model has been proposed as a model of regulation: When an event happens, we tend to regulate our ensuing emotions to return to our baseline (although upregulation may also occur; Gross, 2015). However, we may initially overregulate our emotions, such that we overshoot the baseline. After some time, this overshoot dies out and we eventually reach our resting state.

The (multivariate) damped linear oscillator is defined as follows[6]:

[6] The damped linear oscillator is an example of a *second-order differential equation*, where speed and location of a variable y at time t are both related to changes in speed over time (i.e., acceleration; speeding up or slowing down over time).

$$\frac{d^2\mathbf{y}(t)}{dt^2} = E\mathbf{y}(t) + Z\frac{d\mathbf{y}(t)}{dt}$$

with,

$$\mathrm{E} = \begin{bmatrix} \eta_{11} & \cdots & \eta_{1i'} & \cdots & \eta_{1d} \\ \vdots & \ddots & \vdots & & \vdots \\ \eta_{i1} & \cdots & \eta_{ii'} & \cdots & \eta_{id} \\ \vdots & & \vdots & \ddots & \vdots \\ \eta_{d1} & \cdots & \eta_{di'} & \cdots & \eta_{dd} \end{bmatrix}, \mathrm{Z} = \begin{bmatrix} \zeta_{11} & \cdots & \zeta_{1i'} & \cdots & \zeta_{1d} \\ \vdots & \ddots & \vdots & & \vdots \\ \zeta_{i1} & \cdots & \zeta_{ii'} & \cdots & \zeta_{id} \\ \vdots & & \vdots & \ddots & \vdots \\ \zeta_{d1} & \cdots & \zeta_{di'} & \cdots & \zeta_{dd} \end{bmatrix}$$

In these equations, the matrix E determines the frequency of the oscillations, i.e. how fast one regulates their emotions. Importantly, its diagonal elements capture the self-regulation of the variables (i.e., how fast the variables $\boldsymbol{y}(t)$ are regulated on their own), while the off-diagonal elements capture co-regulation of variables (i.e., how does the regulation of one variable relate to regulation of another). The matrix Z determines the amount of damping on the process, thus controlling how much one overshoots the baseline. In Fig. 10.7, one can see the impact of E and Z on the expected process for a one-dimensional system.

This model has been applied in a number of studies, often in the context of emotion dynamics in couples (e.g., Boker & Laurenceau, 2006; Reed et al., 2015; Steele & Ferrer, 2011), but also in the context of the relation between affective states and psychopathological symptoms (e.g., Hu et al., 2014). It can account for some interesting patterns in the data (although these are not unique, see Strogatz, 2018; Voelkle & Oud, 2013) and allows the estimation of theoretically meaningful parameters. Nevertheless, the model also has its downside. In its current form, it assumes that emotions die out over time. While this is no problematic assumption when examining specific emotional experiences, it may be more difficult to maintain when examining affect dynamics in real life, where a succession of affect-eliciting events may obscure such an (idealized) pattern. In practice, this issue is accounted for by including a measurement model that accounts for different types of noise (see e.g., Boker & Nesselroade, 2002) or by including terms that model perturbations to the system (e.g., Boker & Laurenceau, 2006; Butner et al., 2005).

10.3.2.3 Reservoir Model

Based on the damped linear oscillator, Deboeck and Bergeman (2013) defined the Reservoir Model. It captures the same fluctuations as the damped linear oscillator, but, unlike the latter, takes ceiling and floor effects of measurements into account. The model is based on fluctuations of the water level in a reservoir that is constantly being filled with water (subsequently called the input) while some of the water escapes (subsequently called the output). Depending on the input and output, the

water level in the reservoir may change from being high (when input is greater than output) to low (when output is greater than input) to maintaining a stable value in between (when input and output are relatively equal for a certain amount of time). Based on this physical example, the Reservoir Model is defined as (Deboeck & Bergeman, 2013):

$$dy(t) = \beta y(t)dt + \varepsilon(t)dt$$

where $\beta < 0$ and $\varepsilon(t) \geq 0$. The constraints put on the parameters follow from the derivation of the model, such that β represents the pressure-dependent outflow and $\varepsilon(t)$ represents the inflow. The bigger the outflow, the more easily a buildup of inflow is regulated. To give an example: imagine that during a specific day, stress builds up due to a number of events (i.e., $\varepsilon(t)$ is large). The parameter β then marks the difference between being able to regulate this stress (β is sufficiently large) or having a stress overload (β is too small). Importantly, $\varepsilon(t)$ is itself a function of time, so that the input may change over time: sometimes there is more input (more stressful days) or less input (more relaxing days).

While the model seems promising, it has not yet been applied frequently.

10.3.3 Nonlinear Models

Linear models are a useful tool for investigating affect dynamics. However, evidence suggests that they may fail to capture some fundamental characteristics of affective data, such as V-shaped relationships between variables (e.g., PA-NA; Diener & Iran-Nejad, 1986; Schimmack, 2001; valence-arousal; Kuppens et al., 2012b) and abrupt changes in the temporal dynamics of affect (i.e., phase transitions; Bonsall et al., 2012; Scherer, 2000; Thagard & Nerb, 2002; van de Leemput et al., 2014).

There are several ways to accommodate phenomena that deviate from linear dynamics. First, one may choose to tweak the linear model so that it may be used in specific applications. For example, one may choose to incorporate abrupt changes in the dynamical system by making one or several parameters time- or context-dependent (Boker et al., 2016; Driver & Voelkle, 2018b). These tweaks may seem familiar, as they have also been discussed in the context of nonstationarity (see *Extensions* of *Autoregressive Models*).

Another option is to use *nonlinear* models of affect dynamics. Nonlinear models are models that cannot be rewritten to the form specified in Eq. (10.3). What the nonlinearity looks like, is left to the researcher to decide, and may go from the nonlinear transformation of variables to the inclusion of an interaction term between them. Examples of nonlinear models are

$$d\boldsymbol{y}(t) = \ln\left(\boldsymbol{y}(t)\right)dt$$
$$d\boldsymbol{y}(t) = \boldsymbol{y}^{\boldsymbol{3}}(t)\,dt$$
$$d\boldsymbol{y}(t) = \left(\boldsymbol{y}(t) + \boldsymbol{y}^{\boldsymbol{2}}(t)\right)dt$$

and, to pick the concrete example of a damped nonlinear oscillator (Boker & Graham, 1998):

$$\frac{d^2y(t)}{dt^2} = Ey(t) + Z\frac{dy(t)}{dt} + \Theta y^3(t)$$

The difficulty thus lies not in recognizing what nonlinear models are, but in how these models can be used and interpreted, as they often produce very complex behavior with very few parameters. Despite their great appeal, we caution the interested reader in applying nonlinear models to data without considering (a) what behavior the model produces, (b) whether this behavior is interesting with regards to affect dynamics, and (c) whether other, simpler models can be used instead. This is not to say that nonlinear models cannot be applied to affect dynamics; On the contrary, given that nonlinear tendencies are observed in the affective time series of individuals, this calls to the use of these models. However, researchers should also consider the difficulty in identifying a model that produces much of the behavior we see in the literature, a point to which we will return later (Brown et al., 2013; Sussmann & Zahler, 1978).

10.3.3.1 Catastrophe Theory

Originally conceived of by Thom (1975) and then popularized by Zeeman (e.g., Zeeman, 1976), catastrophe theory quickly gained traction in psychology due to the perceived range of problems that it can deal with (e.g., Flay, 1978; Hartelman et al., 1998). In the domain of emotions, it has been explicitly used by Allen and Carifio (1995), included in theory by Scherer (2000) (albeit speculatively, but see also Sacharin et al., 2012; Sander et al., 2005), and alluded to by Frijda (2007).

In its most basic form, catastrophe theory defines a *potential function* that binds together the variables in which one is interested. Then, the model defines the movement in this potential as (Chow et al., 2015):

$$d\boldsymbol{y}(t) = \frac{\partial V\left(\boldsymbol{y}(t);\boldsymbol{\theta}\right)}{\partial\,\boldsymbol{y}}dt$$

where $V(\boldsymbol{y}(t);\boldsymbol{\theta})$ is the potential function of $\boldsymbol{y}(t)$, given the parameters inside the parameter vector $\boldsymbol{\theta}$. The symbol ∂ denotes the partial derivative. In this context, this means that for the potential function, the derivative is taken with respect to the variables $\boldsymbol{y}(t)$, so that the parameters $\boldsymbol{\theta}$ are considered to be constant. Because of this, the

$\boldsymbol{y}(t)$ are often taken as the dependent variables (or behavioral variables). The values to which the $\boldsymbol{y}(t)$ are regulated then depend on the values of the control parameters in $\boldsymbol{\theta}$. In practice, these control parameters can themselves be functions of some independent variables, so that

$$\theta = \Omega x$$

where Ω is a diagonal matrix consisting of the different weights of the predictors in $\boldsymbol{x}$.

The challenge of catastrophe theory then lies in the construction of the potential function. Luckily, many such models already exist, the most popular one being the cusp catastrophe model (Chow et al., 2015; Hartelman et al., 1998; Scherer, 2000; Zeeman, 1976). Its potential function and partial derivative are:

$$V\left(y(t);,\alpha;,\beta\right) = \frac{1}{4}y^4(t) - \alpha y(t) - \frac{1}{2}\beta y^2(t)$$
$$\frac{\partial V\left(y(t);,\alpha;,\beta\right)}{\partial y(t)} = y^3(t) - \alpha - \beta y(t)$$

It includes one behavioral variable $y(t)$ and two control parameters α and β. To understand how the control parameters work, we refer the reader to Fig. 10.8—for a visualization on what it implies for the potential function, we refer to Fig. 10.9. The black lines represent lines of attractors, meaning that values of $y(t)$ below or above this line are regulated towards it. Once on the line, the system reaches equilibrium and stops moving. Given that there are no sources of stochasticity, the only way to elicit movements in the behavioral variable $y(t)$ is through changes in the

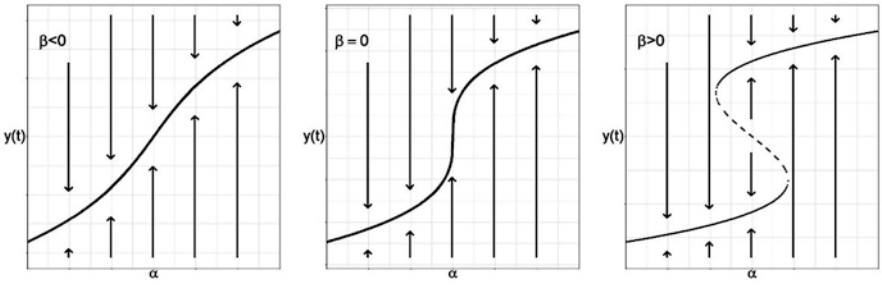

Fig. 10.8 The cusp catastrophe model for different values of α and β. Values of $y(t)$ are regulated towards the solid lines, as indicated by the arrows. Keeping β constant, increasing the value of α goes together with increased values of $y(t)$. When we keep $\beta \leq 0$, increasing the value of α leads to continuous increases in the value to which $y(t)$ is regulated (i.e., the attractor value). When β is greater than 0, a discrete jump in the attractor value can be seen when a certain threshold value of α is exceeded. The emotional state remains on this plane until one decreases the value of α beyond another, separate threshold, a characteristic of the cusp catastrophe model called *hysteresis*. The dotted part in this plot is a line of repellors that can never be reached by the emotional state

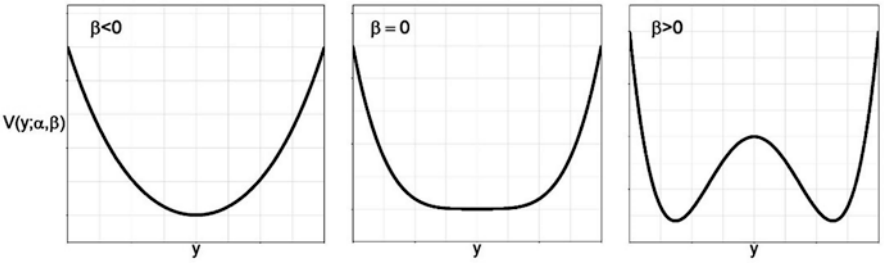

Fig. 10.9 The potential function of the cusp catastrophe model for different values of β, keeping α constant. The affective state $y(t)$ moves towards the local minima of this potential function (i.e., towards the bottom of the wells, which are the attractors), as captured by the differential equation. When β increases, the system goes from having one well or attractor to having two, corresponding to the bifurcation seen in Fig. 10.8

control parameters, which amounts to changes in the predictor variables x. Looking back at the left plot of Fig. 10.8, we can see that increasing the value of α leads to increased values of $y(t)$. But what happens when we increase the value of β? As you can see in the middle and right plot, the potential function creates a fold (a process also known as *bifurcation*; Strogatz, 2018), with a lower region (lower values of $y(t)$), an upper region (higher values of $y(t)$, and a middle region that connects the former two (the dotted line). Note that the middle region consists of repellors, so that $y(t)$ can never be regulated towards it. This implies that $y(t)$ can only be regulated towards the lower and upper region, which produces some interesting behavior.

Imagine that we manipulate α for the higher values of β. Starting out at the lower region and slowly increasing the value of α, we see that $y(t)$ only gradually increases, until suddenly, $y(t)$ shows a discrete jump from the edge of the lower region to the upper region. This behavior is at the heart of why catastrophe theory is popular: Continuous increases in the predictor variables may elicit sudden phase transitions in the dependent variable. Interestingly, decreasing the value of α does not imply a jump back to the lower region at the same threshold as the upwards jump, a characteristic known as *hysteresis*.

To make this a bit more concrete, consider the following example. Take the behavioral variable *y(t)* to be perceived stress during the day, take the value of α to be dependent on physiological and/or emotional arousal, and the value of β to be dependent on suppression. When an individual does not suppress feelings of arousal, then the cusp catastrophe model would predict that with increases of arousal come continuous increases of perceived stress. With this comes the assumption that when arousal decreases, perceived stress will also decrease in a continuous fashion (as shown in the left plot of Fig. 10.8). Now consider the case when an individual suppresses much of the arousal he/she feels (right plot of Fig. 10.8), then one may initially perceive less stress, up until a critical value at which arousal becomes too high and perceived stress abruptly increases (i.e., suddenly the stress becomes overwhelming). Importantly, due to hysteresis, it will not be easy for the individual to

recover from this sudden burst of stress: falling back into the "normal" pattern of stress requires arousal to decrease beyond the previous transition point.

Given our explanation, we hope that the potential of this model is clear. It makes explicit what factors contribute to the creation of the fold (β) and the abrupt change in affect dynamics (α), and how we may attempt to alleviate this state. The model is, however, deterministic in nature, leaving no place to stochastic noise in the system, be it due to internal and external perturbations or due to measurement error. To alleviate this limitation, stochastic versions of catastrophe models have been proposed (see Cobb & Watson, 1980; Wagenmakers et al., 2005).

Despite the frequent use of catastrophe-related terminology in theories of emotion and emotion dynamics, the models themselves have not, to the knowledge of the authors, been directly applied to affect dynamics. It is, however, related to a recently proposed nonlinear model of affect dynamics, to which we turn next.

10.3.3.2 Affective Ising Model

The Affective Ising Model (AIM) is capable of modeling phase transitions, which may occur either due to contextual stimuli or due to random fluctuations (Fig. 10.10). The AIM is a theoretical nonlinear diffusion model that assumes that the emotional life of individuals consists of two pools of binary neurons (i.e., neurons which only have an on- or off-state; Loossens et al., 2020). The state of these two pools determines the amount of PA and NA a person experiences at a given point in time.

Activation in the pools is subject to several forces. More specifically, (a) each neuron has a pool-specific threshold of activation (θ_i) that may be lowered or increased by environmental factors (β_i), and (b) neurons of a given pool that are activated will excite the other neurons of the same pool and inhibit the ones in the other pool (λ_i and λ_{12} respectively).

Instead of describing the time evolution of the binary neurons themselves, the AIM describes the dynamics of the overall activation in the pools. Let y_1 and y_2 denote the average activation of pool 1 (PA) and pool 2 (NA) respectively, then the dynamical equations are given by:

$$dy_1(t) = -\delta \frac{\partial F\left(y_1(t), y_2(t)\right)}{\partial y_1} dt + \sqrt{2\delta}\, dw_1(t)$$

$$dy_2(t) = -\delta \frac{\partial F\left(y_1(t), y_2(t)\right)}{\partial y_2} dt + \sqrt{2\delta}\, dw_2(t)$$

where $F(y_1,y_2)$ is defined as:

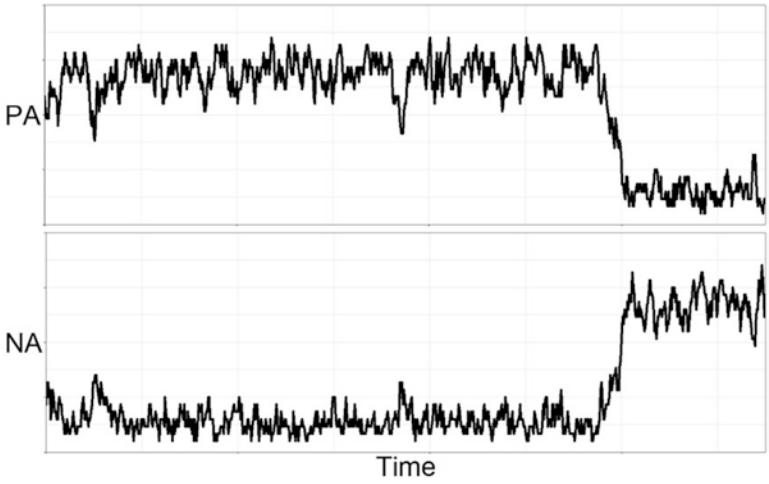

Fig. 10.10 Simulated data of an AIM with two modes. In this figure, it is shown that the AIM can account for discrete shifts in affect dynamics. In this case, the AIM transitions from a high PA-low NA state to a low PA-high NA state

$$F\left(y_1,y_2\right)=\sum_{2}^{i=1}\left(-\lambda_i y_i^2+\left(\theta_i-\beta_i\right)y_i\right)+\lambda_{12}y_1y_2+$$
$$\sum_{2}^{i=1}\nu_i\left(y_i\ln\left(y_i\right)+\left(1-y_i\right)\ln\left(1-y_i\right)\right)$$

where ν_i relates to the number of neurons in a specific pool and δ determines the speed with which the system diffuses towards an equilibrium state (see Loossens et al., 2020 for more on this). The function $F(y_1,y_2)$ is called the *free energy function* and it plays a similar role to the potential function in catastrophe theory models. It can be interpreted as an individual's emotional landscape, through which a the affective state moves. This state will tend towards minima in the landscape (the attractors), but may move upwards due to stochastic fluctuations, so that affect is always evolving.

The AIM induces nonlinearity in two ways. First, it includes logarithmic terms in Eq. (5), making the drift of the AIM nonlinear. Furthermore, the states y_1 and y_2 of the AIM are constrained to fall between 0 (all neurons are inactive) and 1 (all neurons are active), which introduces some boundary effects that are absent in most of the discussed models. As a consequence, the AIM can reproduce some complex phenomena often found in affect dynamics (for some results, see Loossens et al., 2020).

While certainly promising, one clear disadvantage of the model is its limitation to two dimensions (PA and NA), without current alternative to incorporate more than these two dimensions.

10.3.3.3 Chaos

Our discussion of nonlinear models would not be complete without at least braising the topic of *chaos*. Yet, this discussion will stay limited, as chaos has not found its way in affect dynamics despite being used in emotion-related research (e.g., Chow et al., 2016; Fredrickson & Losada, 2005; Losada, 1999).

Chaos refers to deterministic nonlinear systems that are extremely sensitive to initial conditions. With this, we mean that very small differences in a beginning state may lead to large differences in the long run. With this comes another defining feature of chaos, its unpredictable character: Even if there is only small measurement error, your predictions will deviate strongly from the actual behavior of the system. This implies that we can only begin to predict emotional behavior when we are able to measure emotions with no error at all.[7] Consequently, if one decides to use chaotic systems to model affect dynamics, one should also assume that predicting future affect is impossible.

The use of chaotic models in emotion research is limited, and when it has been used, was severely criticized (e.g., Brown et al., 2013; Guastello, 2014 on the studies by Fredrickson & Losada, 2005; Losada, 1999; see also Kellert, 2001). This criticism is primarily directed at the blind use of *fancy* models on data that do not really call for it. For example, Brown et al. (2013) and Guastello (2014) criticized the use of the chaotic Lorenz equations in the studies of Losada (1999) and Fredrickson and Losada (2005), mostly because the latter had no theoretical reasons to use this model in the first place. A same case has also been made for catastrophe theory models, where Sussmann and Zahler (1978) stressed that these models cannot be used on all data that show discrete jumps.

The blind use of such models not only pertains to their predictions, but also to the assumptions that one makes about the underlying process. Chaotic models are deterministic in nature, meaning that no stochasticity is involved at all—neither from inherent randomness of the process nor from unknown internal or external influences on the process. This means that if one is ready to assume such a model is a trustworthy reflection of the emotional system, one also assumes that a select number of variables can be used to describe this system. It seems unlikely that a complex system such as emotions can be described by such a select set of variables, and even if we are ready to assume this, then the question remains whether emotion theorists are comfortable with the notion that emotions evolve in a completely deterministic fashion. We believe few researchers would like to go that far.

[7] If this is the case, we are able to predict the emotional behavior of an individual for eternity, as the system is deterministic.

10.3.4 Limitations

Although continuous-time models may provide some solutions to the problems of discrete-time models, they still suffer from a number of limitations. First, many continuous-time models have no closed-form parametric solutions, meaning that they rely on approximate, numerical methods for their estimation. Unfortunately, these methods are very susceptible to local minima in the parameter space, so that the optimal parameter set is not necessarily identified (Myung, 2003). Matters become even worse for nonlinear models, as they often do not even have an analytic solution to their differential equations (e.g., the AIM; Loossens et al., 2020). This means that the solution of the differential equation has to be approximated with numerical integration methods, which may be difficult to implement and very time-consuming (Strogatz, 2018). Given this limitation, it may become clear why their application to psychological research remains limited (Ryan et al., 2018). Often, continuous-time models require the researcher to use specialized software, or construct it themselves, although attempts have been made to make such software more widely available through e.g. R packages that aid in parameter estimation (*ctsem*; Driver et al., 2017; *dynr*; Ou et al., 2019; *OpenMx*; Boker et al., 2020; Hunter, 2018).

Another limitation pertains to the fact that many of the models that were discussed in this section have originally been created to model the behavior of physical systems (Kellert, 2001). Metaphors like "emotions as a thermostat" or "as a reservoir filling with water" may seem to convey some characteristics of emotions, but this may not necessarily be the case. One should realize that these equations have been designed to model a specific, often simple physical system, and may not always be applicable to the complex systems we try to model with them. This is not to say that these models cannot be used for affect dynamics: Instead, it is a reminder that each model may capture a specific aspect of the emotional system, and that at some point, an integration of such models may be needed.

10.4 Conclusion

In this chapter, we provided the reader with a general overview of some of the computational models that try to capture (and explain) affective fluctuations in individuals. Within the scope of this chapter, we discussed discrete-time and continuous-time models, each with their own strengths and limitations. More than providing a general overview of some computational models, we tried to provide the reader with the means to evaluate the use of these models and interpret their results in a more clear-cut way.

As the focus of our chapter was on describing different models of affect dynamics, some of the topics central to computational modeling have been left undiscussed. We will mention some of these topics and refer the reader to the literature to learn more.

10.4.1 Undiscussed Topics

A first undiscussed topic is the one of parameter estimation. While on some occasions we have commented on the reliability or the difficulty of estimation, we have not touched upon the topic of estimation itself. It is, however, an important topic that has led to a rich literature that evaluates different estimation techniques and their reliability. For the discrete-time topics, some interesting references are Lütkepohl (2005), Rojas (1996), and Sutton and Barto (2018), and for continuous-time models, some examples of how they can be estimated come from Driver and Voelkle (2018a), M. Chen et al. (2018), Chow et al. (2015), Chow et al. (2016), Hu et al. (2014), Oravecz et al. (2011), and Oud and Jansen (2000).

Related to this is the topic of state space models (SSM; Hamilton, 1994a; Harvey, 1989). SSM are models that make a distinction between measurements and process, similar to what structural equation models do. However, the former are more appropriate for use in design with ILD, and we therefore leave the latter undiscussed (see Chow et al., 2010 for a comparison between the two). This way, a set of two equations jointly provide a model for the evolution of a dynamical system over time: A measurement equation to model the observations of the variables and a transition equation to model the latent processes. In the context of the models discussed here, the general SSM for a discrete-time model may be formulated as:

$$\begin{aligned} \boldsymbol{y}_t &= \tau + \Lambda \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t &= \boldsymbol{\alpha} + B\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \end{aligned}$$

where the observations and latent process are contained within $\boldsymbol{y}_t$ and $\boldsymbol{\eta}_t$ resp. Importantly, the factor loading matrix Λ relates the observed variables to the underlying latent processes. $\boldsymbol{\tau}$ and $\boldsymbol{\alpha}$ represent the intercepts of measurement and process, and $\boldsymbol{\varepsilon}$ and $\boldsymbol{\zeta}$ are the measurement error and process noise.

Many of the models that have been discussed in this chapter have originally been formulated within the SSM framework (e.g., fmVAR; Adolf et al., 2017), and SSM techniques are often used to aid in parameter estimation (e.g., Kalman filters; M. Chen et al., 2018; Driver & Voelkle, 2018a). We refer the interested reader to Hamilton (1994a) and Harvey (1989) for a more detailed discussion of this framework.

Another important undiscussed topic is that of *model selection*. In the introduction, we briefly mentioned that computational models could be compared to each other regarding model performance, providing evidence for a given model compared to other models. This is an important analytic step, and often says more than just a simple application of one such models to the data. For example, if we use a VAR model to analyze data, we might get small estimates for the crossregressive effects. If we leave it at this, we cannot infer whether these effects contribute much to the model's fit (i.e., whether they are important enough to interpret). To be able to make such an inference, we can analyze the data using a modified VAR model in which all cross-regressive effects are set to zero. If we then find that this second

model performs better than the first, we have evidence that no temporal relations between the variables exist (at least not for these data).

Many model selection tools exist, of which we mention only a few. A first popular tool of model selection is *cross-validation*. In cross-validation you use your data to assess the predictive performance of your models (Arlot & Celisse, 2010; Bergmeir et al., 2018; Roberts et al., 2017). In time series research, some related techniques are referred to as *forecasting methods*, in which only past data is used to predict future data, which is not the case for cross-validation (Hyndman et al., 2011; Hyndman & Koehler, 2006; Tashman, 2000). A second, a computationally less expensive model comparison tool is the relative measure of fit, which assesses how well a model fits the data while accounting for the complexity of the model (e.g., AIC and BIC; Bengtsson & Cavanaugh, 2006; Masson, 2011; Schwarz, 1978; Vandekerckhove et al., 2015; Wagenmakers & Farrell, 2004). Third, the parametric bootstrap can be used to simulate data and compare these to the real data with the use of some data-driven statistics (Wehrens et al., 2000). Another interesting way to use the parametric bootstrap is to simulate data starting from different models and check whether the data-generating model is also the model that fits these data best. Using this method, the distinguishability and mimicry of different models can be assessed (Navarro et al., 2004; Wagenmakers et al., 2004).

A final undiscussed topic is the one of individual differences in affect dynamics. All models in this chapter have been formulated to be only applicable to one subject. However, computational models need not be limited to this one case, but may be transformed into a multilevel structure to take into account individual differences (Gelman, 2006; Gelman & Hill, 2006). Some models have already been extended to such a multilevel structure, such as the VAR model (Ariens et al., 2020), the OU model (Driver & Voelkle, 2018a; Oravecz et al., 2011), and the damped linear oscillator (Hu et al., 2014). While very useful, multilevel extensions are not always straightforward and require some additional thought on the modeler's side, often making them difficult to implement.

10.4.2 Final Note

We would like to end this chapter on a final note. While computational models may certainly help progress the affect dynamics field, it is clear that each model suffers from its own limitations. Moreover, computational models are often simplifications of the processes in which we are interested. In light of these limitations, one may become skeptic about their use. To battle this skepticism, we want to remind the reader of another quote of the late Box (1979) (see also Box, 1976, p. 202): "All models are wrong but some are useful."

Appendix 1: Properties of the VAR

Properties of the AR Model

Given the AR model (repeated here):

$$y_j = \delta + \varphi y_{j-1} + \varepsilon_j.$$

we can define some properties of the process. These properties are defined and mathematically derived below.

Predictions. Given a first observation y_0 collected at time t_0, we are able to predict the next measurement y_1, as:

$$\begin{aligned} \langle y_1 | y_0 \rangle &= \delta + \varphi y_0 + \varepsilon_1 \\ &= \delta + \varphi y_0 \end{aligned}$$

where $\langle \cdot \rangle$ denotes the time-dependent expected value (i.e., $E[.]$). Note that the innovations do not play a role in the expectation of y_1, given that their expected value is equal to 0.

Using the same principle, we can also make predictions about observations further in the future. For instance, the expectation of y_2 conditional on the observation y_0 is given by:

$$\begin{aligned} \langle y_2 | y_0 \rangle &= \delta + \varphi y_1 \\ &= \delta + \varphi(\delta + \varphi y_0) \\ &= (1+\varphi)\delta + \varphi^2 y_0 \end{aligned}$$

In general, the prediction of a future observation y_j conditional on y_0 is:

$$\langle y_j | y_0 \rangle = \left(\sum_{k=0}^{j-1} \varphi^k \right) \delta + \varphi^j y_0 \tag{10.7}$$

Baseline. Since the magnitude of φ^j shrinks as j increases, in the long-time limit, it holds that:

$$lim_{j \to \infty} \left(\sum_{j-1}^{k=0} \varphi^k \right) = \frac{1}{1-\varphi}$$

As a result, the predictions $\langle y_j | y_0 \rangle$ converge towards a fixed point:

$$\begin{aligned} lim_{j\to\infty}(\langle y_1 \mid y_0 \rangle) &= lim_{j\to\infty}\left(\left(\sum_{k=0}^{j-1}\varphi^k\right)\delta + \varphi^j y_0\right) \\ &= \frac{1}{1-\varphi}\delta + 0 \\ &= \frac{\delta}{1-\varphi} \\ &= \mu \end{aligned} \tag{10.8}$$

This fixed point μ can be considered the emotional baseline (i.e., the dotted line in Fig. 10.1) and represents the emotional state to which the emotional state is expected to evolve to. As such coincides with the end state of a regulation process (provided nothing happens to disrupt the regulation process).

It also represents the state that will be visited the most by the individual over longer periods of time. For that reason, it coincides with the mean of the distribution of observations $\{y_j \mid \mathrm{j},\ldots,\mathrm{N}\}$ for sufficiently large N. Because of this, the baseline is also referred to as the *stationary mean*. The term stationary is used to stress that the baseline is independent of time.

When an AR(1) process is only observed during a short period of time during which the emotional state is still relaxing (i.e., converging) towards the baseline, then the mean of the observations will differ from the stationary mean. Only when measurements have been collected for a sufficiently long period of time will the mean of the data distribution coincide with the stationary mean.

Uncertainty. Until now, we were only concerned with point-predictions of future observations. However, we can also compute the uncertainty that is associated with these predictions. For this, we realize that the observation y_1 is normally distributed with mean $\delta + \varphi y_0$ (the prediction) and variance σ_ε^2:

$$y_1 \mid y_0 \sim N\left(\delta + \varphi y_0, \sigma_\varepsilon^2\right)$$

Because of stochasticity, uncertainty about predictions typically grows the further in the future you go. It can be shown that the future observation y_j, given observation y_0, is normally distributed with the mean being the point-prediction in Eq. (10.7) and variance given by:

$$\sigma_j^2 = \sum_{j-1}^{k=0} \varphi^{2k}\sigma_\varepsilon^2 \tag{10.9}$$

where in the long-time limit:

$$lim_{j\to\infty}\left(\sum_{j-1}^{k=0}\varphi^{2k}\right) = \frac{1}{1-\varphi^2}$$

so that the variance of the uncertainty distribution in the long-time limit converges to

$$\sigma^2 = \frac{\sigma_\varepsilon^2}{1-\varphi^2} \tag{10.10}$$

Like the stationary mean, this variance is time-independent and thus called the *stationary variance*.

Autocovariance. An AR model relies on the assumption that measurements y_j at time t_j are related to measurements y_{j-1} at time t_{j-1}, i.e. that there is a time-dependence between measurements. The extent to which this relationship holds is expressed by the autocovariance. The autocovariance at lag-p σ_p is defined as:

$$\sigma_p = \left\langle \left(y_{j+p} - \mu\right)\left(y_j - \mu\right)\right\rangle$$

To compute the autocovariance of the AR process, we first reformulate the model in terms of the baseline μ. To do so, we substitute δ for $(1 - \varphi)\mu$ (see Eq. (10.8)) to obtain:

$$\begin{aligned} y_j &= (1-\varphi)\,\mu + \varphi y_{j-1} + \varepsilon_j \\ &= \mu - \varphi\mu + \varphi y_{j-1} + \varepsilon_j \end{aligned}$$

Then, by rearranging the terms, we can write

$$y_j - \mu = \varphi\left(y_{j-1} - \mu\right) + \varepsilon_j .$$

Setting the innovations to zero (they do not correlate with anything), we find (see Eq. (10.7))

$$\begin{aligned} \sigma_p &= \left\langle \left(y_{j+p} - \mu\right)\left(y_j - \mu\right)\right\rangle \left(\text{Def.autocovariance}\right) \\ &= \left\langle \varphi^p \left(y_j - \mu\right)\left(y_j - \mu\right)\right\rangle \left(\text{Generalization previous property}\right) \\ &= \left\langle \varphi^p \left(y_j - \mu\right)^2 \right\rangle \\ &= \varphi^p \sigma^2 \end{aligned} \tag{10.11}$$

Here we have used the fact that the centered variable $y_j - \mu$ have the same stationary variance (Eq. (10.10)) as the variable y_j themselves. If we standardize the measurements so that $\sigma^2 = 1$, we obtain the autocorrelation:

$$\rho(p) = \varphi^p$$

From this expression, it can be seen that the autoregressive coefficient φ of the AR model corresponds to the autocorrelation between measurements at lag 1.

Properties of the VAR Model

We can generalize the properties of the AR model to fit the *d*-dimensional VAR model (repeated here):

$$\boldsymbol{y}_j = \boldsymbol{\delta} + \Phi \boldsymbol{y}_{j-1} + \boldsymbol{\varepsilon}_j$$
$$\boldsymbol{\varepsilon}_j \sim N\left(\mathbf{0}, \Sigma_\varepsilon\right)$$

Predictions. Just like for the AR model, the prediction of a future observation conditional on the observation y_0 is given by (see Eq. (10.7))

$$\left\langle y_j \middle| \mathrm{y}_0 \right\rangle = \left(\sum_{k=0}^{j-1} \Phi^k \right) \boldsymbol{\delta} + \Phi^j \boldsymbol{y}_0 \tag{10.12}$$

Importantly, this equation results in a vector that contains all expectation values for all *d* variables of the model.

Baseline. Using a similar reasoning as for the AR model (see Eq. (10.8)), but this time using matrices instead of scalars, it can be shown that the predictions of the VAR model Eq. (10.12) converge to the baseline:

$$\boldsymbol{\mu} = \left(I_d - \Phi\right)^{-1} \boldsymbol{\delta}$$

where I_d is the *d*-dimensional identity matrix.

Uncertainty. An expression similar to Eq. (10.9) can be obtained for the growing uncertainty of the VAR model:

$$\Sigma_j = \sum_{j-1}^{k=0} \Phi^k \Sigma_\varepsilon \left(\Phi^k\right)^T$$

For stable transition matrices Φ, this covariance matrix becomes constant in the long-time limit. This stationary covariance is given by

$$\Sigma = \sum_{\infty}^{k=0} \Phi^k \Sigma_\varepsilon \left(\Phi^k\right)^T$$

and is a solution of the discrete-time Lyapunov equation

$$\Sigma - \Phi \Sigma \Phi^T = \Sigma_\varepsilon$$

Given the transition matrix Φ and the covariance Σ_ε of the innovations, this Lyapunov equation enables us to compute the stationary covariance without having to compute an infinite sum.

Autocovariance. The autocovariance of the VAR model is similar to the autocovariance of the AR (see Eq. (10.11), namely

$$\begin{aligned}\Sigma_p &= \left\langle \left(y_{j+p} - \mu\right)\left(y_j - \mu\right)^T \right\rangle \\ &= \Phi^p \Sigma\end{aligned}$$

Appendix 2: Autocorrelation of Bivariate VAR

If we take a bivariate VAR model with the intercepts $\boldsymbol{\delta} = \mathbf{0}$, then we can compute the autocovariance as:

$$\begin{aligned}\sigma_{tt-1} &= \left(y_{1t} y_{1t-1}\right) \\ &= \left(\delta_1 + \varphi_{11} y_{1t-1} + \varphi_{12} y_{2t-1} + \varepsilon_{1t}\right) y_{1t-1} \\ &= \delta_1 y_{1t-1} + \varphi_{11} y_{1t-1}^2 + \varphi_{12} y_{2t-1} y_{1t-1} + \varepsilon_{1t} \\ &= \delta_1 y_{1t-1} + \varphi_{11} y_{1t-1}^2 + \varphi_{12} y_{2t-1} y_{1t-1} + \varepsilon_{1t} \\ &= 0 + \varphi_{11}\sigma_1^2 + \varphi_{12}\sigma_{12} + 0 \\ &= \varphi_{11}\sigma_1^2 + \varphi_{12}\sigma_{12}\end{aligned}$$

We can compute the autocorrelation as:

$$\begin{aligned}\rho_{tt-1} &= \frac{\sigma_{tt-1}}{\sigma_{1t}\sigma_{1t-1}} \\ &= \frac{\sigma_{tt-1}}{\sigma_1^2} \\ &= \frac{\varphi_{11}\sigma_1^2 + \varphi_{12}\sigma_{12}}{\sigma_1^2} \\ &= \varphi_{11} + \frac{\varphi_{12}\sigma_{12}}{\sigma_1^2}\end{aligned}$$

More generally, it holds that for a variable y_i:

$$\begin{aligned}\sigma_{tt-1} &= y_{it}y_{it-1} \\ &= \left(\delta_i + \varphi_{ii}y_{it-1} + \sum_{\substack{i'=1 \\ i'\neq i}}^{d}\varphi_{ii'}y_{i't-1} + \varepsilon_{it}\right)y_{it-1} \\ &= \varphi_{ii}\sigma_i^2 + \sum_{\substack{i'=1 \\ i'\neq i}}^{d}\varphi_{ii'}\sigma_{ii'}\end{aligned}$$

and:

$$\rho_{tt-1} = \varphi_{ii} + \sum_{\substack{i'=1 \\ i'\neq i}}^{d}\frac{\varphi_{ii'}\sigma_{ii'}}{\sigma_i^2}$$

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